Problem Set 3

Exercise 1:

The demand curve for a product is given by the following:

$$Q_x^d = 1{,}200 - 3P_x - 0.1P_z \quad , where\ P_z = \$300$$

a) What is the own price elasticity of demand when $P_x = \$140$? Is the demand elastic or inelastic at this price? What would happen to the firm's revenue if it decided to charge a price below \$140?

Solution 1a):

Calculate quantity demand (Q_x^d) at the given prices:

$Q_x^d = 1{,}200 - 3P_x - 0.1P_z$

$Q_x^d = 1{,}200 - 3 * 140 - 0.1 * 300$

$Q_x^d = 1{,}200 - 420 - 30$

$Q_x^d = 750$

Own price elasticity of demand:

$$E_{Q_x,P_x} = \frac{\%\Delta Q_x}{\%\Delta P_x} = \frac{\partial Q_x^d}{\partial \Delta P_x} * \frac{P_x}{Q_x^d}$$

$$E_{Q_x,P_x} = -3 * \frac{P_x}{Q_x^d} = -3 * \frac{140}{750} = -0.56$$

Since the own-price elasticity is less than one in absolute terms, i.e., $|E_{Q_x,P_x}| = |-0.56| < 1$, demand is inelastic at this price.

If the firm charged a lower price, total revenue would decrease.

b) What is the own price elasticity of demand when $P_x = \$240$? Is demand elastic or inelastic at this price? What would happen to the firm's revenue if it decided to charge a price above \$240?

<u>Solution 1b):</u>

<u>Calculate quantity demand (Q_x^d) at the given prices:</u>

$Q_x^d = 1{,}200 - 3P_x - 0.1P_z$

$Q_x^d = 1{,}200 - 3 * 240 - 0.1 * 300$

$Q_x^d = 1{,}200 - 720 - 30$

$Q_x^d = 450$

<u>Own price elasticity of demand:</u>

$$E_{Q_x,P_x} = \frac{\%\Delta Q_x}{\%\Delta P_x} = \frac{\partial Q_x^d}{\partial P_x} * \frac{P_x}{Q_x^d}$$

$$E_{Q_x,P_x} = -3 * \frac{P_x}{Q_x^d} = -3 * \frac{240}{450} = -1.6$$

Since the own-price elasticity is greater than one in absolute terms, i.e., $|E_{Q_x,P_x}| = |-1.6| < 1$, demand is elastic at this price.

If the firm charged a higher price, total revenue would decrease.

c) What is the cross-price elasticity of demand between good X and good Z when $P_x = \$140$? Are goods X and Z substitutes or complements?

Solution 1c):

Calculate quantity demand (Q_x^d) at the given prices:

$Q_x^d = 1{,}200 - 3P_x - 0.1P_z$

$Q_x^d = 1{,}200 - 3 * 140 - 0.1 * 300$

$Q_x^d = 1{,}200 - 420 - 30$

$Q_x^d = 750$

Cross-price elasticity of demand:

$$E_{Q_x,P_z} = \frac{\%\Delta Q_x}{\%\Delta P_z} = \frac{\partial Q_x^d}{\partial P_z} * \frac{P_z}{Q_x^d}$$

$$E_{Q_x,P_z} = -0.1 * \frac{P_z}{Q_x^d} = -0.1 * \frac{300}{750} = -0.04$$

Since the cross-price elasticity is negative, i.e., $E_{Q_x,P_z} = -0.04 < 1$, goods X and Z are complements. This implies that if the price of good Z increases, the demand for good X decreases and vice versa.

Exercise 2:

Suppose the demand function for a firm's product is given by the following:

$$\ln Q_x^d = 7 - 1.5 \ln P_x + 2 \ln P_y - 0.5 \ln M + \ln A$$

, where $P_x = \$15$, $P_y = \$6$, $M = \$40{,}000$ (income) , and $A = \$350$ (advertising)

a) Determine the own price elasticity of demand, and state whether demand is elastic, inelastic, or unitary elastic.

Solution 2a):

Own price elasticity of demand:

The own price elasticity of demand is simply the coefficient of $\ln P_x$, which is:

$$E_{Q_x, P_x} = -1.5$$

Since the own price elasticity is greater than one in absolute terms, i.e., $|E_{Q_x,P_x}| = |-1.5| > 1$, demand is elastic.

b) Determine the cross-price elasticity of demand between good X and good Y, and state whether these two goods are substitutes or complements.

Solution 2b):

Cross-price elasticity of demand:

The cross-price elasticity of demand is simply the coefficient of $\ln P_y$, which is:

$$E_{Q_x, P_y} = 2$$

Since the cross-price elasticity is positive, goods X and Y are subtitutes. This implies that the demand for good Xwill increase as the price of good Yincreases, and vice versa.

c) Determine the income elasticity of demand, and state whether good X is a normal or inferior good.

Solution 2c):

Income elasticity of demand:

The income elasticity of demand is simply the coefficient of M, which is:

$$E_{Q_x,M} = -0.5$$

Since the income elasticity of demand is negative, good X is an inferior good. This implies that the demand for good X increases as income decreases, and vice versa.

d) Determine the own advertising elasticity of demand.

Solution 2d):

Advertising elasticity of demand:

The advertising elasticity of demand is simply the coefficient of A, which is:

$$E_{Q_x,A} = 1$$

The advertising elasticity of demand is positive, implying that an increase in advertising increases the demand for good X,

Exercise 3:

Suppose you are the manager of a firm that receives revenues of $40,000 per year from product X and $90,000 per year from product Y.

The own price elasticity of demand for product X is – 1.5 and the cross-price elasticity of demand between product Y and X is –1.8., i.e., $E_{Q_x,P_x} = -1.5$ and $E_{Q_x,P_y} = -1.8$

How much will your firm's revenues (i.e., revenues from both products) change if you increase the price of product X by 2%?

Solution 3):

Firm's Total Revenues:

$$\underline{TR = R_x + R_y}$$

$$\underline{TR = Q_x P_x + Q_y P_y}$$

Change in Revenue Formula elasticity of demand:

$$\Delta TR = [R_x * \left(1 + E_{Q_x,P_x}\right) + R_y * E_{Q_y,P_x}] * \frac{\Delta P_x}{P_x}$$

Substitute given values:

$$\Delta TR = \left[40{,}000 * \left(1 + (-1.5)\right) + \$90{,}000 * (-1.8)\right] * 0.02$$

$$\Delta TR = [40{,}000 - 60{,}000 - 162{,}000] * 0.02$$

$$\Delta TR = -3{,}640$$

A 2% increase in the price of good X would cause total revenues from both products to decrease by $3,540.

Exercise 4:

A quant jock from your firm used a linear demand specification to estimate the demand for its product and sent you a hard copy of the results. Use the information presented below to answer the accompanying questions.

a) Based on these estimates, write an equation that summarizes the demand for the firm's product.

Solution 4a):

$$Q^d = \beta_0 + \beta_1\, x_1 + \beta_2\, x_2 + \cdots$$

$$Q^d = \beta_{intercept} + \beta_{price}\, Price_x + \beta_{income}\, Income$$

$$Q^d = 58.87 - 1.64P_x + 1.11M$$

b) Which regression coefficients are statistically significant at the 5 percent level?

Solution 4b):

Only the coefficients for the intercept and Income are statistically significant at the 5 percent level (or better), as the corresponding p-values are $0.00 < 0.05$

The coefficient for the price is statistically significant at the 10% level, as $p = 0.6 < 0.1$

c) Comment on how well the regression line fits the data

Solution 4c):

The R-Square (R^2) is quite low, indicating that the model explains only 14% of the total variation in demand for X. The adjusted R-Square is only marginally lower (13 percent), suggesting that the R-Square is not the result of an excessive number of estimated coefficients relative to the sample size.

The F-statistic, however, suggests that the overall regression is statistically significant at beterr that the 5 percent level.

Exercise 5:

The demand function for good X is given by the following:

$$\ln Q_x^d = a + b \ln P_x + c \ln M + e$$

, where P_x is the price of good X and M is income.

Least square regression reveals coefficient estimates of: $\hat{a} = 7.42$, $\hat{b} = -2.18$, and $\hat{c} = 0.34$.

a) If $M = 55{,}000$ and $P_x = 4.39$, compute the own price elasticity of demand based on these estimates. Determine whether demand is elastic or inelastic.

<u>Solution 5a):</u>

<u>Own price elasticity of demand:</u>

As in 2a), the own price elasticity of demand is simply the coefficient (estimate) of $\ln P_x$, which is:

$$E_{Q_x,P_x} = \hat{b} = -2.18$$

Since the own price elasticity is greater than one in absolute terms, i.e., $|E_{Q_x,P_x}| = |-2.18| > 1$, demand is elastic.

b) If $M = 55{,}000$ and $P_x = 4.39$, compute the income elasticity of demand based on these estimates. Determine whether good X is a normal or inferior good.

Solution 5b):

Income elasticity of demand:

As in 2c), the income elasticity of demand is simply the coefficient (estimate) of M, which is:

$$E_{Q_x,M} = \hat{c} = 0.34$$

Since the income elasticity of demand is positive, good X is a normal good. This implies that the demand for good X increases as income increases.

Exercise 6:

Suppose you are a division manager at Toyota. If your marketing department estimates that the semiannual demand for the Highlander is $Q_H^d = 150{,}000 - 1.5P_H$, what price should you charge in order to maximize revenues from sales of the Highlander?

Solution 6):

To maximize revenues, Toyota should charge the price that makes demand unit elastic.

Using the own price elasticity of demand, this implies that:

$$E_{Q_H,P_H} = \frac{\partial Q_H^d}{\partial P_H} * \frac{P_H}{Q_H^d} = -1$$

$$E_{Q_H,P_H} = -1.5 * \frac{P_H}{(150{,}000 - 1.5P_H)} = -1$$

Now, solving for P_H:

$$-1.5 * \frac{P_H}{(150{,}000 - 1.5P_H)} = -1$$

$$-1.5\ P_H = -1\ (150{,}000 - 1.5P_H)$$

$$-1.5\ P_H = -150{,}000 + 1.5\ P_H$$

$$-3\ P_H = -150{,}000$$

$$P_H^* = 50{,}000$$

Thus, in order to maximize revenues from the sales of the Highlander, Toyota should charge a price of $50,000