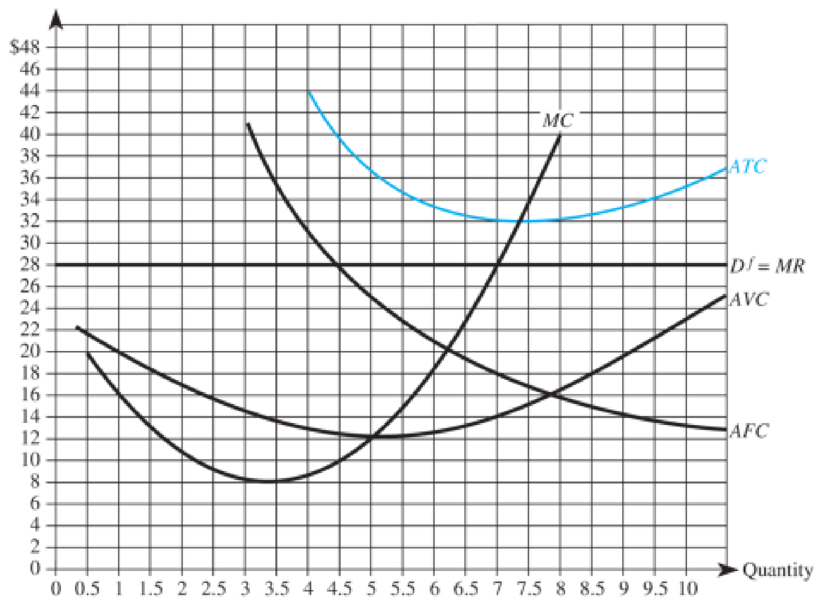
Problem Set 6

Exercise 1:

The following graph summarizes the demand and costs for a firm that operates in a perfectly competitive market:



a) What level of output should this firm produce in the short run?

<u>Solution 1a):</u>

In the short run, the firm produces (maximizes profits) at the point where the marginal cost are equal to the marginal revenue, i.e., $MC = MR$

This occurs at a quantity of $Q = 7$ units.

b) What price should this firm charge in the short run?

Solution 1b):

In the short run the firm produces where $MC = MR$. Hence, the firm should charge a price of $P = \$28 = D^f = MR$.

c) What is the firm's total cost at this level of output?

Solution 1c):

Total Costs:

The total cost is the sum of fixed costs and variable cost.

At a quantity of $Q = 7$ (short term equilibrium) the average fixed cost are $AFC(7) = 14$ and average variable cost are: $AVC(7) = 18$.

Thus, the total cost are:

$$TC(Q) = FC(Q) + VC(Q)$$

$$TC(Q) = Q * AFC(Q) + Q * AVC(Q)$$

$$TC(7) = 7 * 14 + 7 * 18$$

$$TC(7) = 98 + 126$$

$$TC(7) = 224$$

d) What is the firm's total variable cost at this level of output?

Solution 1d):

Variable Cost:

$$VC(Q) = Q * AVC(Q)$$

In the short term, the firm produces $Q = 7$ units of output:

$$VC(7) = 7 * 14$$

$$VC(7) = 98$$

e) What is the firm's fixed cost at this level of output?

Solution 1e):

Fixed Cost:

The fixed cost are given by the difference between total cost and variable cost:

$$FC = TC(Q) - VC(Q)$$

The optimal output level is $\mathrm{Q}^* = 7$ (solution 1a):

$$\mathrm{FC} = \mathrm{TC}(7) - \mathrm{VC}(7)$$

$$\mathrm{FC} = 224 - 98$$

$$\mathrm{FC} = 126$$

f) What is the firm's profit if it produces this level of output?

Solution 1f):

Profits:

$$\pi(Q^*) = R(Q^*) - TC(Q^*)$$

$$\pi(Q^*) = p * Q^* - [FC + VC(Q^*)]$$

$$\pi(7) = 28 * 7 - 224$$

$$\pi(7) = 196 - 224$$

$$\pi(7) = -28$$

The firm is earning a loss of $28, i.e., it incurs negative profits.

g) What is the firm's profit if it shuts down?

Solution 1g):

If the firm shuts down it will not produce any output. Hence, the firm's revenues and variable cost will be equal to zero. However, the firm still faces fixed costs of $\mathrm{FC} = 126$

$$\pi(0) = R(0) - TC(0)$$

$$\pi(0) = R(0) - [FC + VC(0)]$$

$$\pi(0) = 0 - 126 - 0$$

$$\pi(0) = -126$$

Thus, when shutting down, the firm will incur losses equal to its fixed costs.

h) In the long run, should this firm continue to operate or shut down?

Solution 1h):

In the long term the firm should shut down, as it incurs negative profits.

Exercise 2:

A firm sells its product in a perfectly competitive market where other firms charge a price of \$90 per unit. The firm's total costs are given by:

$$C(Q) = 50 + 10Q + 2Q^2$$

a) How much output should the firm produce in the short run?

Solution 2a):

Marginal Revenue:

$$MR(Q) = \frac{\partial R(Q)}{\partial Q} = \frac{\partial (Q * p)}{\partial Q} = p = 90$$

Marginal Cost

$$MC = \frac{\partial C(Q)}{\partial Q} = 10 + 4Q$$

Optimal Output:

Optimal output occurs where the marginal cost are equal to the marginal revenue, i.e., $MC = MR$

$$MC = MR$$

$$10 + 4Q = 90$$

$$4Q = 80$$

$$Q^* = 20$$

b) What price should the firm charge in the short run?

Solution 2b):

In the short run, the firm should charge the market price of $90 per unit. This is equal to the form’s marginal revenue.

c) What are the firm's short-run profits?

Solution 2c):

Profits:

$$\pi(Q^*) = R(Q^*) - C(Q^*)$$

$$\pi(Q^*) = Q^* * p - [50 + 10Q^* + 2Q^{*2}]$$

$$\pi(20) = 20 * 90 - [50 + 10 * 20 + 2 * 20^2]$$

$$\pi(20) = 1800 - [50 + 200 + 800]$$

$$\pi(20) = 1800 - 1050$$

$$\pi^*(20) = 750$$

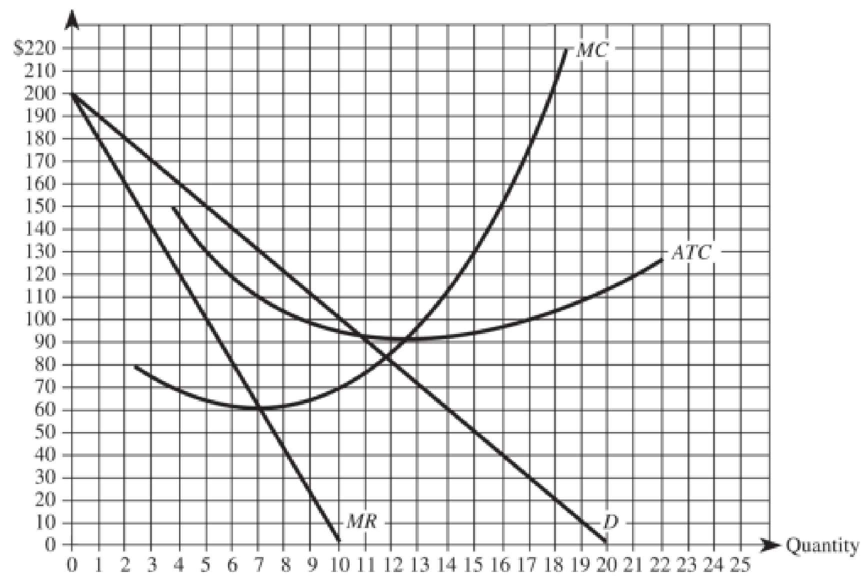
d) What adjustments should be anticipated in the long run?

Solution 2d):

The firm needs to anticipate that in the long-run, new firms will enter the market, i.e., entry will occur. Consequently, the market market price will fall and the firm should plan to reduce its output. In the long-run, the firm's economic profits will shrink to zero.

Exercise 3:

The following graph summarizes the demand and costs for a firm that operates in a monopolistically competitive market:



a) What is the firm's optimal output?

Solution 3a):

The monopolistic firm will produce at the point where the marginal cost are equal to the marginal revenue, i.e., $MC = MR$

This occurs at a quantity of $Q^M = 7$ units.

b) What is the firm's optimal price?

Solution 3b):

At a quantity of $Q^M = 7$ the monopolistic firm will charge a price of $p^M = \$130$. Executing its market power, the firm will charge a price equal to the demand function at the firm's optimal quantity, i.e., $p^M = D(Q^M)$.

c) What are the firm's maximum profits?

Solution 3c):

Profits:

$$\pi^M(Q^M) = R(Q^M) - C(Q^M)$$

$$\pi^M(Q^M) = p^M * Q^M - ATC(Q^M) * Q^M$$

$$\pi^M(7) = 130 * 7 - 110 * 7$$

$$\pi^M(7) = 910 - 770$$

$$\pi^M(7) = 140$$

d) What adjustments should the manager be anticipating?

Solution 3d):

In the long-term, new firms will enter the market, i.e., monopolistically competitive market. Thus, the firm's demand will decrease over time and it's economic profits will shrink to zero.

Exercise 4:

Suppose you are the manager of a monopoly, and your demand and cost functions are given by the following:

Demand: $P(Q) = 300 - 3Q$

Cost: $C(Q) = 1{,}500 + 2Q^2$

a) What price-quantity combination maximizes your firm's profits?

<u>Solution 4a):</u>

<u>Revenues:</u>

$$R(Q) = P(Q) * Q$$

$$R(Q) = (300 - 3Q) * Q$$

$$R(Q) = 300Q - 3Q^2$$

<u>Marginal Revenue:</u>

$$MR(Q) = \frac{\partial R(Q)}{\partial Q} = 300 - 6Q$$

<u>Marginal Cost</u>

$$MC = \frac{\partial C(Q)}{\partial Q} = 4Q$$

Optimal Output:

Optimal output occurs where $MC = MR$

$$MC = MR$$

$$4Q = 300 - 6Q$$

$$10Q = 300$$

$$Q^* = 30$$

Optimal Price:

$$P^*(Q^*) = 300 - 3Q^*$$

$$P^*(30) = 300 - 3 * 30$$

$$P^*(30) = 210$$

b) Calculate the maximum profits.

Solution 4b):

Profits:

$$\pi(Q^*) = R(Q^*) - C(Q^*)$$

$$\pi(Q^*) = P^* * Q^* - C(Q^*)$$

$$\pi(30) = 210 * 30 - [1{,}500 + 2 * 30^2]$$

$$\pi(30) = 6{,}300 - 3{,}300$$

$$\pi(30) = 3{,}000$$

c) Is demand elastic, inelastic, or unit elastic at the profit-maximizing price-quantity combination?

Solution 4c):

Demand:

$$P(Q) = 300 - 3Q \;\Leftrightarrow\; Q(P) = 100 - \tfrac{1}{3}P$$

Price elasticity of demand:

$$E_{Q,P} = \frac{\partial Q(P)}{\partial P} * \frac{P}{Q}$$

$$E_{Q,P} = -\frac{1}{3} * \frac{P^*}{Q^*}$$

$$E_{Q,P} = -\frac{1}{3} * \frac{210}{30}$$

$$E_{Q,P} = -\frac{70}{3}$$

Since the price elasticity is greater than one in absolute terms, i.e. $|E_{Q,P}| = \left|-\frac{70}{3}\right| > 1$, demand is elastic.

d) What price-quantity combination maximizes revenue?

Solution 4d):

Revenues are maximized when the firm's marginal revenues are equal to zero:

Revenue maximizing output:

$$MR(Q) = 300 - 6Q = 0$$

$$6Q = 300$$

$$Q^{**} = 50$$

Revenue maximizing price:

$$P(Q^{**}) = 300 - 3 * Q^{**}$$

$$P(50) = 300 - 3 * 50$$

$$P^{**} = 150$$

e) Calculate the maximum revenues.

Solution 4e):

Maximum Revenues:

$$R(Q^{**}) = P^{**} * Q^{**}$$

$$R(50) = 150 * 50$$

$$R(50) = 7{,}500$$

f) Is demand elastic, inelastic, or unit elastic at the revenue-maximizing price-quantity combination?

Solution 4f):

Demand:

$$P(Q) = 300 - 3Q \;\Leftrightarrow\; Q(P) = 100 - \tfrac{1}{3}P$$

Price elasticity of demand:

$$E_{Q,P} = \frac{\partial Q(P^{**})}{\partial P} * \frac{P^{**}}{Q^{**}}$$

$$E_{Q,P} = -\frac{1}{3} * \frac{150}{50}$$

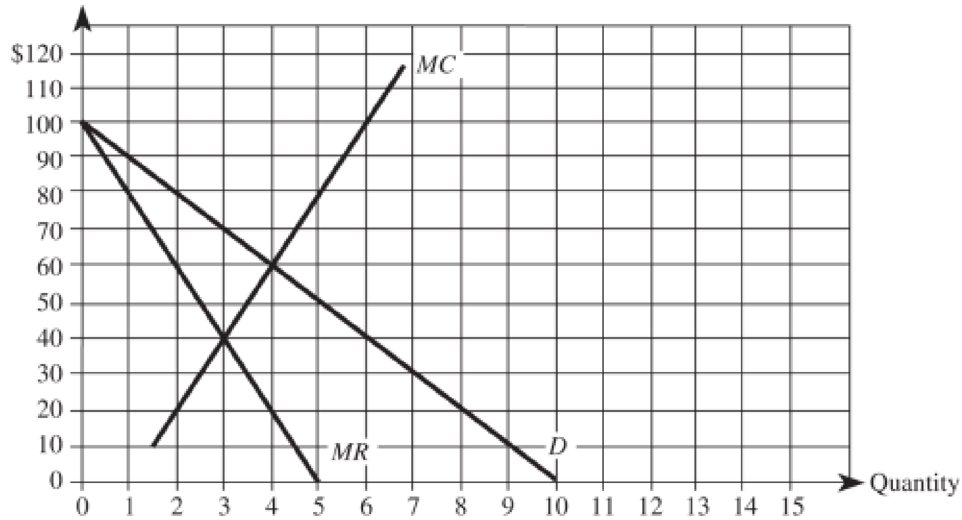
$$E_{Q,P} = -\frac{1}{3} * 3$$

$$E_{Q,P} = -1$$

Since the price elasticity is equal to one in absolute terms, i.e. $|E_{Q,P}| = |-1| = 1$, demand is unit elastic.

Exercise 5:

The accompanying diagram shows the demand, marginal revenue, and marginal cost of a monopolist:



a) Determine the profit-maximizing output and price.

<u>Solution 5a):</u>

The monopolistic firm will produce at the point where the marginal cost are equal to the marginal revenue, i.e., $MC = MR$

This occurs at a quantity of $Q^M = 3$ units.

At a quantity of $Q^M = 3$ the monopolistic firm will charge a price of $p^M = \$70$. Executing its market power, the firm will charge a price equal to the demand function at the firm's optimal quantity, i.e., $p^M = D(Q^M)$.

b) What price and output would prevail if this firm's product were sold by price-taking firms in a perfectly competitive market?

Solution 5b):

Under perfect competition, the market equilibrium will occur where the supply curve and demand curve intersect, i.e. where $MC = D$:

Thus, under perfect competitive market conditions, the firm would produce $Q^* = 4$ units with a market price of $P^* = 60$.

c) Calculate the deadweight loss of this monopoly.

Solution 5c):

Deadweight Loss (DWL):

$$DWL = \frac{1}{2}[(70 - 40) * (5 - 4)]$$

$$DWL = \frac{1}{2}[30 * 1]$$

$$DWL = 15$$