Problem Set 2

Exercise 1:

Suppose the supply function for product X is given by: $Q_X^S = -30 + 2P_X - 4P_Z$

a) How much of product X is produced when $P_X = \$600$ and $P_Z = \$60$?

Solution 1a):

Insert $P_X = \$600$ and $P_Z = \$60$ into the supply function:

$Q_X^S = -30 + 2 * 600 - 4 * 60$

$Q_X^S = -30 + 1{,}200 - 240$

$Q_X^S = 930$

b) How much of product X is produced when $P_X = \$80$ and $P_Z = \$60$?

Solution 1b):

Insert $P_X = \$80$ and $P_Z = \$60$ into the supply function:

$Q_X^S = -30 + 2 * 80 - 4 * 60$

$Q_X^S = -30 + 160 - 240$

$Q_X^S = -110$

Notably, having a negative output in supply is impossible. Thus, the quantity supplied is zero, i.e., $Q_X^S = 0 \;\; if \; P_X = \$80 \; and \; P_Z = \$60$.

c) Suppose that $P_Z = \$60$. Determine the supply function and inverse supply function for good X. Graph the inverse supply function.

Solution 1c):

Insert $P_Z = \$60$ into the supply function:

$$Q_X^S = -30 + 2P_X - 4 * 60$$

$$Q_X^S = -30 + 2P_X - 240$$

$$Q_X^S = 2P_X - 270$$

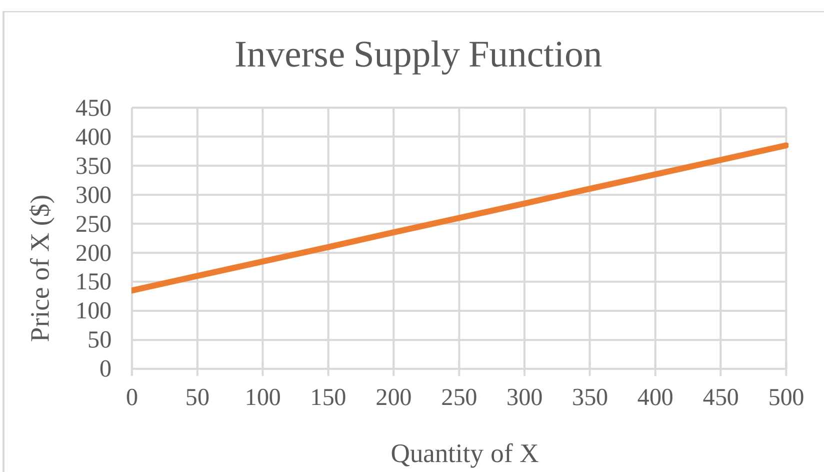
To obtain the inverse supply equation, simply solve for P_X:

$$Q_X^S = 2P_X - 270$$

$$2P_X = Q_X^S + 270$$

$$P_X = \frac{1}{2}Q_X^S + 135$$

The inverse supply function is graphed below:



Exercise 2:

Suppose the demand function for good X is given by:

$$Q_X^D = 6000 - \frac{1}{2}P_X - P_Y + 9P_Z + \frac{1}{10}M$$

Research shows that the prices of related goods are given by: $P_Y = \$6{,}500$ and $P_Z = \$100$. The average income of individuals consuming this product is: $M = \$70{,}000$.

a) Indicate whether goods Y and Z are substitutes or complements for good X.

Solution 2a):

Cross price elasticity of demand:

$$E_{Q_X,P_Y} = \alpha_Y \frac{P_Y}{Q_X}$$

$$E_{Q_X,P_Y} = -1 * \frac{P_Y}{Q_X^D} \quad < \quad \mathbf{0}$$

Good Y is a complement good for good X since the cross-price elasticity is negative, i.e., $E_{Q_X,P_Y} < 0$¨

This means that the demand for good X decreases as the price for good Y increases and vice versa (e.g., toner and printer)

$$E_{Q_X,P_Z} = \alpha_Y \frac{P_Z}{Q_X}$$

$$E_{Q_X,P_Z} = (+)\, 9 * \frac{P_Z}{Q_X^D} \quad > \quad \mathbf{0}$$

Good Z is a substitute good for good X since the cross-price elasticity is positive, i.e., $E_{Q_X,P_Y} > 0$

This means that the demand for good X will increase as the price for good Z increases and vice versa (e.g., butter and margarine)

b) Is X an inferior or a normal good?

Solution 2b):

Income elasticity:

$$E_{Q_X,M} = \alpha_M \frac{M}{Q_X^D}$$

$$E_{Q_X,M} = (+) \frac{1}{10} \frac{M}{Q_X^D} \quad > \quad \mathbf{0}$$

Good X is considered a normal good, as the income elasticity is positive, i.e., $E_{Q_X,M} > 0$

The demand for good X increases as the income (M) increases and vice versa (e.g., high tech products, cars, home services…)

For inferior foods, the demand for a good X would deacrese as income (M) increases and vice versa, (e.g., secondhand clothing, used cars…)

c) How many units of good X will be purchased when $P_X = \$5{,}230$?

Solution 2c):

Insert in all known variables in demand function:

$$Q_X^D = 6000 - \frac{1}{2}P_X - P_Y + 9P_Z + \frac{1}{10}M$$

$$Q_X^D = 6000 - \frac{1}{2} * 5{,}230 - 6{,}500 + 9 * 100 + \frac{1}{10} * 70{,}000$$

$$Q_X^D = 6000 - 2{,}615 - 6{,}500 + 900 + 7{,}000$$

$$Q_X^D = 4{,}785$$

d) Determine the demand function and inverse demand function for good X.

Solution 2d):

For the given income and prices of other goods, the demand function for good X is:

$$Q_X^D = 6000 - \frac{1}{2}P_X - P_Y + 9P_Z + \frac{1}{10}M$$

$$Q_X^D = 6000 - \frac{1}{2} * P_X - 6{,}500 + 9 * 100 + \frac{1}{10} * 70{,}000$$

$$Q_X^D = 6000 - \frac{1}{2}P_X - 6{,}500 + 900 + 7{,}000$$

$$Q_X^D = 7{,}400 - \frac{1}{2}P_X$$

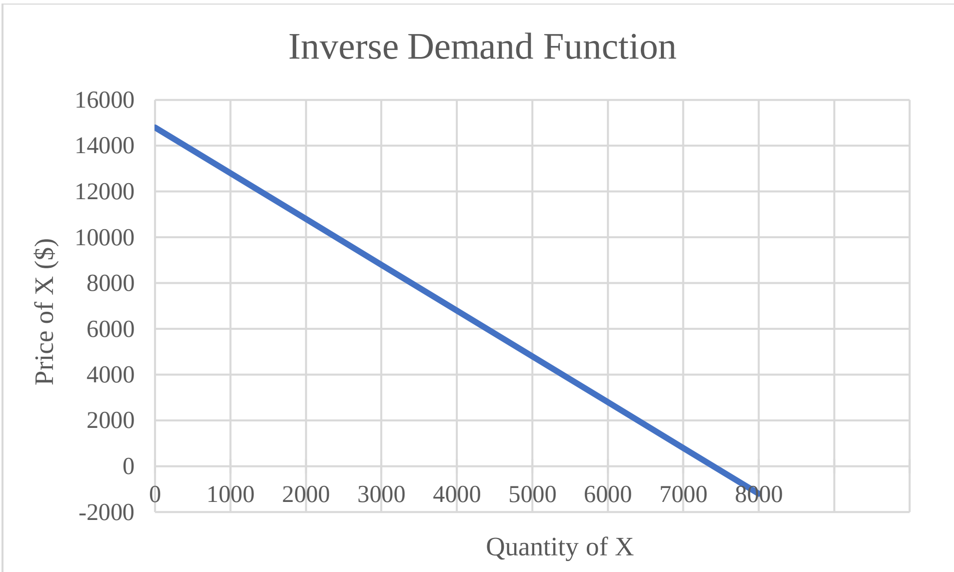
To find the inverse demand function, solve for the price P_X:

$$Q_X^D = 7{,}400 - \frac{1}{2}P_X$$

$$\frac{1}{2}P_X = 7{,}400 - Q_X^D$$

$$P_X = 14{,}800 - 2Q_X^D$$

The inverse demand function is graphed in the figure below:



Exercise 3:

The demand curve for product X us given by the following function:

$$Q_X^D = 300 - 2P_X$$

a) Find the inverse demand curve.

<u>Solution 3a):</u>

<u>To find the inverse demand function, solve for the price P_X:</u>

$Q_X^D = 300 - 2P_X$

$2P_X = 300 - Q_X^D$

$P_X = 150 - \frac{1}{2}Q_X^D$

b) How much consumer surplus (CS) of consumers receive when $P_X = \$45$?

<u>Solution 3b):</u>

Find quantity demand if $P_X = \$45$:

$Q_X^D = 300 - 2P_X$

$Q_X^D = 300 - 2 * 45$

$Q_X^D = 300 - 90$

$Q_X^D = 210$

From 3a) we know that the vertical intercept of the inverse demand curve is 150.

The consumer surplus is calculated by:

$CS = \frac{1}{2}\left[P_X^{\max} \quad - P_X\right] * Q_X^D$

$CS = \frac{1}{2}[150 - 45] * 210$

$CS = \frac{1}{2} * 105 * 210$

$CS = 52.5 * 210$

$CS = 11{,}025$

c) How much consumer surplus (CS) do consumers receive when $P_X = \$30$?

Solution 3c):

Find quantity demand if $P_X = \$30$:

$Q_X^D = 300 - 2P_X$

$Q_X^D = 300 - 2 * 30$

$Q_X^D = 300 - 60$

$Q_X^D = 240$

From 3a) we know that the vertical intercept of the inverse demand curve is 150.

The consumer surplus is calculated by:

$CS = \frac{1}{2}\left[P_X^{\max} \quad - P_X\right] * Q_X^D$

$CS = \frac{1}{2}[150 - 30] * 240$

$CS = \frac{1}{2} * 120 * 240$

$CS = 60 * 240$

$CS = 14{,}400$

d) In general, what happens to the level of consumer surplus (CS) as the price of a good falls?

Solution 3d):

So long as the law of demand holds, a decrease in price leads to an increase in consumer surplus, and vice versa.

In general, there is an inverse relationship between the price of a product and consumer surplus.

$$CS = \frac{1}{2}\left[P_X^{\max} \quad - P_X\right] * Q_X^D$$

If $P_X \uparrow \ \rightarrow \ Q_X^D \downarrow \ \rightarrow \ CS \downarrow$

If $P_X \downarrow \ \rightarrow \ Q_X^D \uparrow \ \rightarrow \ CS \uparrow$

Exercise 4:

Suppose the demand and supply are given by the following equations:

$$Q_X^D = 14 - \frac{1}{2}P_X \qquad \text{and} \qquad Q_X^S = \frac{1}{4}P_X - 1$$

a) Determine the equilibrium price (P_X^*) and quantity (Q_X^*)

Solution 4a):

To find the equilibrium price set $Q_X^D = Q_X^S$ and solve for P_X

$$Q_X^D = Q_X^S$$

$$14 - \frac{1}{2}P_X = \frac{1}{4}P_X - 1$$

$$\frac{1}{4}P_X + \frac{1}{2}P_X = 14 + 1$$

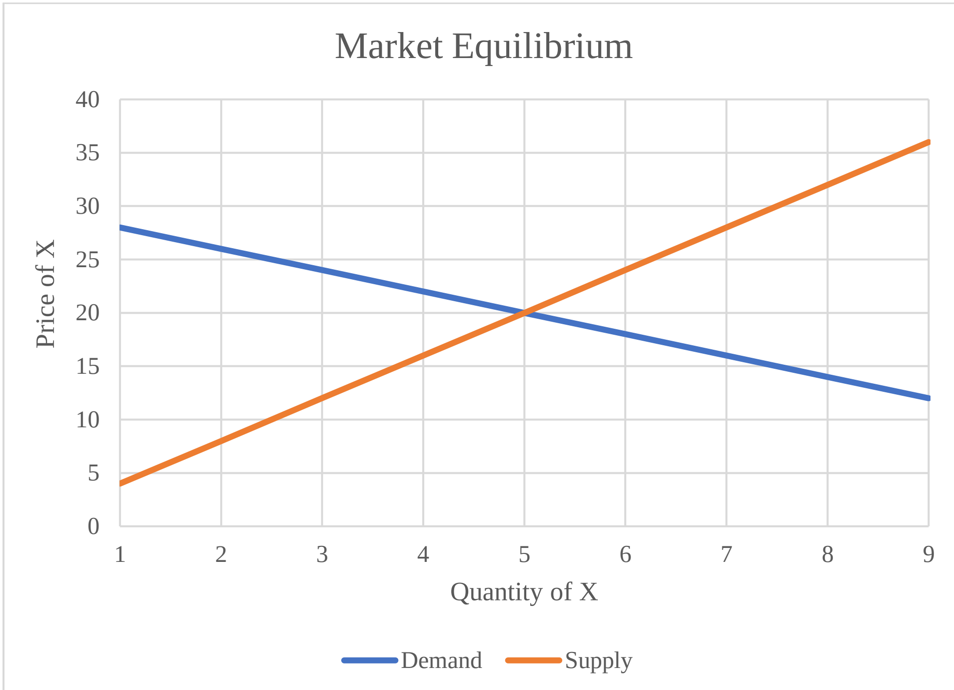
$$\frac{3}{4}P_X = 15$$

$$P_X^* = 20$$

To find the equilibrium quantity, put P_X^* into the demand or supply function and solve for Q_X^*:

Demand Function	Supply Function
$Q_X^D = 14 - \frac{1}{2}P_X^*$ $Q_X^D = 14 - \frac{1}{2}20$ $Q_X^D = 14 - 10$ $Q_X^* = 4$	$Q_X^S = \frac{1}{4}P_X^* - 1$ $Q_X^S = \frac{1}{4}20 - 1$ $Q_X^S = 5 - 1$ $Q_X^* = 4$

The equilibrium is shown in the figure below:



b) Suppose a \$12 excise tax is imposed on the good. Determine the new equilibrium price and quantity

Solution 4b):

A \$12 excise tax shifts the supply curve up by the amount of the tax.

Mathematically that means that the intercept of the inverse supply function increases by \$12.

Inverse supply function before tax:

$Q_X^S = \frac{1}{4}P_X - 1$

$\frac{1}{4}P_X = 1 + Q_X^S$

$P_X = 4 + 4Q_X^S$

Inverse demand function after tax:

$P_X^{tax} = 4 + tax + 4Q_X^S$

$P_X^{tax} = 4 + 12 + 4Q_X^S$

$P_X^{tax} = 16 + 4Q_X^S$

After Tax quantity supplied:

$P_X^{tax} = 16 + 4Q_X^S$

$4Q_X^S = P_X - 16$

$Q_X^{S,tax} = \frac{1}{4}P_X - 4$

To find the new equilibrium price, we equate quantity demand to after-tax quantity supplied:

$$Q_X^D = Q_X^{S,tax}$$

$$14 - \frac{1}{2}P_X = \frac{1}{4}P_X - 4$$

$$\frac{1}{4}P_X + \frac{1}{2}P_X = 14 + 4$$

$$\frac{3}{4}P_X = 18$$

$$P_X^* = 24$$

<u>To find the equilibrium quantity, put P_X^* into the demand or supply function and solve for Q_X^*:</u>

Demand Function	Supply Function
$Q_X^D = 14 - \frac{1}{2}P_X^*$ $Q_X^D = 14 - \frac{1}{2}24$ $Q_X^D = 14 - 12$ $Q_X^{tax\,*} = 2$	$Q_X^{S,tax} = \frac{1}{4}P_X^* - 4$ $Q_X^{S,tax} = \frac{1}{4}24 - 4$ $Q_X^{S,tax} = 6 - 4$ $Q_X^{tax\,*} = 2$

c) How much tax revenue does the government earn with the $12 tax?

<u>Solution 4c):</u>

After imposing the tax, a total of $Q_X^{tax\,*} = 2$ units are sold.

<u>With a tax rate of $12 per unit, total tax revenues (TR) are:</u>

$$TR = tax * Q_X^{tax\,*}$$

$$TR = 12 * 2$$

$$TR = 24$$

Exercise 5:

Suppose the supply curve for product X is given by:

$$Q_X^S = -520 + 20P_X$$

a) Find the inverse supply curve.

<u>Solution 5a):</u>

<u>Inverse supply curve:</u>

$Q_X^S = -520 + 20P_X$

$20P_X = Q_X^S + 520$

$P_X = 26 + \frac{1}{20}Q_X^S$

b) How much surplus do producers receive when $Q_X^S = 400$ and $Q_X^S = 1{,}200$?

Solution 5b):

Producer Surplus (PS):

$$PS = \frac{1}{2}\left[P_X - P_X^{\min}\right] * Q_X^S$$

For $Q_X^S = 400$:	For $Q_X^S = 1{,}200$:
$PS = \frac{1}{2}\left[26 + \frac{1}{20} Q_X^S - P_X^{\min}\right] * Q_X^S$	$PS = \frac{1}{2}\left[26 + \frac{1}{20} Q_X^S - P_X^{\min}\right] * Q_X^S$
$PS = \frac{1}{2}\left[26 + \frac{1}{20} 400 - 26\right] * 400$	$PS = \frac{1}{2}\left[26 + \frac{1}{20} 1{,}200 - 26\right] * 1{,}200$
$PS = \frac{1}{2}[26 + 20 - 26] * 400$	$PS = \frac{1}{2}[26 + 60 - 26] * 1{,}200$
$PS = \frac{1}{2}[46 - 26] * 400$	$PS = \frac{1}{2}[86 - 26] * 1{,}200$
$PS = \frac{1}{2} 20 * 400$	$PS = \frac{1}{2} 60 * 1{,}200$
$PS = 4{,}000$	$PS = 36{,}000$