Problem Set 4

Exercise 1:

A firm can manufacture a product according to the following production function:

$$Q = F(K, L) = K^{\frac{3}{4}} * L^{\frac{1}{4}}$$

a) Calculate the average product of labor (AP_L), when the level of capital (K) is fixed at $K = 81$ units and the firm uses 16 units of labor ($L = 16$). How does the average product of labor change when the firm uses 256 units of labor?

<u>Solution 1a):</u>

<u>Average product of Labor:</u>

$$AP_L = \frac{Q}{L} = \frac{F(K, L)}{L}$$

a.1) For $K = 81 \quad and \quad L = 16$

$$AP_L = \frac{K^{\frac{3}{4}} * L^{\frac{1}{4}}}{L} = \frac{81^{\frac{3}{4}} * 16^{\frac{1}{4}}}{16}$$

$$AP_L = \frac{27 * 2}{16} = \frac{54}{16}$$

$$AP_L = 3.375$$

Thus, on average, one worker produces 3.375 units of output.

a.2) For $K = 81 \quad and \quad L = 256$

$$AP_L = \frac{K^{\frac{3}{4}} * L^{\frac{1}{4}}}{L} = \frac{81^{\frac{3}{4}} * 256^{\frac{1}{4}}}{256}$$

$$AP_L = \frac{27 * 2}{256} = \frac{54}{256}$$

$$AP_L = 0.422$$

If the firm uses 256 units of labor, on average, one worker produces 0.422 units of output.

b) Find an expression for the marginal product of labor (MP_L), when the amount of capital is fixed at 81 units. Then, illustrate that the marginal product of labor depends on the amount of labor hired by calculating the marginal product of labor for $L = 16$ and $L = 81$ units of labor.

Solution 1b):

Marginal product of Labor:

$$MP_L = \frac{\partial Q}{\partial L} = \frac{\partial F(K, L)}{\partial L}$$

$$MP_L = \frac{\partial \, [K^{\frac{3}{4}} * L^{\frac{1}{4}}]}{\partial L}$$

$$MP_L = \frac{1}{4} * K^{\frac{3}{4}} * L^{-\frac{3}{4}}$$

a.1) For $K = 81 \quad and \quad L = 16$

$$MP_L = \frac{1}{4} * K^{\frac{3}{4}} * L^{-\frac{3}{4}}$$

$$MP_L = \frac{1}{4} * 81^{\frac{3}{4}} * 16^{-\frac{3}{4}}$$

$$MP_L = \frac{1}{4} * 27 * 0.125$$

$$MP_L = 0.844$$

Thus, with $K = 81 \quad and \quad L = 16$, adding one more unit labor increases output by 0.844.

a.2) For $K = 81 \quad and \quad L = 81$

$$MP_L = \frac{1}{4} * K^{\frac{3}{4}} * L^{-\frac{3}{4}}$$

$$MP_L = \frac{1}{4} * 81^{\frac{3}{4}} * 81^{-\frac{3}{4}}$$

$$MP_L = \frac{1}{4} * 27 * 0.04$$

$$MP_L = 0.25$$

Thus, with $K = 81 \quad and \quad L = 81$, adding one more unit labor increases output by 0.25.

c) Suppose capital is fixed at 81 units. If the firm can sell its output at a price of $200 per unit of output and can hire labor at $50 per unit of labor, how many units of labor should the firm hire in order to maximize profits?

Solution 1c):

Value Marginal Product:

Labor: $VMP_L = P * MP_L$ → *benefit to the firm from each unit of labor*

Capital: $VMP_K = P * MP_K$ → *benefit to the firm from each unit of capital*

Equate VMP_L with wage ($w = 50$):

$$VMP_L = P * MP_L = w$$

$$P * \frac{1}{4} * K^{\frac{3}{4}} * L^{-\frac{3}{4}} = w$$

$$200 * \frac{1}{4} * 81^{\frac{3}{4}} * L^{-\frac{3}{4}} = 50$$

$$200 * \frac{1}{4} * 27 * L^{-\frac{3}{4}} = 50$$

$$200 * 6.75 * L^{-\frac{3}{4}} = 50$$

$$1{,}350 * L^{-\frac{3}{4}} = 50$$

$$L^{-\frac{3}{4}} = \frac{50}{1{,}350}$$

$$\frac{1}{\sqrt[4]{L^3}} = \frac{50}{1{,}350}$$

$$\frac{1}{\frac{50}{1{,}350}} = \sqrt[4]{L^3}$$

$$L^3 = \left[\frac{1}{\frac{50}{1{,}350}}\right]^4$$

$$L = \sqrt[3]{\left[\frac{1}{\frac{50}{1{,}350}}\right]^4}$$

$$L = 81$$

Thus, hiring 81 units of labor maximizes the firm's profits.

Exercise 2:

Explain the difference between the law of diminishing marginal returns and the law of diminishing marginal rate of substitution.

<u>Solution 2):</u>

<u>Law of Diminishing Marginal Returns:</u>

- ➔ Marginal productivity declines due to increased input usage, holding all inputs constant.

<u>Marginal Rate of Technical Substitution (MRTS):</u>

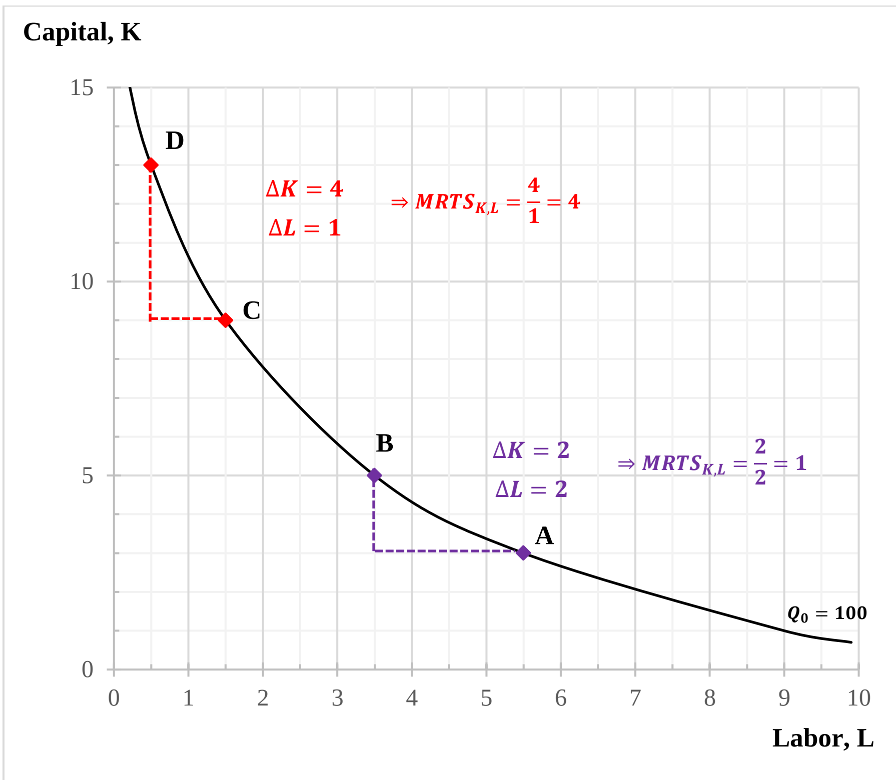
- ➔ The rate at which a producer can substitute between two inputs and maintain the same level of output

$$MRTS_{K,L} = \frac{MP_L}{MP_K}$$

Law of diminishing Marginal Rate of Technical Substitution (MRTS):

➔ When a producer uses less of an input, increasingly more of the other input must be employed to produce the same level of output

Marginal Rate of Technical Substitution (MRTS):



- ***A → B***:
  - The producer substitutes 2 units of capital (K) for 2 units of labor (L) and still produces a total quantity of $Q_0 = 100$

- ***C → D***:
  - The producer has to substitute 4 units of capital (K) for one unit of labor (L) to maintain $Q_0 = 100$

Exercise 3:

An economist estimated that the cost function of a single-product firm is:

$$C(Q) = 100 + 20Q + 15Q^2 + 10Q^3$$

Based on this information, determine:

a) The fixed cost of producing 10 units of output.

Solution 3a):

Fixed Cost:

$$FC(Q) = 100$$

$$FC(10) = 100$$

b) The variable cost of producing 10 units of output.

Solution 3b):

Variable Cost:

$$VC(Q) = 20Q + 15Q^2 + 10Q^3$$

$$VC(10) = 20 * 10 + 15 * 10^2 + 10 * 10^3$$

$$VC(10) = 200 + 1{,}500 + 10{,}000$$

$$VC(10) = 11{,}700$$

c) The total cost of producing 10 units of output.

Solution 3c):

Total Cost:

$$TC(Q) = FC(Q) + VC(Q)$$

$$TC(Q) = 100 + 20Q + 15Q^2 + 10Q^3$$

$$TC(10) = 100 + 11{,}700$$

$$TC(10) = 11{,}800$$

d) The average fixed cost of producing 10 units of output.

Solution 3d):

Average Fixed Cost:

$$AFC(Q) = \frac{FC(Q)}{Q}$$

$$AFC(10) = \frac{100}{10}$$

$$AFC(10) = 10$$

e) The average variable costr5 of producing 10 units of output.

Solution 3e):

Average Variable Cost:

$$AVC(Q) = \frac{VC(Q)}{Q}$$

$$AVC(Q) = \frac{20Q + 15Q^2 + 10Q^3}{Q}$$

$$AVC(10) = \frac{11{,}700}{10}$$

$$AVC(10) = 1{,}170$$

f) The average total cost of producing 10 units of output.

Solution 3f):

Average Total Cost:

$$ATC(Q) = \frac{TC(Q)}{Q}$$

$$ATC(Q) = \frac{100 + 20Q + 15Q^2 + 10Q^3}{Q}$$

$$ATC(10) = \frac{11{,}800}{10}$$

$$ATC(10) = 1{,}180$$

g) The marginal cost when $Q = 10$.

Solution 3g):

Marginal Cost:

$$MC(Q) = \frac{\partial\, TC(Q)}{\partial\, Q}$$

$$MC(Q) = 20 + 30Q + 30Q^2$$

$$MC(10) = 20 + 30 * 10 + 30 * 10^2$$

$$MC(10) = 20 + 300 + 3{,}000$$

$$MC(10) = 3{,}320$$

Exercise 4:

A multi-product firm's cost function was recently estimates as the following:

$$C(q_1, q_2) = 90 - 0.5q_1q_2 + 0.4q_1^2 + 0.3q_2^2$$

a) Are there economies of scope in producing 10 units of product 1 and 10 units of product 2 ($q_1 = q_2 = 10$)?

Solution 4a):

Economies of Scope:

- A firm's unit cost to produce a product will decline as the variety of its products increases.
- Economies of scope exist when the total cost of producing q_1 and $\mathrm{q}_2$together is less than producing q_1 and q_2 separately.

$$C(q_1, 0) + C(0, q_2) > C(q_1, q_2)$$

$$C(q_1, 0) + C(0, q_2) - C(q_1, q_2) > 0$$

$$f + (q_1)^2 + f + (q_2)^2 - [f + \alpha\, q_1q_2 + (q_1)^2 + (q_2)^2] > 0$$

$$f + \alpha\, q_1q_2 > 0$$

For $q_1 = q_2 = 10$

$$f + \alpha\, q_1 q_2 > 0$$

$$90 - 0.5\, q_1 q_2 > 0$$

$$90 - 0.5 * 10 * 10 > 0$$

$$90 - 50 > 0$$

$$40 > 0$$

Yes, economies of scope exist when producing 10 units of both products, as total cost of producing both products together is less than producing them separately.

b) Are there cost complementarities in producing products 1 and 2?

Solution 4b):

Cost Complementarity:

➔ Exists when the marginal cost of producing one unit of output is reduced when the output of another product is increased.

$$\frac{\partial\, MC_1(q_1, q_2)}{\partial\, q_2} < 0$$

Marginal Cost of product1:

$$MC_1(q_1, q_2) = \frac{\partial\, C(q_1, q_2)}{\partial\, q_1} = -0.5 q_2 + 0.8 q_1$$

$$\frac{\partial\, MC_1(q_1, q_2)}{\partial\, q_2} = -0.5 < 0$$

Yes, there are cost complementarities. An increase in q_2 reduces the marginal cost of producing product 1.

c) Suppose the division selling product 2 is floundering and another company has made an offer to buy the exclusive rights to produce product 2. How would the sale of the rights to produce product 2 change the firm's marginal cost of producing product 1?

Solution 4c):

Marginal Cost of product1 with $q_2 = 0$:

$$MC_1(q_1, 0) = \frac{\partial\, C(q_1, 0)}{\partial\, q_1} = 0.8q_1$$

Compare marginal cost of product 1 before and after.

$$MC_1(q_1, q_2) < MC_1(q_1, 0)$$

If the division sales the rights of producing product 2, then the marginal cost of producing product 1increases.

Exercise 5:

Explain the difference between fixed costs, sunk costs, and variable costs. Provide an example that illustrates that these costs are, in general, different.

<u>Solution 5):</u>

- **Fixed costs** are associated with fixed inputs and do not change when output changes.
- **Variable costs** are costs associated with variable inputs and do change when output changes.
- **Sunk costs** are costs that are forever lost once they have been paid.