Fuzzy Modeling via Sector Nonlinearity Concept

Hiroshi Ohtake and Kazuo Tanaka
Department of Mechanical Engineering
and Intelligent Systems
The University of Electro-Communications
1-5-1 Chofugaoka, Chofu, Tokyo 182-8585 Japan
Email hohtake@yama.mce.uec.ac.jp
ktanaka@mce.uec.ac.jp

Hua O. Wang
Department of Electrical and Computer Engineering
Duke University
Durham, NC 27708-0291 USA
Email hua@ee.duke.edu

Abstract

This paper presents a new fuzzy modeling technique via the so-called sector nonlinearity concept. To fully take advantage of the sector nonlinearity concept, we propose a new type of Takagi-Sugeno fuzzy model and develop an algorithm to identify model parameters. The algorithm consists of two steps. The purpose of the first step is to determine sector coefficients from input-output data. The second part identifies membership functions from the determined sector coefficients and the input-output data. Identification examples illustrate the utility of this approach.

1. Introduction

A number of fuzzy modeling techniques [1, 2, 3, 4, 5, 6, 7] , which identify a fuzzy model from input-output data of a nonlinear system, have been proposed. Some of these techniques [3, 4, 5, 6] identify a Takagi-Sugeno fuzzy model (T-S fuzzy model) from input-output data. A key feature of the T-S fuzzy model [3] is that its consequent parts are represented by local linear equations. The model techniques identify some local linear models and construct a global nonlinear model by fuzzily blending them. This paper presents a fuzzy modeling technique via the so-called sector nonlinearity concept that is completely different from the local linear approximation. The main feature is that the sector nonlinearity concept guarantees to exactly represent the dynamics of any smooth nonlinear functions globally or at least semi-globally. To fully use the advantage, we propose a new type of fuzzy model and develop a fuzzy modeling technique to identify the fuzzy model from input-output data.

Section 2 presents a new type of Takagi-Sugeno fuzzy model. Section 3 addresses the sector nonlinearity concept both in the single variable system and in the multivariable system. In Section 4, we propose a method of finding multi-dimensional sectors, that is, a method of determining coefficients of multi-dimensional sectors, from input-output data. In addition, multi-dimensional membership functions are identified using neural networks from the determined sector coefficients and the input-output data. Finally, Section 5 illustrates the utility of our approach through identification examples.

2. New Takagi-Sugeno Model

The Takagi-Sugeno fuzzy model is of the following form.

$$\text{Rule } i: \ \text{IF } x_1 \text{ is } M_{1i} \text{ and} \cdots \text{and } x_n \text{ is } M_{ni}$$
$$\text{THEN } y_i = a_{1i}x_1 + \cdots + a_{ni}x_n, \qquad (1)$$

where $i = 1, 2, \cdots, r$ and r is the number of rules. The fuzzy reasoning (defuzzification) process is described by

$$y = \sum_{i=1}^{r} h_i(x) \left\{ a_{1i}x_1 + \cdots + a_{ni}x_n \right\}, \qquad (2)$$

where $x = [x_1, \cdots, x_n]^T$ and

$$h_i(x) = \prod_{j=1}^{n} M_{ji}(x_j) \quad i = 1, 2, \cdots, r.$$

$M_{ji}(x_j)$ denotes the grade of membership of x_j in M_{ji}.

To fully take the advantage of sector nonlinearity concept, we propose a fuzzy model which consists of local T-S fuzzy models represented in each region.

Region p :

$$\textit{Rule } 1: \ \text{IF } (x_1, \cdots, x_n) \text{ is } h_{1p}$$
$$\text{THEN } y_{1p} = a_{1p1}x_1 + \cdots + a_{np1}x_n, \qquad (3)$$
$$\textit{Rule } 2: \ \text{IF } (x_1, \cdots, x_n) \text{ is } h_{2p}$$
$$\text{THEN } y_{2p} = a_{1p2}x_1 + \cdots + a_{np2}x_n, \qquad (4)$$

where $p = 1, 2, \cdots, Q$ and $Q = 2^{n-1}$. Each region basically corresponds to each quadrant. The local T-S fuzzy model in each region has two rules. The fuzzy reasoning process is defined as

$$y = \sum_{p=1}^{Q} \sum_{i=1}^{2} v_p(x) h_{ip}(x) \times \left\{ a_{1pi}x_1 + \cdots + a_{npi}x_n \right\}, \qquad (5)$$

0-7803-7078-3/01/$10.00 (C)2001 IEEE.

where

$$v_p(x) = \begin{cases} 1 & (x \in p), \\ 0 & (x \notin p). \end{cases} \tag{6}$$

Note from (6) that rules in different regions never fire simultaneously.

Remark 1 *The necessity of dividing every quadrant into regions will be addressed in Section 3.2.1.*

Remark 2 *If the number of input variables, i.e., n, is determined, $v_p(x)$ can be automatically defined. Therefore, the fuzzy modeling problem proposed here reduces to an identification problem of consequent parameters $a_{jpi}(j = 1, 2, \cdots, n)$ and multi-dimensional membership functions $h_{ip}(x)$.*

To construct multi-dimensional membership functions, neural networks are employed in this algorithm. The learned neural networks directly become membership functions. The membership functions may be constructed by using other techniques instead of neural networks. Thus, the use of neural networks to identify the membership functions is not our subject. We emphasize that the subject of this paper is to provide a fuzzy modeling technique via the sector nonlinearity concept.

3. Sector nonlinearity concept

3.1. Single variable system case

Consider a nonlinear function with single variable.

$$y = f(x), \tag{7}$$

where $f(0) = 0$ and f is known. Figure 1 illustrates the sector nonlinearity concept. The model construction based on the sector nonlinearity guarantees to exactly represent $f(x)$ with the sector (straight lines: a_1x and a_2x) globally or semi-globally. From a_1, a_2 and $f(x)$, membership functions $h_1(x)$ and $h_2(x)$ are constructed as

$$h_1(x) = \frac{f(x) - a_2x}{(a_1 - a_2)x},\ h_2(x) = \frac{a_1x - f(x)}{(a_1 - a_2)x},$$

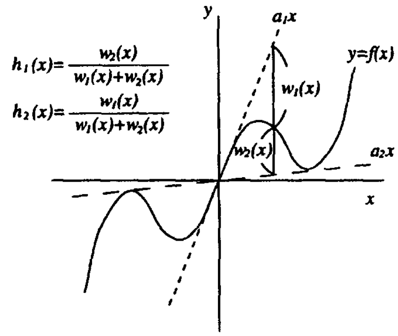
where $h_1(x) + h_2(x) = 1$ and $h_1(x), h_2(x) \geq 0$ for all x. From these membership functions, a_1 and a_2, the output y is reconstructed by

$$\begin{aligned} y &= h_1(x)a_1x + h_2(x)a_2x \\ &= \sum_{i=1}^{2} h_i(x)a_ix. \end{aligned} \tag{8}$$

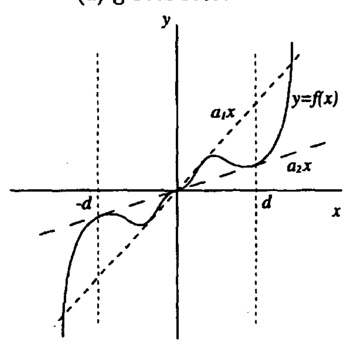
As shown in Figure 1 (a), to exactly represent the dynamics (7) with (8), the constructed sector is required to exactly cover the dynamics. Even if it is difficult to globally cover the dynamics, it is possible to semi-globally cover as shown in Figure 1 (b).

Remark 3 *We assumed in (7) that $f(0) = 0$. However, even if $f(0) \neq 0$, the sector nonlinearity concept can be utilized after a coordinate transformation. Figure 2 shows an example for the coordinate transformation.*

Remark 4 *In [8], the sector nonlinearity concept for only single variable system case is considered. This paper discusses the sector nonlinearity concept for multi-variable system case. The most important observation is that for the multi-variable system case we need to develop a new type of Takagi-Sugeno fuzzy model to construct multi-dimensional sectors. More importantly, we need to consider a compatibility condition for multi-dimensional sectors.*



(a) global sector.



(b) semi-global sector for $x \in [-d \;\; d\,]$.

Figure 1. Concept of sector nonlinearity.

3.2. Multi-variable system case

We newly propose sector nonlinearity concept based on multi-dimensional sectors.

3.2.1 Construction of multi-dimensional sectors

Consider a nonlinear function with n variables.

$$y = f(x) = f(x_1, x_2, \cdots, x_n), \tag{9}$$

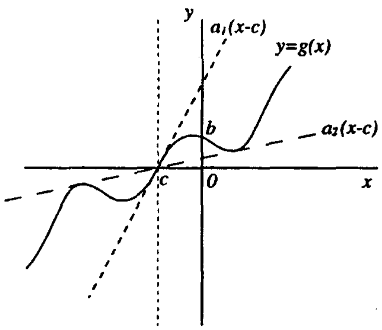


Figure 2. Construction of new sector via transformation ($f(0) \neq 0$).

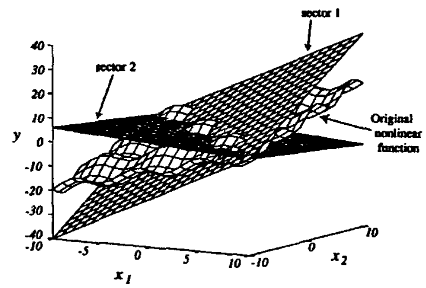


Figure 3. Limitation of global sector.

where $f(0, 0, \cdots, 0) = 0$ and f is known. The sector nonlinearity concept can be applied even if $f(0, 0, \cdots, 0) \neq 0$ as noted in *Remark 3*. A sector for the function (9) is constructed by n dimensional linear functions

$$y = a_1x_1 + a_2x_2 + \cdots + a_nx_n. \quad (10)$$

As described above, in the single variable system case, it is possible to exactly represent the nonlinear function (7) with the sector (a_1x and a_2x) globally or semi-globally. However, in the multi-variable system case, it is generally impossible to exactly represent the nonlinear function (9) with a sector (linear hyperplanes represented by (10)). *Example 1* illustrates the problem.

Example 1 *Consider a nonlinear function with two variables.*

$$y = f(x_1, x_2) = x_1 + 3\sin x_1 + x_2 - 2\sin x_2. \quad (11)$$

Figure 3 shows the case where it is impossible to exactly cover the function via the sector (planes). It is impossible to exactly cover the function via the sector even if any coefficients of a_1 and a_2 are selected.

As illustrated in *Example 1*, in the multi-variable system case, it is impossible to exactly cover any function via a sector. To overcome the problem, we consider each quadrant on inputs space as an (local) area. In each (local) area, any function can be globally or semi-globally covered via a sector. Therefore, the model structure described by (3) and (4) is required to represent the nonlinear dynamics (9).

Definition 1 *Consider a quadrant*

$$x_1 \geq 0,\ x_2 \leq 0,\ x_3 \leq 0,\ x_4 \geq 0, \cdots, x_n \geq 0.$$

We define the quadrant as

$$R(s_1, s_2, s_3, s_4, \cdots, s_n),$$

$$s_1 = 1,\ s_4, s_5, \cdots, s_n = 1,\ s_2, s_3 = 0$$

or

$$R(1, 0, 0, 1, \cdots, 1),$$

where

$$\begin{cases} s_j = 1 & (x_j \geq 0), \\ s_j = 0 & (x_j \leq 0). \end{cases}$$

Definition 2 *Using the definition mentioned above, the first and second quadrants in single variable system are represented as $R(1)$ and $R(0)$, respectively. In each symmetric quadrant area with respect to the origin, a common sector can be employed to cover a function. In the two variables system shown in Figure 3, the function can be covered via a common sector in $R(1,1)$ and $R(0,0)$. Thus, in areas which combines symmetric quadrants with respect to the origin, it is possible to cover (9) via a common sector. In our approach, we define the areas as a region.*

Using these definitions, each region in an n variables system is represented as

$$\begin{gathered} R(1,1,1,\cdots,1,1,1) \cup R(0,0,0,\cdots,0,0,0), \\ R(0,1,1,\cdots,1,1,1) \cup R(1,0,0,\cdots,0,0,0), \\ R(1,0,1,\cdots,1,1,1) \cup R(0,1,0,\cdots,0,0,0), \\ R(0,0,1,\cdots,1,1,1) \cup R(1,1,0,\cdots,0,0,0), \\ \vdots \\ R(0,0,0,\cdots,0,0,1) \cup R(1,1,1,\cdots,1,1,0). \end{gathered}$$

The total number of regions is $Q = 2^{n-1}$. In this paper, R^* denotes the symmetric quadrant of a quadrant R.

Remark 5 *There are two rules in each region. The total number of region is 2^{n-1}. Therefore, there are totally 2^n rules in the new fuzzy model (3) and (4). Regions which do not include any input data at all may exist. For the case, we can ignore the regions. That is, the number of rules in the new fuzzy model can decrease. For example, if all the input data for the nonlinear function (9) exist at one region $x_1, x_2, \cdots, x_n \geq 0$, the number of regions is one. Therefore, the number of rules is two.*

Next, we present a technique to construct multi-dimensional sectors. In the multi-variable system case, it is impossible to construct a multi-dimensional sector uniquely. However, by assuming some constraints, a multi-dimensional sector can be uniquely constructed.

Consider a region p.

$$R(s_1,\cdots,s_{p-1},s_p,\cdots,s_n)$$
$$\cup R^*(s_1,\cdots,s_{p-1},s_p,\cdots,s_n),$$
$$s_1,\cdots,s_{p-1}=0,\ s_p,\cdots,s_n=1.$$

After partially differentiating the function (9) with respect to $x_1 \sim x_n$, we find the maximum and minimum values in the region p.

$$\left.\begin{array}{l} \overline{a}_{1p}=\max\limits_{x_1,\cdots,x_n}\dfrac{\partial f}{\partial x_1},\ \underline{a}_{1p}=\min\limits_{x_1,\cdots,x_n}\dfrac{\partial f}{\partial x_1} \\ \overline{a}_{2p}=\max\limits_{x_1,\cdots,x_n}\dfrac{\partial f}{\partial x_2},\ \underline{a}_{2p}=\min\limits_{x_1,\cdots,x_n}\dfrac{\partial f}{\partial x_2} \\ \quad\vdots \qquad\qquad ,\qquad \vdots \\ \overline{a}_{np}=\max\limits_{x_1,\cdots,x_n}\dfrac{\partial f}{\partial x_n},\ \underline{a}_{np}=\min\limits_{x_1,\cdots,x_n}\dfrac{\partial f}{\partial x_n} \end{array}\right\}. \qquad (12)$$

When sector coefficients, i.e., $a_{1p1},\cdots,a_{np1}$ and $a_{1p2},\cdots,a_{np2}$ in (3) and (4), are chosen as follows:

$$a_{jp1}\equiv\begin{cases}\overline{a}_{jp} & (s_j=1)\\ \underline{a}_{jp} & (s_j=0)\end{cases},\quad a_{jp2}\equiv\begin{cases}\underline{a}_{jp} & (s_j=1)\\ \overline{a}_{jp} & (s_j=0)\end{cases},$$

where $j=1,2,\cdots,n$, (13) and (14) become a sector, i.e., consequent equations in (3) and (4), that covers (9) in the region p.

$$\begin{aligned} y_{1p} &= \underline{a}_{1p}x_1+\underline{a}_{2p}x_2+\cdots+\underline{a}_{(p-1)p}x_{p-1} \\ &\quad +\overline{a}_{pp}x_p+\cdots+\overline{a}_{np}x_n \qquad (13) \\ y_{2p} &= \overline{a}_{1p}x_1+\overline{a}_{2p}x_2+\cdots+\overline{a}_{(p-1)p}x_{p-1} \\ &\quad +\underline{a}_{pp}x_p+\cdots+\underline{a}_{np}x_n. \qquad (14) \end{aligned}$$

To obtain (12), the nonlinear function (9) should be known. Based on the idea mentioned above, Section 4.1 will describe a technique to find sectors from input-output data.

3.2.2 Compatibility condition of sectors on region boundary

Since each region corresponds to each quadrant, each region is partitioned by each axis. For example, the area

$$x_1=0,\ x_2=0,\ x_3>0,\ \cdots x_n>0$$

belongs to the following four regions.

$$\begin{array}{l} \text{Region 1}:\ R(1,1,1,\cdots,1)\cup R(0,0,0,\cdots,0),\\ \text{Region 2}:\ R(0,1,1,\cdots,1)\cup R(1,0,0,\cdots,0),\\ \text{Region 3}:\ R(1,0,1,\cdots,1)\cup R(0,1,0,\cdots,0),\\ \text{Region 4}:\ R(0,0,1,\cdots,1)\cup R(1,1,0,\cdots,0).\end{array}$$

If outputs of local T-S fuzzy models in the above area are different, the nonlinear function (9) reconstructed by the fuzzy model should be able to be discontinuous. To avoid the phenomenon, we consider a compatibility condition of sectors on region boundary. To satisfy a compatibility condition, all the other coefficients excepting coefficients for x_1 and x_2, i.e., $\overline{a}_{1p}$, $\underline{a}_{1p}$, $\overline{a}_{2p}$, $\underline{a}_{2p}$ $(p=1,2,3,4)$, should be equal each other.

$$\begin{array}{l} \overline{a}_{31}=\overline{a}_{32}=\overline{a}_{33}=\overline{a}_{34},\ \underline{a}_{31}=\underline{a}_{32}=\underline{a}_{33}=\underline{a}_{34},\\ \overline{a}_{41}=\overline{a}_{42}=\overline{a}_{43}=\overline{a}_{44},\ \underline{a}_{41}=\underline{a}_{42}=\underline{a}_{43}=\underline{a}_{44},\\ \qquad\vdots \qquad\qquad\qquad ,\qquad\qquad \vdots\\ \overline{a}_{n1}=\overline{a}_{n2}=\overline{a}_{n3}=\overline{a}_{n4},\ \underline{a}_{n1}=\underline{a}_{n2}=\underline{a}_{n3}=\underline{a}_{n4}.\end{array}$$

However, since sector coefficients are independently obtained in each region from (12), (13) and (14), it is very difficult to satisfy the compatibility condition. Therefore, we adjust sector coefficients from the viewpoint of the compatibility as follows:

$$\left.\begin{array}{lcl} \overline{a}_1=\max\limits_p\overline{a}_{1p} & , & \underline{a}_1=\min\limits_p\underline{a}_{1p}\\ \overline{a}_2=\max\limits_p\overline{a}_{2p} & , & \underline{a}_2=\min\limits_p\underline{a}_{2p}\\ \quad\vdots & , & \quad\vdots\\ \overline{a}_n=\max\limits_p\overline{a}_{np} & , & \underline{a}_n=\min\limits_p\underline{a}_{np}\end{array}\right\}, \qquad (15)$$

where $p=1,2,\cdots,Q$. By employing (15) instead of (12), the compatibility condition is satisfied. Note that the sector in each region still covers the nonlinear function.

3.2.3 Construction of multi-dimensional membership functions

In the region p, the nonlinear function (9) is reconstructed by

$$f(\boldsymbol{x})=h_{1p}(\boldsymbol{x})\cdot y_{1p}(\boldsymbol{x})+h_{2p}(\boldsymbol{x})\cdot y_{2p}(\boldsymbol{x}), \qquad (16)$$

where $h_{1p}(\boldsymbol{x})$ and $h_{2p}(\boldsymbol{x})$ are membership functions for the sector (13) and (14), i.e., for the local T-S fuzzy model in region p. In (16), the following constraint exists.

$$h_{1p}(\boldsymbol{x})\ge 0,\ h_{2p}(\boldsymbol{x})\ge 0,\ h_{1p}(\boldsymbol{x})+h_{2p}(\boldsymbol{x})=1. \qquad (17)$$

From (16) and (17), each membership function is calculated as

$$h_{1p}(\boldsymbol{x})=\frac{f(\boldsymbol{x})-y_{2p}(\boldsymbol{x})}{y_{1p}(\boldsymbol{x})-y_{2p}(\boldsymbol{x})}, \qquad (18)$$

$$h_{2p}(\boldsymbol{x})=\frac{y_{1p}(\boldsymbol{x})-f(\boldsymbol{x})}{y_{1p}(\boldsymbol{x})-y_{2p}(\boldsymbol{x})}. \qquad (19)$$

Note that (18) and (19) satisfy the constraint.

4. Modeling algorithm

This section presents a fuzzy modeling technique using input-output data. For simplicity, we show the case where identification data belong to the region p, i.e.,

$$R(s_1,\cdots,s_{p-1},s_p,\cdots,s_n)$$
$$\cup R^*(s_1,\cdots,s_{p-1},s_p,\cdots,s_n),$$

where $s_1,\cdots,s_{p-1}=0$ and $s_p,\cdots,s_n=1$. Assume that input-output data at time instance k are given by

$$\boldsymbol{x}_p[k]=[x_{1p}[k],x_{2p}[k],\cdots,x_{np}[k]],\ y_p[k],$$

where $k=1,2,\cdots,m$ and $m>1$.

4.1. Construction of sectors from input-output data

In the multi-variable system case, it is generally impossible to construct a multi-dimensional sector uniquely. By assuming the following constraint, a multi-dimensional sector is uniquely constructed.

$$|a_{1p}| = |a_{2p}| = \cdots = |a_{np}| = |A_p|.$$

A sector based on the constraint is constructed by the following linear function.

$$y_p = A_p \left\{(-1)^{1+s_1} x_1 + \cdots + (-1)^{1+s_{p-1}} x_{p-1} + (-1)^{1+s_p} x_p + \cdots + (-1)^{1+s_n} x_n \right\}. \quad (20)$$

Candidates $A_p[k]$ for sector coefficients are calculated from input-output data as follows:

$$A_p[k] = \frac{y_p[k]}{(-1)^{1+s_1} x_{1p}[k] + \cdots + (-1)^{1+s_n} x_{np}[k]}, \quad (21)$$

where $k = 1, 2, \cdots, m$. From the candidates, sector coefficients are determined as follows:

$$\overline{A}_p = \max_k A_p[k], \ \underline{A}_p = \min_k A_p[k]. \quad (22)$$

Then, sector in the region p is

$$y_{1p} = \overline{A}_p \left\{(-1)^{1+s_1} x_1 + \cdots + (-1)^{1+s_{p-1}} x_{p-1} + (-1)^{1+s_p} x_p + \cdots + (-1)^{1+s_n} x_n \right\}, \quad (23)$$

$$y_{2p} = \underline{A}_p \left\{(-1)^{1+s_1} x_1 + \cdots + (-1)^{1+s_{p-1}} x_{p-1} + (-1)^{1+s_p} x_p + \cdots + (-1)^{1+s_n} x_n \right\}. \quad (24)$$

Next, we discuss the compatibility condition of sectors on region boundaries. Consider the intersection area between Region α and Region β:

$$\text{Region } \alpha: \ R(\alpha_1, \cdots, \alpha_n) \cup R^*(\alpha_1, \cdots, \alpha_n),$$

$$\text{Region } \beta: \ R(\beta_1, \cdots, \beta_n) \cup R^*(\beta_1, \cdots, \beta_n).$$

Sector coefficients in the region α are $\overline{A}_\alpha$ and $\underline{A}_\alpha$, and sector coefficients in the region β are $\overline{A}_\beta$ and $\underline{A}_\beta$. If $R(\alpha_1, \cdots, \alpha_n) \cap R(\beta_1, \cdots, \beta_n) \neq \phi$ (,i.e., $R^*(\alpha_1, \cdots, \alpha_n) \cap R^*(\beta_1, \cdots, \beta_n) \neq \phi$), then the compatibility conditions are $\overline{A}_\alpha = \overline{A}_\beta$ and $\underline{A}_\alpha = \underline{A}_\beta$. Conversely, if $R(\alpha_1, \cdots, \alpha_n) \cap R^*(\beta_1, \cdots, \beta_n) \neq \phi$ (,i.e., $R^*(\alpha_1, \cdots, \alpha_n) \cap R(\beta_1, \cdots, \beta_n) \neq \phi$), then the compatibility conditions are $|\overline{A}_\alpha| = |\underline{A}_\beta|$ and $|\underline{A}_\alpha| = |\overline{A}_\beta|$. Therefore, the compatibility conditions on all the region boundaries are

$$|\overline{A}_\alpha| = |\overline{A}_\beta| = |\underline{A}_\alpha| = |\underline{A}_\beta|, \ \forall \alpha, \beta. \quad (25)$$

However, since sector coefficients are independently obtained in each region, it is very difficult to satisfy the compatibility condition (25). Therefore, we modify sector coefficients from the viewpoint of the compatibility as follows:

$$\left.\begin{array}{l} \overline{A} = \max\left(|\overline{A}_1|, |\underline{A}_1|, |\overline{A}_2|, |\underline{A}_2|, \cdots, |\overline{A}_Q|, |\underline{A}_Q|\right) \\ \underline{A} = -\overline{A} \end{array}\right\}. \quad (26)$$

By employing (26) instead of (22), the compatibility condition is satisfied. Note that the sector in each region still covers all the data.

4.2. Identification of multi-dimensional membership functions

A remained problem is to identify membership functions for the sector (23) and (24). Note that membership functions for a multi-dimensional sector are multi-dimensional. From the input-output data, membership values are calculated as follows:

$$h_{1p}(x_p[k]) = \frac{y_p[k] - y_{2p}[k]}{y_{1p}[k] - y_{2p}[k]}, \quad (27)$$

$$h_{2p}(x_p[k]) = \frac{y_{1p}[k] - y_p[k]}{y_{1p}[k] - y_{2p}[k]}, \quad (28)$$

where

$$y_{1p}[k] = \overline{A}_p \{ (-1)^{1+s_1} x_{1p}[k] + \cdots + (-1)^{1+s_{p-1}} x_{(p-1)p}[k] + (-1)^{1+s_p} x_{pp}[k] + \cdots + (-1)^{1+s_n} x_{np}[k] \},$$

$$y_{2p}[k] = \underline{A}_p \{ (-1)^{1+s_1} x_{1p}[k] + \cdots + (-1)^{1+s_{p-1}} x_{(p-1)p}[k] + (-1)^{1+s_p} x_{pp}[k] + \cdots + (-1)^{1+s_n} x_{np}[k] \},$$

and $k = 1, 2, \cdots, m$. Since $h_{1p}(x_p[k]) + h_{2p}(x_p[k]) = 1$, it is sufficient to calculate either $h_{1p}(x_p[k])$ or $h_{2p}(x_p[k])$. Neural networks are employed to obtain the relations between the input data and the membership values. Inputs and outputs of neural networks are the input data and the membership values, respectively. The back propagation algorithm is employed to determine the multi-dimensional membership functions. The following sigmoid function is used as transfer functions in neurons.

$$g(x) = \frac{1}{1 + e^{-x}} \quad (29)$$

Note from (29) that the output range of the neural networks becomes [0 1]. Therefore, the learned neural networks are directly utilized as membership functions. Membership functions h_{1p} and h_{2p} constructed by neural networks are multi-dimensional (nonlinear) functions.

The modeling algorithm can be summarized below

- Step1 Calculate sector coefficients $\overline{A}_p$, $\underline{A}_p$ in each region by using (21) and (22) from the data $(x_p[k], y_p[k])$ $(k = 1, \cdots, m)$ for the region p.
- Step2 Determine compatible sector coefficients $\overline{A}$ and $\underline{A}$ by using (26).
- Step3 Calculate membership values $h_{1p}(x_p[k])$ and $h_{2p}(x_p[k])$ by using (27) and (28) from the data $(x_p[k], y_p[k])$ $(k = 1, \cdots, m)$ for the region p and sector determined in Step2.
- Step4 Identify membership functions by neural networks from sector determined in Step2 and membership values calculated in Step3.

Table 1. Model evaluations in Example 1

	MAE for I.D.	MAE for E.D.
This method	1.830×10^{-5}	3.591×10^{-4}
Other method [2]	0.0250	0.0279

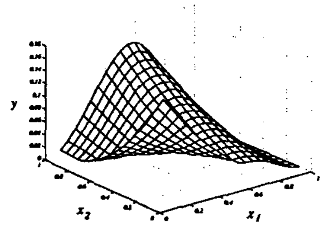


Figure 4. Original output in (30).

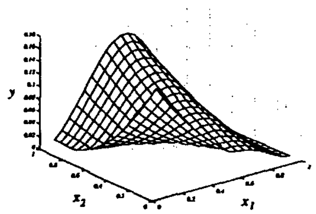


Figure 5. Output of identified fuzzy model.

5. Application examples

Application example 1 *Consider the following two-variables function:*

$$y = max \left\{ \begin{array}{l} (x_1(1-x_1)(x_1+x_2))^2 \\ \left(\frac{1}{2}(1-x_1)(1-x_2)\right)^2 \end{array} \right. , \quad (30)$$

where $0 \leq x_1 \leq 1$ and $0 \leq x_2 \leq 1$. The number of identification data (I.D.) and evaluation data (E.D.) are 300 samples, respectively. These are randomly generated.

Table 1 shows the identification results (mean absolute error: MAE) of our approach and other approach [2]. The results show the utility of our approach. Figures 4 and 5 show the outputs of (30) and the identified fuzzy model, respectively.

Application example 2 *Consider the following six-variables function:*

$$\begin{aligned} y &= (1 + x_1^{0.5} + x_2^{-1})^2 \\ &\quad + 10 \sin \frac{\pi(x_3 + x_4)}{4} + e^{1+x_5+x_6}. \quad (31) \end{aligned}$$

We use the identification data (I.D.) and evaluation data (E.D.) shown in [1]. The modeling result is shown in Table 2. In [1], not only model parameters but also model structure were identified, where a dummy variable x_7 were added to the original variables $x_1, x_2, \cdots, x_6$. As a result of the structure identification, x_7 was removed in [1]. Since a subject of this paper is not to identify a model structure, in our approach, we use only

Table 2. Model evaluations in Example 2

	MRE for I.D.	MRE for E.D.
This method	0.0002707	0.534
Other method [1]	2.27	4.41

$x_1, x_2, \cdots, x_6$ in the parameter identification. Table 2 shows the identification result.

6. Conclusions

We have proposed a fuzzy modeling technique via the sector nonlinearity concept. Identification examples have illustrated the utility of our fuzzy modeling technique using multi-dimensional sectors and membership functions.

References

[1] S. Matsushita et al., "A Hierarchical Fuzzy Modeling Method Using Multiple Submodels," *Journal of Japan Society for Fuzzy Theory and Systems*, Vol.8, No.3, pp.488-498, 1996.

[2] K. Tachibana et al., "A Proposal of Fuzzy Modeling Method with Generation/Deletion of Membership Functions," *Journal of Japan Society for Fuzzy Theory and Systems*, Vol.11, No.6, pp.1078-1088, 1999.

[3] T. Takagi and M. Sugeno, "Fuzzy Identification of Systems and Its Applications to Modeling and Control," *IEEE Transactions on Systems, Man and Cybernetics*, Vol.15, pp.116-132, 1985.

[4] M. Sugeno and G. T. Kang, "Structure Identification of Fuzzy model," *Fuzzy Sets and Systems*, Vol.18, pp.329-346, 1986.

[5] G. T. Kang and M. Sugeno, "Fuzzy Modeling," *Transactions of the Society of Instrument and Control Engineers*, Vol.23, No.6, pp.650-652, 1987.

[6] S. Horikawa, T. Furuhashi and Y. Uchikawa, "On Fuzzy Modeling Using Fuzzy Neural Networks with the Back-Propagation Algorithm," *IEEE Transactions on Neural Network*, Vol.3, No. 5, pp.801-806, 1992.

[7] L. Wang and R. Langari, "Building Sugeno-Type Models Using Fuzzy Discretization and Orthogonal Parameter Estimation Techniques," *IEEE Transactions on Fuzzy Systems*, Vol.3, No.4, pp.454-458, 1995.

[8] S. Kawamoto et al., "An Approach to Stability Analysis of Second Order Fuzzy Systems," *Proceeding of First IEEE International Conference on Fuzzy Systems*, Vol.1, pp.1427-1434, 1992.

[9] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis*, JOHN WILEY & SONS, INC., 2000.