Introduction

To begin answering this question we need to first define three relations that an arbitrary point can have to the other 7 points in a cube. The first one I will call *directly connected*, three of these exists and it is the closest points. The second one is *diagonally connected*, there are also three of these and they can be found by crossing a the cubes sides. The last and final one I will call *long diagonally connected*, now with this one it only exists one and it is the one where the connection goes through the cubes centre.

To enumerate all linearly separable functions we only need to figure out the amount of linearly separable functions for $k = \{0, 1, 2, 3, 4\}$, where k is the amount of points that is labeled as 1. We may reduce the problem in such a way due to symmetry, the amount of linearly inseparable functions for $k = 1$ will be the same for $k = 7$ since we get the same pattern but only reversed.

k=0 This case only yields one possibility and that pattern is linearly separable.

k=1 In this case we have 8 possible patterns, of which all are linearly separable.

k=2 Now we have $8!/6!2! = 28$ possible patterns. If the two positive points in a cube are *directly connected* the pattern will be linearly separable. However if they are *diagonally connected* or *long diagonally connected* the pattern is not linearly separable. So in this case we have 12 linearly separable patterns since a cube consists of 12 straight lines.

k=3 Here we get $8!/5!3! = 56$ possible patterns. Of these we can find three different symmetries. The first one we get by choosing to connect an arbitrary point to two of it's *directly connected* neighbors. This pattern is linearly separable and we can calculate that there exists $3 \cdot 8 = 24$ since each of the eight points has three possibilities for creating this symmetry. The second one we get by choosing to connect an arbitrary point with two other points that are *diagonally connected*, this symmetry is not linearly separable. The third and final one we get by connecting an arbitrary point with the point which it is *long diagonally connected* with and then choosing the third point arbitrarily, the third one doesn't really matter. The third symmetry is also not linearly separable. So we get 24 linearly separable for $k = 3$.

k=4 In this situation we have $8!/4!4! = 70$ possible functions. There are multiple symmetries which is quite hard to define without having two different symmetries overlapping each other, so I will only explain the two symmetries that gives us linearly separable functions. The first one we get by connecting an arbitrary point to all of its *directly connected* points, this pattern basically forms an edge in the cube. The second one we get by first choosing an arbitrary point and the choosing two *directly connected* to the first one and lastly choosing the point that is directly connected to both of the points chosen in the second step. This symmetry becomes the side of a cube. Since a cube have six edges and eight sides we can conclude that we have $6 + 8 = 14$ linearly separable functions when $k = 4$.

Conclusion

Now we may conclude that there exists:

$$2 \cdot 1 + 2 \cdot 8 + 2 \cdot 12 + 2 \cdot 24 + 14 = 104$$

linearly separable Boolean functions in three dimensions.