Coloured Petri Nets

Modelling and Validation of Concurrent Systems

Chapter 3: CPN ML Programming

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```
colset PACKETS  = list PACKET;
var packets : PACKETS;
fun member (e,l) =
    let
       fun equal x = (e=x)
    in
       exists (equal,l)
    end;
```

CPN ML programming language

- Based on the functional programming language Standard ML.
- CPN ML extends the Standard ML environment with:
  - Constructs for defining colour sets and declaring variables.
  - Concept of multi-sets and associated functions and operators.
- Standard ML plays a major role in CPN modelling and CPN Tools:
  - Provides the expressiveness required to model data and data manipulation as found in typical industrial projects.
  - Used to implement simulation, state space analysis, and performance analysis in CPN Tools.
  - Supports a flexible and open architecture that makes it possible to develop extensions and prototypes in CPN Tools.

Why Standard ML?

- Formal definition of CP-nets uses types, variables, and evaluation of expressions, which are basic concepts from functional programming.
- Patterns in functional programming languages provide an elegant way of implementing enabling inference.
- Standard ML is based on the lambda-calculus which has a formal syntax and semantics. This implies that CPN Tools get an expressive and sound formal foundation.
- Standard ML is supported by mature compilers and associated documentation.

Functional programming and CPN ML

- Computation proceeds by evaluation of expressions not by executing statements making modifications to memory locations.

- Strong typing means that all expressions have a type that can be determined at compile time. This eliminates many run-time errors.
- Types of expressions are inferred by the type system rather than being declared by the user.

- Functions are first-order values and is treated in the same way as basic types such as integers, Booleans, and strings.
- Functions can be polymorphic and hence operate on different types of values.
- Recursion is used to express iterative constructs.

Simple colour sets

- A set of basic types for defining simple colour sets:

Integers	- **int**:	{..., ~2 , ~1 , 0 , 1 , 2 ,...}
Strings	- **string**:	{ ″a″ , ″abc″ ,...}
Booleans	- **bool**:	{true,false}
Unit	- **unit**:	{()}

- Standard colour set definitions:

```
colset INT    = int;
colset STRING = string;
colset BOOL   = bool;
colset UNIT   = unit;
```

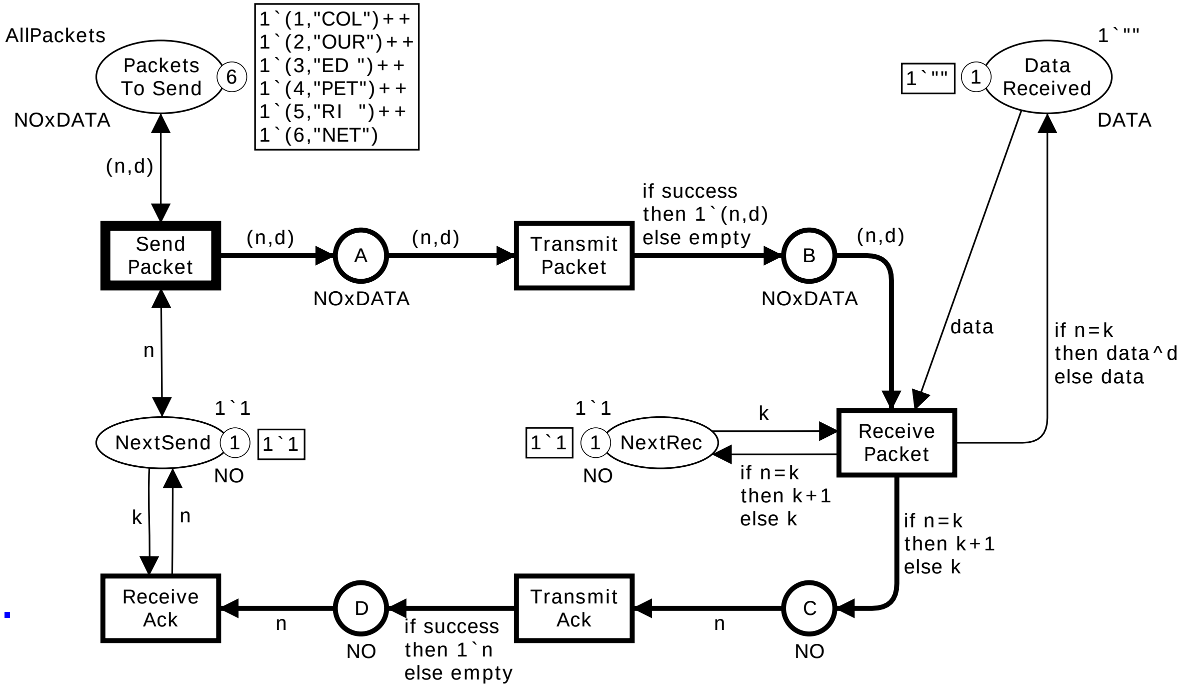
- Two other kinds of simple colour sets:
  - enumeration colour sets.
  - indexed colour sets.

Structured colour sets

- Structured colours sets are defined using colour set constructors:
  - Products
  - Records
  - Unions
  - Lists
  - Subsets

```
colset NOxDATA  = product NO * DATA;
colset DATAPACK = record seq:NO * data:DATA;
colset PACKET   = union Data:DATAPACK + Ack:ACKPACK;
colset PACKETS  = list PACKET;
```

Simple protocol



- The previous versions use products to represent data packets.

- We will now develop a new version where:
  - Data packets are modelled as a record colour set.
  - Data packets and acknowledgement packets are modelled by a common union colour set.
  - We have duplication of packets – in addition to loss and successful transmission.

Revised colour set definitions

- Old definitions:

```
colset DATA    = string;
colset NO      = int;
colset NOxDATA = product NO * DATA;
```

- New definitions:

Record field names

```
colset DATAPACK = record seq : NO * data : DATA;
colset ACKPACK  = NO;
colset PACKET   = union Data : DATAPACK + Ack : ACKPACK;
```

Data constructors

Enumeration colour set (with three explicitly specified data values)

```
colset RESULT   = with success | failure | duplicate;
```

Example values

- Record colour set:

```
colset DATAPACK = record seq : NO * data : DATA;
```

```
{seq=1,data="COL"}   {data="COL",seq=1,}
```

Same data value

- Union colour set:

Data constructors

```
colset PACKET = union Data : DATAPACK + Ack : ACKPACK;
```

```
Data{seq=1,data="COL"}
```

← Data packet

```
Ack(2)
```

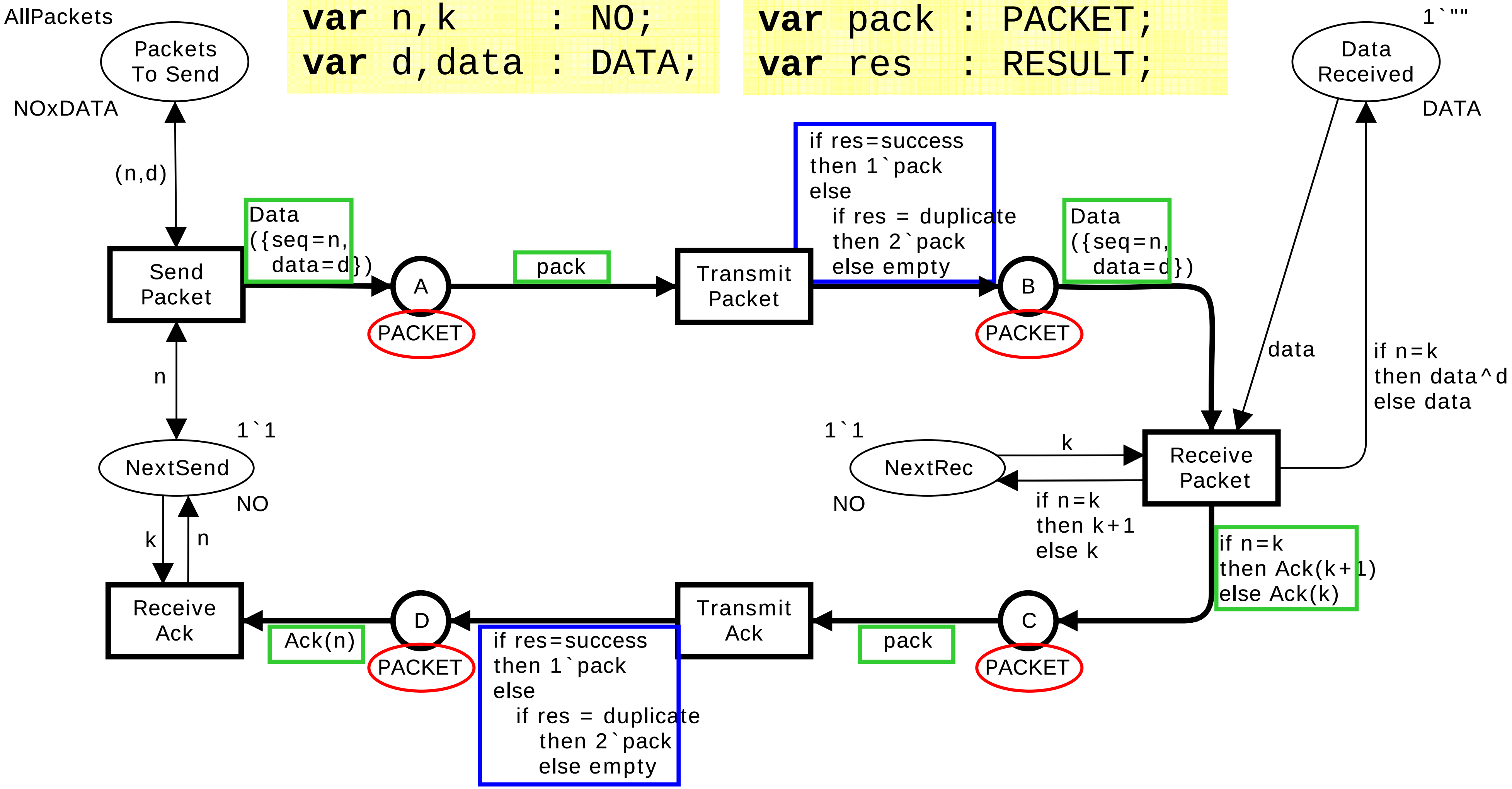
← Acknowledgement packet

```
colset ACKPACK  = NO;
```

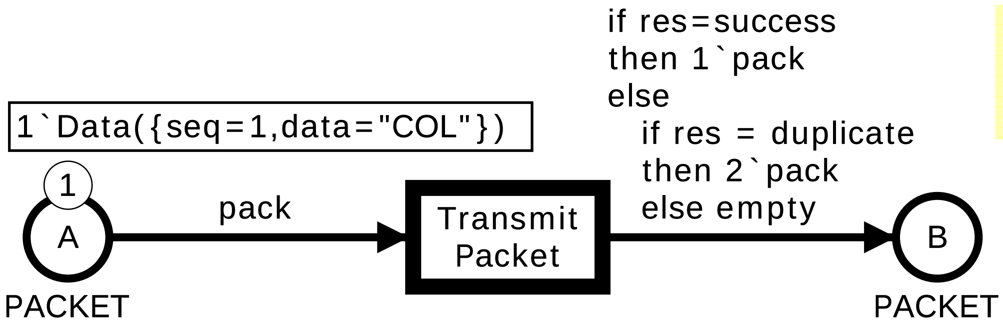
Revised CPN model

```
var n,k    : NO;
var d,data : DATA;
```

```
var pack : PACKET;
var res  : RESULT;
```



Transmit Packet transition

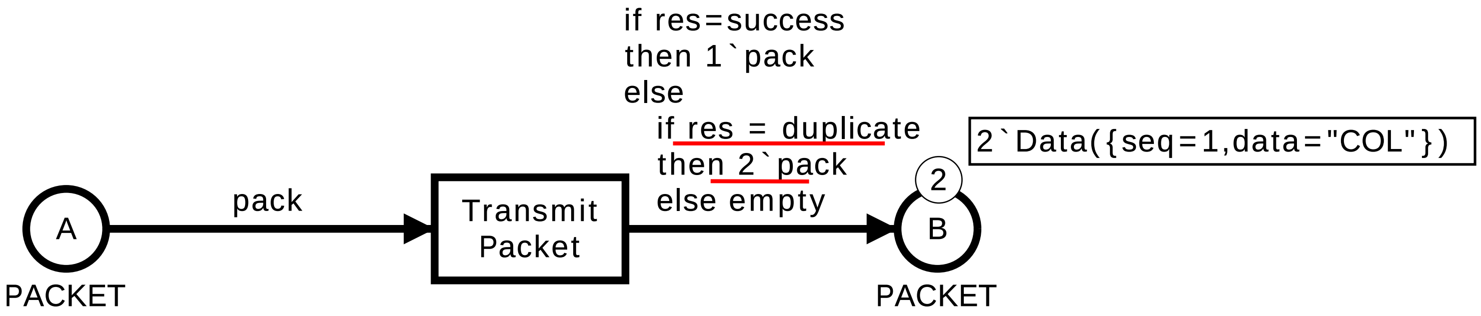


```
var pack : PACKET;
var res  : RESULT;
```

b^{+} = <pack=Data({seq=1,data="COL"}), res=success>

b^{-} = <pack=Data({seq=1,data="COL"}), res=failure>

b^{++} = <pack=Data({seq=1,data="COL"}), res=duplicate>



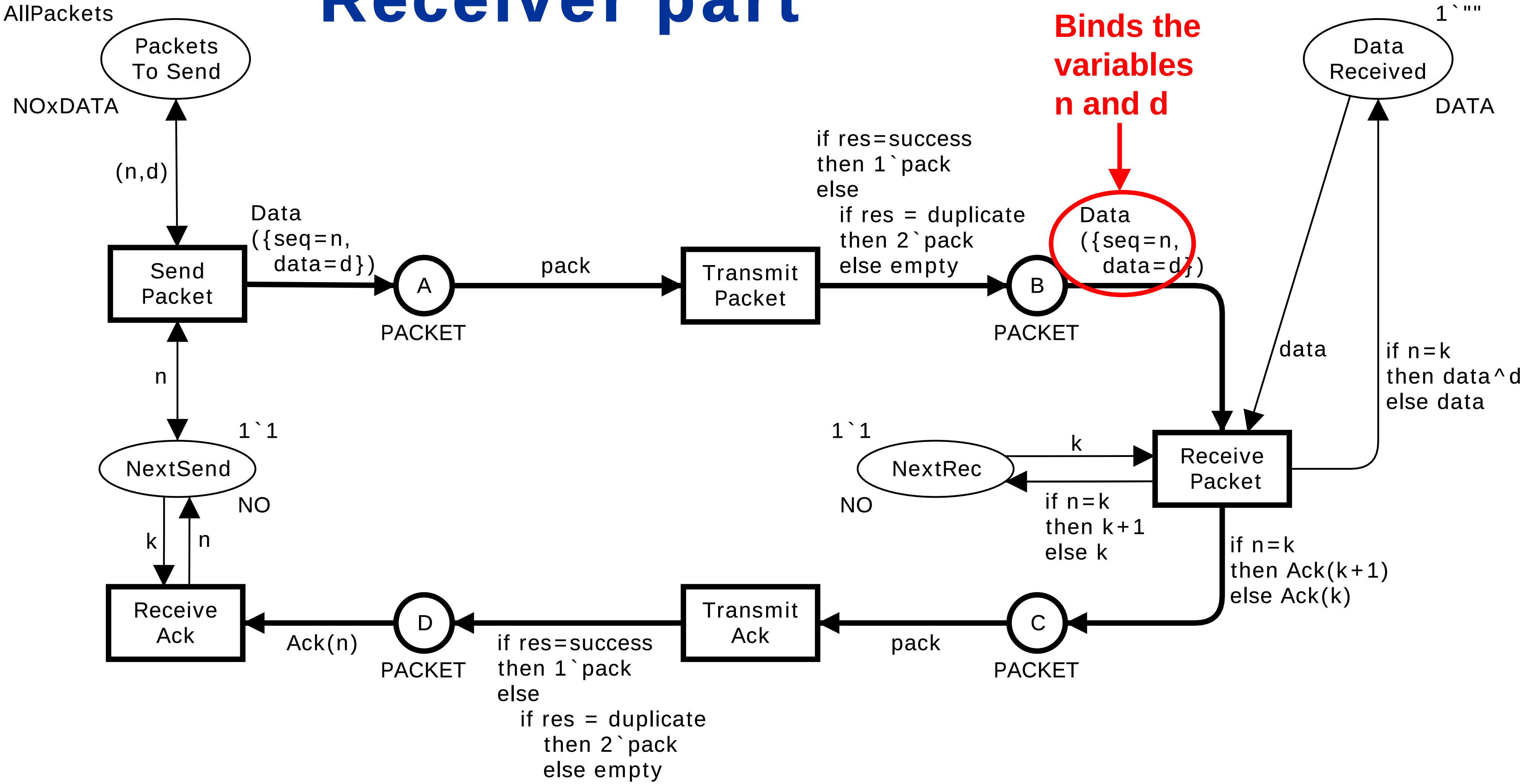
Tuples and records

- Tuple components and record fields can be accessed using the family of # operators.

- Examples:

`#seq {seq=1,data="COL"}`	1	Records
`#data {seq=1,data="COL"}`	"COL"	
`#1 (3,"ED ")`	3	Products
`#2 (3,"ED ")`	"ED "	

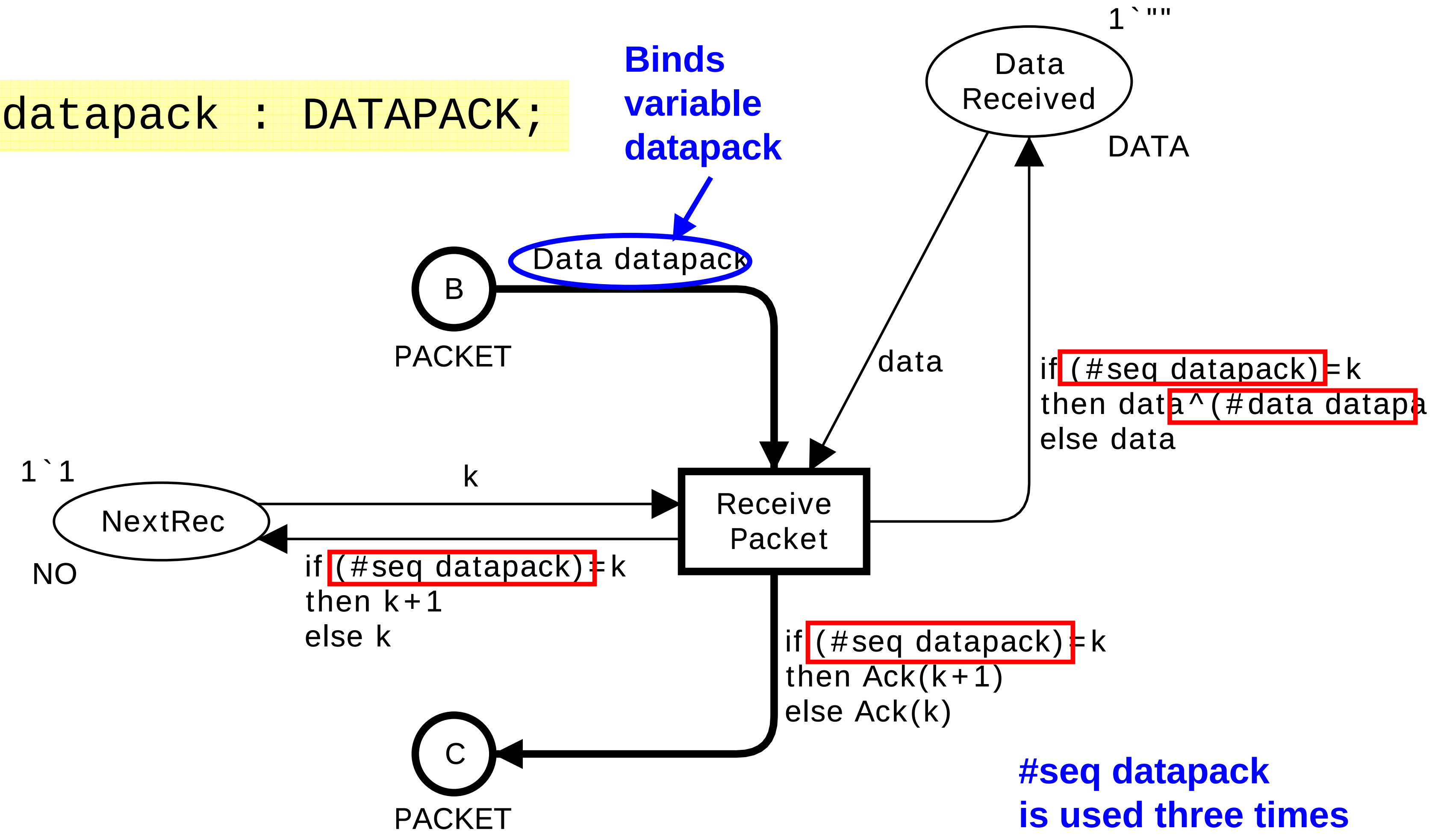
Receiver part



Binds the variables n and d

First variant of receiver

```
var datapack : DATAPACK;
```

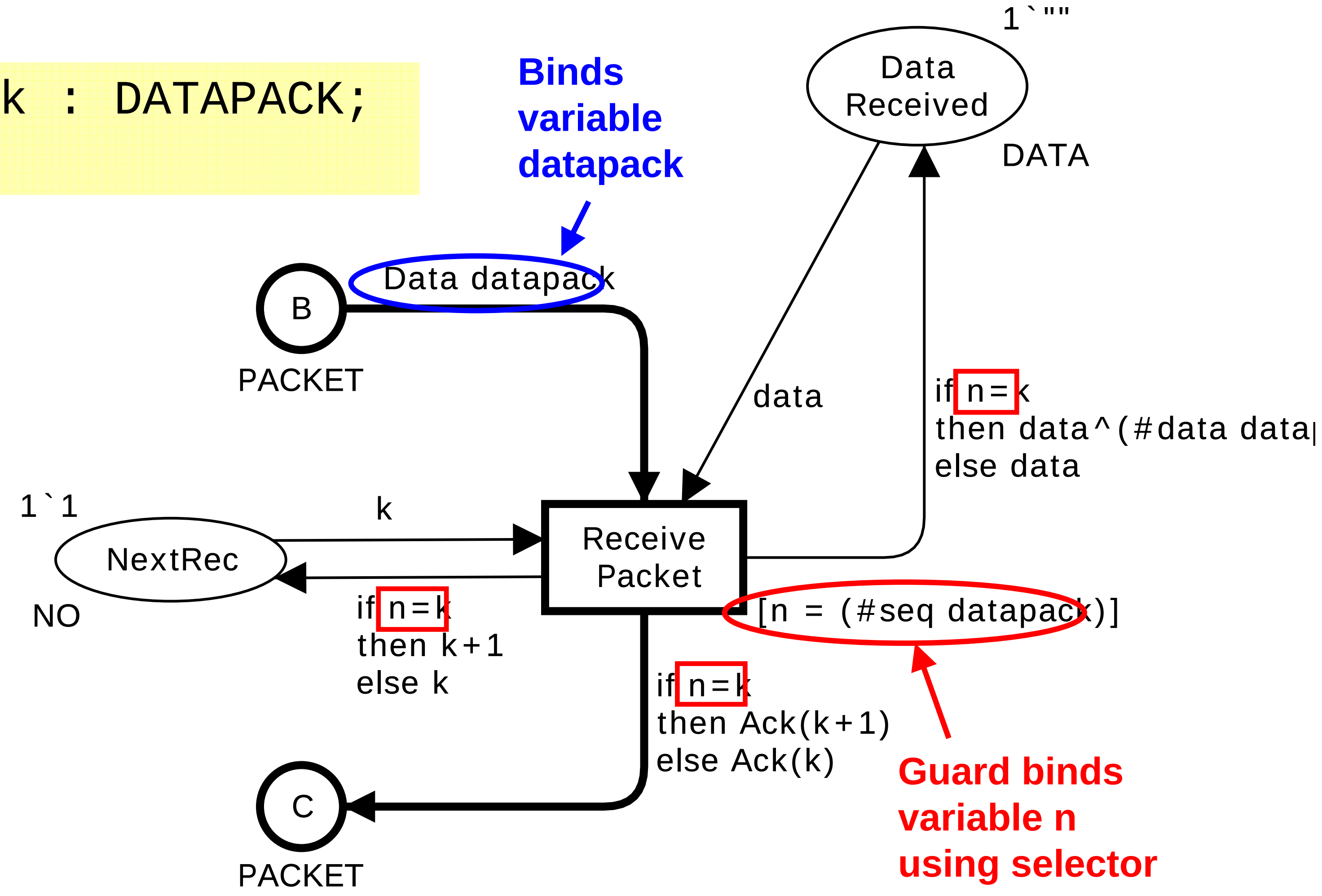


Binds variable datapack

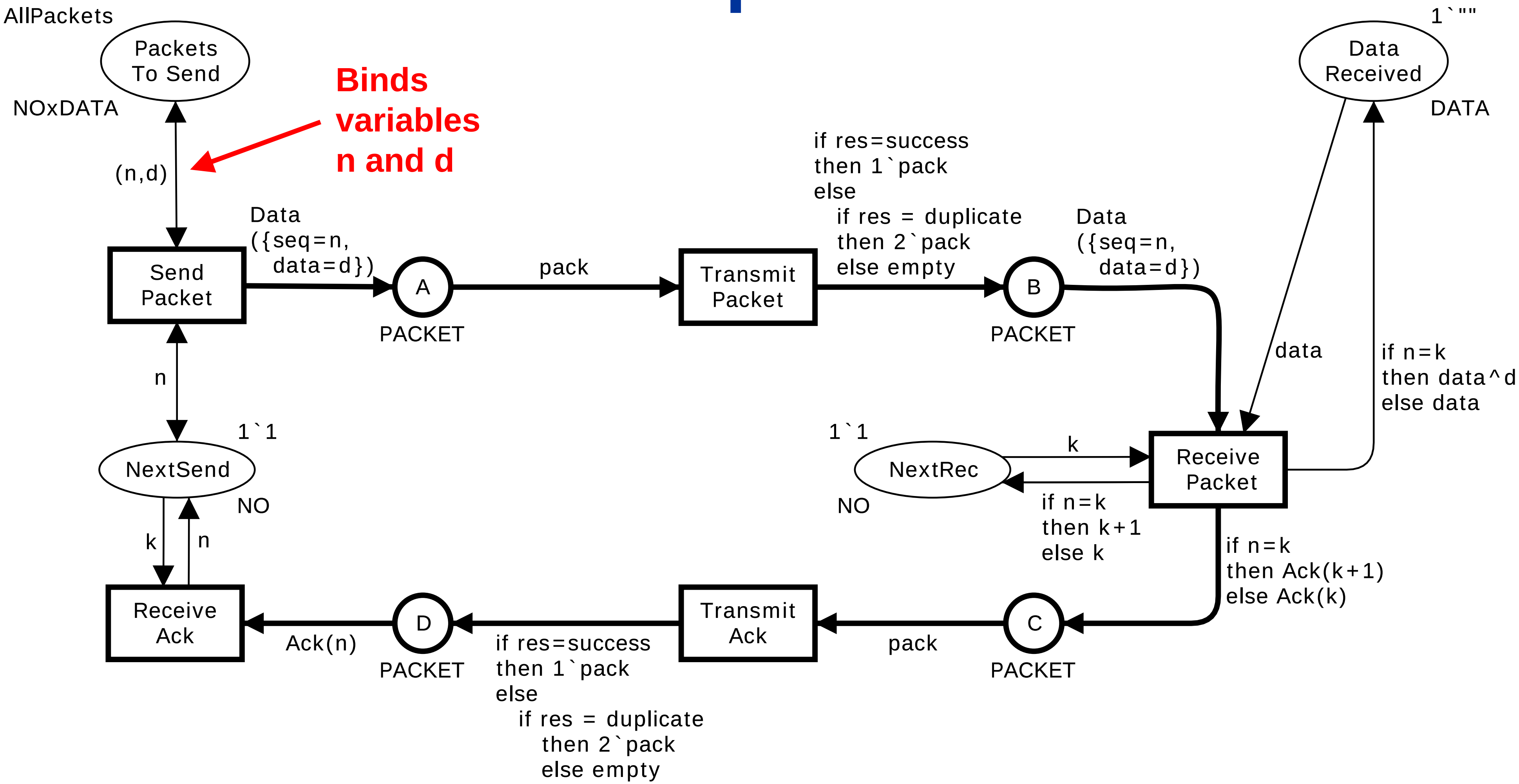
#seq datapack is used three times

Second variant of the receiver

```
var datapack : DATAPACK;
var n : NO;
```

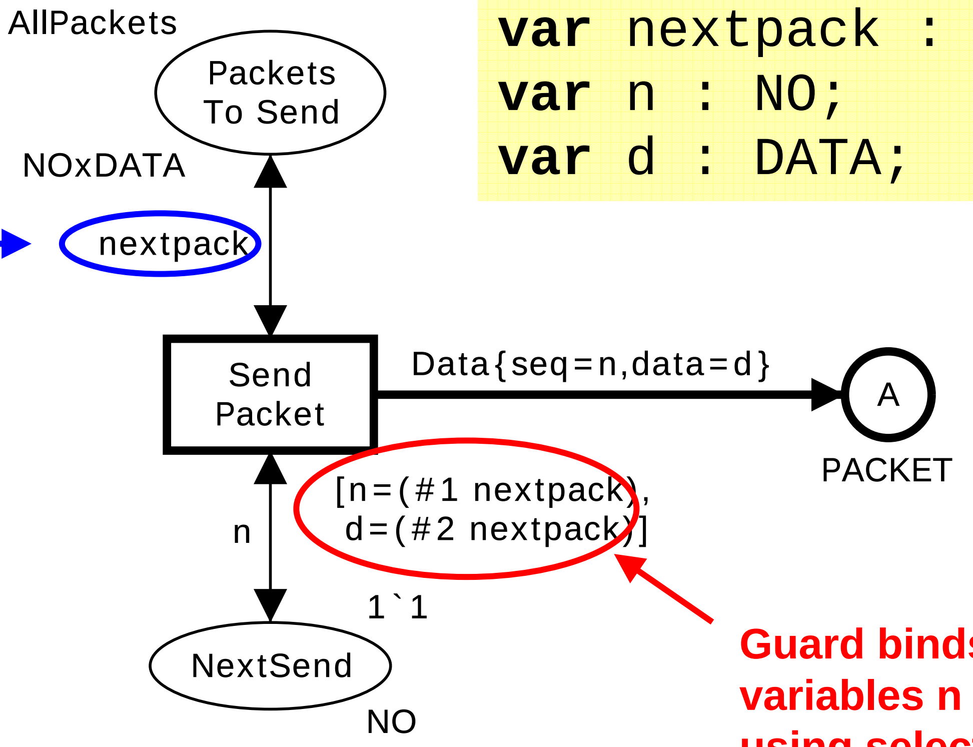


Sender part



Binds variables n and d

Variant of the sender



```
var nextpack : NOxDATA;
var n : NO;
var d : DATA;
```

Binds variable nextpack

Guard binds variables n and d using selectors

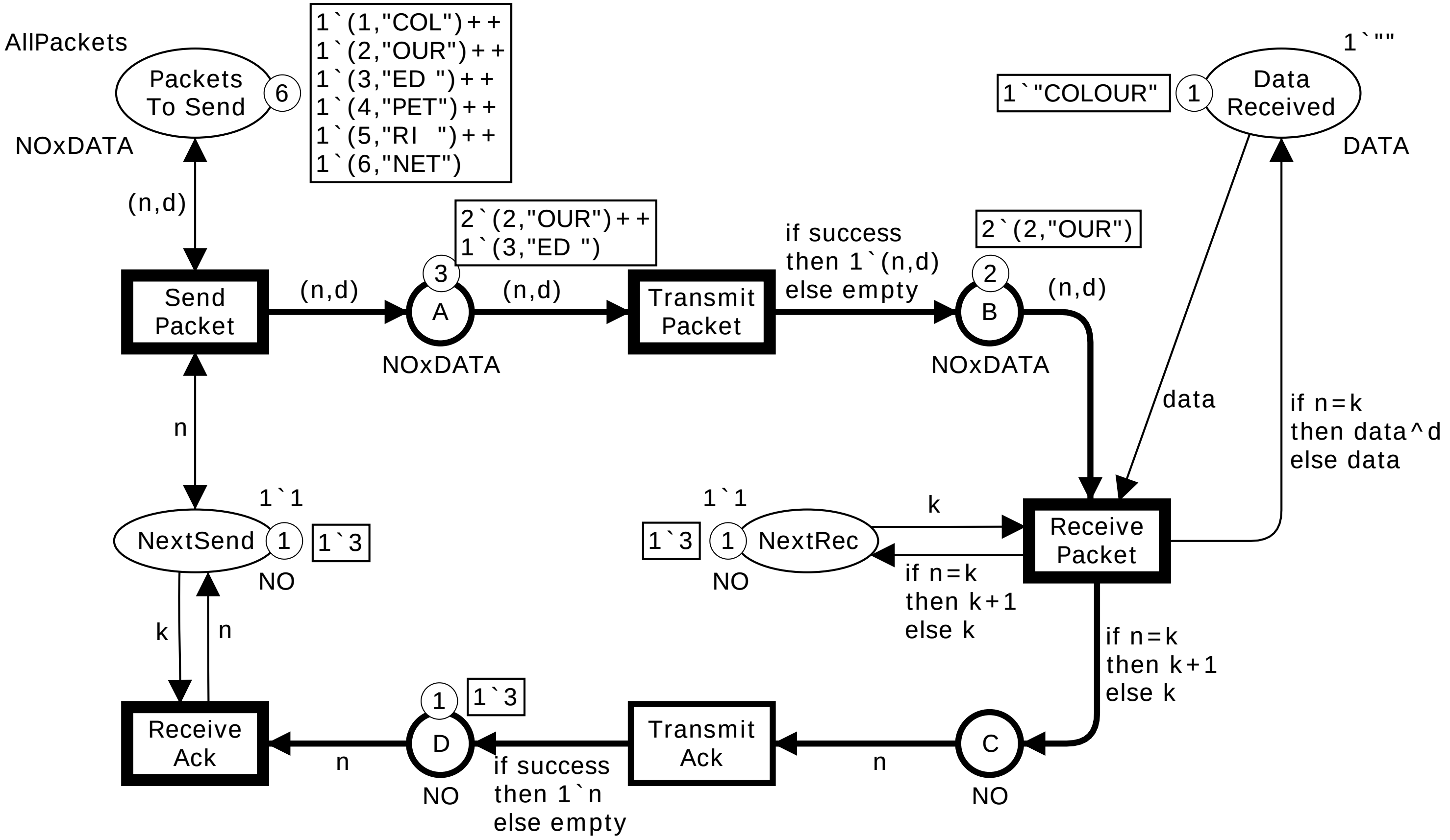
Products or records?

- There is a always a choice between using product or record colour sets.

- Products may give shorter net inscriptions, because we avoid the selector names used in records.
- Records may give more readable net inscriptions due to the mnemonic selector names. The same effect can often be achieved for products by using variables with mnemonic names, e.g. (seq,data).

- As a rule of thumb we do not recommend using products with more than 4-5 components. In such cases it is better to use records.

Overtaking possible



- We will develop a new version where overtaking of data packets and acknowledgements is impossible.

List colour sets

- Colour set definitions:

```
colset DATAPACKS = list NOxDATA;
colset ACKPACKS  = list NO;
```

- Example values:

```
[(1,"COL"),(1,"COL"),(2,"OUR")]
```

← Three data packets

```
[2,2,3,3]
```

← Four acknowledgement packets

```
[]
```

← Empty list (polymorphic)

List concatenation (^^)

- Application:

```
[(1,"COL"),(1,"COL")]^^[(2,"OUR"),(3,"ED ")]
```

List

List

- Result:

```
[(1,"COL"),(1,"COL"),(2,"OUR"),(3,"ED ")]
```

List

List construction (::)

- Application:

```
(1,"COL")::[(1,"COL"),(2,"OUR")]
```

Element

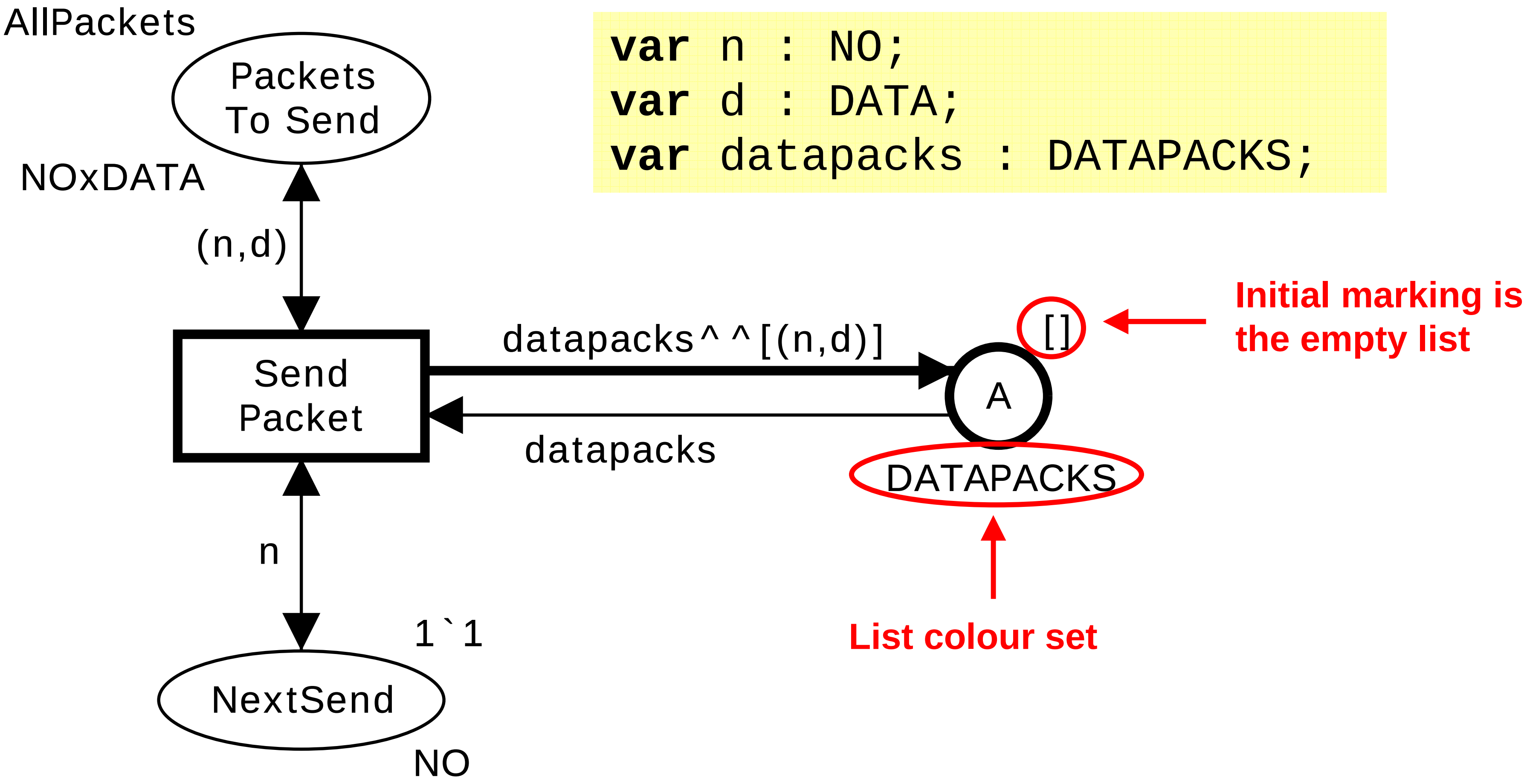
List

- Result:

```
[(1,"COL"),(1,"COL"),(2,"OUR")]
```

List

Revised SendPacket

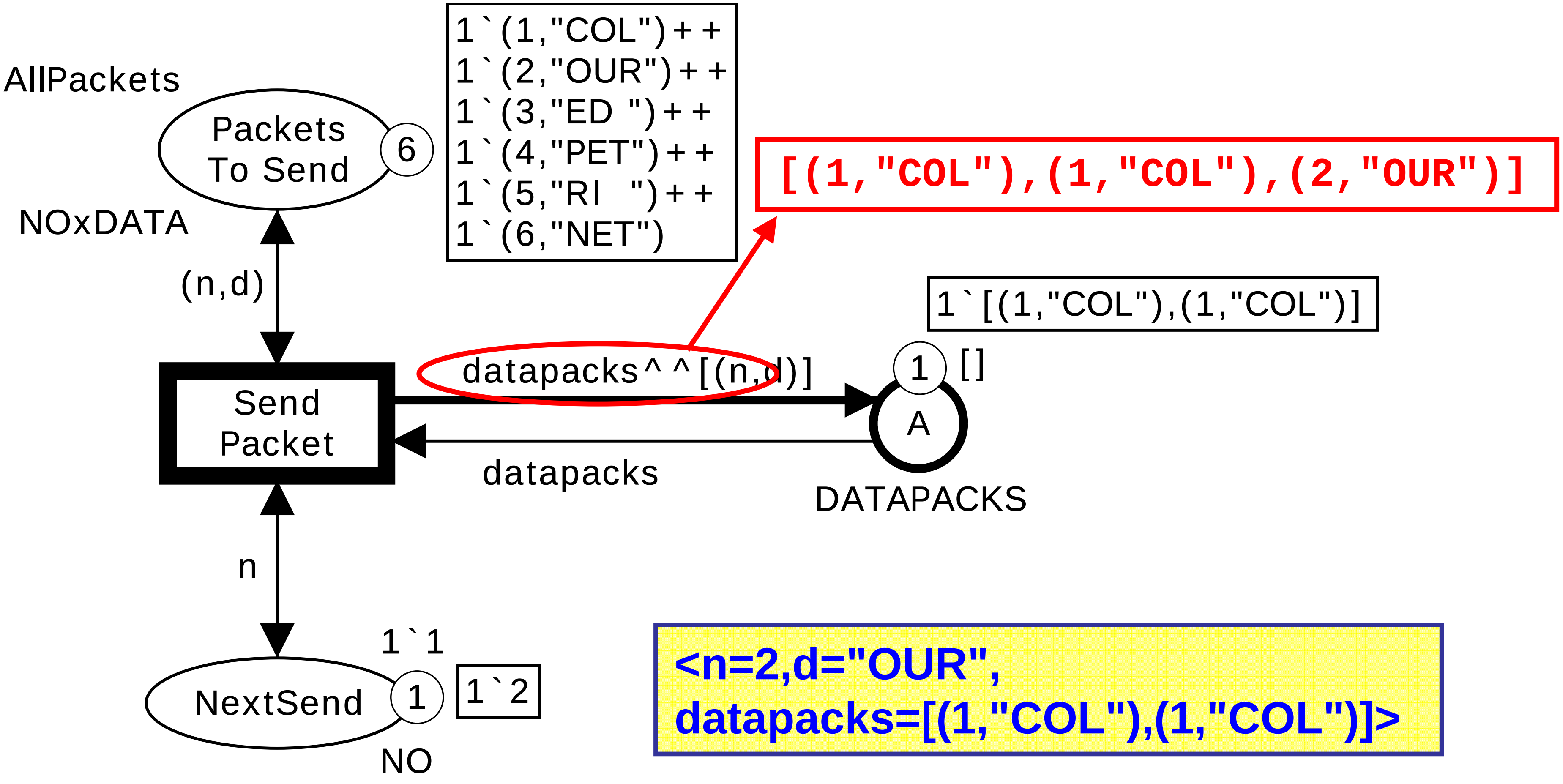


```
var n : NO;
var d : DATA;
var datapacks : DATAPACKS;
```

Initial marking is the empty list

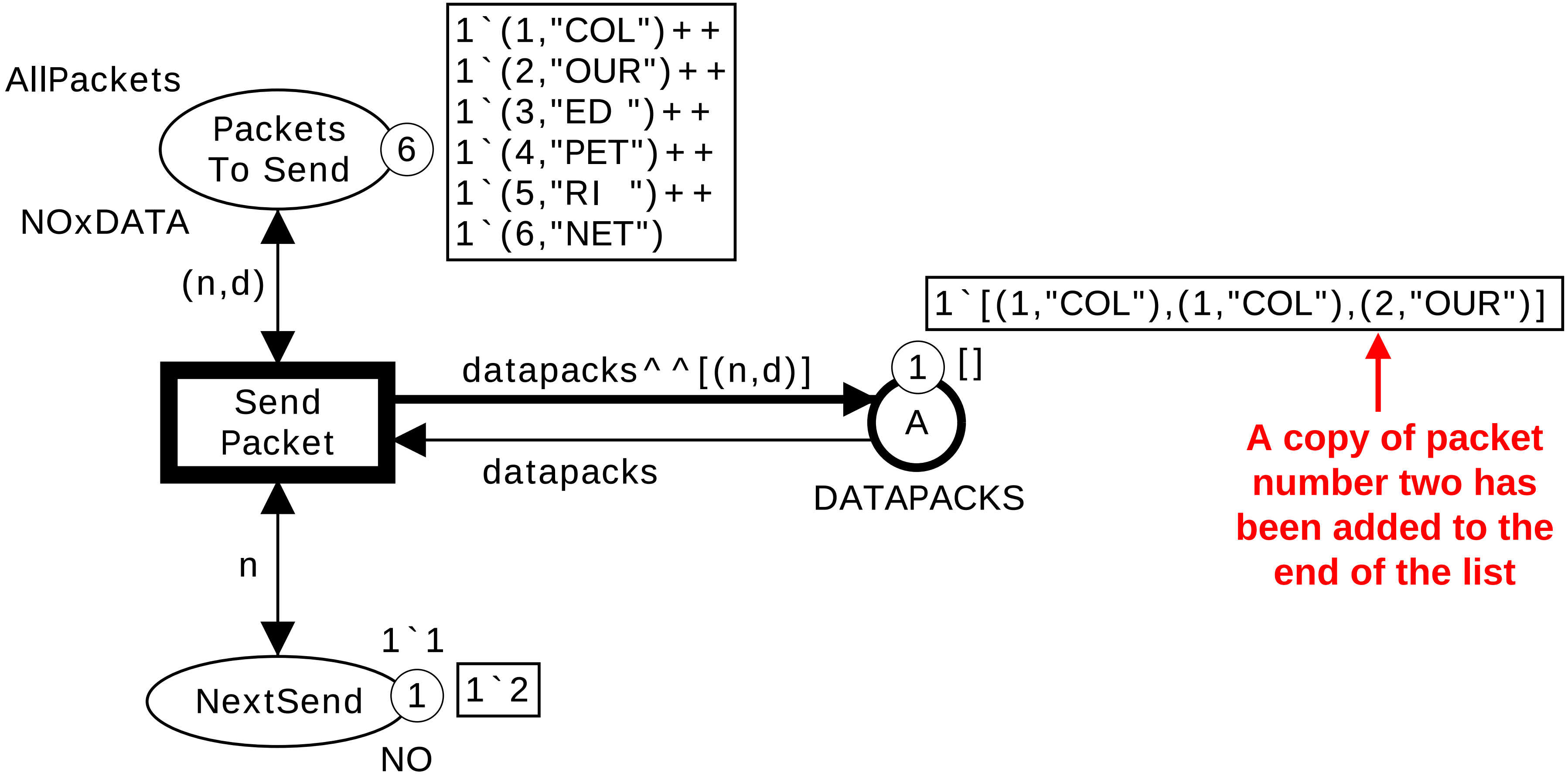
List colour set

Enabling of SendPacket



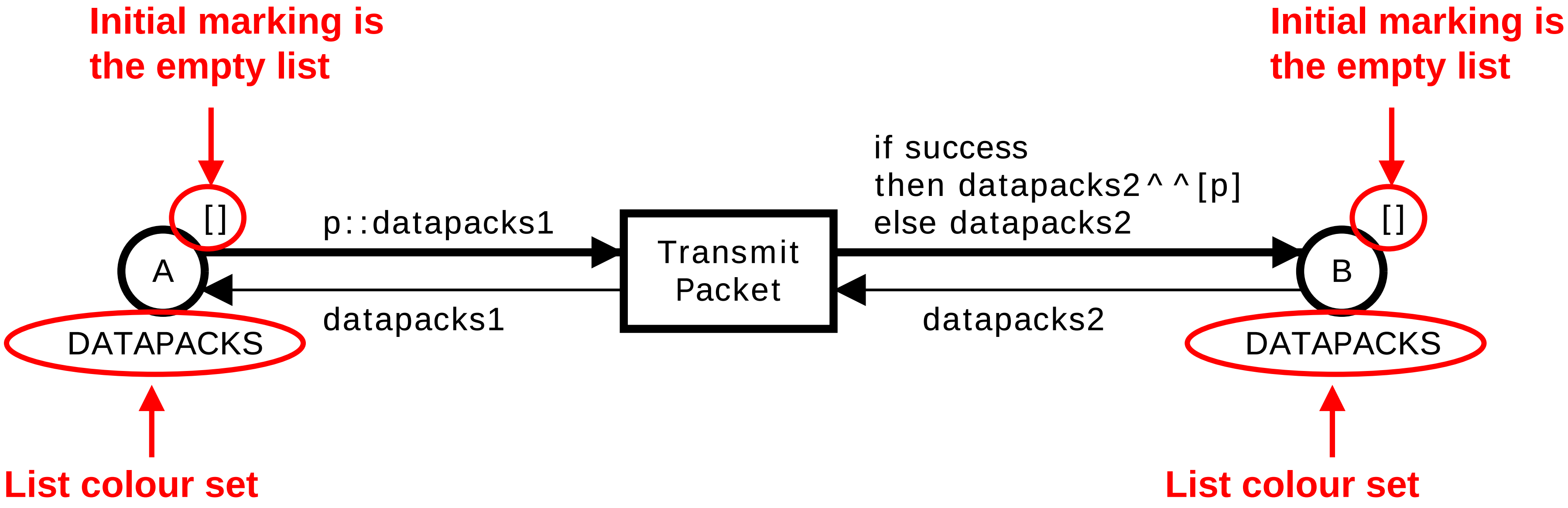
**<n=2,d="OUR",
datapacks=[(1,"COL"),(1,"COL")]>**

Occurrence of SendPacket



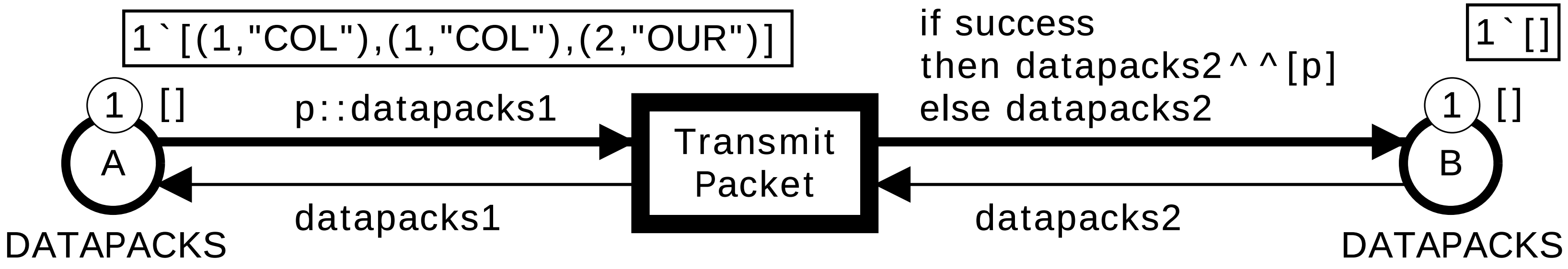
A copy of packet number two has been added to the end of the list

Revised TransmitPacket



```
var p       : NOxDATA;
var success : BOOL;
var datapacks1,datapacks2 : DATAPACKS;
```

Enabling of TransmitPacket

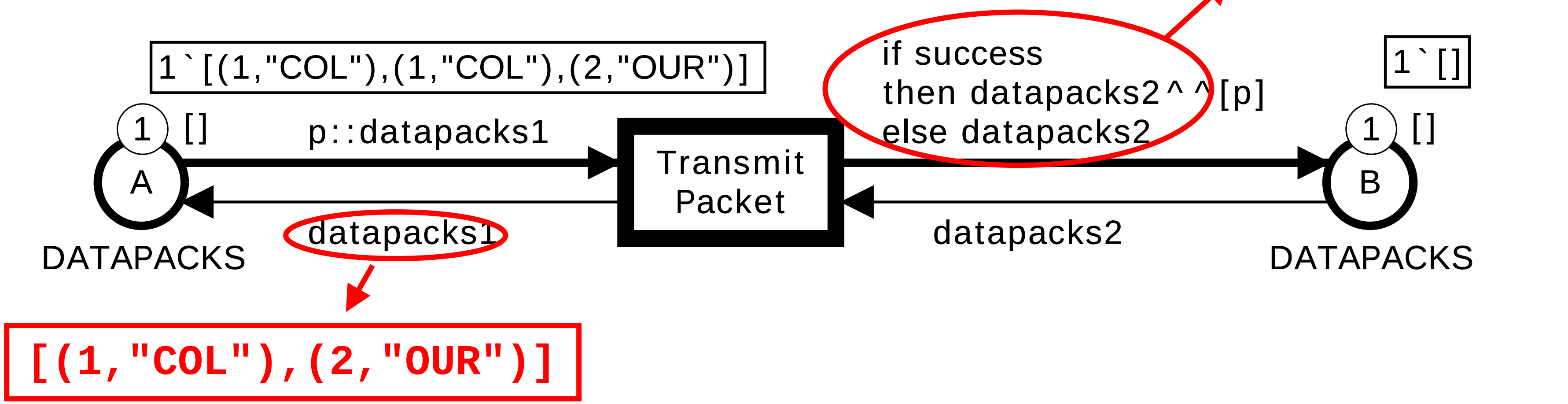


b^+ = <p=(1,"COL"),datapacks1=[(1,"COL"),(2,"OUR")],
success=true,datapacks2=[]>

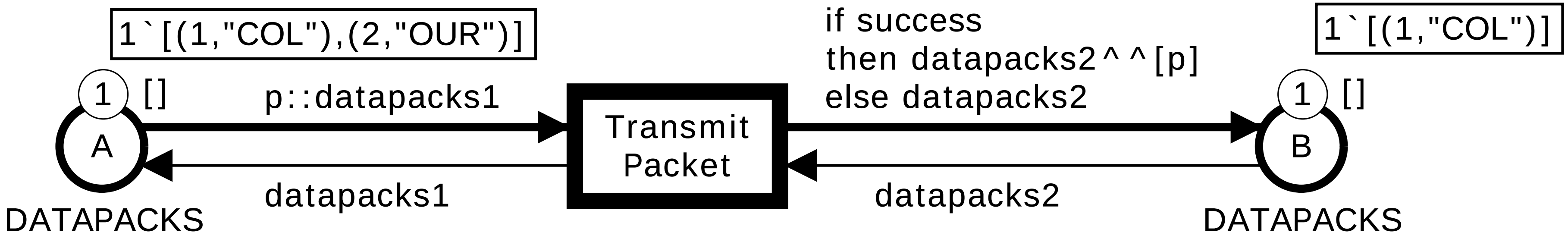
b^- = <p=(1,"COL")},datapacks1=[(1,"COL"),(2,"OUR")],
success=false,datapacks2=[]>

Successful transmission

[(1,"COL")]

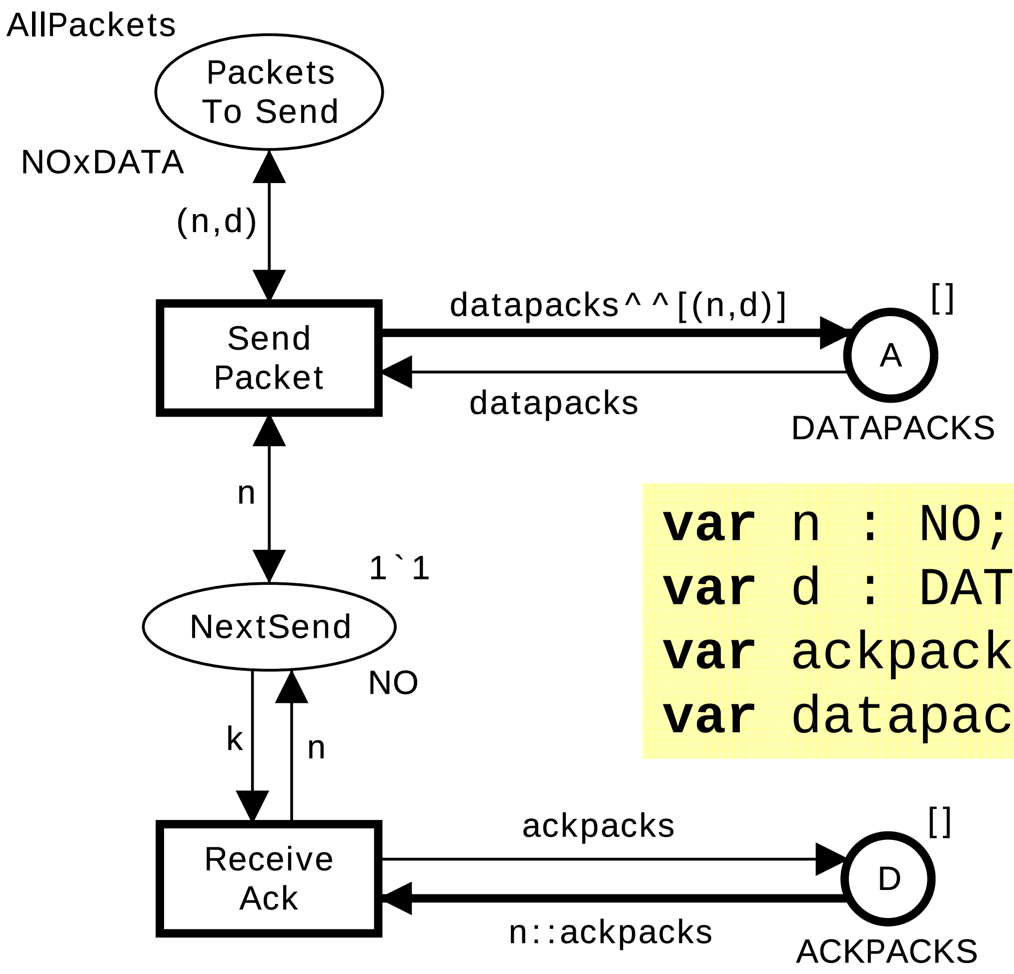


b^+ = <p=(1,"COL"),datapacks1=[(1,"COL"),(2,"OUR")], success=true,datapacks2=[]>



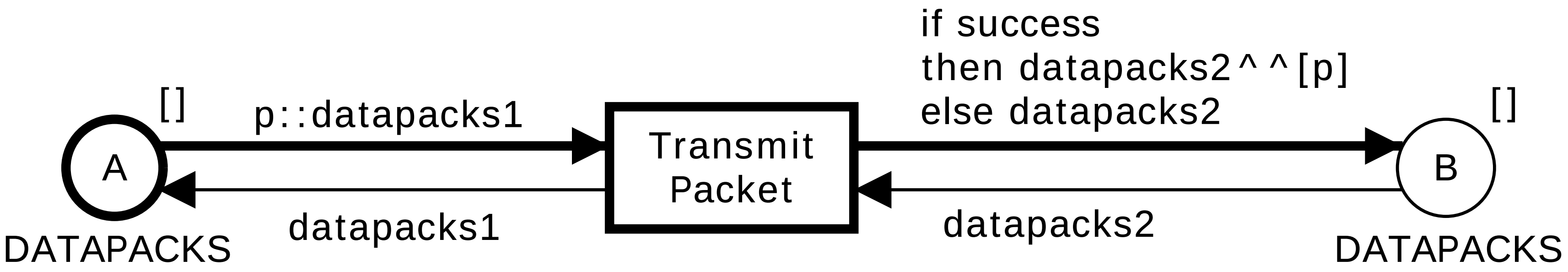
The first element from the A-list has been moved to the end of the B-list

Revised sender



```
var n : NO;
var d : DATA;
var ackpacks : ACKPACKS;
var datapacks : DATAPACKS;
```

Revised network



if success
then datapacks2^^[p]
else datapacks2

[]

p::datapacks1

[]

A

Transmit Packet

B

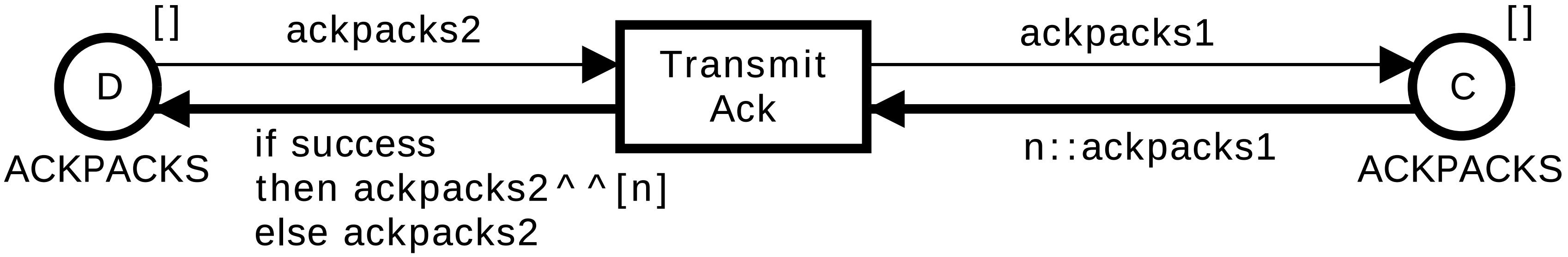
datapacks1

datapacks2

DATAPACKS

DATAPACKS

```
var n : NO;
var p : DATAPACK;
var success : BOOL;
var ackpacks1,ackpacks2 : ACKPACKS;
var datapacks1,datapacks2 : DATAPACKS;
```



[]

ackpacks2

ackpacks1

[]

D

Transmit Ack

C

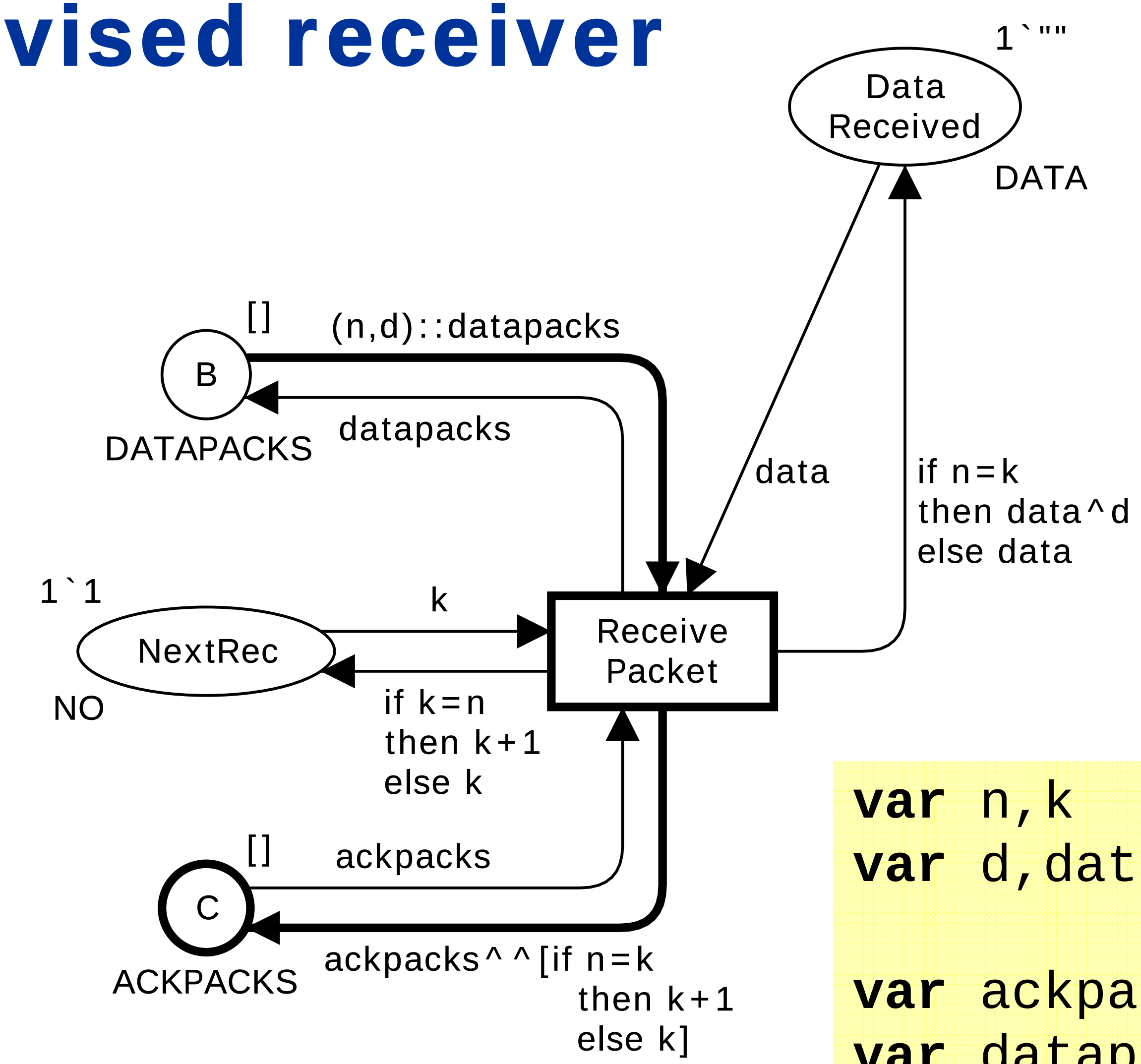
ACKPACKS

if success
then ackpacks2^^[n]
else ackpacks2

n::ackpacks1

ACKPACKS

Revised receiver



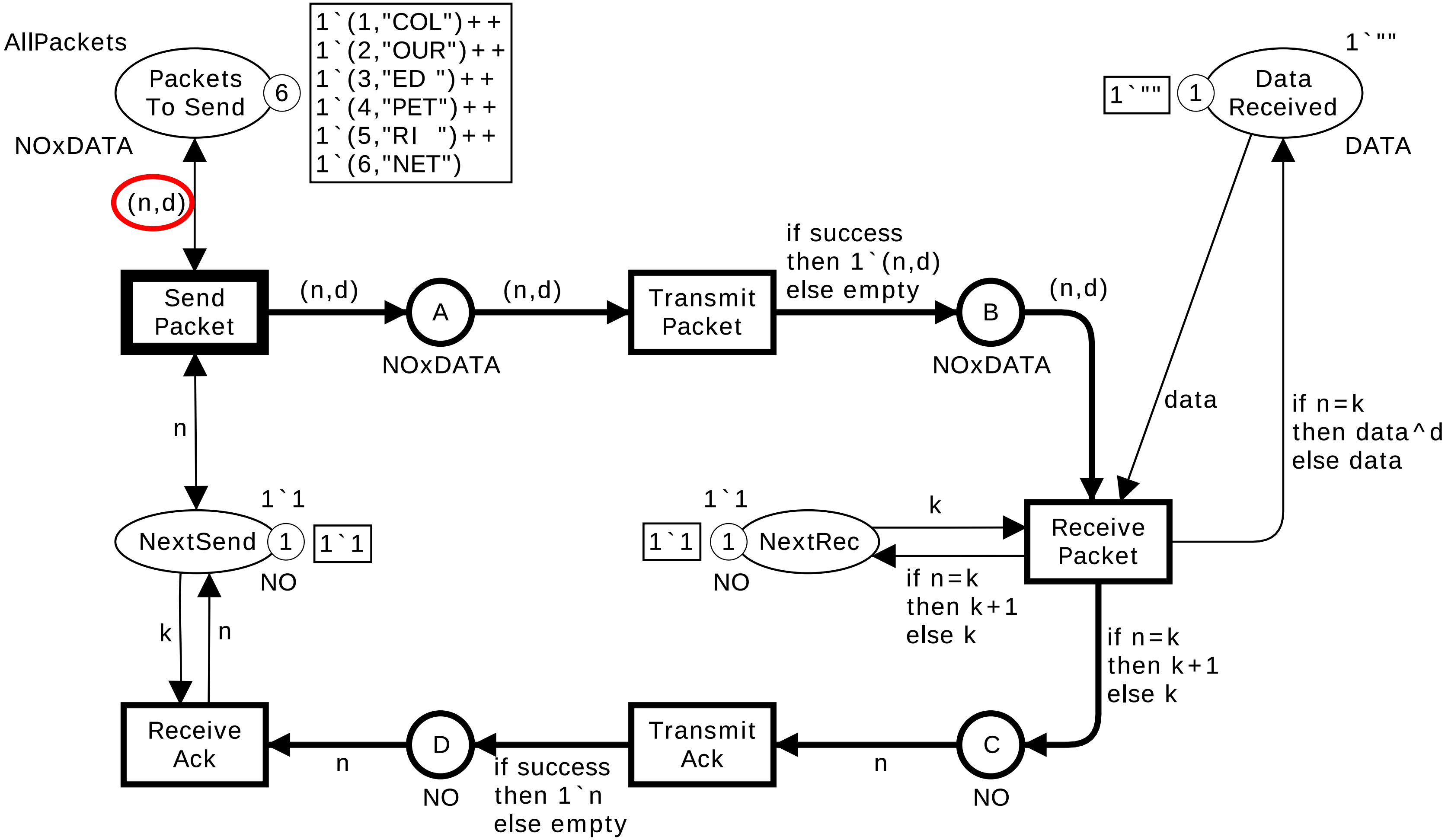
```
var n,k       : NO;
var d,data    : DATA;

var ackpacks : ACKPACKS;
var datapacks: DATAPACKS;
```

Expressions and types

- The complete set of Standard ML expressions can be used in net inscriptions provided that they have the proper type:
  - The type of an arc expression must be equal to the colour set of the place connected to the arc (or a multi-set over the colour set of the place).
  - The type of an initial marking must be equal to the colour set of the place (or a multi-set over the colour set of the place).
  - A guard must be a Boolean expression (or a list of Boolean expressions).
- The CPN ML type system checks that all net inscriptions are type consistent and satisfies the above type constraints.
- This is done by automatically inferring the types of expressions.

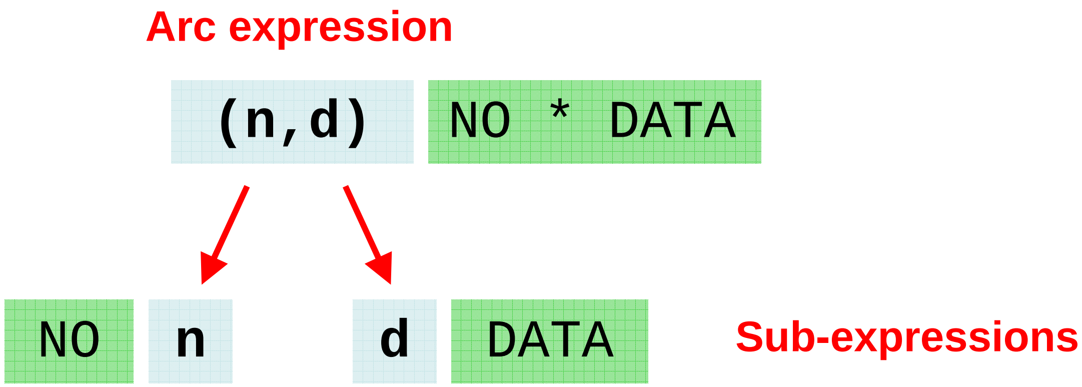
Example of type checking



Type checking of (n,d)

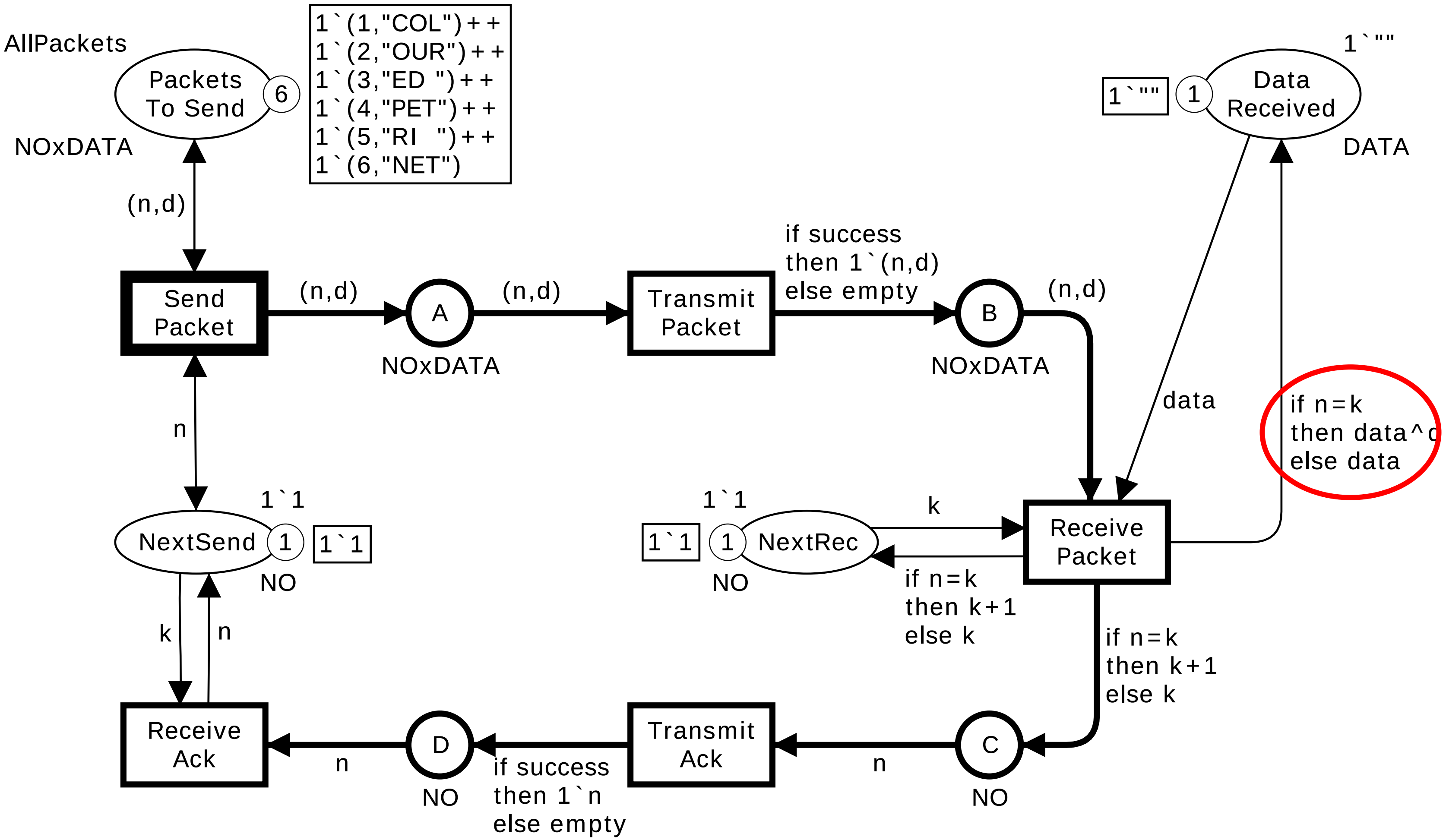
```
colset NOxDATA = product NO * DATA;
```

```
var n : NO;
var d : DATA;
```



- (n,d) is type consistent and of type NO * DATA.
- NO * DATA matches NOxDATA which is the colour set of the connected place.

Second example of type checking



Type checking of if expression

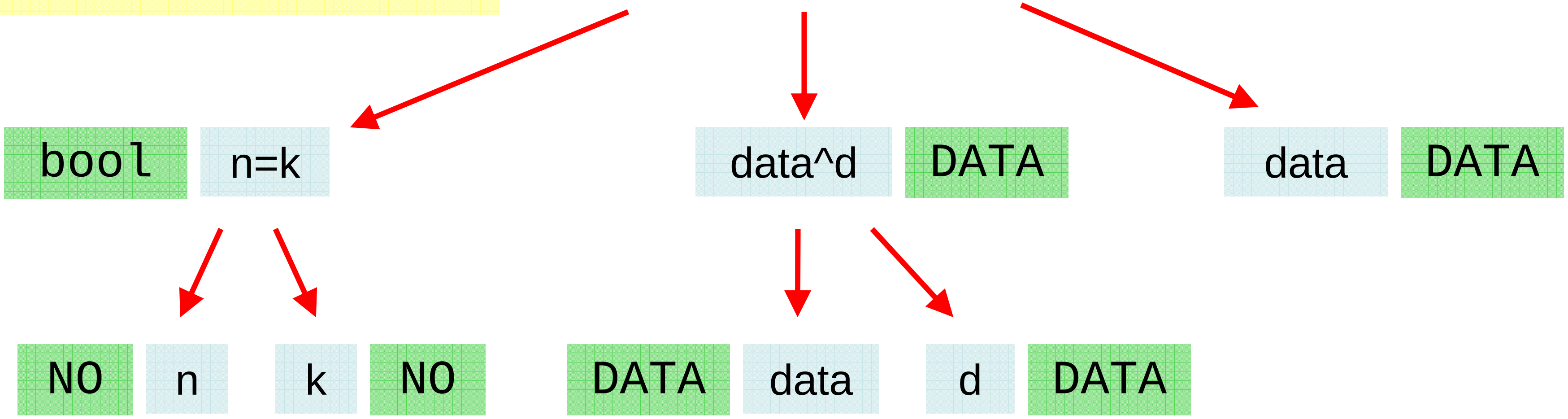
```
colset DATA = string;
```

```
var n,k : NO;
var d,data : DATA;
```

Arc expression

```
if n=k
then data^d
else data
```

DATA



bool n=k

data^d DATA

data DATA

NO n

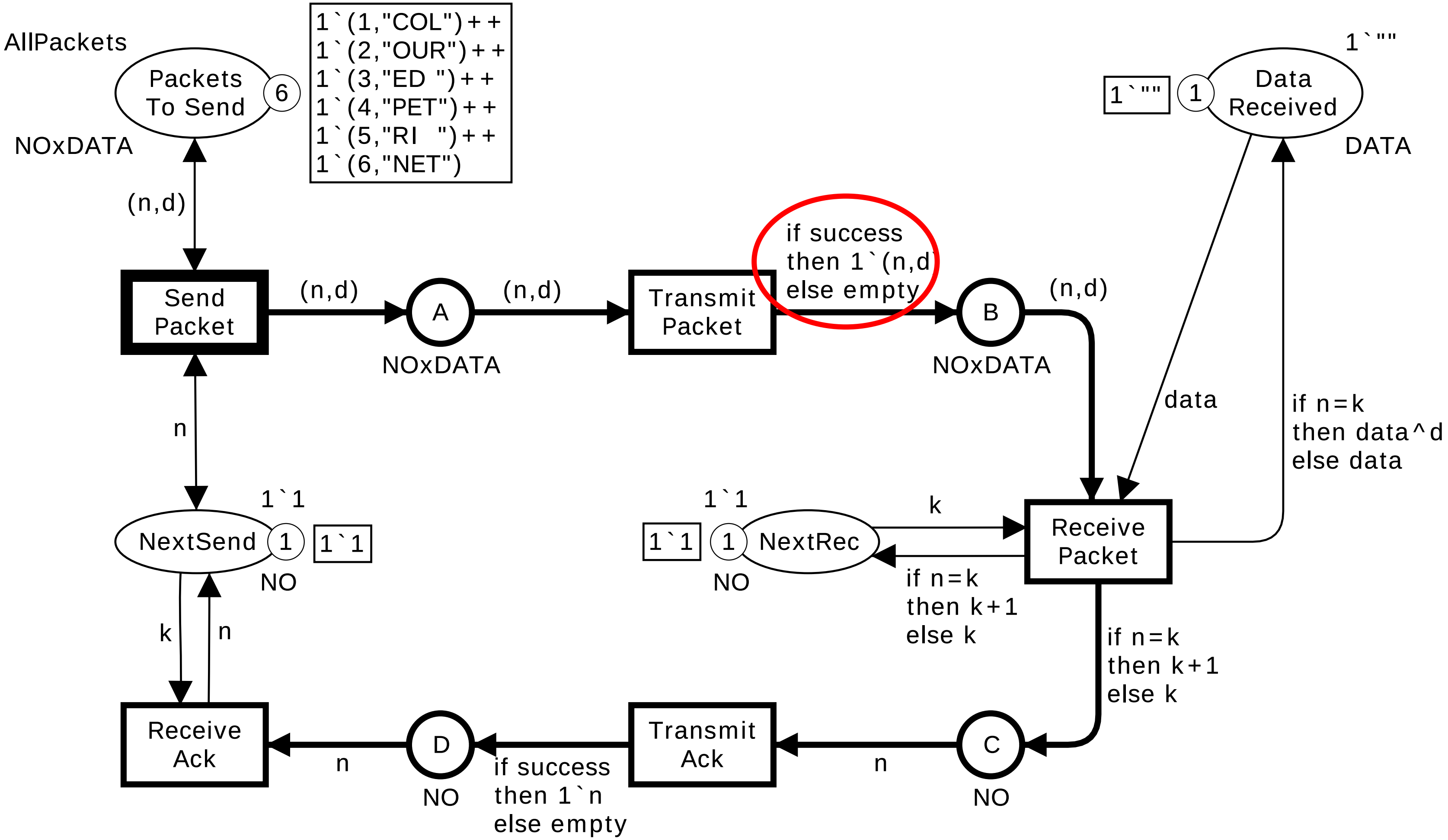
k NO

DATA data

d DATA

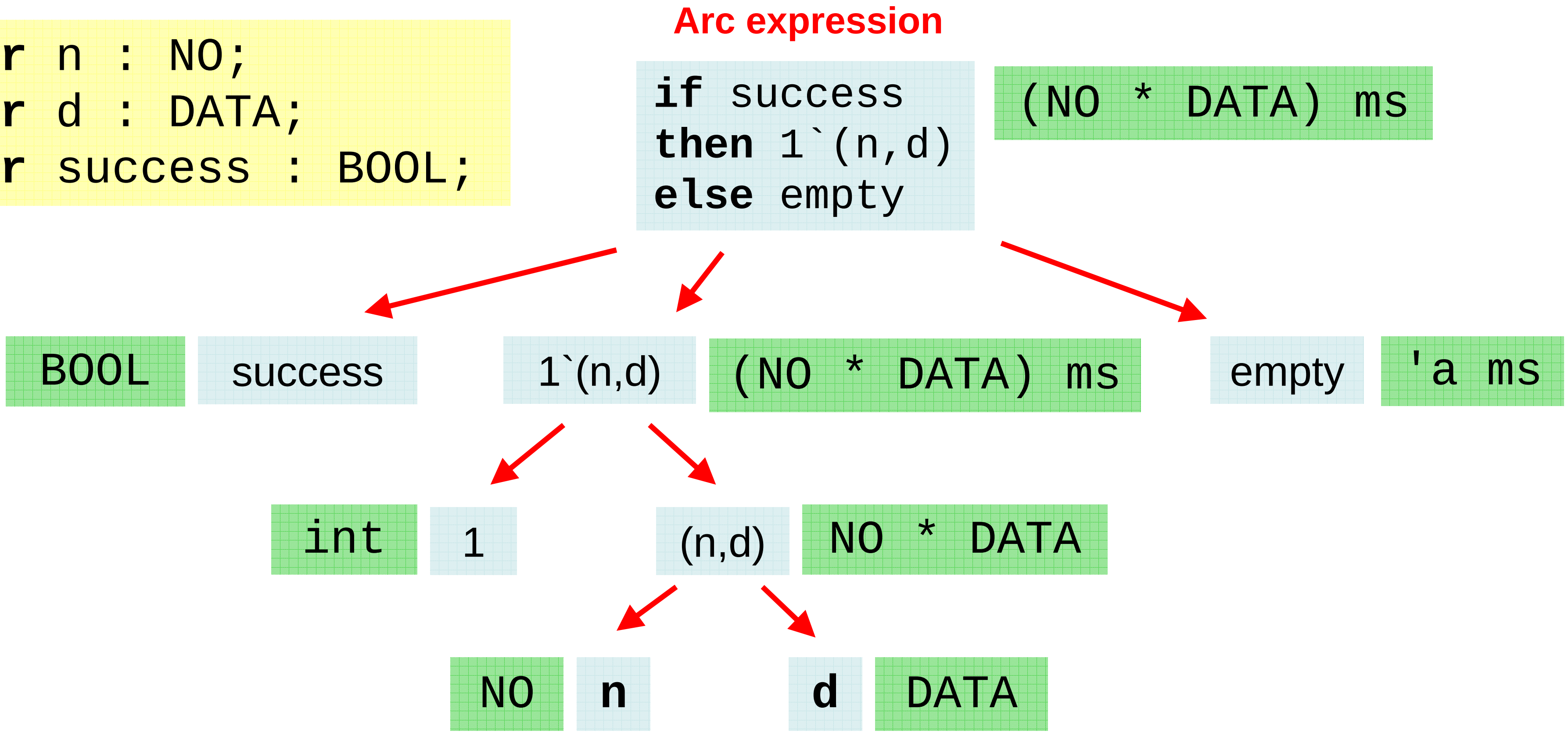
- If expression is type consistent and of type DATA (which is the colour set of the connected place).

Third example of type checking



Type checking of if expression

```
var n : NO;
var d : DATA;
var success : BOOL;
```



Arc expression

```
if success
then 1`(n,d)
else empty
```

(NO * DATA) ms

BOOL success

1`(n,d) (NO * DATA) ms

empty 'a ms

int 1

(n,d) NO * DATA

NO n

d DATA

- If expression is type consistent and of type NO * DATA ms (multi-sets over the colour set of the connected place).

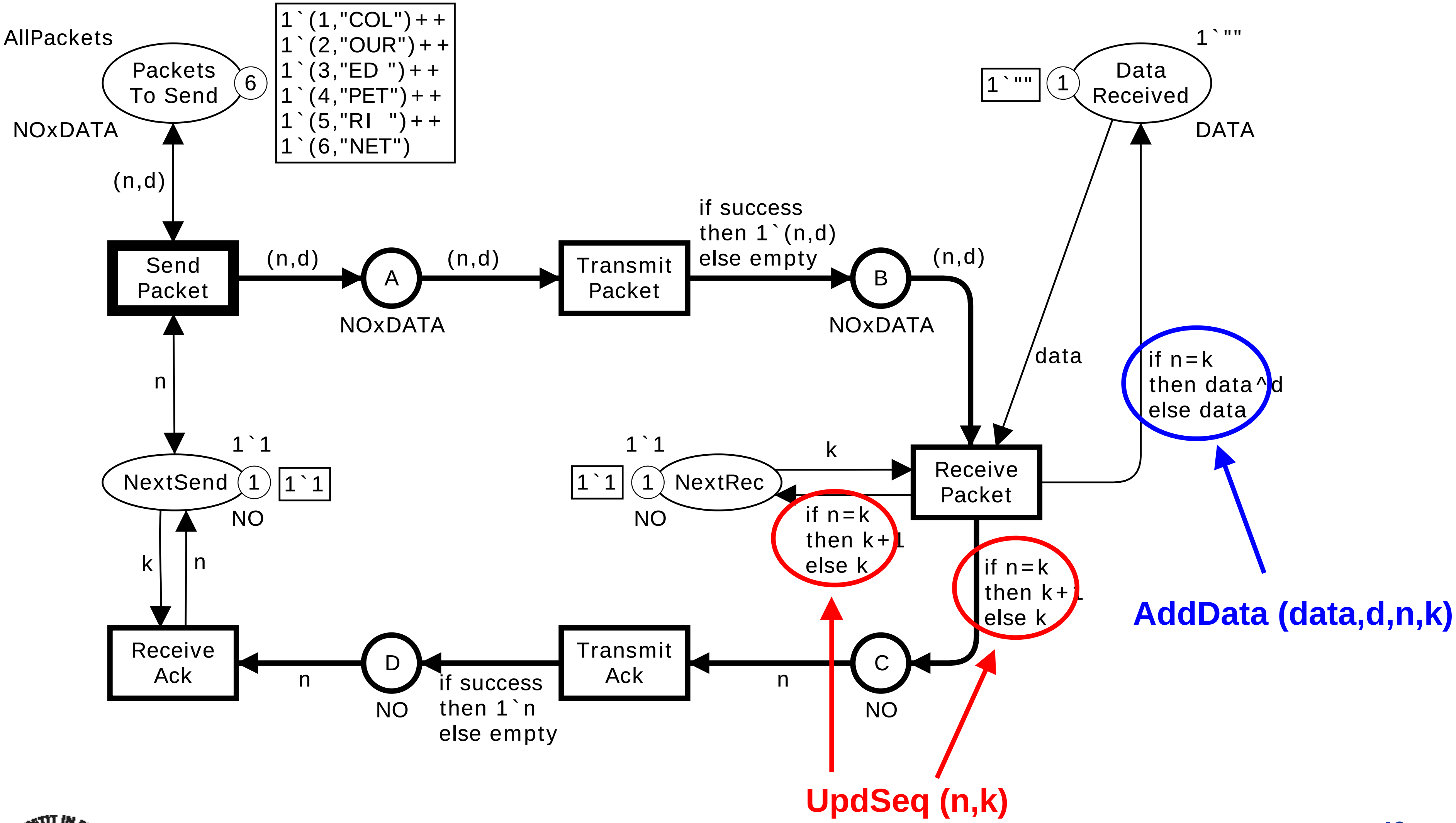
Functions

- Functions can be used in all kinds of the net expressions:
  - Guards.
  - Arc expressions.
  - Initial markings.

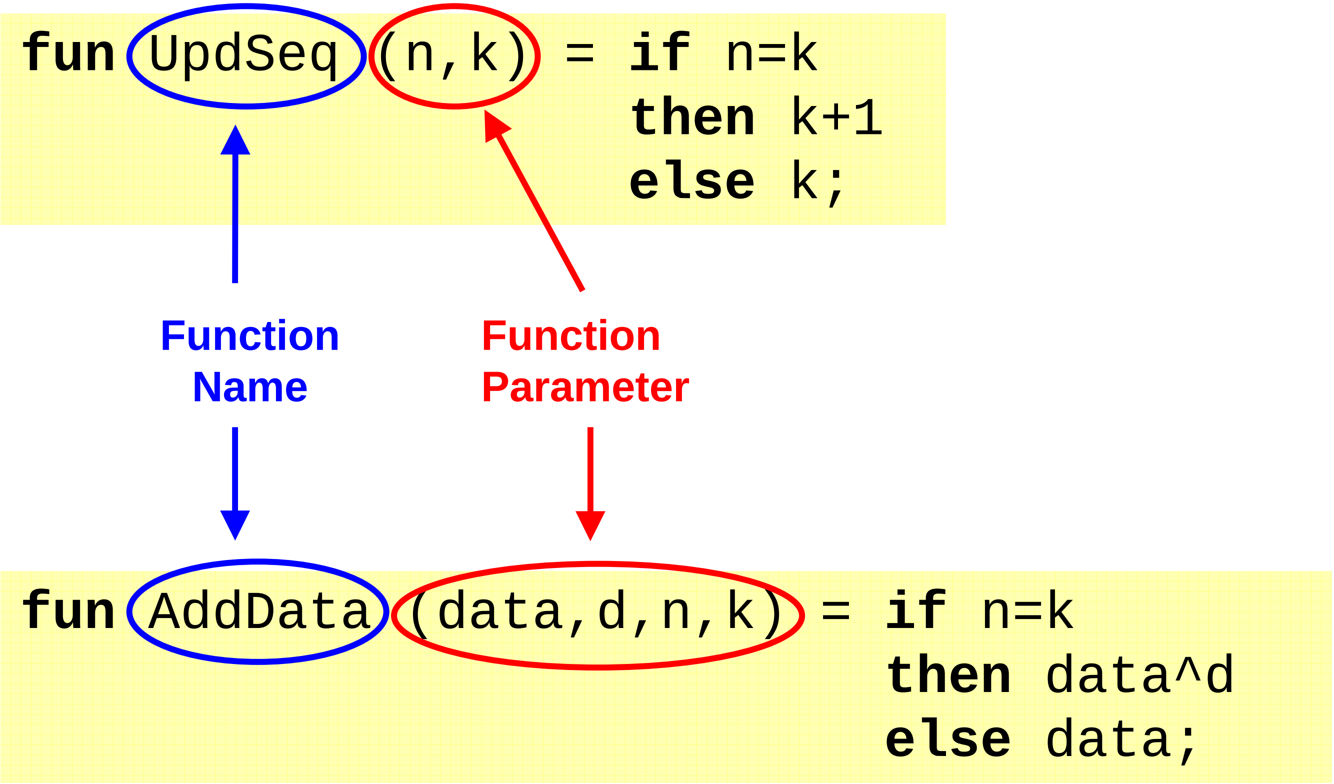
- Functions are used when:
  - Complex expressions takes up too much space in the graphical representation.
  - Same functionality is required in different parts of the model.

- Functions make CPN models easier to read and maintain.

Simple protocol



Definition of two functions



```
fun UpdSeq (n,k) = if n=k
                   then k+1
                   else k;
```

Function Name

Function Parameter

```
fun AddData (data,d,n,k) = if n=k
                           then data^d
                           else data;
```

- All functions in Standard ML take a single parameter which may be a tuple.

Inference of function type

```
fun UpdSeq (n,k) = if n=k
                   then k+1
                   else k;
```

n : INT

k : INT

```
int * int -> int
```

Function evaluates to an integer

- The variables n and k are local to the function definition.
- They should not be confused with the variables n and k of type NO used as arguments in the function call.

Inference of function type

```
fun AddData (data,d,n,k) = if n=k
                           then data^d
                           else data;
```

n and k must have the same type

data : string
d : string

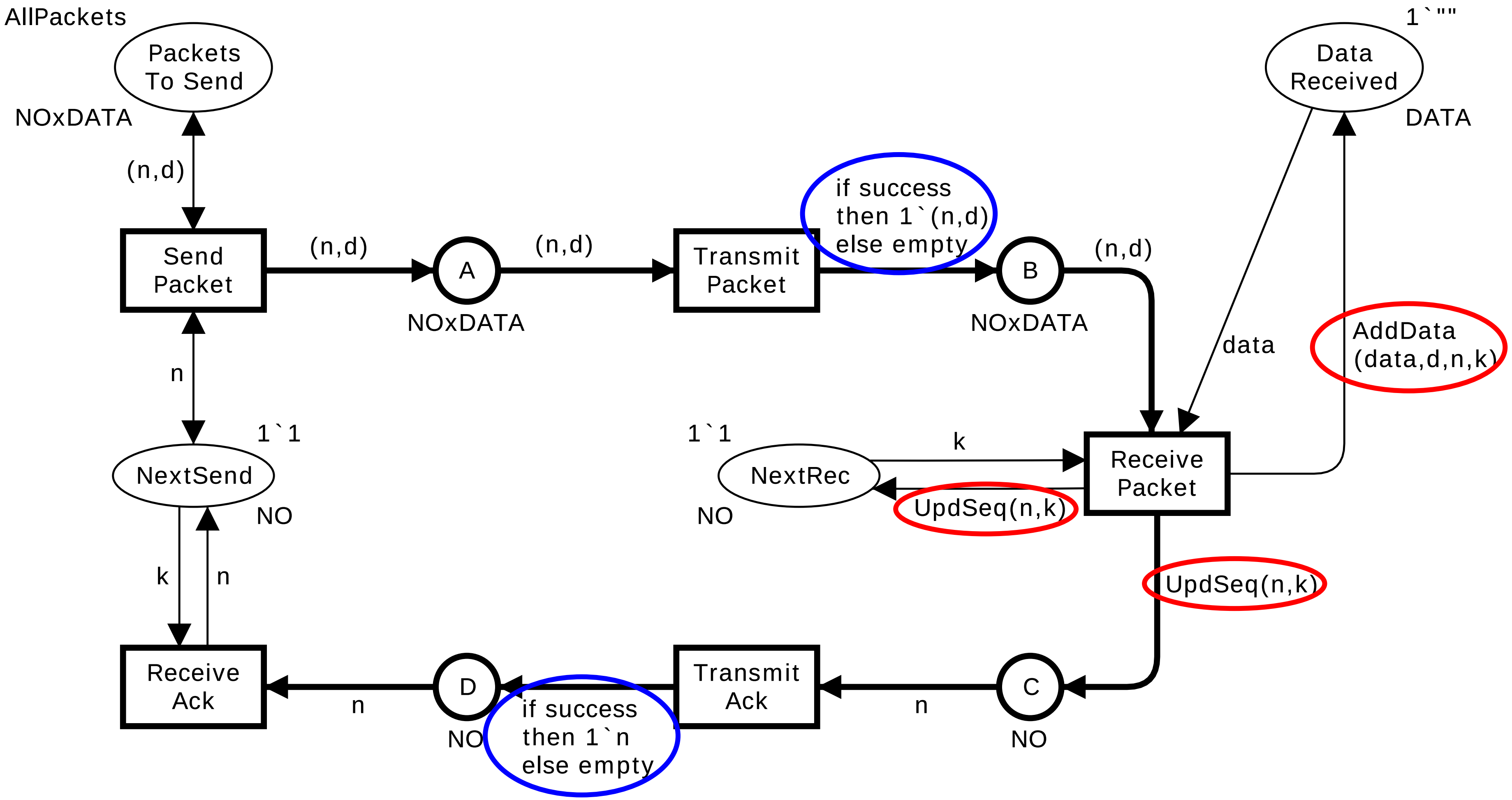
Function evaluates to a string

```
string * string * ''a * ''a -> string
```

Type variable:
Some type with equality operation

- Polymorphic function.
- Can be called with different types of arguments.

CPN model with functions



Exploiting polymorphism

```
fun Transmit (success,pack) = if success
                              then 1`pack
                              else empty;
```

success : bool

```
bool * 'a -> 'a ms
```

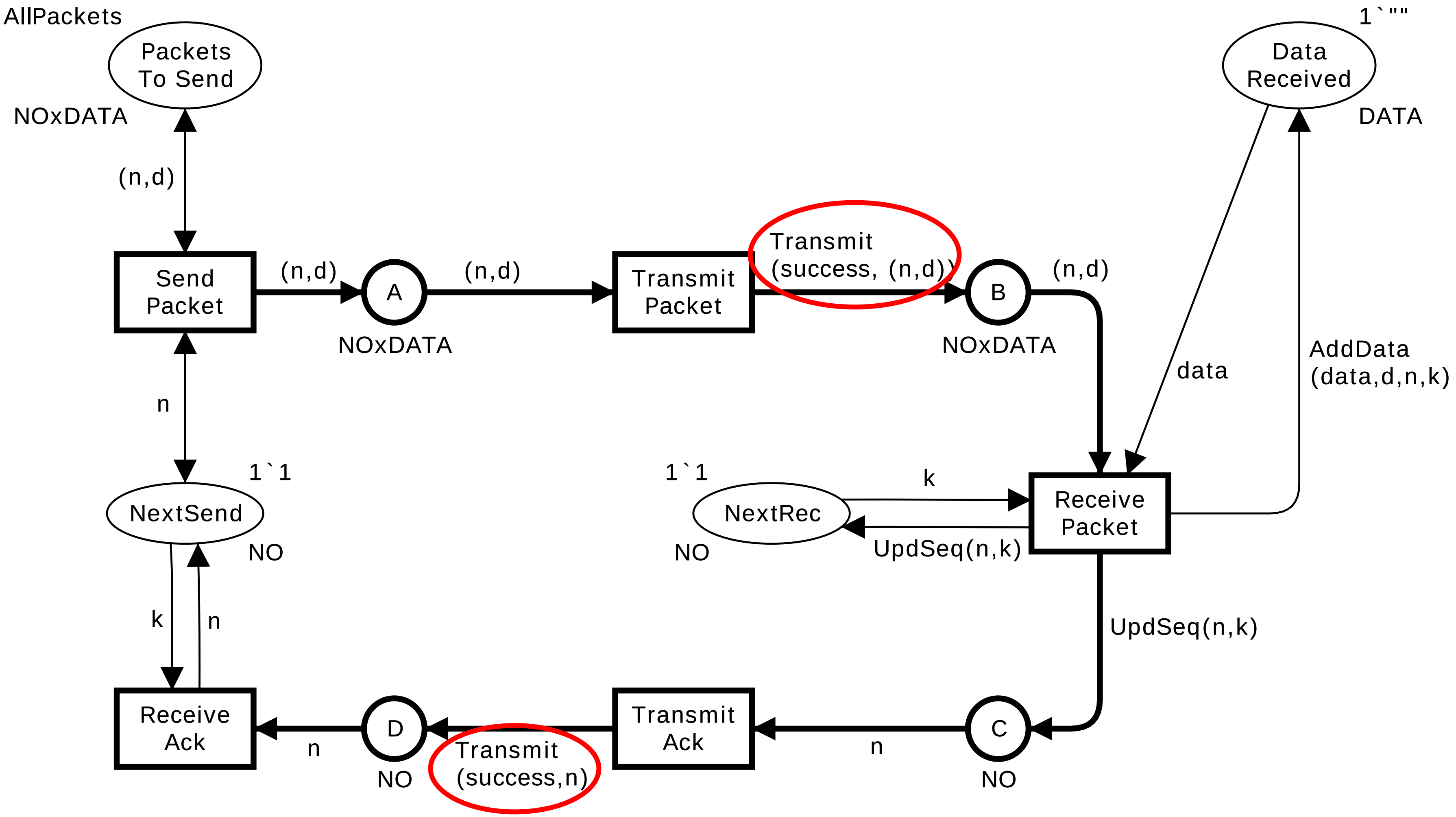
Function evaluates to a multi-set over the type of pack

Multi-set

Type variable: Some type where equality operation not required

- Polymorphic function.
- Can be called with different types of arguments:
  - Transmit (success,(n,d)) ← To transmit data packets
  - Transmit (success,n) ← To transmit acknowledgments

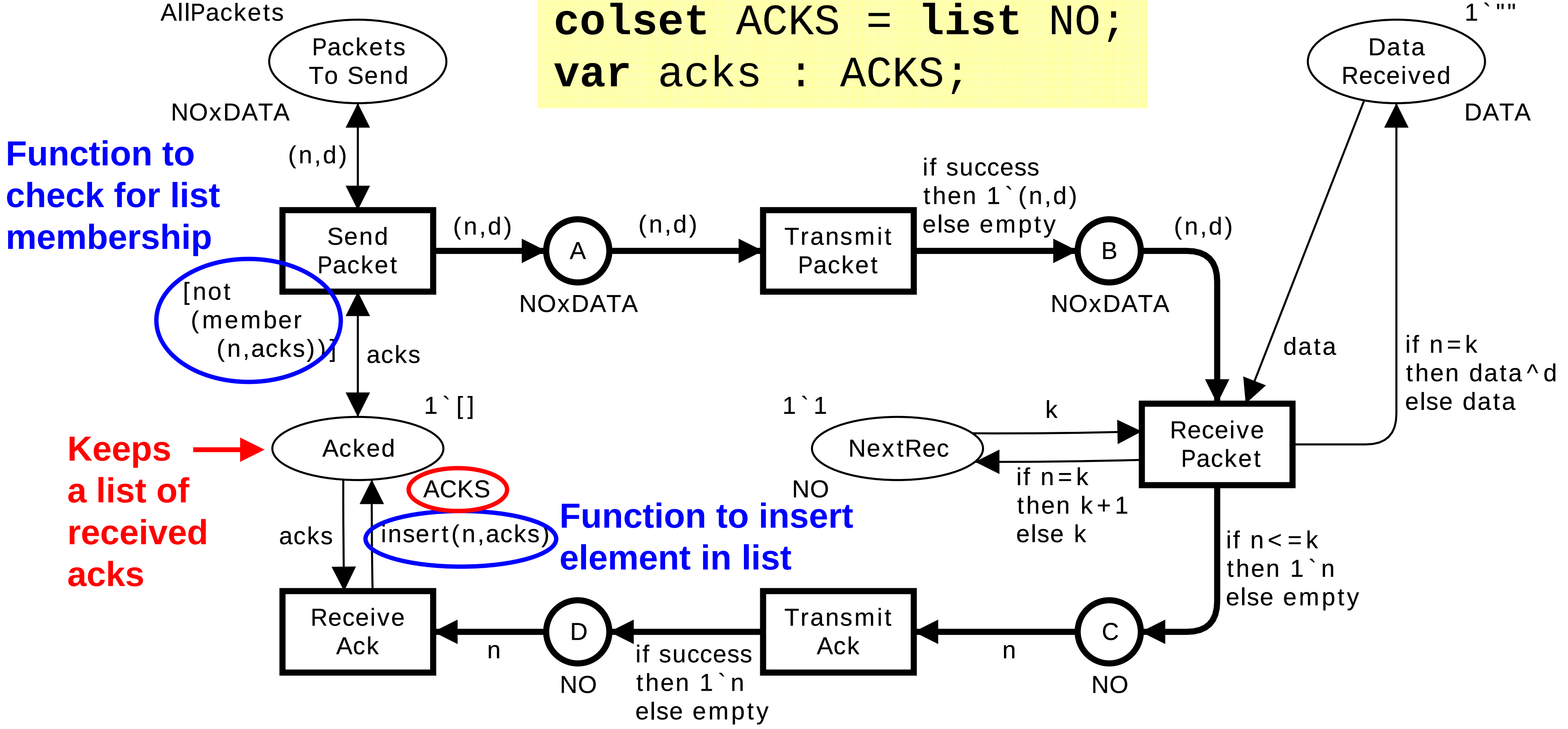
CPN model with polymorphic function



Revised protocol

- Sender can send any unacknowledged data packet.

```
colset ACKS = list NO;
var acks : ACKS;
```



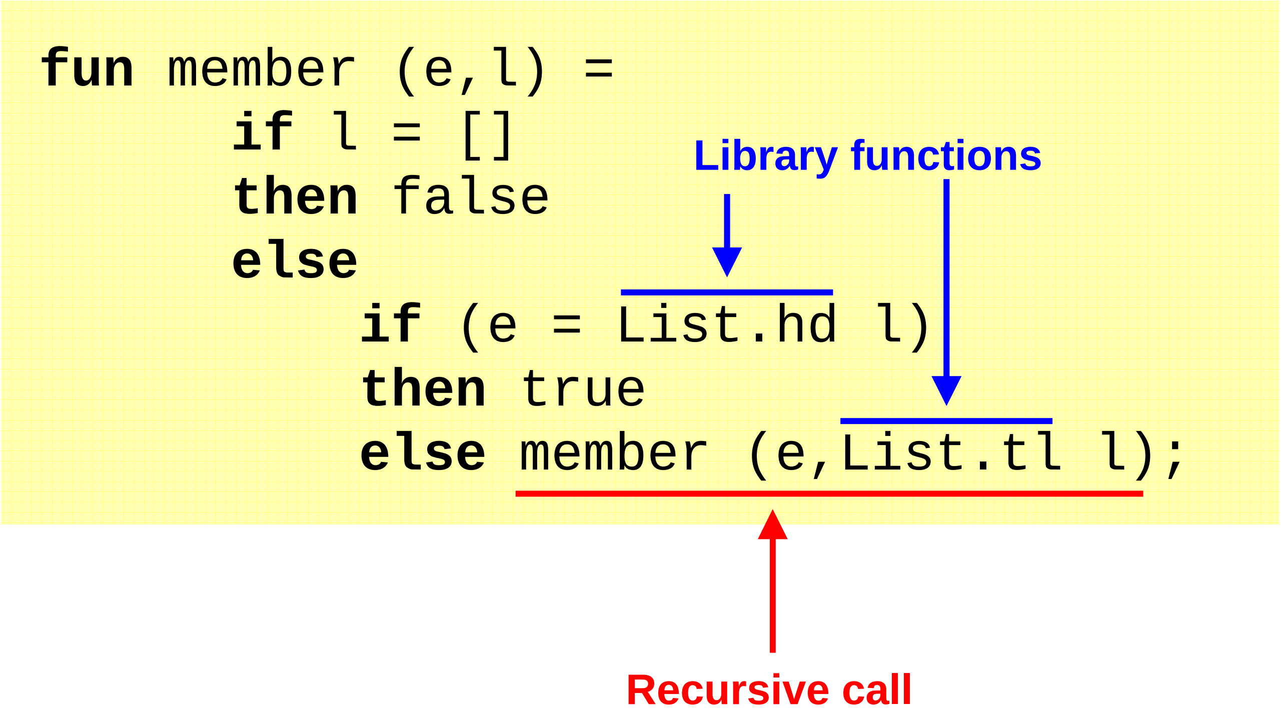
Function to check for list membership

Keeps a list of received acks

Function to insert element in list

Function member

- Checks whether the element e is present in the list l.



```
fun member (e,l) =
      if l = []
      then false
      else
          if (e = List.hd l)
          then true
          else member (e,List.tl l);
```

Library functions

Recursive call

Function insert

- Inserts the element e in the list l if it is not already present.

```
fun insert (e,l) =
      if member (e,l)
      then l
      else e::l;
```

Uses the
member function

Local environments

- Can be introduced using a let expression:

Comments

```
fun member (e,l) =
    if l = []
    then false (* if list empty, e is not a member *)
    else       (* list is not empty *)
       let
           (* extract head and tail of the list *)
           val head = List.hd l
           val tail = List.tl l
       in
           if e = head
           then true   (* e was equal to the head *)
           else member (e,tail) (* check the tail *)
       end;
```

Even short ML functions can be tricky to read and understand. Hence it is a very good idea to use comments.

Higher-order functions

- Member is a special case of determining whether there exist an element in the list **l** satisfying a Boolean predicate **p**:

```
fun exists (p,l) =      (''a -> bool) * ''a list -> bool
    if l = []
    then false
    else
       if p (List.hd l)
       then true
       else exists (p,List.tl l);
```

```
fun member (e,l) =      ''a * ''a list -> bool
    let
       fun equal x = (e=x)
    in
       exists (equal,l)
    end;
```

Anonymous and curried functions

- Anonymous functions are specified without an explicit name:

```
fn x => (e=x);
```

```
fun member (e,l)  = exists (fn x => (e=x),l);
```

- Curried functions take their parameters one at a time:

```
fun equal e x = (e=x);
```

```
''a -> ''a -> bool
```

```
equal e;
```

```
''a -> bool
```

```
fun member (e,l)  = exists (equal e,l);
```

Patterns in function applications

- Expressions are built from constants, constructors, and variables.
- Can be matched with arguments to bind values to the variables.

```
fun member (e,l) =
    if l = []
    then false
    else
       if (e = List.hd l)
       then true
       else member (e,List.tl l);
```

Pattern

```
member (2,[1,3,4])
```

Function call

- The argument **(2,[1,3,4])** is matched with the pattern **(e,l)**.

Patterns in function definitions

Not used

Matches the empty list

```
fun member (e,[])    = false
 |  member (e,x::l) =
    if (x = e)
          then true
          else member (e,l);
```

Matches a non-empty list

Wilcard (matches everything)

```
fun member (_,[])    = false
 |  member (e,x::l) =
    if (x = e)
          then true
          else member (e,l);
```

Patterns in case expressions

- Case expressions can be used instead of nested if expressions.

```
case res of
    success   => 1`p
  | duplicate => 2`p
  | failure   => empty;
```

Three patterns

```
if res = success
then 1`pack
else if res = duplicate
     then 2`pack
     else empty;
```

- Alternative:

```
(case res of
    success => 1
  | duplicate => 2
  | failure => 0)`pack
```

Common patterns pitfalls

- Redundant match:

```
case res of
    _            => empty
  | success    => 1`p
  | duplicate => 2'p;
```

Warning!

Programming error:

- Everything will match the first clause.
- The other clauses will never be used.

- Non-exhaustive match:

```
fun member (e,x::l) =
    if (x = e)
    then true
    else member (e,l);
```

NO:

- Recursion will end with a call involving the empty list.

Warning! – Is it wise to ignore the warning?

Patterns in records

```
colset DATAPACK = record seq:NO * data:DATA;
```

```
fun ExtractData (datapack : DATAPACK) = #data datapack;
```

- Pattern match:

```
fun ExtractData ({seq=n,data=d}) = d;
```

- Pattern match without explicit local variables:

```
fun ExtractData ({seq,data}) = data;
```

Records with many fields

```
colset DATAPACK = record seq:NO * data:DATA * ……;
```

- Extract data:

```
fun ExtractData ({data,...} : DATAPACK) = data;
```

Wildcard symbol

- Update data:

```
DATAPACK.set_data r d
```

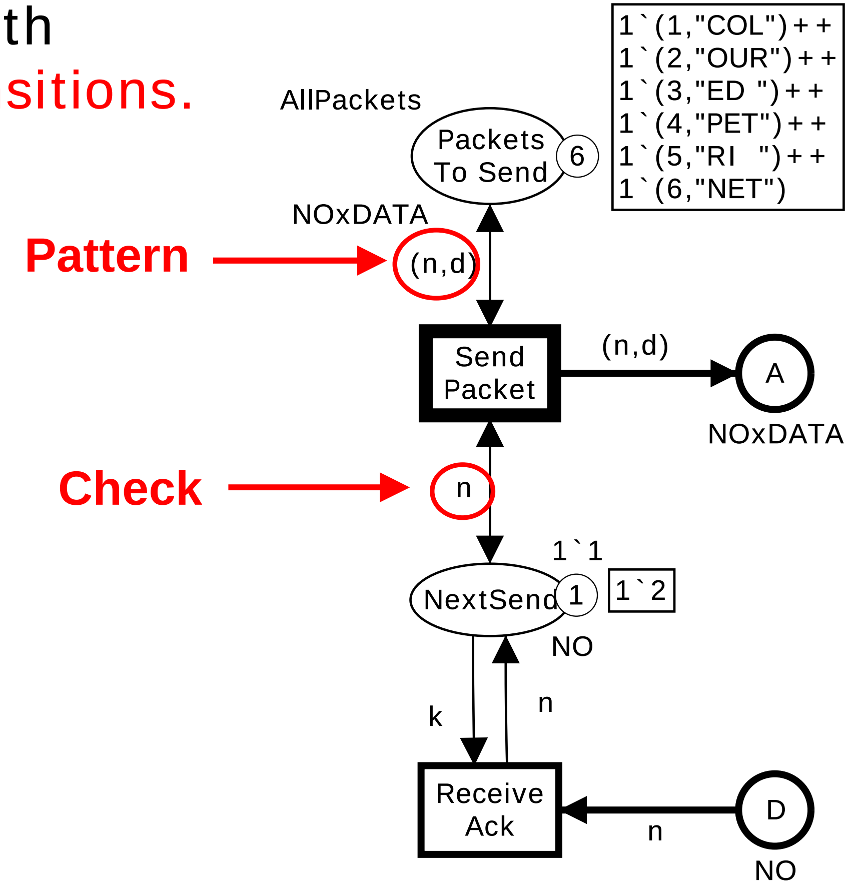
Library function

Patterns and enabling inference

- Patterns are exploited when calculating the set of enabled binding elements in a marking.
- Token values are matched with patterns on input arcs of transitions.

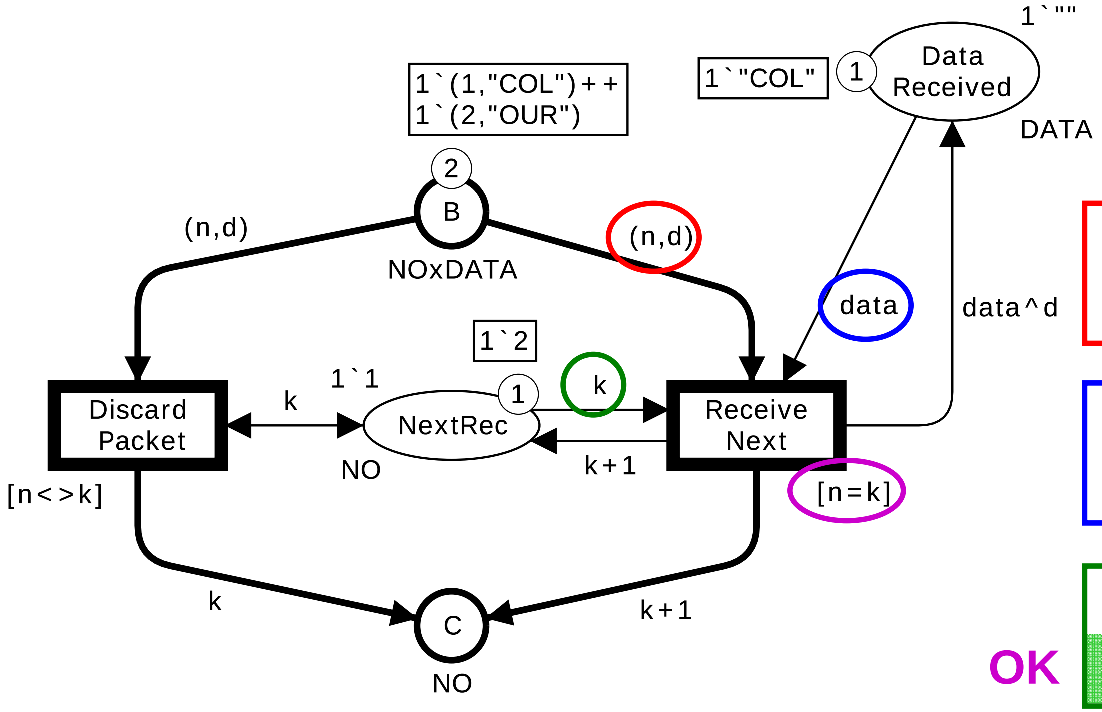
Candidate binding elements:

<n=1,d="COL">
<n=2,d="OUR">
<n=3,d="ED ">
<n=4,d="PET">
<n=5,d="RI ">
<n=6,d="NET">



Enabling inference example

- We may have to use patterns in different input arc expressions to bind all variables.



<n=1,d="COL",k=?,data=?>
<n=2,d="OUR",k=?,data=?>

<n=1,d="COL",k=?,data="COL">
<n=2,d="OUR",k=?,data="COL">

<n=1,d="COL",k=2,data="COL">
OK <n=2,d="OUR",k=2,data="COL">