Homework 2

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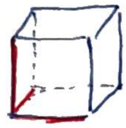
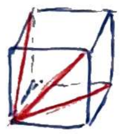
How many linerly seperable Boolean functions are there in 3 dimensions?

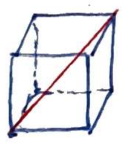
To begin answering this question we need to first define three relations that one point can have to another:



Directly connected, each point in a cube is directly connected to it's three closest points.



Diagonal connections are the ones that crosses the sides diagonally. Each point have three diagonal connections



Long diagonal connections are the ones that passes throung the center of the cube.

To enumerate all the linearly seperable functions we only need to figure out the amount of linearly seperable functions for $K = \{0, 1, 2, 3, 4\}$, where K is how many of the 8 poits that is equal to 1. We can reduce the problem in such away since finding the amont of linearly seperable functions for K is the same for 8-K, due to the fact that we can just flip the values and end up with the same situation.

$K = 0$, yields only one possibility which is linearly seperable

$K = 1$, yields $\binom{8}{1} = 8$ different patterns, of which all are linerly separable

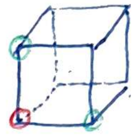
$K = 2$, yields $\binom{8}{2} = \frac{8 \cdot 7}{2} = 28$ different patterns. If the chosen points are diagonal or long-diagonal connection the function can't be linearly seperable. So only the possibilities where the two points are connected directly gives us a linerly seperable function, there are 12 of these.

$K=3$, Now we have $\binom{8}{3} = \frac{8\cdot7\cdot6}{3\cdot2} = 56$ patterns. Of these patterns, we can find three symmetries.

The first one being if we choose an arbitrary point and connect it with 2 out of the 3 poinst it is directly connected with. Then we get a right-angle shape where the arbitrary choosen point is in the corner. Each point have $\binom{3}{2}=3$ possible variation of this pattern. Multiplying this with the amount of points:

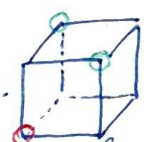
$$3\cdot 8 = 24$$

gives us the total amount of these patterns, All of these patterns are directly connected.



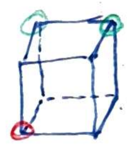
Example of symmetry 1

The second unique symmetry we get by choosing an arbitrary point and choosing 2 of its diagonally connected points. We can clerly see that this want give us a linerly seperable function. Thus all of thes possibilities are linearly inseperable.



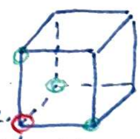
Example of symmetry 2

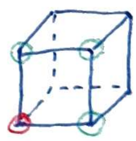
The third pattern is created with a connection from an arbritrary point through its long diagonal connection and then selecting a third point, The third point doesn't matter since we can see that just two points being connected through the long diagonal makes the pattern inseperable.



Example of symmetry 3

$K=4$, In this situation we have $\binom{8}{4} = \frac{8\cdot7\cdot6\cdot5}{4\cdot3\cdot2} = 70$ functions. We only have two symmetries that yields linerly seperable functions. We get them by either connecting an arbitrary point with all its directly connected points or by connecting it to two of them and one diagonal.
So we get this edge pattern and a side pattern. If we would do it any other way we would get something linerly inseperable.





Since a cube have 6 sides and 8 edges, we can conclude that there is $6+8=14$ linerly seperable patterns when $K=4$.

Conclusion:
There are $2(1)+2(8)+2(12)+2(24)+14 = 104$ linerly seperable 3-dimentional Boolean functions.