Problem Set 1

Exercise 1:

What is the maximum amount you would pay for an asset that generated an income of $200,000 at the end of each of four years if the opportunity cost of using funds is 5%?

Solution 1):

Present Value of Future Cash Flows:

Formula:

$$PV = \sum_{n=1}^{N} \frac{CF}{(1+r)^n}$$

Apply formula:

$$PV = \sum_{n=1}^{4} \frac{200{,}000}{(1+0.05)^n}$$

$$PV = \frac{200{,}000}{(1+0.05)} + \frac{200{,}000}{(1+0.05)^2} + \frac{200{,}000}{(1+0.05)^3} + \frac{200{,}000}{(1+0.05)^4}$$

$$PV = 709{,}190.1$$

Exercise 2:

A firm's current profits are $200,000. These profits are expected to grow indefinitely at a constant annual rate of 5%.

If the firm's opportunity cost of funds Is 7¤%, determine the present value of the firm at…

a) … the instant before it pays out the current profits as dividends.

Solution 2a):

Present Value of a Firm:

Assumptions:

- A firm's current profits (π_0) have not yet been paid out as dividends to stockholders.
- The firm's profits are expected to grow at a constant rate of g percent each year.
- The profit growth (g) is less than the opportunity costs/interest rate (r), i.e., $g < r$

Formula:

$$PV_{Firm} = \sum_{n=0}^{N} \frac{\pi_0(1+g)^n}{(1+r)^n}$$

$$PV_{Firm} = \frac{\pi_0(1+g)^0}{(1+r)^0} + \frac{\pi_0(1+g)^1}{(1+r)^1} + \frac{\pi_0(1+g)^2}{(1+r)^2} + \frac{\pi_0(1+g)^3}{(1+r)^3} + \cdots$$

$$PV_{Firm} = \pi_0 + \frac{\pi_0(1+g)^1}{(1+r)^1} + \frac{\pi_0(1+g)^2}{(1+r)^2} + \frac{\pi_0(1+g)^3}{(1+r)^3} + \cdots$$

$$PV_{Firm} = \pi_0 \frac{(1+r)}{(r-g)}$$

Apply formula:

$PV_{Firm} = \pi_0 \frac{(1+r)}{(r-g)}$

$PV_{Firm} = 200{,}000 * \frac{(1+0.07)}{(0.07-0.05)}$

$PV_{Firm} = 200{,}000 * \frac{1.07}{0.02}$

$PV_{Firm} = 200{,}000 * 53.5$

$PV_{Firm} = 10{,}700{,}000$

b) … the instant after it pays out current profits as dividends.

Solution 2b):

PV of a firm after paying out dividends

- The value of the firm immediately after its current profit have been paid out as dividends:

Formula:

$$PV_{Firm}^{Ex-Dividend} = PV_{Firm} - \pi_0$$

$$PV_{Firm}^{Ex-Dividend} = \sum_{n=0}^{N} \frac{\pi_0(1+g)^n}{(1+r)^n} - \pi_0$$

$$PV_{Firm}^{Ex-Dividend} = \pi_0 \frac{(1+r)}{(r-g)} - \pi_0$$

$$PV_{Firm}^{Ex-Dividend} = \pi_0 \frac{(1+g)}{(r-g)}$$

Apply formula:

$$PV_{Firm}^{Ex-Dividend} = 200{,}000 * \frac{(1+0.05)}{(0.07-0.05)}$$

$$PV_{Firm}^{Ex-Dividend} = 200{,}000 * \frac{1.05}{0.02}$$

$$PV_{Firm}^{Ex-Dividend} = 200{,}000 * 52.5$$

$$PV_{Firm}^{Ex-Dividend} = 10{,}500{,}000$$

Exercise 3:

What is the value of a preferred stock that pays a perpetual dividend of $150 at the end of each year when the interest rate is 8%?

Solution 3):

Present Value of Perpetuity:

Formula:

$$PV_{Perpetuity} = \sum_{n=1}^{\infty} \frac{CF_n}{(1+r)^n}$$

$$PV_{Perpetuity} = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} + \cdots$$

$$PV_{Perpetuity} = \frac{CF}{r}$$

Apply formula:

$$PV_{Perpetuity} = \frac{perpetual\ divident}{interest\ rate}$$

$$PV_{Perpetuity} = \frac{150}{0.08}$$

$$PV_{Perpetuity} = 1{,}875$$

Exercise 4:

Suppose the total benefit $B(Q)$ derived from a continuous decision (Q) and the corresponding total cost $C(Q)$, is given by:

$$B(Q) = 10Q - Q^2 \qquad ; \qquad C(Q) = 2 + Q^2$$

The corresponding marginal benefits $[MB(Q)]$ and marginal costs $[MC(Q)]$ are:

$$MB(Q) = \frac{\partial B(Q)}{\partial Q} = 10 - 2Q \qquad ; \qquad MC(Q) = \frac{\partial C(Q)}{\partial Q} = 2Q$$

a) What is the net benefit when $Q = 2$ and $Q = 5$?

Solution 4a):

Net Benefit (NB):

$$NB(Q) = B(Q) - C(Q)$$

$$NB(Q) = (10Q - Q^2) - (2 + Q^2)$$

For $\boldsymbol{Q} = \mathbf{2}$	For $\boldsymbol{Q} = \mathbf{5}$
$NB(2) = B(2) - C(2)$ $NB(2) = (10 * 2 - 2^2) - (2 + 2^2)$ $NB(2) = (20 - 4) - (2 + 4)$ $NB(2) = 16 - 6$ $NB(2) = 10$	$NB(5) = B(5) - C(5)$ $NB(5) = (10 * 5 - 5^2) - (2 + 5^2)$ $NB(5) = (50 - 25) - (2 + 25)$ $NB(5) = 25 - 27$ $NB(5) = -2$

b) What is the marginal benefit (MB) when $Q = 2$ and $Q = 5$?

Solution 4b):

Marginal Benefit (MB):

$$MB(Q) = \frac{\partial B(Q)}{\partial Q} = 10 - 2Q$$

For $\boldsymbol{Q = 2}$	**For $\boldsymbol{Q = 5}$**
$MB(2) = 10 - 2 * 2$ $MB(2) = 10 - 4$ $MB(2) = 6$	$MB(5) = 10 - 2 * 5$ $MB(5) = 10 - 10$ $MB(5) = 0$

c) What level of Q maximizes total benefit?

Solution 4c):

Total benefit is maximized when the marginal benefits are equal to zero:

$$MB(Q) = 0 = 10 - 2Q$$

Solve for Q:

$$2Q = 10$$

$$Q^* = 5$$

d) What are the total costs when $Q = 2$ and $Q = 5$?

Solution 4d):

Total Costs:

$$C(Q) = 2 + Q^2$$

For $Q = 2$	**For $Q = 5$**
$C(2) = 2 + 2^2$ $C(2) = 2 + 4$ $C(2) = 6$	$C(5) = 2 + 5^2$ $C(5) = 2 + 25$ $C(5) = 27$

e) What is the marginal cost when $Q = 2$ and $Q = 5$?

Solution 4e):

Marginal Cost (MC):

$$MC(Q) = \frac{\partial C(Q)}{\partial Q} = 2Q$$

For $Q = 2$	**For $Q = 5$**
$MC(2) = 2 * 2$ $MC(2) = 4$	$MC(5) = 2 * 5$ $MC(5) = 10$

f) What level of Q maximizes total cost?

Solution 4f):

Total costs are maximized when marginal costs are equal to zero:

$$MC(Q) = 0 = 2Q$$

$$2Q = 0$$

$$Q^* = 0$$

g) What level of Q maximizes net benefits?

Solution 4g):

Net benefits are maximized when marginal benefits equal marginal costs:

$$MC(Q) = MB(Q)$$

$$2Q = 10 - 2Q$$

$$4Q = 10$$

$$Q^* = 2.5$$

Exercise 5:

In a coffee shop, suppose that the total benefit of selling Q cups of coffee is given by:

$$B(Q) = 5 + 30Q - 0.05Q^2$$

The costs for making Q cups of coffee is:

$$C(Q) = 10 + 0.1Q$$

Consequently, the marginal benefit and marginal costs per cup of coffee are:

$$MB(Q) = \frac{\partial B(Q)}{\partial Q} = 30 - 0.1Q \quad ; \qquad MC(Q) = \frac{\partial C(Q)}{\partial Q} = 0.1$$

a) Write out the equation for the net benefits (NB):

Solution 5a):

Net Benefit (NB):

$NB(Q) = B(Q) - C(Q)$

$NB(Q) = (5 + 30Q - 0.05Q^2) - (10 + 0.1Q)$

$NB(Q) = -5 + 29.9Q - 0.05Q^2$

b) What are the net benefits when the shop sells 100 cups of coffee?

Solution 5b):

Net Benefit (NB) if $Q = 100$:

$NB(100) = -5 + 29.9 * 100 - 0.05 * 100^2$

$NB(100) = -5 + 2{,}990 - 0.05 * 10{,}000$

$NB(100) = -5 + 2{,}990 - 500$

$NB(100) = 2{,}485$

c) Write out the marginal net benefits (MNB)

Solution 5c):

Marginal Net Benefits (MNB):

$MNB(Q) = \frac{\partial NB(Q)}{\partial Q} = 29.9 - 0.1Q$ m

Alternatively:

$MNB(Q) = MB(Q) - MC(Q)$

$MNB(Q) = 30 - 0.1Q - 0{,}1$

$MNB(Q) = 29.9 - 0.1Q$

d) What are the marginal net benefits (MNB) when the shop sells 100 cups of coffee?

Solution 5d):

Marginal Net Benefits (MNB) if $Q = 100$:

$MNB(100) = 29.9 - 0.1 * 100$

$MNB(100) = 29.9 - 10$

$MNB(100) = 19.9$

e) How many cups of coffee should the shop sell to maximize net benefits?

Solution 5e):

The shop maximizes net benefits where the marginal net benefits are qual to zero:

$MNB(Q) = MB(Q) - MC(Q) = 0$

$MC(Q) = MB(Q)$

$0.1 = 30 - 0.1Q$

$0.1Q = 29.9$

$Q^{*} = 299$

f) When they sold a certain amount of coffee that maximize the net benefit, what is the value of marginal net benefits (MNB)

Solution 5f):

Marginal Net Benefits (MNB) with maximized net benefits, i.e. Q^*

$MNB(Q^*) = MB(Q^*) - MC(Q^*)$

$MNB(299) = 30 - 0.1 * 299 - 0.1$

$MNB(299) = 0.1 - 0.1$

$MNB(299) = 0$