Present value of a firm:

Assumptions:

- A firm's current profits () have not yet been paid out as dividends to stockholders
- The firm's profits are expected to grow at a constant rate of g percent each year
- The profit growth is less than the interest rate (g < i)

$$PV_{Firm} = \pi_0 + \frac{\pi_0(1+g)}{(1+i)} + \frac{\pi_0(1+g)^2}{(1+i)^2} + \frac{\pi_0(1+g)^3}{(1+i)^3} + \cdots$$

$$= \pi_0 \frac{(1+i)}{(i-g)}$$

PV of a firm after paying out dividends

- The value of the firm immediately after its current profit have been paid out as dividends:

$$PV_{Firm}^{Ex-Dividend} = PV_{Firm} - \pi_0$$

$$= \pi_0 \frac{(1+g)}{(i-g)}$$

Present value of indefinitely lived assets:

Assumption:

- An asset generates a cash flow of CF_0 today, CF_1 one year from today, CF_2 two years from today, and so on for an indefinite time period.

$$PV_{Asset} = CF_0 + \frac{CF_1}{(1+i)} + \frac{CF_2}{(1+i)^2} + \frac{CF_3}{(1+i)^3} + \frac{CF_4}{(1+i)^4} + \cdots$$

Present Value of a perpetuity:

Assumption:

- Current cash flow is zero ($CF_0 = 0$) and all future cash flows are identical ($CF_1 = CF_2 = \cdots$)

$$PV_{Perpetuity} = \frac{CF}{(1+i)} + \frac{CF}{(1+i)^2} + \frac{CF}{(1+i)^3} + \frac{CF}{(1+i)^4} + \cdots$$

$$= \frac{CF}{i}$$

Source: Baye, M.R., 2009. Managerial economics and business strategy, 6th ed. ed. McGraw-Hill Irwin, Boston (pp. 14-16).

Derivation of Formulas

1) PV of a firm

$$PV_{Firm} = \pi_0 + \frac{\pi_0(1+g)}{(1+i)} + \frac{\pi_0(1+g)^2}{(1+i)^2} + \frac{\pi_0(1+g)^3}{(1+i)^3} + \cdots \quad (1)$$

a) *Multiply both sides with $\frac{(1+i)}{(1+g)}$:*

$$\frac{PV_{Firm}(1+i)}{(1+g)} = \pi_0 * \frac{(1+i)}{(1+g)} + \pi_0 + \frac{\pi_0(1+g)}{(1+i)} + \frac{\pi_0(1+g)^2}{(1+i)^2} + \cdots \quad (2)$$

b) *Subtract first equation from second equation (2)-(1):*

$$\frac{PV_{Firm}(1+i)}{(1+g)} - PV_{Firm} = \left[\pi_0 \frac{(1+i)}{(1+g)} + \pi_0 + \frac{\pi_0(1+g)}{(1+i)} + \frac{\pi_0(1+g)^2}{(1+i)^2} + \cdots\right] - \left[\pi_0 + \frac{\pi_0(1+g)}{(1+i)} + \frac{\pi_0(1+g)^2}{(1+i)^2} + \frac{\pi_0(1+g)^3}{(1+i)^3} + \cdots\right]$$

$$\frac{PV_{Firm}(1+i)}{(1+g)} - PV_{Firm} = \pi_0 \frac{(1+i)}{(1+g)}$$

c) *Solve for PV_{Firm}:*

$$PV_{Firm} * \left[\frac{(1+i)}{(1+g)} - 1\right] = \pi_0 \frac{(1+i)}{(1+g)}$$

$$PV_{Firm} * \left[\frac{(1+i)}{(1+g)} - \frac{(1+g)}{(1+g)}\right] = \pi_0 \frac{(1+i)}{(1+g)}$$

$$PV_{Firm} * \left[\frac{(1+i-1-g)}{(1+g)}\right] = \pi_0 \frac{(1+i)}{(1+g)}$$

$$PV_{Firm} * \left[\frac{(i-g)}{(1+g)}\right] = \pi_0 \frac{(1+i)}{(1+g)}$$

$$PV_{Firm} * (i-g) = \pi_0(1+i)$$

$$\boldsymbol{PV_{Firm} = \pi_0 \frac{(1+i)}{(i-g)}}$$

Source: Baye, M.R., 2009. Managerial economics and business strategy, 6th ed. ed. McGraw-Hill Irwin, Boston (pp. 14-16).

2) PV of a firm after paying out dividends

$$PV_{Firm}^{Ex-Dividend} = PV_{Firm} - \pi_0$$

$$= \pi_0 \frac{(1+i)}{(i-g)} - \pi_0$$

$$= \pi_0 \left(\frac{(1+i)}{(i-g)} - 1\right)$$

$$= \pi_0 \left(\frac{(1+i)}{(i-g)} - \frac{(i-g)}{(i-g)}\right)$$

$$= \pi_0 \left(\frac{(1+i-i+g)}{(i-g)}\right)$$

$$= \boldsymbol{\pi_0 \frac{(1+g)}{(i-g)}}$$

Source: Baye, M.R., 2009. Managerial economics and business strategy, 6th ed. ed. McGraw-Hill Irwin, Boston (pp. 14-16).

3) Present Value of a perpetuity:

$$PV_{Perpetuity} = \frac{CF}{(1+i)} + \frac{CF}{(1+i)^2} + \frac{CF}{(1+i)^3} + \frac{CF}{(1+i)^4} + \cdots \quad (1)$$

a) *Divide both sides by* $(1 + i)$:

$$\frac{PV_{Perpetuity}}{(1+i)} = \frac{CF}{(1+i)^2} + \frac{CF}{(1+i)^3} + \frac{CF}{(1+i)^4} + \cdots \quad (2)$$

b) *Subtract second equation from first equation (1)-(2):*

$$PV_{Perpetuity} - \frac{PV_{Perpetuity}}{(1+i)} = \left[\frac{CF}{(1+i)} + \frac{CF}{(1+i)^2} + \frac{CF}{(1+i)^3} + \frac{CF}{(1+i)^4} + \cdots\right] - \left[\frac{CF}{(1+i)^2} + \frac{CF}{(1+i)^3} + \frac{CF}{(1+i)^4} + \cdots\right]$$

$$PV_{Perpetuity} - \frac{PV_{Perpetuity}}{(1+i)} = \frac{CF}{(1+i)}$$

c) *Solve for* $PV_{Perpetuity}$:

$$PV_{Perpetuity} = \frac{CF}{(1+i)} + \frac{PV_{Perpetuity}}{(1+i)}$$

$$PV_{Perpetuity} * (1 + i) = CF + PV_{Perpetuity}$$

$$PV_{Perpetuity} + PV_{Perpetuity} * i = CF + PV_{Perpetuity}$$

$$PV_{Perpetuity} * i = CF$$

$$\boldsymbol{PV_{Perpetuity} = \frac{CF}{i}}$$

Source: Baye, M.R., 2009. Managerial economics and business strategy, 6th ed. ed. McGraw-Hill Irwin, Boston (pp. 14-16).