INTRODUCTORY ECONOMICS: LECTURE 3
Quantitative Demand Analysis

Highlights

- *Price elasticity of demand*
- *Factors affecting price elasticity*
- *Price elasticity and total revenue*
- *Cross-price elasticity of demand*
- *Substitutes/complements*
- *Income elasticity of demand*
- *Normal/inferior good*
- *A taste of econometrics*

One Observation in Demand

- One observation: Some demand curves are steeper; some are flatter.
- What does it mean to have the demand curve be steeper or flatter?
- We need a **unit-free** measure for responsiveness of demand to price.

The Elasticity Concept

- **Elasticity**
  - A measure of the responsiveness of one variable to changes in another variable; the percentage change in one variable that arises due to a given percentage change in another variable.

The Elasticity Concept

- The elasticity between two variables, G and S, is mathematically expressed as:

$$E_{G,S} = \frac{\%\Delta G}{\%\Delta S}$$

- When a functional relationship exists, like $G = f(S)$, the elasticity is:

$$E_{G,S} = \frac{dG}{dS}\frac{S}{G}$$

Price Elasticity of Demand

- **(Own) Price elasticity of demand**
  - Measures the responsiveness of a percentage change in the quantity demanded of good *X* to a percentage change in its price.

$$E_{Q_X{}^d, P_X} = \frac{\%\Delta Q_X{}^d}{\%\Delta P_X} = \frac{dQ_X}{dP_X}\frac{P_X}{Q_X}$$

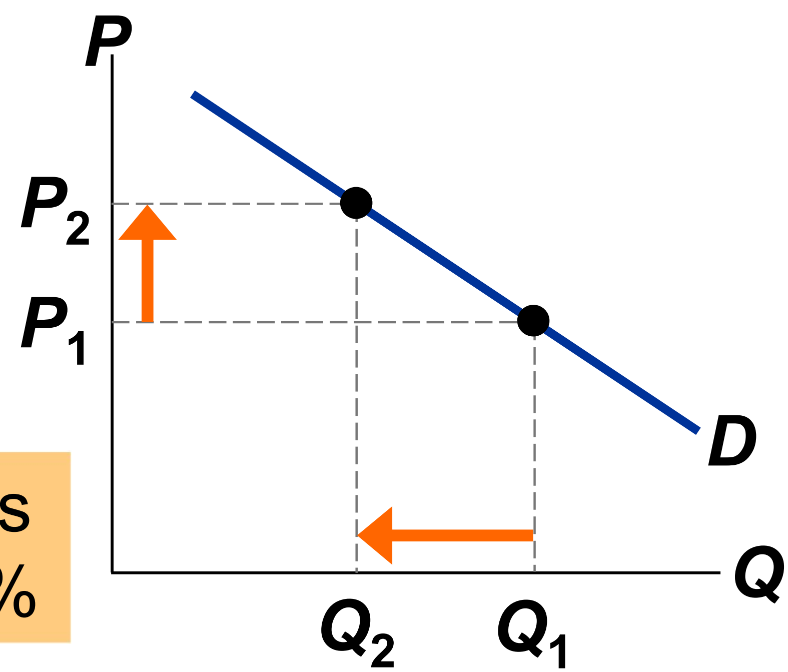
<u>Example:</u>

Price elasticity of demand equals

$$\frac{-15\%}{10\%} = -1.5$$

P rises by 10%

Q falls by 15%



Price Elasticity of Demand

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$$E_{Q_X{}^d, P_X} = \frac{\%\Delta Q_X{}^d}{\%\Delta P_X} = \frac{dQ_X}{dP_X}\frac{P_X}{Q_X}$$

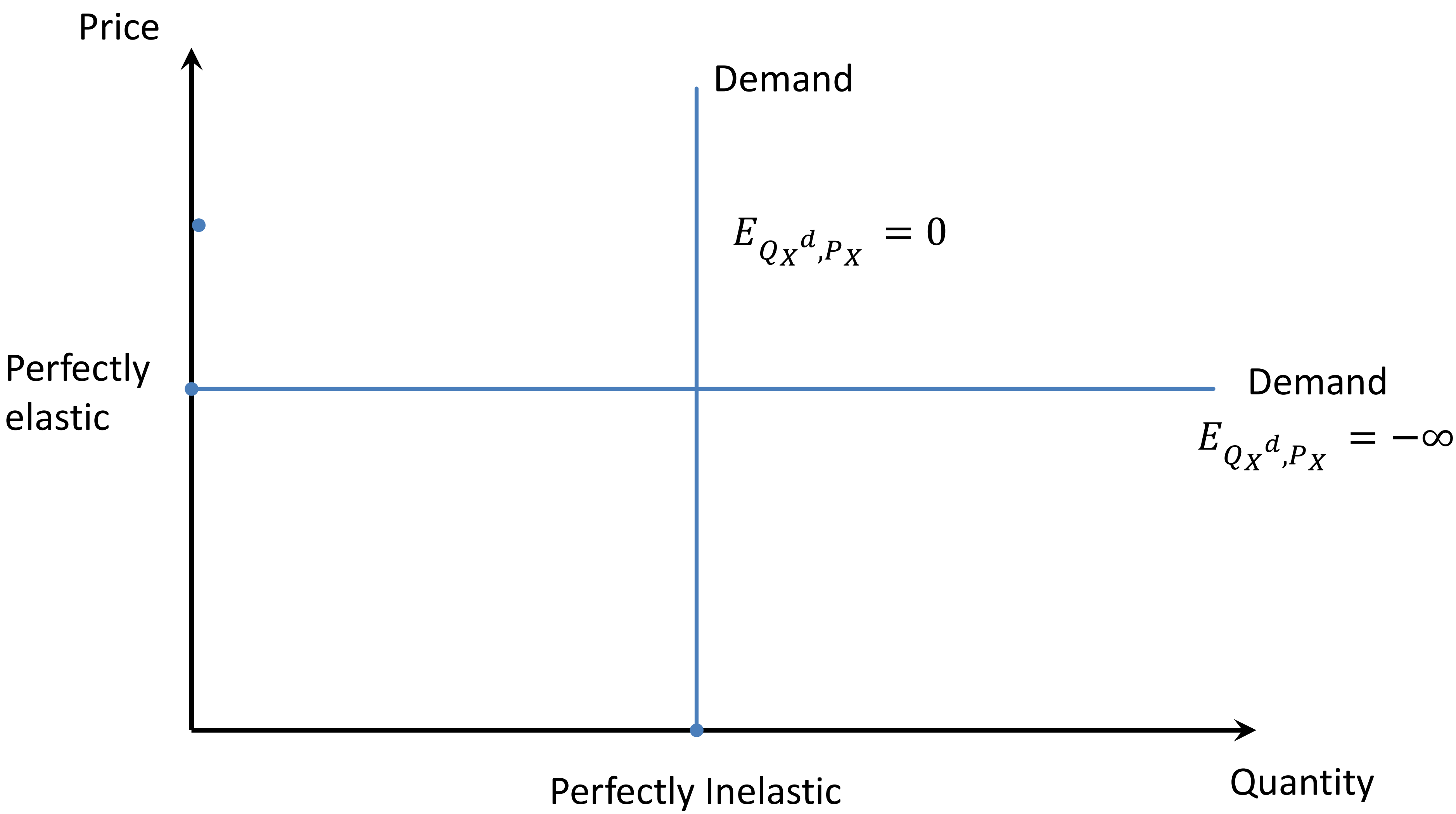
  - Sign: negative by law of demand.
  - Magnitude of absolute value relative to unity:
    - $\left|E_{Q_X{}^d, P_X}\right| > 1$: Elastic.
    - $\left|E_{Q_X{}^d, P_X}\right| < 1$: Inelastic.
    - $\left|E_{Q_X{}^d, P_X}\right| = 1$: Unitary elastic.

Price Elasticity of Demand

- The price elasticity of demand is closely related to the slope of the demand curve.

- Rule of thumb:
  The flatter the curve, the bigger the elasticity. The steeper the curve, the smaller the elasticity.

Perfectly Elastic and Inelastic Demand



Discussions: Examples?

Price Elasticity of Demand in Reality

Table 3–2

Selected Own Price Elasticities

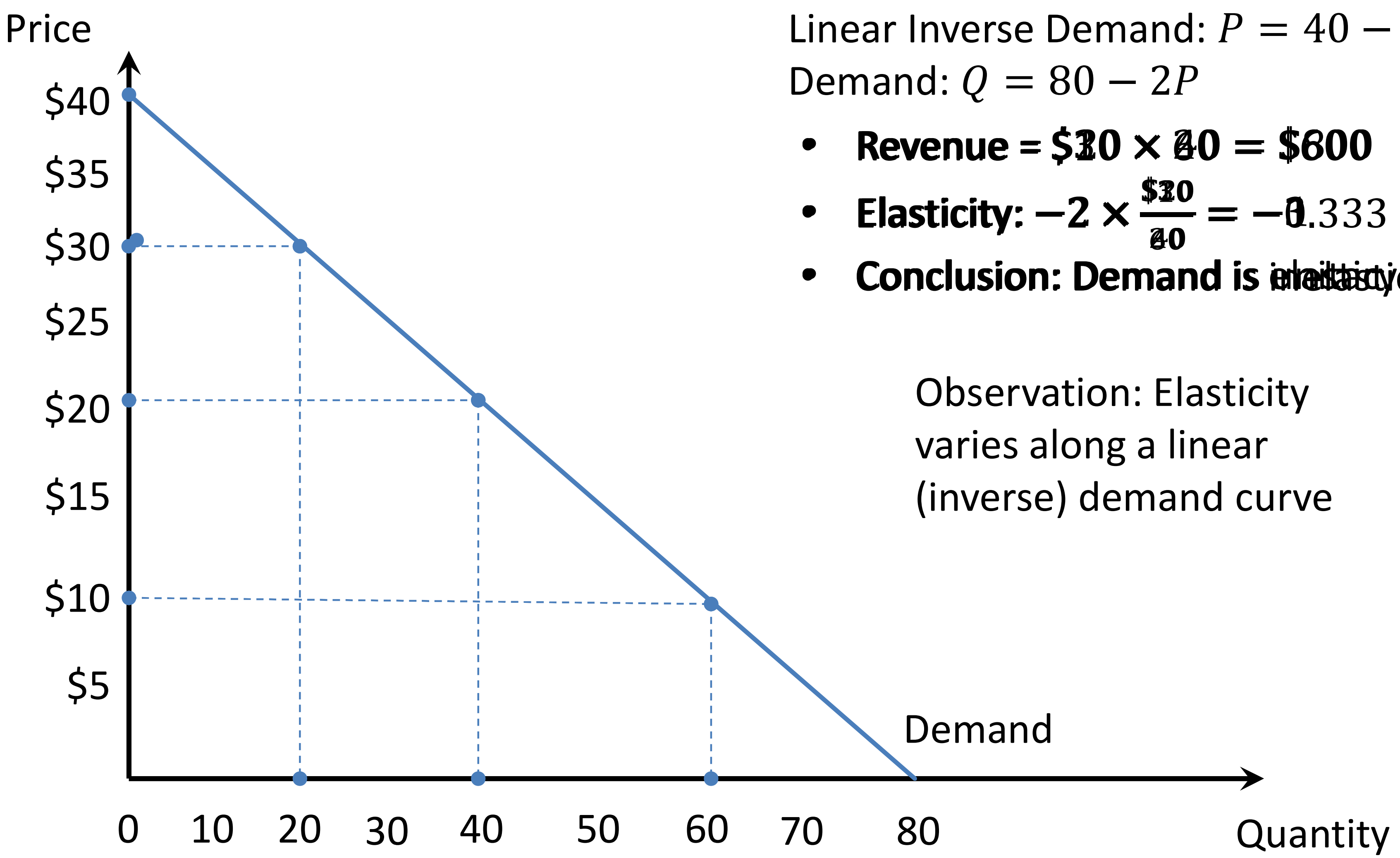
Market	Own Price Elasticity
Transportation	−0.6
Motor vehicles	−1.4
Motorcycles and bicycles	−2.3
Food	−0.7
Cereal	−1.5
Clothing	−0.9
Women's clothing	−1.2

SOURCES: M. R. Baye, D. W. Jansen, and J. W. Lee, "Advertising Effects in Complete Demand Systems," *Applied Economics* 24 (1992), pp. 1087–96; W. S. Comanor and T. A. Wilson, *Advertising and Market Power* (Cambridge, MA: Harvard University Press, 1974).

Factors Affecting Price Elasticity

- Factors can impact the own price elasticity of demand:
  - Availability of consumption substitutes
    - How broadly or narrowly the good is defined; e.g., “Blue Jeans” vs. “Clothing”
  - Time horizon (short-run or long-run)
    - e.g., If gasoline price increases, …
  - Whether the good is a necessity or a luxury
    - e.g., Insulin vs. Caribbean Cruises
  - Expenditure share of consumers’ budgets
    - e.g., food vs. transport

Linear Demand, Elasticity, and Revenue



Linear Inverse Demand: $P = 40 - 0.5Q$

Demand: $Q = 80 - 2P$

- **Revenue = $\$30 \times 20 = \$600$ / $\$20 \times 40 = \$800$ / $\$10 \times 60 = \$600$**
- **Elasticity: $-2 \times \frac{\$30}{20} = -3$ / $-2 \times \frac{\$20}{40} = -1$ / $-2 \times \frac{\$10}{60} = -0.333$**
- **Conclusion: Demand is elastic. / unitary elastic. / inelastic.**

Observation: Elasticity varies along a linear (inverse) demand curve

Price Elasticity and Total Revenue

- Continuing our scenario, if you raise your price, would your revenue rise or fall?

$$\text{Revenue} = \boldsymbol{P} \times \boldsymbol{Q}$$

- A price increase has two effects on revenue:
  - Higher ***P*** means more revenue on each unit you sell.
  - But you sell fewer units (lower ***Q***), due to Law of Demand.
- Which of these two effects is bigger?
  It depends on the price elasticity of demand.

Price Elasticity and Total Revenue

$$\text{Price elasticity of demand} = \frac{\text{Percentage change in } \boldsymbol{Q}}{\text{Percentage change in } \boldsymbol{P}}$$

Revenue = ***P*** x ***Q***

- If demand is elastic, then

  price elast. of demand > 1

  % change in ***Q*** > % change in ***P***

- The fall in revenue from lower ***Q*** is greater than the increase in revenue from higher ***P***, so revenue falls.

Price Elasticity and Total Revenue

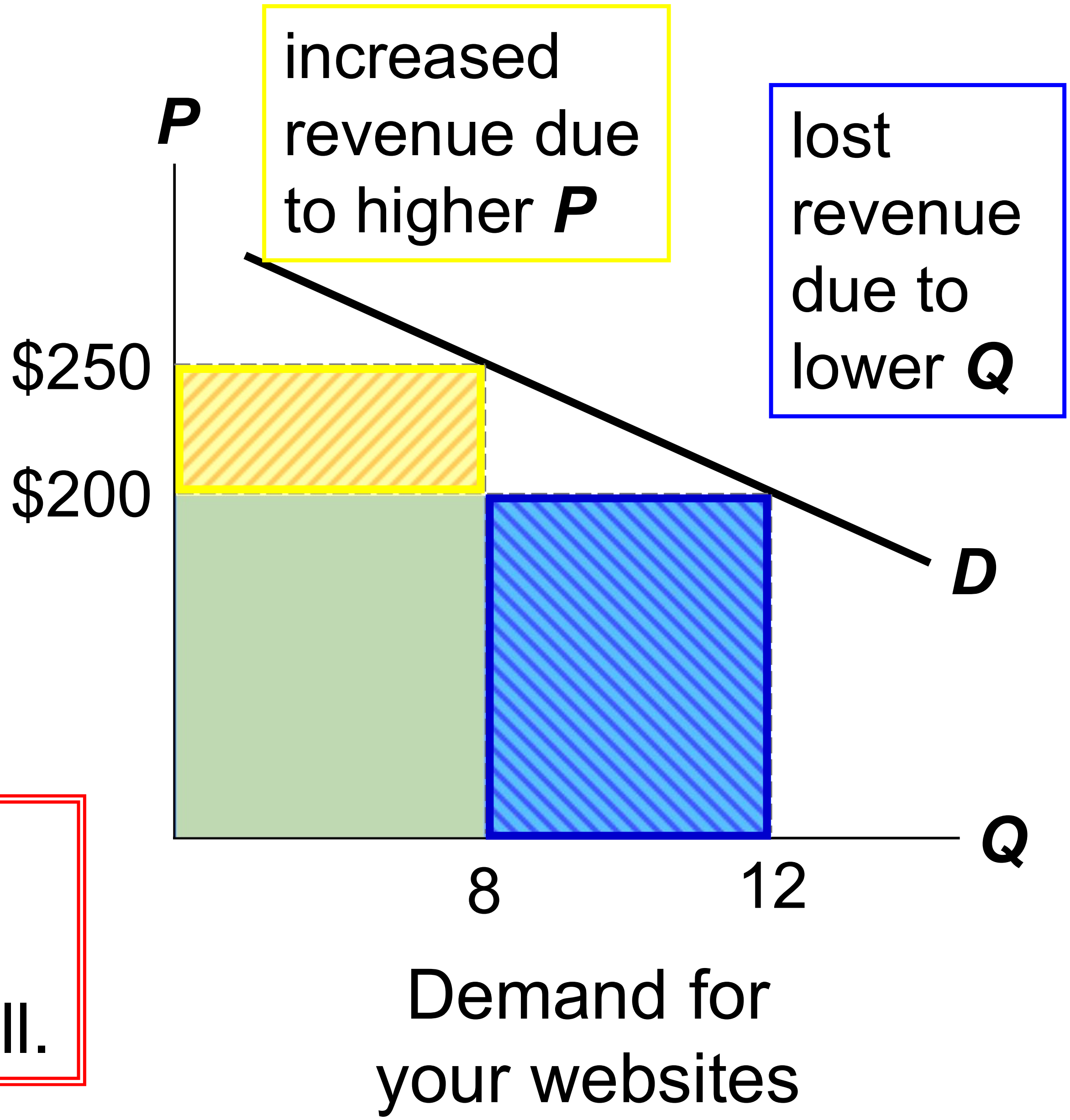
Scenario: You design websites for local businesses.

With elastic demand:

If ***P*** = \$200,
Q = 12 and
revenue = \$2400.

If ***P*** = \$250,
Q = 8 and
revenue = \$2000.

When ***D*** is elastic,
a price increase
causes revenue to fall.



Price Elasticity and Total Revenue

$$\text{Price elasticity of demand} = \frac{\text{Percentage change in } \boldsymbol{Q}}{\text{Percentage change in } \boldsymbol{P}}$$

Revenue = ***P*** x ***Q***

- If demand is inelastic, then
  price elast. of demand < 1
  % change in ***Q*** < % change in ***P***
- The fall in revenue from lower ***Q*** is smaller than the increase in revenue from higher ***P***, so revenue rises.

Price Elasticity and Total Revenue

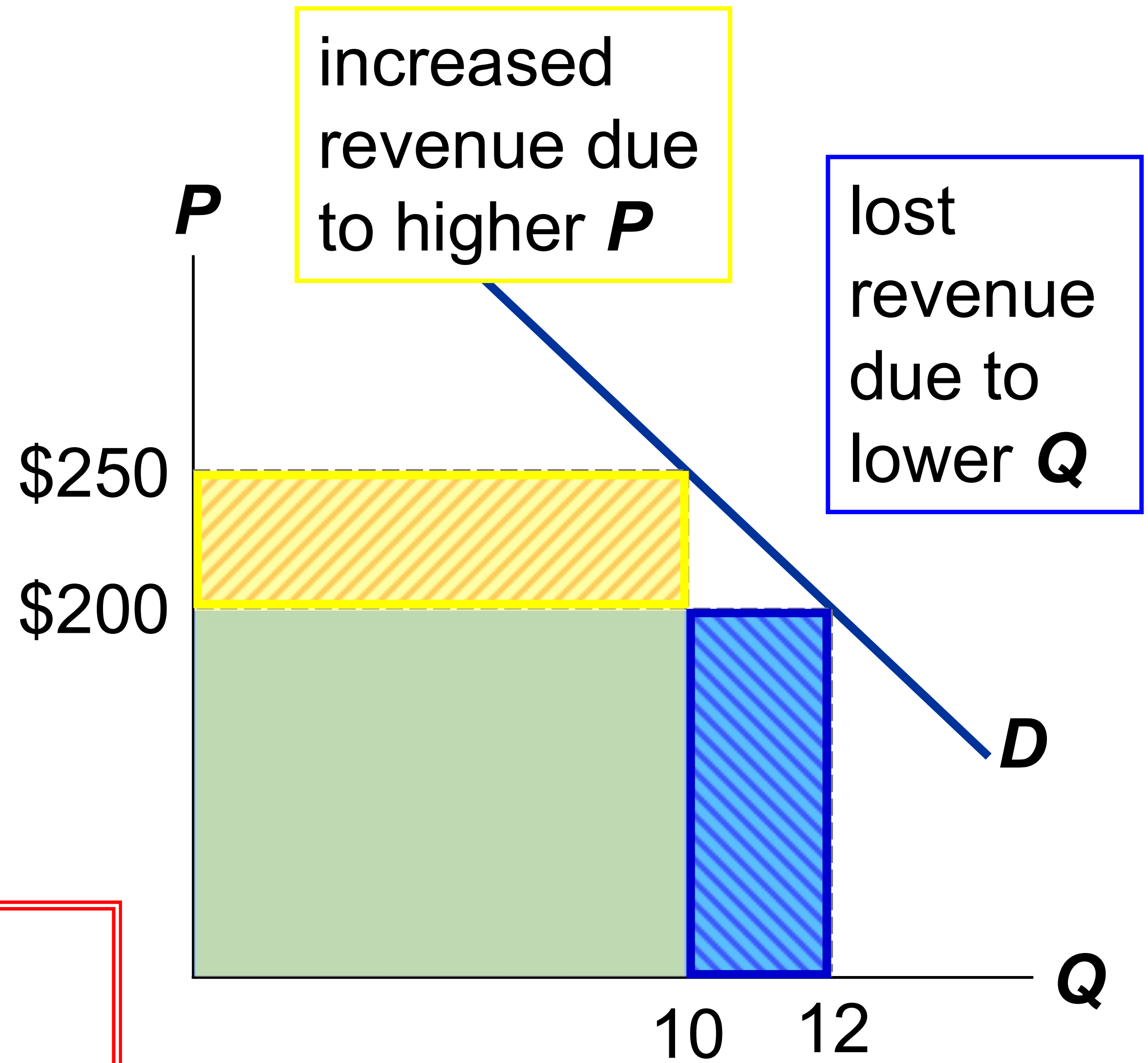
Now, demand is inelastic:

elasticity = 0.82

If ***P*** = \$200,
Q = 12 and
revenue = \$2400.

If ***P*** = \$250,
Q = 10 and
revenue = \$2500.

When ***D*** is inelastic, a price increase causes revenue to rise.



Demand for your websites

Total Revenue

- When demand is elastic:
  - A price increase (decrease) leads to a decrease (increase) in total revenue.
- When demand is inelastic:
  - A price increase (decrease) leads to an increase (decrease) in total revenue.
- When demand is unitary elastic:
  - Total revenue is maximized.

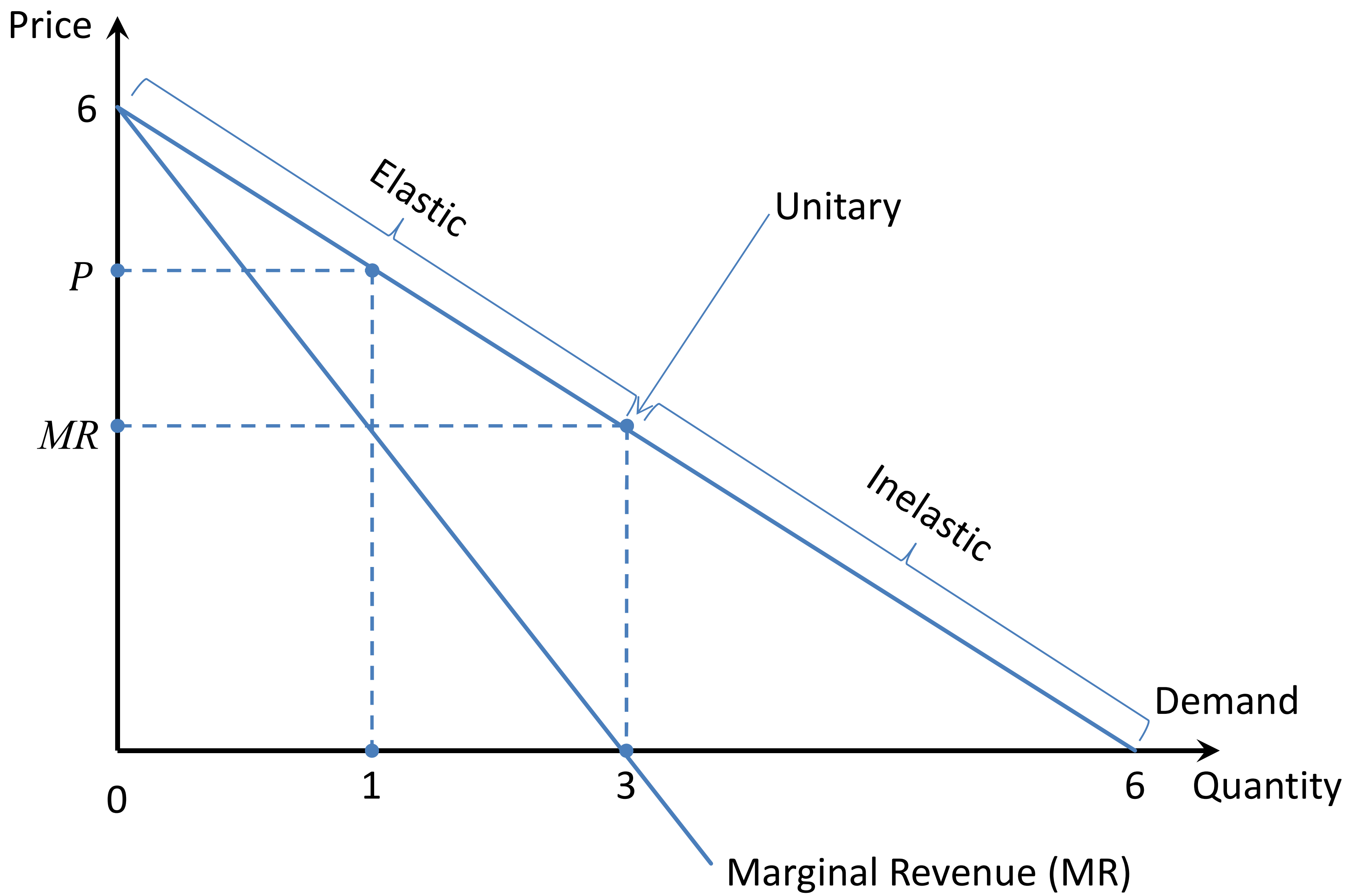
Marginal Revenue and the Own Price Elasticity of Demand

- The *marginal revenue* can be derived from a market demand curve.
  - Marginal revenue measures the additional revenue due to a change in output.
- This link relates *marginal revenue* to the own price elasticity of demand as follows:

$$MR = P\left(\frac{1+E}{E}\right)$$

  - When $-\infty < E < -1$ then, $MR > 0$.
  - When $E = -1$ then, $MR = 0$.
  - When $-1 < E < 0$ then, $MR < 0$.

Demand and Marginal Revenue



Cross-Price Elasticity

- Cross-price elasticity
  - Measures responsiveness of a percent change in demand for good X due to a percent change in the price of good Y.

$$E_{{Q_X}^d, P_Y} = \frac{\%\Delta {Q_X}^d}{\%\Delta P_Y}$$

  - If $E_{{Q_X}^d, P_Y} > 0$, then X and Y are substitutes.
  - If $E_{{Q_X}^d, P_Y} < 0$, then X and Y are complements.

Cross-Price Elasticity in the News

"As Gas Costs Soar, Buyers Flock to Small Cars"
-*New York Times*, 5/2/2008

"Gas Prices Drive Students to Online Courses"
-*Chronicle of Higher Education*, 7/8/2008

"Gas prices knock bicycle sales, repairs into higher gear"
-*Associated Press*, 5/11/2008

"Camel demand soars in India"
(as a substitute for "gas-guzzling tractors")
-*Financial Times*, 5/2/2008

"High gas prices drive farmer to switch to mules"
-*Associated Press*, 5/21/2008

Cross-Price Elasticity in Action

- Suppose it is estimated that the cross-price elasticity of demand between clothing and food is -0.18. If the price of food is projected to increase by 10 percent, by how much will demand for clothing change?

$$-0.18 = \frac{\%\Delta Q_{Clothing}{}^{d}}{10} \Rightarrow \%\Delta Q_{Clothing}{}^{d} = -1.8$$

  – That is, demand for clothing is expected to decline by 1.8 percent when the price of food increases 10 percent.

Cross-Price Elasticity in Action

- Suppose a restaurant earns \$4,000 per week in revenues from hamburger sales (*X*) and \$2,000 per week from soda sales (*Y*).
- If the own price elasticity for burgers is $E_{Q_X,P_X} = -1.5$ and the cross-price elasticity of demand between sodas and hamburgers is $E_{Q_Y,P_X} = -4.0$, what would happen to the firm's total revenues if it reduced the price of hamburgers by 1 percent?

$$\Delta R = [\$4{,}000(1 - 1.5) + \$2{,}000(-4.0)](-1\%)$$
$$= \$100$$

  – That is, lowering the price of hamburgers 1 percent increases total revenue by \$100.

Income Elasticity

- **Income elasticity**
  - Measures responsiveness of a percent change in demand for good X due to a percent change in income.

$$E_{{Q_X}^d,M} = \frac{\%\Delta {Q_X}^d}{\%\Delta M}$$

  - If $E_{{Q_X}^d,M} > 0$, then X is a ***normal good***.
  - If $E_{{Q_X}^d,M} < 0$, then X is an ***inferior good***.

Income Elasticity in Action

- Suppose that the income elasticity of demand for transportation is estimated to be 1.80. If income is projected to decrease by 15 percent,
- what is the impact on the demand for transportation?

$$1.8 = \frac{\%\Delta {Q_X}^d}{-15}$$

  – Demand for transportation will decline by 27 percent.
- is transportation a normal or inferior good?
  – Since demand decreases as income declines, transportation is a normal good.

Other Elasticities

- ***(Own) advertising elasticity*** of demand for good *X* is the ratio of the percentage change in the consumption of *X* to the percentage change in advertising spent on *X*.
- ***Cross-advertising elasticity*** between goods *X* and *Y* would measure the percentage change in the consumption of *X* that results from a 1 percent change in advertising toward *Y*.

Elasticities for Linear Demand Functions

- From a linear demand function, we can easily compute various elasticities.
- Given a linear demand function:

$$Q_X{}^d = \alpha_0 + \alpha_X P_X + \alpha_Y P_Y + \alpha_M M + \alpha_H P_H$$

  – Own price elasticity: $\alpha_X \frac{P_X}{Q_X{}^d}$.

  – Cross price elasticity: $\alpha_Y \frac{P_Y}{Q_X{}^d}$.

  – Income elasticity: $\alpha_M \frac{M}{Q_X{}^d}$.

Elasticities for Linear Demand Functions In Action

The daily demand for Invigorated PED shoes is estimated to be:

$$Q_X{}^d = 100 - 3P_X + 4P_Y - 0.01M + 2P_{A_X}$$

Suppose good *X* sells at $25 a pair, good *Y* sells at $35, the company utilizes 50 units of advertising, and average consumer income is $20,000. Calculate the own price, cross-price and income elasticities of demand.

– $Q_X{}^d = 100 - 3(\$25) + 4(\$35) - 0.01(\$20{,}000) + 2(50) = 65$ units.

– Own price elasticity: $-3(\frac{25}{65}) = -1.15$.

– Cross-price elasticity: $4(\frac{35}{65}) = 2.15$.

– Income elasticity: $-0.01(\frac{20{,}000}{65}) = -3.08$.

Elasticities for Nonlinear Demand Functions

- One non-linear demand function is the log-linear demand function:

$$\ln {Q_X}^d = \beta_0 + \beta_X \ln P_X + \beta_Y \ln P_Y + \beta_M \ln M + \beta_H \ln H$$

  - Own price elasticity: β_X.
  - Cross price elasticity: β_Y.
  - Income elasticity: β_M.

Elasticities for Nonlinear Demand Functions In Action

An analyst for a major apparel company estimates that the demand for its raincoats is given by

$$ln\ Q_X{}^d = 10 - 1.2 \ln P_X + 3 \ln R\ - 2 \ln A_Y$$

where R denotes the daily amount of rainfall and A_Y the level of advertising on good Y. What would be the impact on demand of a 10 percent increase in the daily amount of rainfall?

$$E_{Q_X{}^d,R} = \beta_R = 3. \text{ So, } E_{Q_X{}^d,R} = \frac{\%\Delta Q_X{}^d}{\%\Delta R} \Rightarrow 3 = \frac{\%\Delta Q_X{}^d}{10}.$$

A 10 percent increase in rainfall will lead to a 30 percent increase in the demand for raincoats.

Econometric/Regression Analysis

- How does one obtain information on the demand function?
  - Published studies
  - Hire consultant
  - Statistical technique called regression analysis using data on quantity, price, income and other important variables.

An Introduction to Econometrics

- Econometrics fills a gap between being a "student of economics" and being a "practicing economist"
  - It lets you tell your employer:
    - "I can predict the sales of your product"
    - "I can estimate the effect on your sales if your competition lowers its price by $1 per unit"
    - "I can test whether your new ad campaign is actually increasing your sales"
  - Helps you develop "intuition" about how things work and is invaluable if you go to graduate school

An Introduction to Econometrics

Econometrics is about how we can use theory and data from economics, business, and the social sciences, along with tools from statistics, to answer ‘‘how much’’ questions.

- Every day, decision-makers face ‘‘how much’’:
  - The owner of a local Pizza Hut must decide how much advertising space to purchase in the local newspaper, and thus must estimate the relationship between advertising and sales
  - A public transportation council in Melbourne, Australia, must decide how an increase in fares for public transportation (trams, trains, and buses) will affect the number of travelers who switch to car or bike, and the effect of this switch on revenue going to public transportation

An Introduction to Econometrics

- In economics we express our ideas about relationships between economic variables using the mathematical concept of a function

$$Consumption = f(Income)$$

$$Q^d = f(P, P^s, P^C, INC)$$

- An econometric model consists of a systematic part and a random and unpredictable component *e* that we will call a **random error**

$$Q^d = f(P, P^s, P^c, INC) + e$$

$$f(P, P^s, P^c, INC) = \beta_1 + \beta_2 P + \beta_3 P^s + \beta_4 P^c + \beta_5 INC$$

$$Q^d = \beta_1 + \beta_2 P + \beta_3 P^s + \beta_4 P^c + \beta_5 INC + e$$

An Introduction to Econometrics

- **Econometric model of criminal activity**
  - The functional form has to be specified
  - Variables may have to be approximated by other quantities

Measure of criminal activity

Wage for legal employment

Other income

Frequency of prior arrests

Unobserved determinants of criminal activity

$$crime = \beta_0 + \beta_1 wage_m + \beta_2 othinc + \beta_3 freqarr$$
$$+ \beta_4 freqconv + \beta_5 avgsen + \beta_6 age + u$$

Frequency of conviction

Average sentence length after conviction

Age

e.g. moral character, wage in criminal activity, family background ...

An Introduction to Econometrics

- **Econometric model of job training and worker productivity**

$$wage = \beta_0 + \beta_1 educ + \beta_2 exper + \beta_3 training + u$$

Hourly wage

Years of formal education

Years of work-force experience

Weeks spent in job training

Unobserved deter-minants of the wage

e.g. innate ability, quality of education, family background …

- **Econometric models may be used for hypothesis testing**
  - For example, the parameter β_3 represents effect of training on wage
  - How large is this effect? Is it different from zero?

A Simple Regression Model

- **Definition of the simple linear regression model**

"Explains variable y in terms of variable x"

Intercept

Slope parameter

$$y = \beta_0 + \beta_1 x + u$$

Dependent variable,
explained variable,
response variable,...

Independent variable,
explanatory variable,
regressor,...

Error term,
disturbance,
unobservables,...

A Simple Regression Model

- **Interpretation of the simple linear regression model**

"Studies how y varies with changes in x:"

$$\frac{\partial y}{\partial x} = \beta_1 \quad \text{as long as} \quad \frac{\partial u}{\partial x} = 0$$

By how much does the dependent variable change if the independent variable is increased by one unit?

Interpretation only correct if all other things remain equal when the indepen-dent variable is increased by one unit

- **The simple linear regression model is rarely applicable in prac-tice but its discussion is useful for pedagogical reasons**

A Simple Regression Model

- **Example: A simple wage equation**

$$wage = \beta_0 + \beta_1 educ + u$$

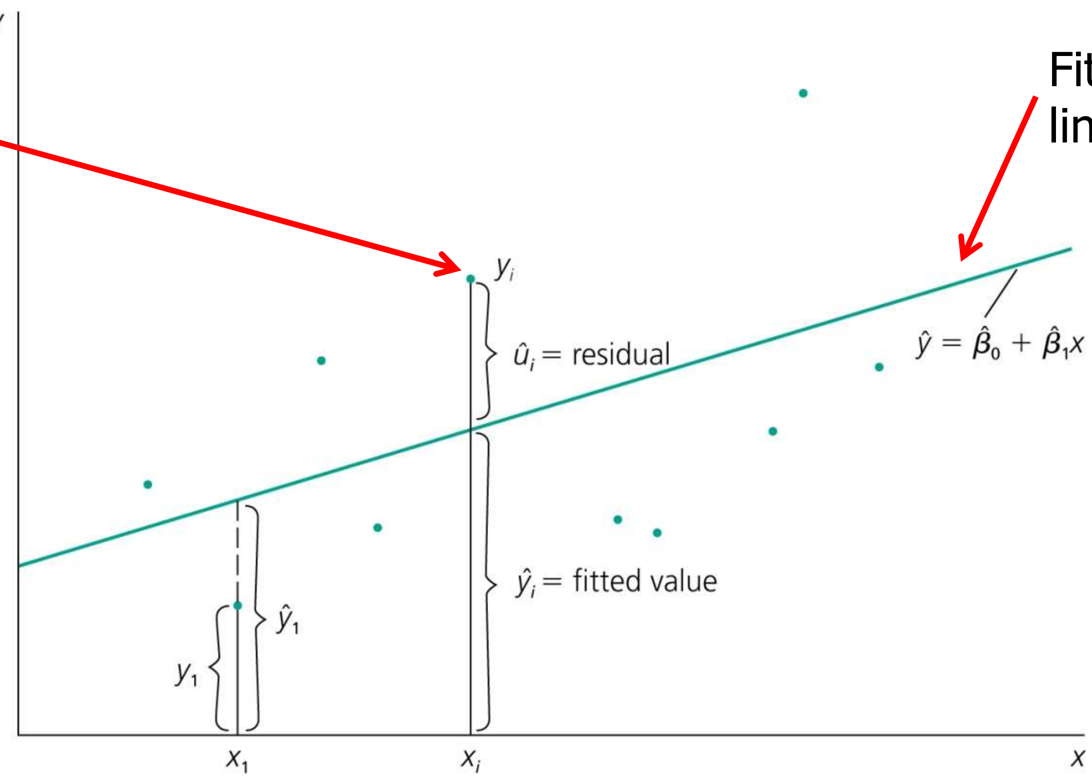
Labor force experience, tenure with current employer, work ethic, intelligence …

Measures the change in hourly wage given another year of education, holding all other factors fixed

A Simple Regression Model

- **Fit as good as possible a regression line through the data points:**

For example, the i-th data point (x_i, y_i)



A Simple Regression Model

- **What does „as good as possible" mean?**
- **Regression residuals**

$$\widehat{u}_i = y_i - \widehat{y}_i = y_i - \widehat{\beta}_0 - \widehat{\beta}_1 x_i$$

- **Minimize sum of squared regression residuals**

$$\min \sum_{i=1}^{n} \widehat{u}_i^2 \quad \rightarrow \quad \widehat{\beta}_0, \widehat{\beta}_1$$

- **Ordinary Least Squares (OLS) estimates**

$$\widehat{\beta}_1 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}, \qquad \widehat{\beta}_0 = \bar{y} - \widehat{\beta}_1 \bar{x}$$

Fit of the Regression

- **Goodness-of-Fit**

„How well does the explanatory variable explain the dependent variable?"

- **Measures of Variation**

$$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2 \qquad SSE = \sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2 \qquad SSR = \sum_{i=1}^{n} \hat{u}_i^2$$

Total sum of squares, represents total variation in dependent variable

Explained sum of squares, represents variation explained by regression

Residual sum of squares, represents variation not explained by regression

Fit of the Regression

- **Decomposition of total variation**

$$SST = SSE + SSR$$

Total variation

Explained part

Unexplained part

- **Goodness-of-fit measure (R-squared)**

$$R^2 = \frac{SSE}{SST} = 1 - \frac{SSR}{SST}$$

R-squared measures the fraction of the total variation that is explained by the regression

Evaluating Statistical Significance

- ***Standard error***
  - The estimated standard deviations of the regression coefficients are called „standard errors". They measure how precisely the regression coefficients are estimated.

$$se(\hat{\beta}_1) = \sqrt{\widehat{Var}(\hat{\beta}_1)} = \sqrt{\hat{\sigma}^2 / SST_x}$$

$$se(\hat{\beta}_0) = \sqrt{\widehat{Var}(\hat{\beta}_0)} = \sqrt{\hat{\sigma}^2 n^{-1} \sum_{i=1}^{n} x_i^2 / SST_x}$$

- **t-statistic (or t-ratio)**

$$t_{\hat{\beta}_j} = \frac{\hat{\beta}_j}{se(\hat{\beta}_j)}$$

  - When $|t| > 1.96$, we are 95 percent confident the true parameter is in the regression is not zero.

Excel and Least Squares Estimates

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.87
R Square	0.75
Adjusted R Square	0.72
Standard Error	112.22
Observations	10.00

$se\ (\widehat{a}) = 243.97$

$se(\widehat{b}) = 0.53$

$t_{\hat{a}} = |6.69| > 1.96$, the intercept is different from zero.

$t_{\hat{b}} = |-4.89| > 1.96$, the intercept is different from zero.

ANOVA

	Df	*SS*	*MS*	*F*	*Significance F*
Regression	1	301470.89	301470.89	23.94	0.0012
Residual	8	100751.61	12593.95		
Total	9	402222.50			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	1631.47	243.97	6.69	0.0002	1068.87	2194.07
Price	-2.60	0.53	-4.89	0.0012	-3.82	-1.37

Take-home messages

- **Price elasticity of demand** equals percentage change in $\boldsymbol{Q}^{d}$ divided by percentage change in $\boldsymbol{P}$. When it's less than one, demand is "inelastic." When greater than one, demand is "elastic."
- **Demand is less elastic** in the short run, for broadly defined goods, or for goods with few close substitutes, for goods with small share in expenditure..
- When **demand** is inelastic, **total revenue** rises when price rises. When demand is elastic, total revenue falls when price rises.

Take-home messages

- The **cross-price elasticity of demand** measures how much demand for one good responds to changes in the price of another good.
- The **income elasticity of demand** measures how much quantity demanded responds to changes in buyers' incomes.