INTRODUCTORY ECONOMICS: LECTURE 4

The Production Process and Costs

Highlights

- *Production function and marginal product*
- *Marginal rate of technical substitutions*
- *Cost-minimizing input rule*
- *Cost function: Marginal cost vs Avg. cost*
- *Fixed costs vs sunk costs*
- *Long-run cost curve*
- *Economies of scale*
- *Economies of scope*

The Production Function

- Mathematical function that defines the maximum amount of output that can be produced with a given set of inputs.

$$Q = F(K, L)$$

  - Q is the level of output.
  - K is the quantity of capital input.
  - L is the quantity of labor input.

Short-Run versus Long-Run Decisions: Fixed and Variable Inputs

- Short-run
  - Period of time where some factors of production (inputs) are ***fixed***, and constrain a manager's decisions.
- Long-run
  - Period of time over which all factors of production (inputs) are ***variable***, and can be adjusted by a manager.

Measures of Productivity

- **Total product (*TP*)**
  - Maximum level of output that can be produced with a given amount of inputs.
- **Average product (*AP*)**
  - A measure of the output produced per unit of input.
    - Average product of labor: $AP_L = \frac{Q}{L}$
    - Average product of capital: $AP_K = \frac{Q}{K}$
- **Marginal product (*MP*)**
  - The change in total product (output) attributable to the last unit of an input.
    - Marginal product of labor: $MP_L = \frac{\Delta Q}{\Delta L}$
    - Marginal product of capital: $MP_K = \frac{\Delta Q}{\Delta K}$

Measures of Productivity in Action

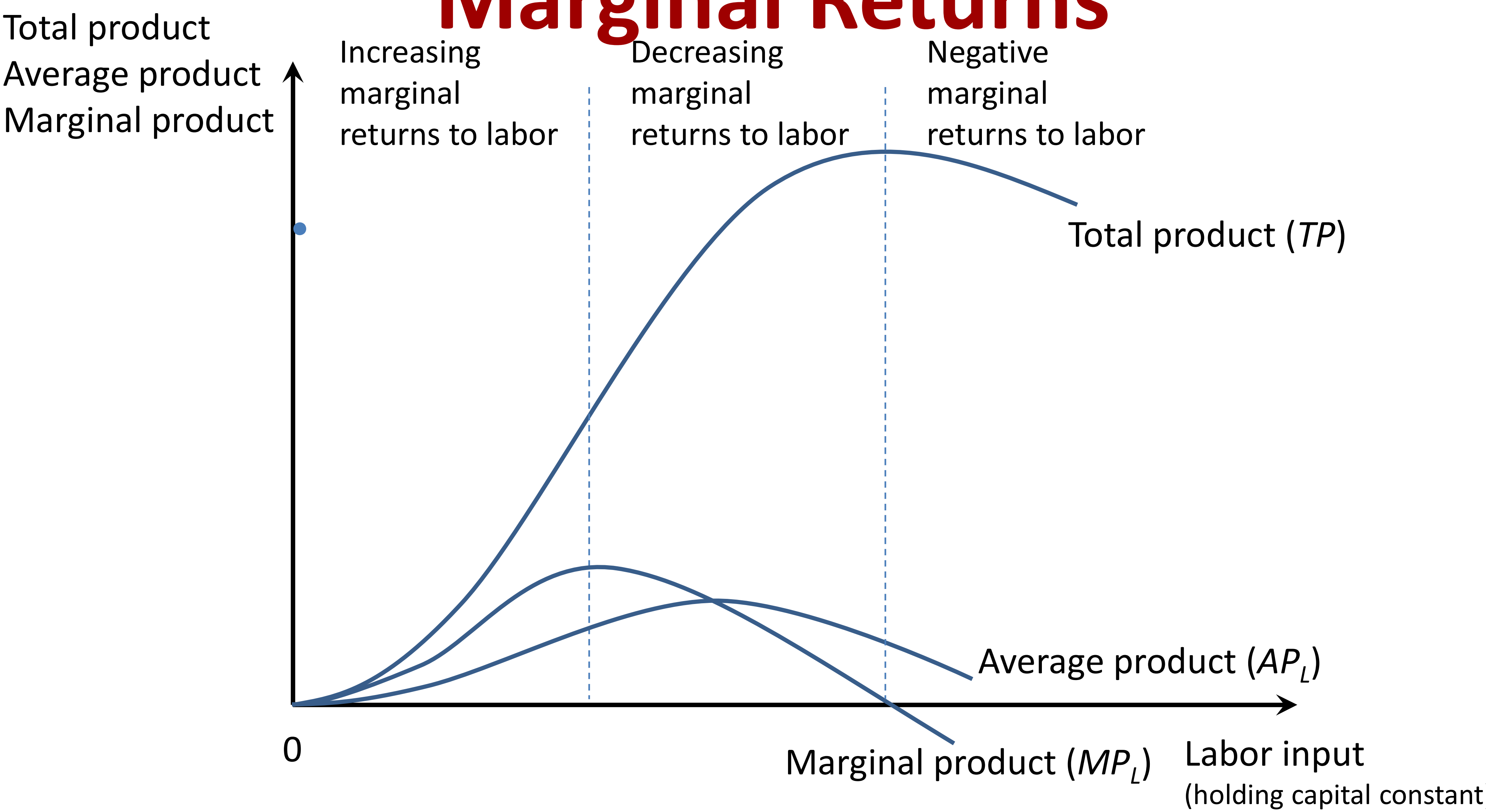
- Consider the following production function when 5 units of labor and 10 units of capital are combined produce: $Q = F(10{,}5) = 150$.
- Compute the average product of labor.

$$AP_L = \frac{150}{5} = 30 \text{ units per worker}$$

- Compute the average product of capital.

$$AP_L = \frac{150}{10} = 15 \text{ units capital unit}$$

Increasing, Decreasing, and Negative Marginal Returns



Algebraic Forms of Production Functions

- Commonly used algebraic production function forms:
  - ***Linear***: Assumes a perfect linear relationship between all inputs and total output

    $Q = F(K, L) = aK + bL$, where a and b are constants.
  - ***Leontief***: Assumes that inputs are used in fixed proportions

    $Q = F(K, L) = \min\{aK, bL\}$, where a and b are constants.
  - ***Cobb-Douglas***: Assumes some degree of substitutability among inputs

    $Q = F(K, L) = K^a L^b$, where a and b are constants.

Algebraic Forms of Production Functions in Action

- Suppose that a firm's estimated production function is:

$$Q = 3K + 6L$$

- How much output is produced when 3 units of capital and 7 units of labor are employed?

$$Q = F(3,7) = 3(3) + 6(7) = 51 \text{ units}$$

Algebraic Measures of Productivity

- Given the commonly used algebraic production function forms, we can compute the measures of productivity as follows:
  - *Linear*:
    - Marginal products: $MP_K = a$ and $MP_L = b$
    - Average products: $AP_K = \frac{aK+bL}{K}$ and $AP_L = \frac{aK+bL}{L}$
  - *Cobb-Douglas*:
    - Marginal products: $MP_K = aK^{a-1}L^b$ and $MP_L = bK^aL^{b-1}$
    - Average products: $AP_K = \frac{K^aL^b}{K}$ and $AP_L = \frac{K^aL^b}{L}$

Algebraic Measures of Productivity in Action

- Suppose that a firm produces output according to the production function

$$Q = F(1, L) = (1)^{1/4} L^{3/4}$$

- Which is the fixed input?
  - Capital is the fixed input.
- What is the marginal product of labor when 16 units of labor is hired?

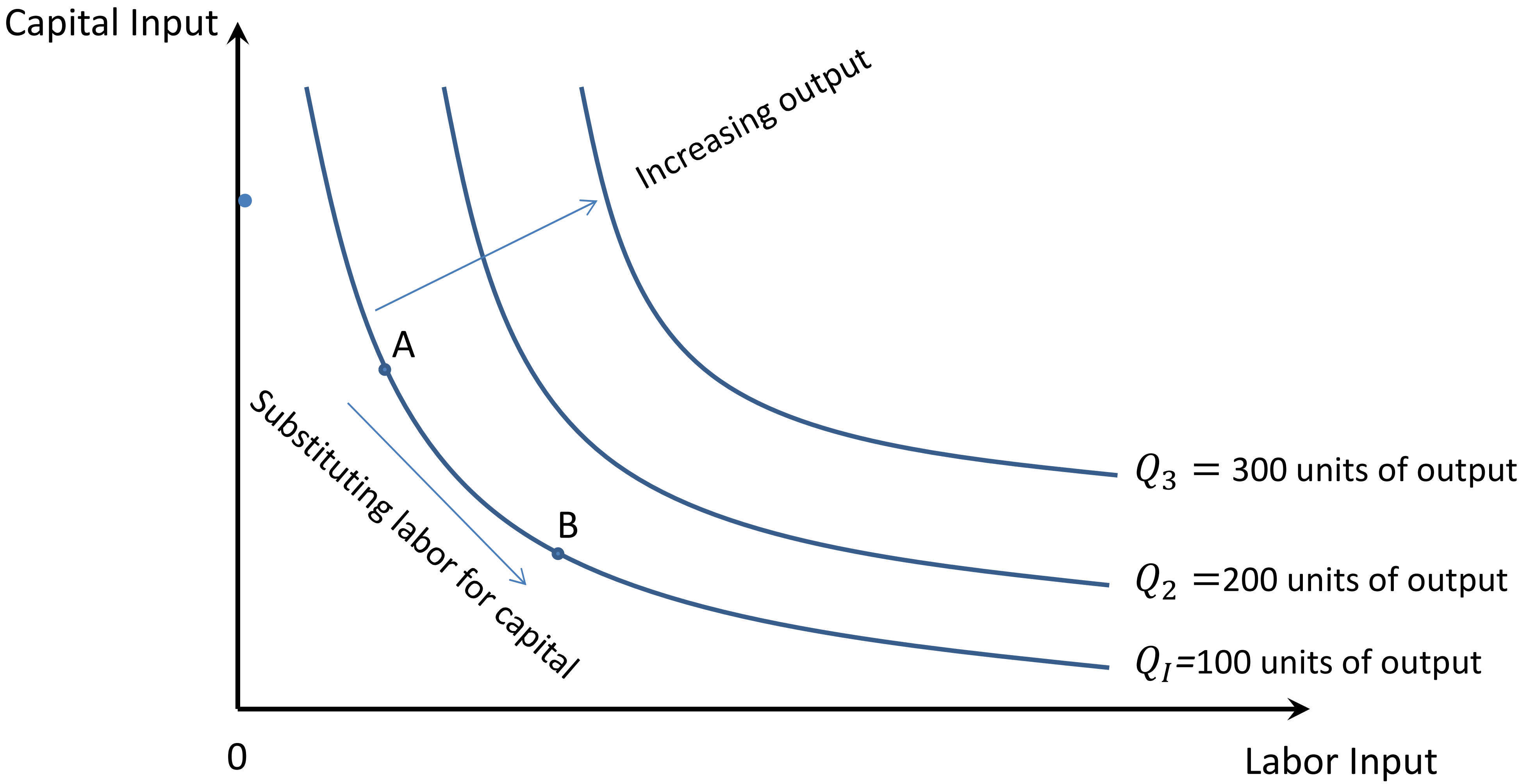
$$MP_L = 1 \times \frac{3}{4} L^{-\frac{1}{4}} = 1 \times \frac{3}{4} (16)^{-\frac{1}{4}} = \frac{3}{8}$$

Isoquants and Marginal Rate of Technical Substitution

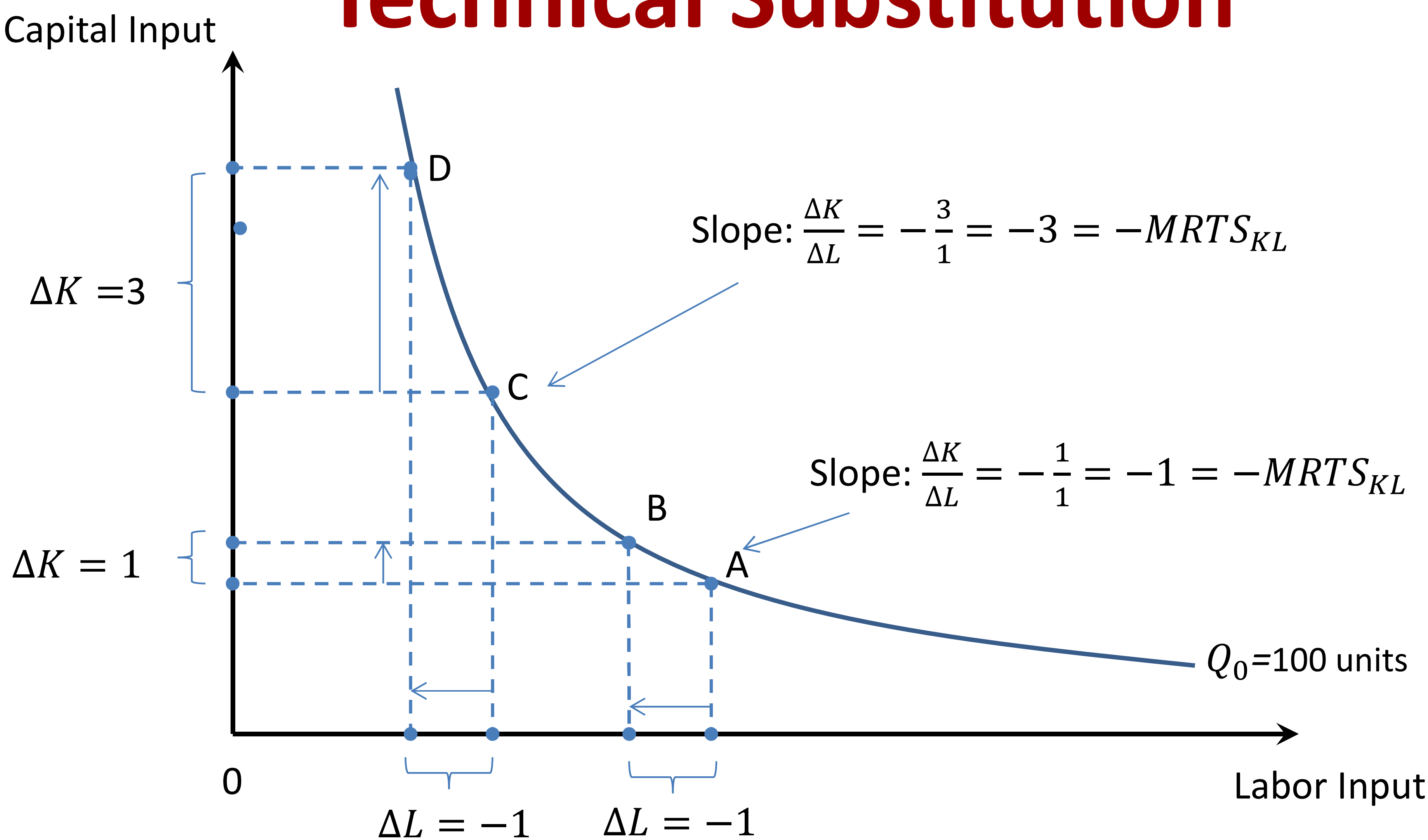
- ***Isoquants*** capture the tradeoff between combinations of inputs that yield the same output in the long run, when all inputs are variable.
- **Marginal rate of technical substitutions (MRTS)**
  - The rate at which a producer can substitute between two inputs and maintain the same level of output.
  - Absolute value of the slope of the isoquant.

$$MRTS_{KS} = \frac{MP_L}{MP_K}$$

Isoquants and Marginal Rate of Technical Substitution in Action



Diminishing Marginal Rate of Technical Substitution



Isocost and Changes in Isocost Lines

- **Isocost**
  - Combination of inputs that yield cost the same cost.

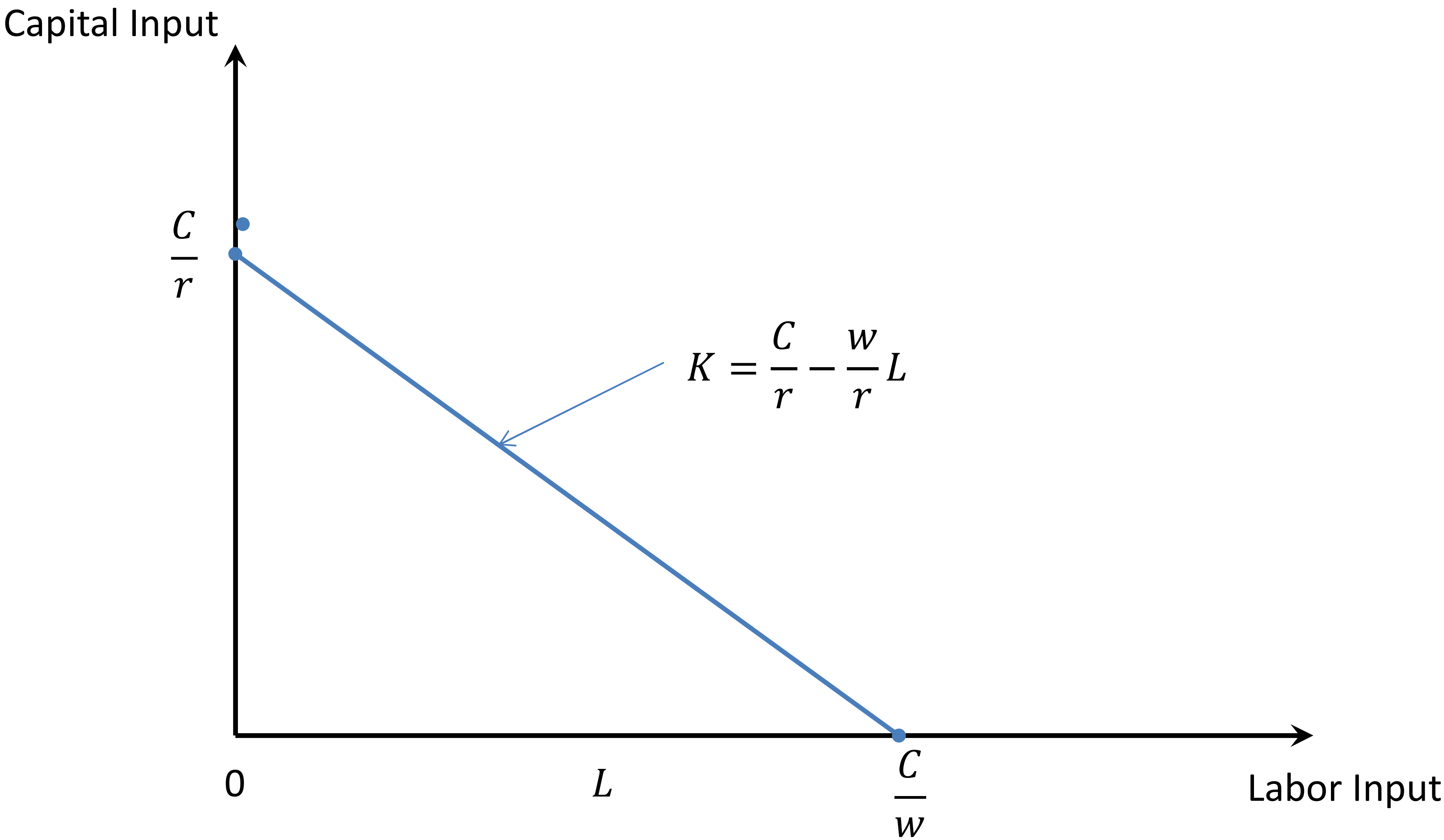
$$wL + rK = C$$

  or, re-arranging to the intercept-slope formulation:

$$K = \frac{C}{r} - \frac{w}{r}L$$

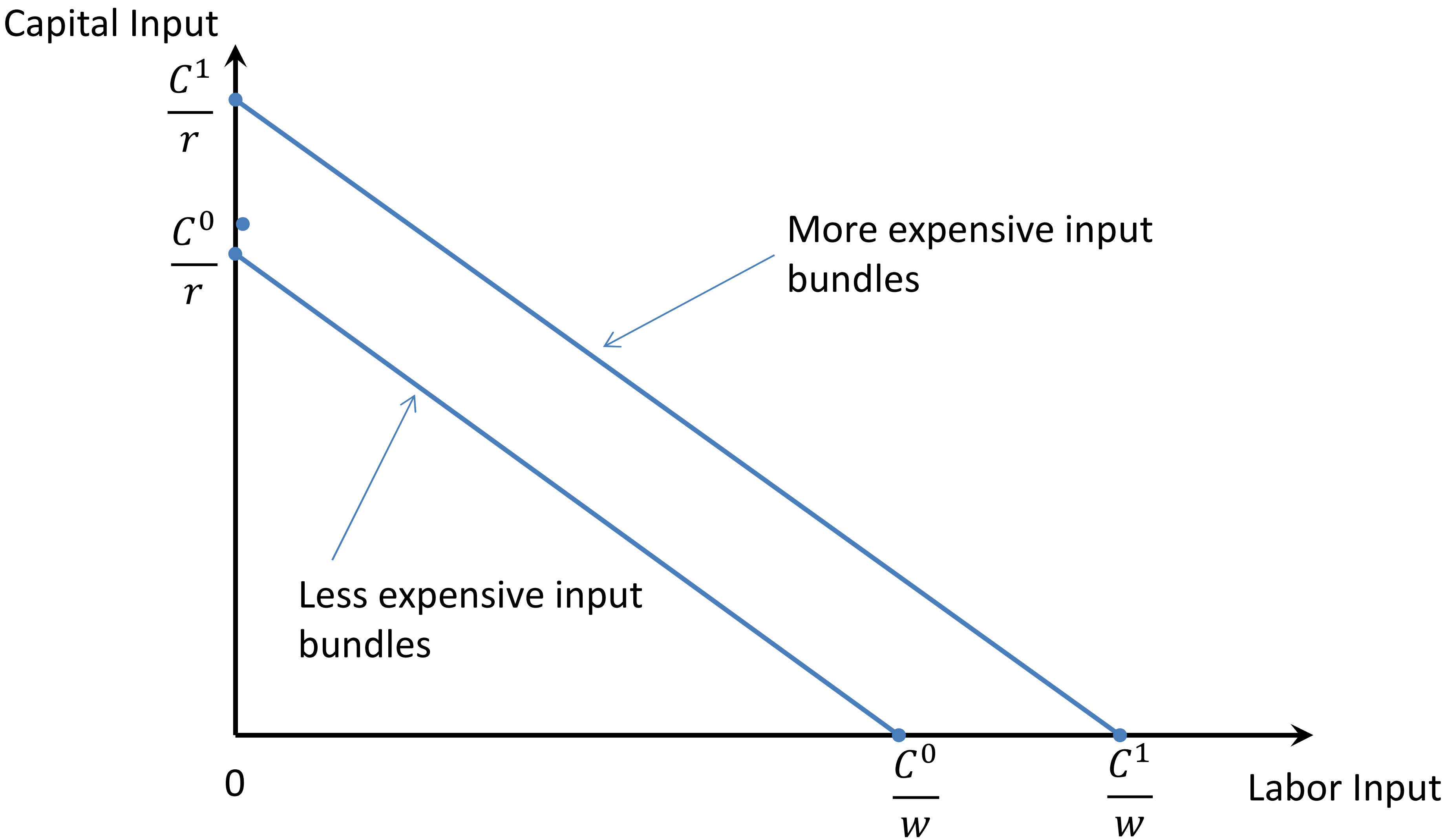
- **Changes in isocosts**
  - For given input prices, isocosts farther from the origin are associated with higher costs.
  - Changes in input prices change the slopes of isocost lines.

Isocosts

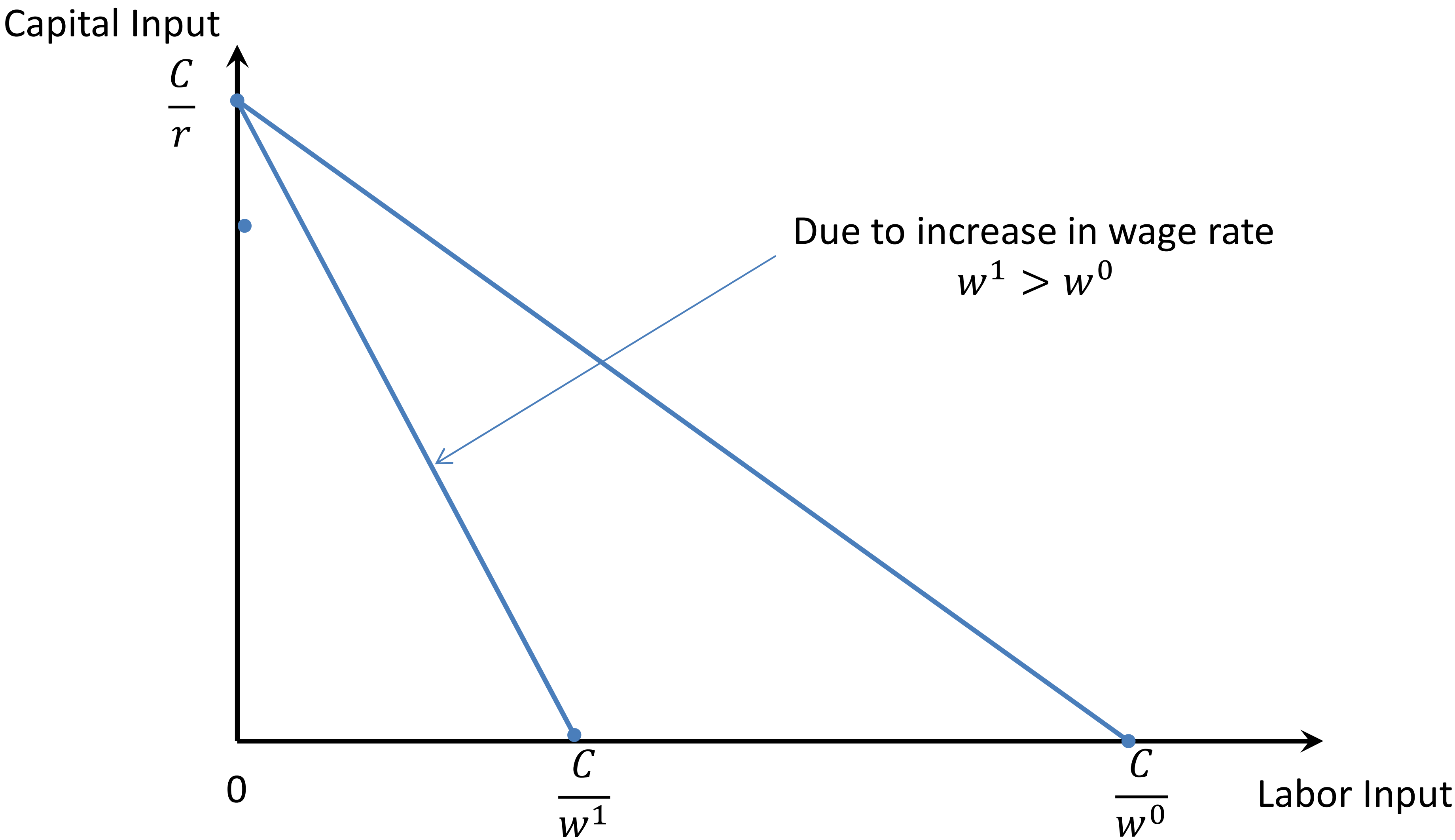


Capital Input
$\frac{C}{r}$
$K = \frac{C}{r} - \frac{w}{r}L$
0
L
$\frac{C}{w}$
Labor Input

Changes in the Isocosts



Changes in the Isocost Line



Cost Minimization and the Cost-Minimizing Input Rule

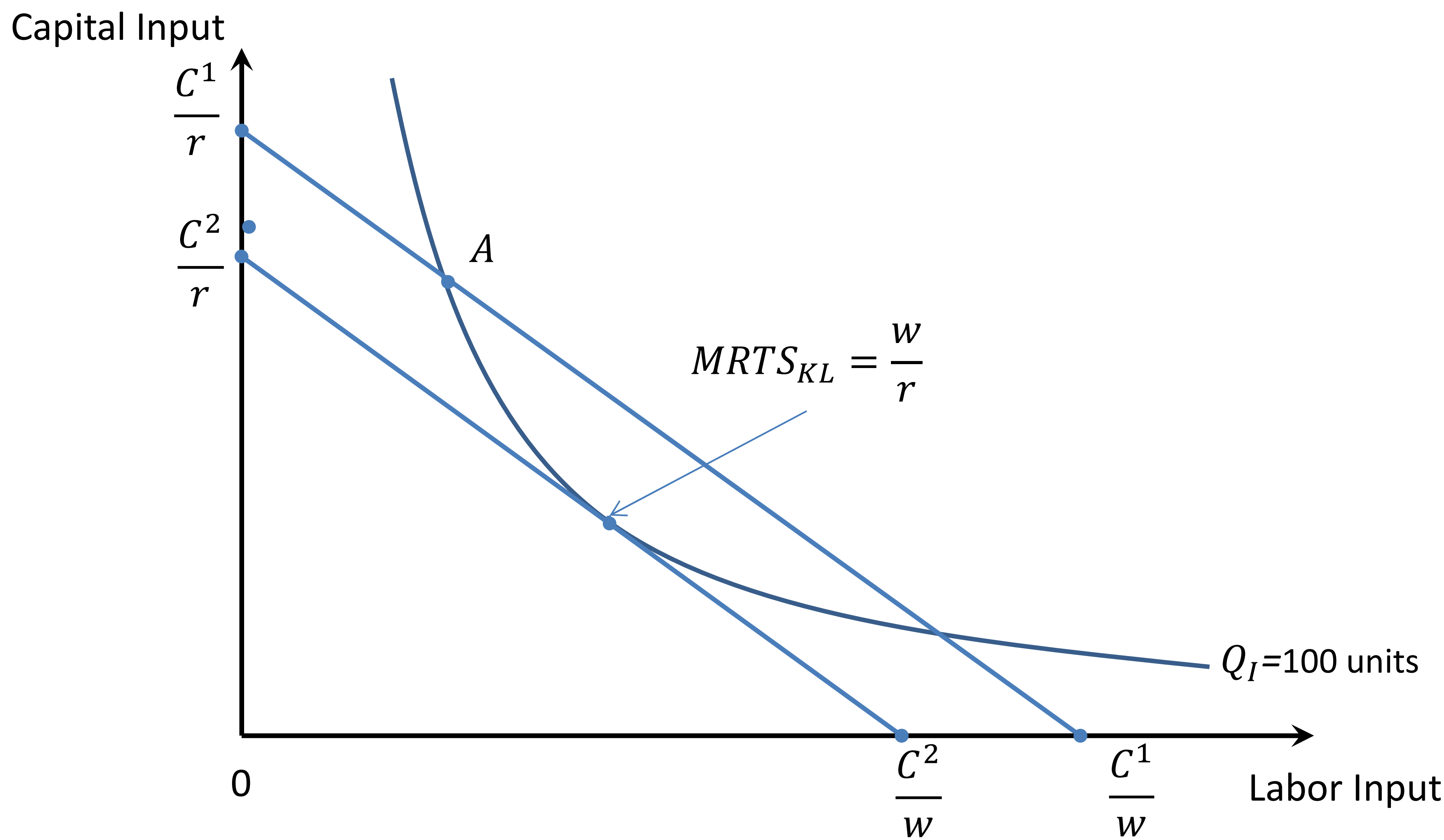
- **Cost minimization**
  - Producing at the lowest possible cost.
- **Cost-minimizing input rule**
  - Produce at a given level of output where the marginal product per dollar spent is equal for all input:

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

  - Equivalently, a firm should employ inputs such that the marginal rate of technical substitution equals the ratio of input prices:

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

Cost-Minimization Input Rule in Action



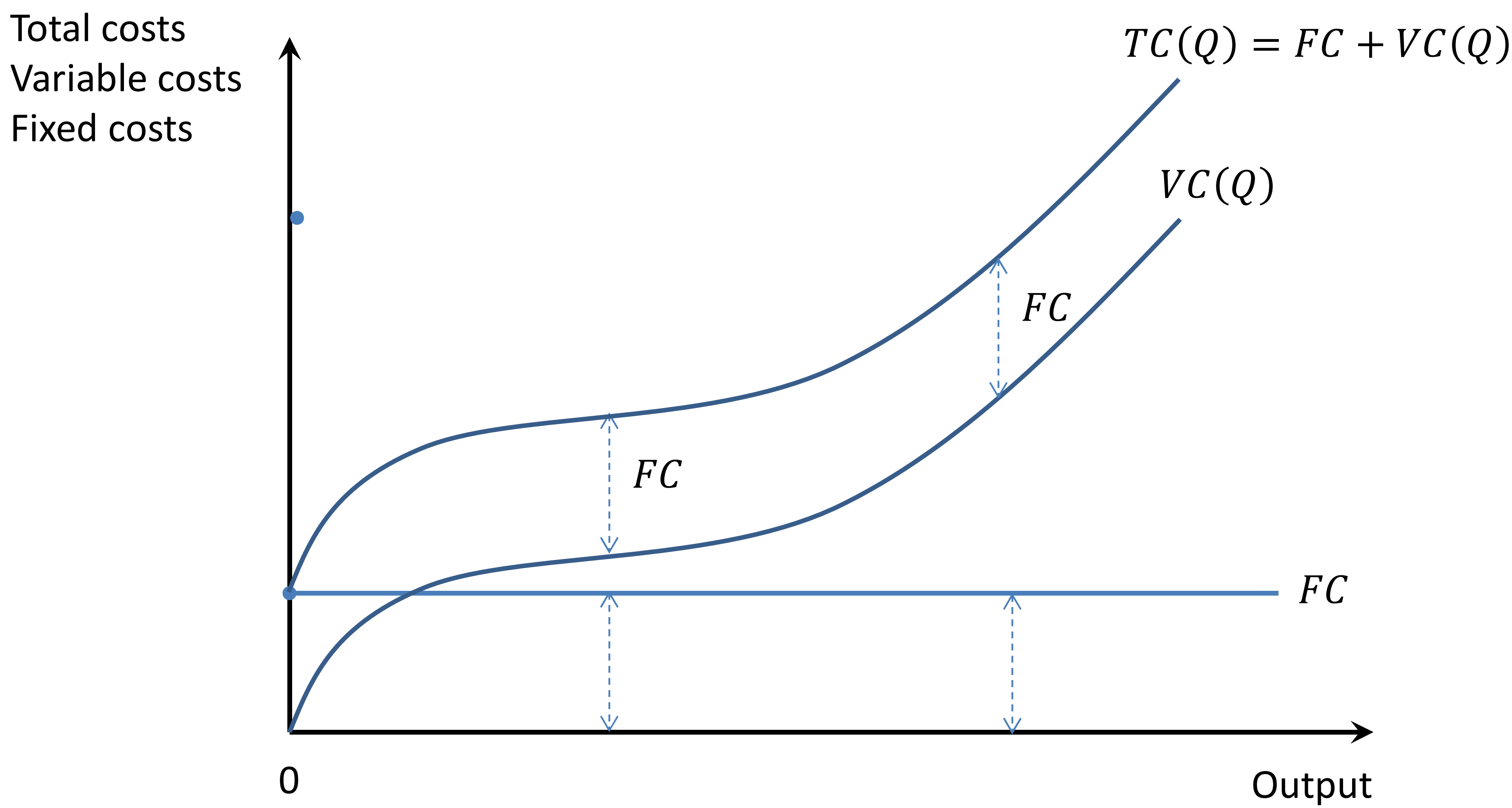
Optimal Input Substitution

- To minimize the cost of producing a given level of output, the firm should use less of an input and more of other inputs when that input's price rises.

The Cost Function

- Mathematical relationship that relates cost to the cost-minimizing output associated with an isoquant.
- Short-run costs
  - **Fixed costs ($\boldsymbol{FC}$)**: do not change with changes in output; include the costs of fixed inputs used in production
  - **Variable costs [$\boldsymbol{VC(Q)}$]**: costs that change with changes in outputs; include the costs of inputs that vary with output
  - **Total costs:** $TC(Q) = FC + VC(Q)$
- Long-run costs
  - All costs are variable
  - No fixed costs

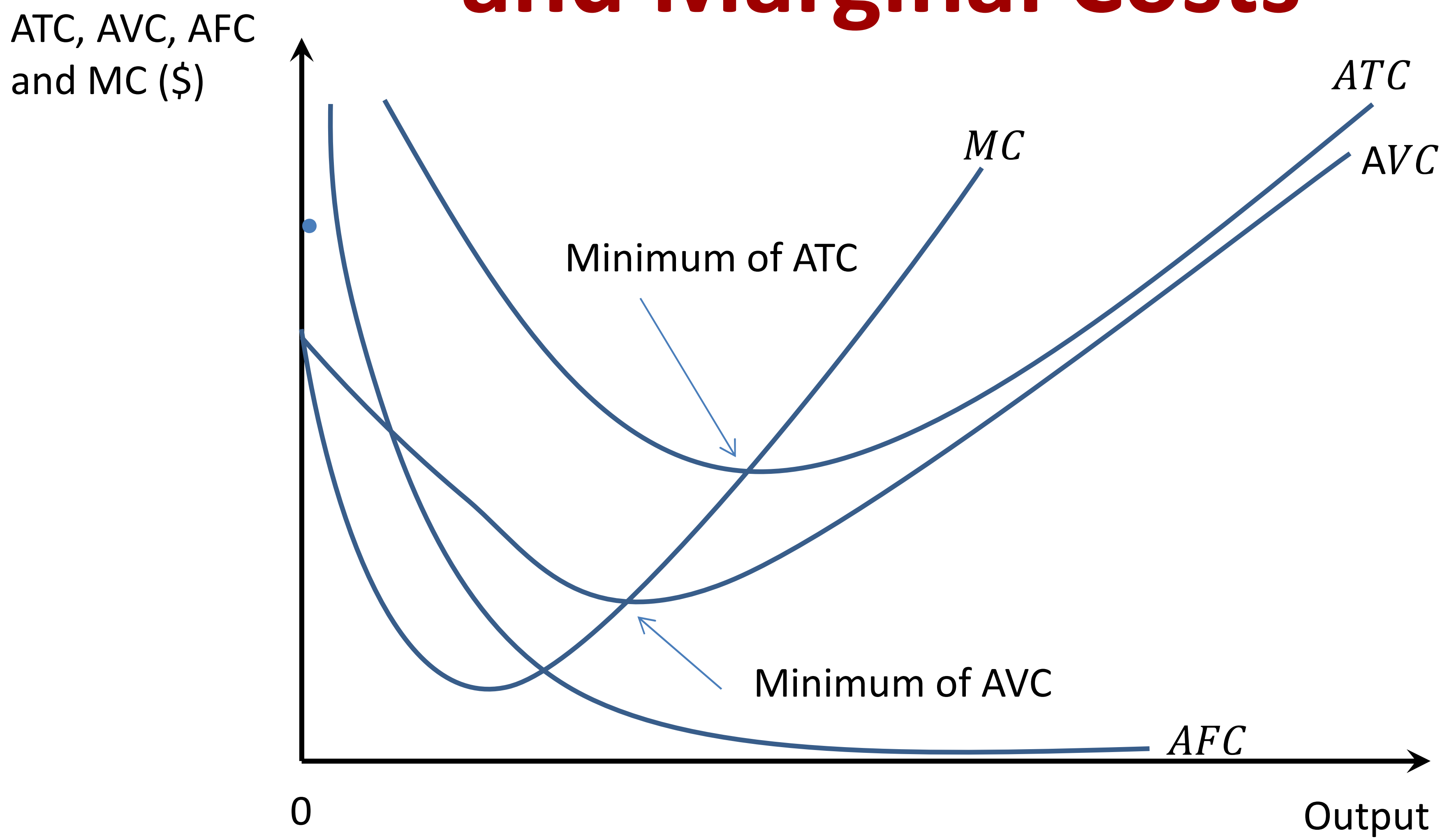
Short-Run Costs



Average and Marginal Costs

- Average costs
  - **Average fixed cost**: $AFC = \frac{FC}{Q}$
  - **Average variable costs:** $AVC = \frac{VC(Q)}{Q}$
  - **Average total cost:** $ATC = \frac{C(Q)}{Q}$
- **Marginal cost (*MC*)**
  - The (incremental) cost of producing an additional unit of output.
  - $MC = \frac{\Delta C}{\Delta Q}$

The Relationship between Average and Marginal Costs



Fixed and Sunk Costs

- **Fixed costs**
  - Cost that does not change with output.
- **Sunk cost**
  - Cost that is forever lost after it has been paid.

- **Irrelevance of Sunk Costs**
  - A decision maker should ignore sunk costs to maximize profits or minimize loses.

Algebraic Forms of Cost Functions

- The cubic cost function: costs are a cubic function of output; provides a reasonable approximation to virtually any cost function.

  $$C(Q) - F + aQ + bQ^2 + cQ^3$$

  where *a*, *b*, *c*, and *f* are constants and *f* represents fixed costs

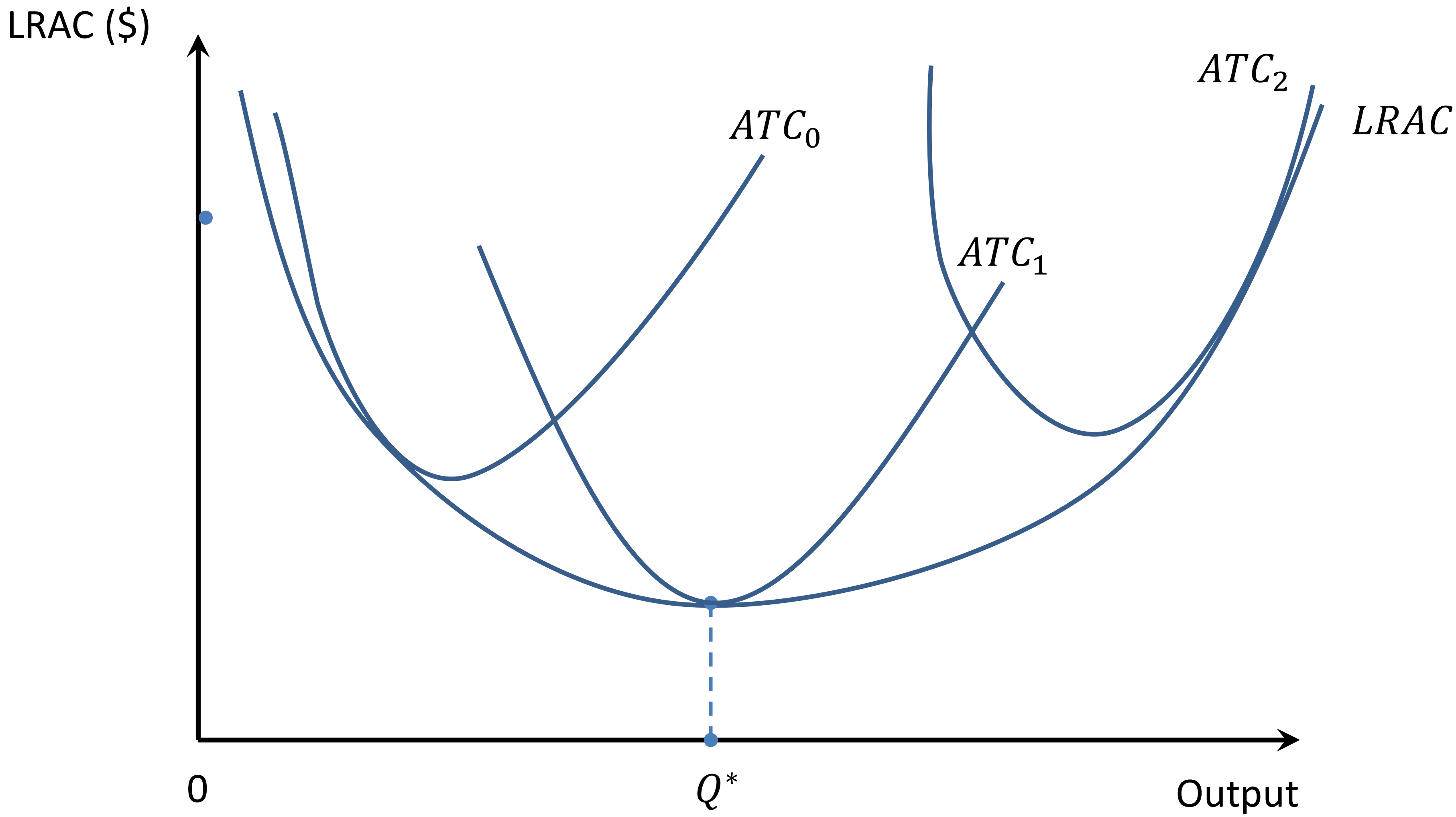
- Marginal cost function is:

  $$MC(Q) = a + 2bQ + 3cQ^2$$

Long-Run Costs

- In the long run, all costs are variable since a manager is free to adjust levels of all inputs.
- **Long-run average cost curve**
  - A curve that defines the minimum average cost of producing alternative levels of output allowing for optimal selection of both fixed and variable factors of production.

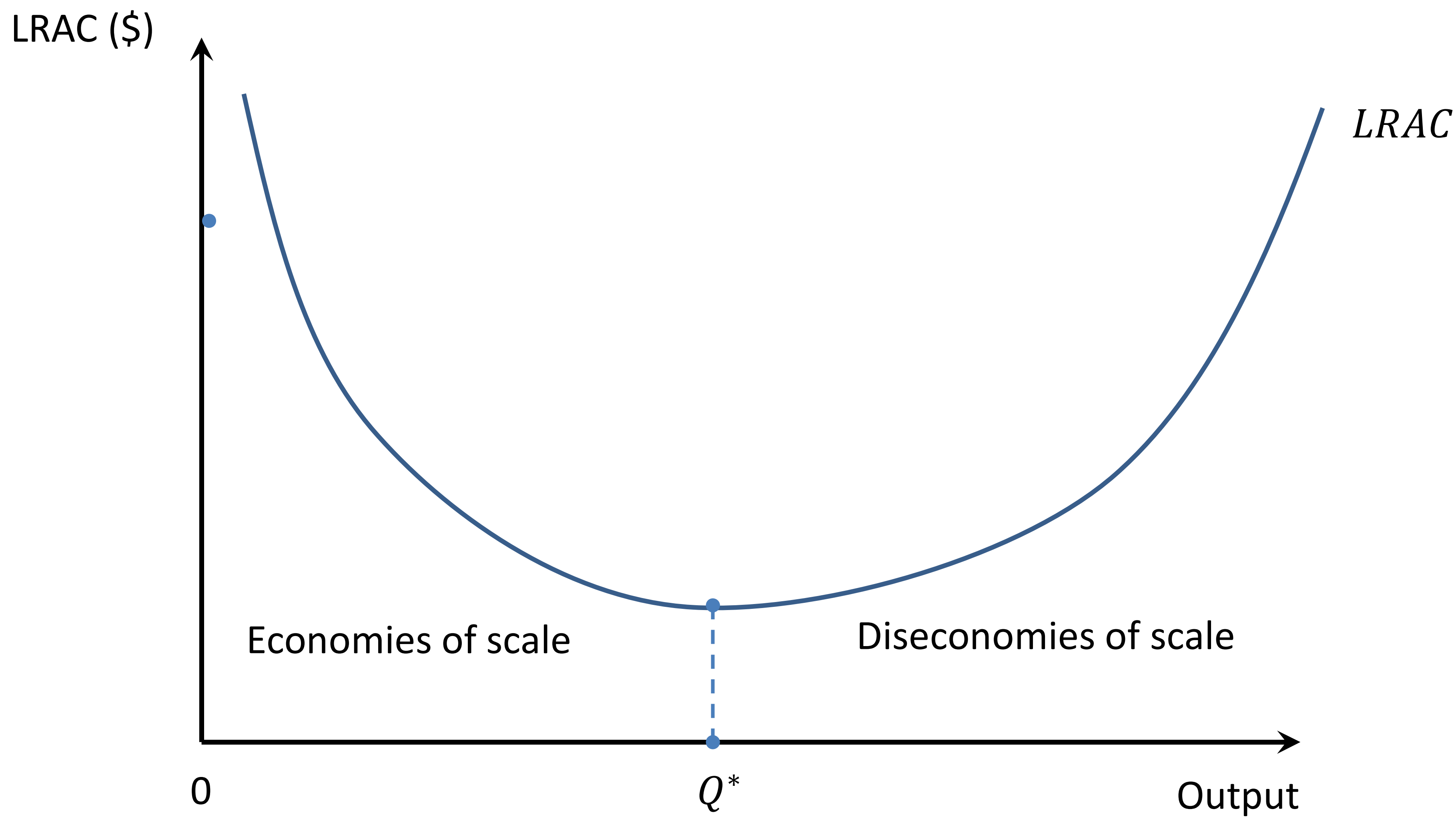
Long-Run Average Cost



Economies of Scale

- **Economies of scale**
  - Declining portion of the long-run average cost curve as output increase.
- **Diseconomies of scale**
  - Rising portion of the long-run average cost curve as output increases.
- **Constant returns to scale**
  - Portion of the long-run average cost curve that remains constant as output increases.

Economies and Diseconomies of Scale



Multiple-Output Cost Function

- **Economies of scope**
  - Exist when the total cost of producing Q_1 and Q_2 together is less than the total cost of producing each of the type of output separately.

$$C(Q_1, 0) + C(0, Q_2) > C(Q_1, Q_2)$$

- **Cost complementarity**
  - Exist when the marginal cost of producing one type of output decreases when the output of another good is increased.

$$\frac{\Delta MC_1(Q_1, Q_2)}{\Delta Q_2} < 0$$

Algebraic Form for a Multiproduct Cost Function

$$C(Q_1, Q_2) = f + aQ_1Q_2 + (Q_1)^2 + (Q_2)^2$$

- For this cost function:

  $MC_1 = aQ_2 + 2Q_1$

  - When a < 0, an increase in Q_2 reduces the marginal cost of producing product 1.
  - If a < 0, this cost function exhibits cost complementarity
  - If a > 0, there are no cost complementarities
  - Exhibits economies of scope whenever $f - aQ_1Q_2 > 0$

Take-home Message

- The **production function** shows the relationship between output and inputs.
- **Marginal product usually diminishes** as the input increases.
- A firm should employ inputs such that the **marginal rate of technical substitution equals the ratio of input prices**.
- Variable costs vary with output; fixed costs do not.
- The **marginal cost curve** intersects the **average cost curve** at minimum average cost.
- Economies of scale: LR average cost falls as ***Q*** rises.