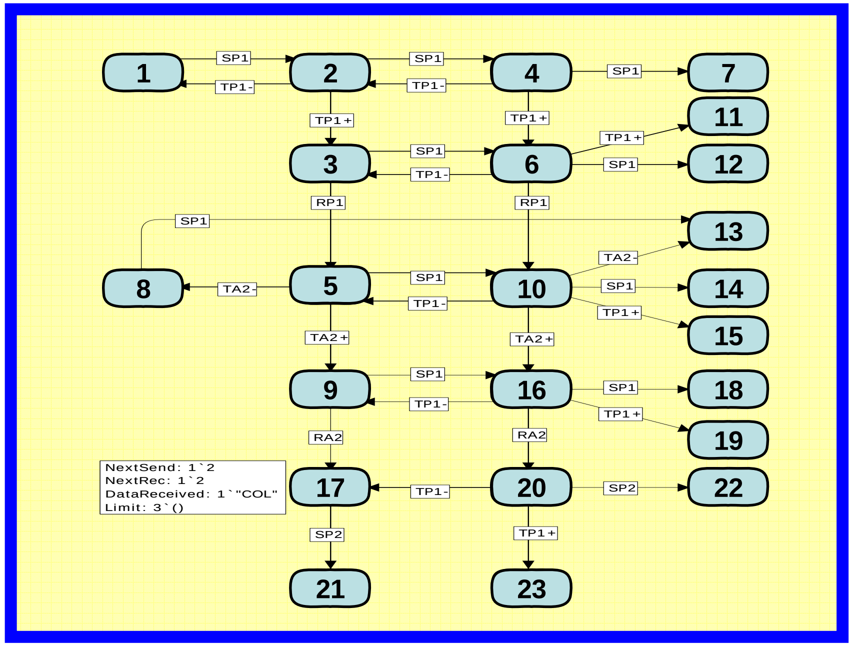
Coloured Petri Nets

Modelling and Validation of Concurrent Systems

Chapter 7: State Spaces and Behavioural Properties

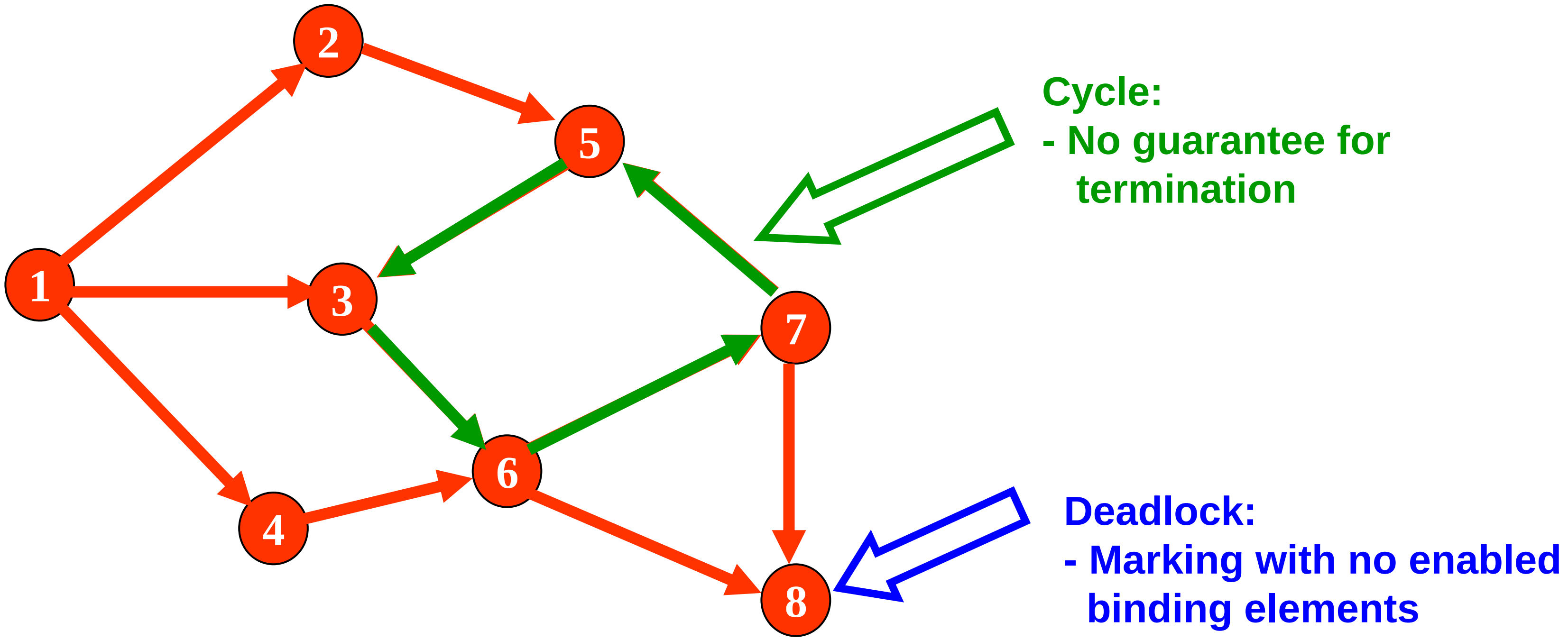
Kurt Jensen &
Lars Michael Kristensen

{kjensen,lmkristensen}
@daimi.au.dk



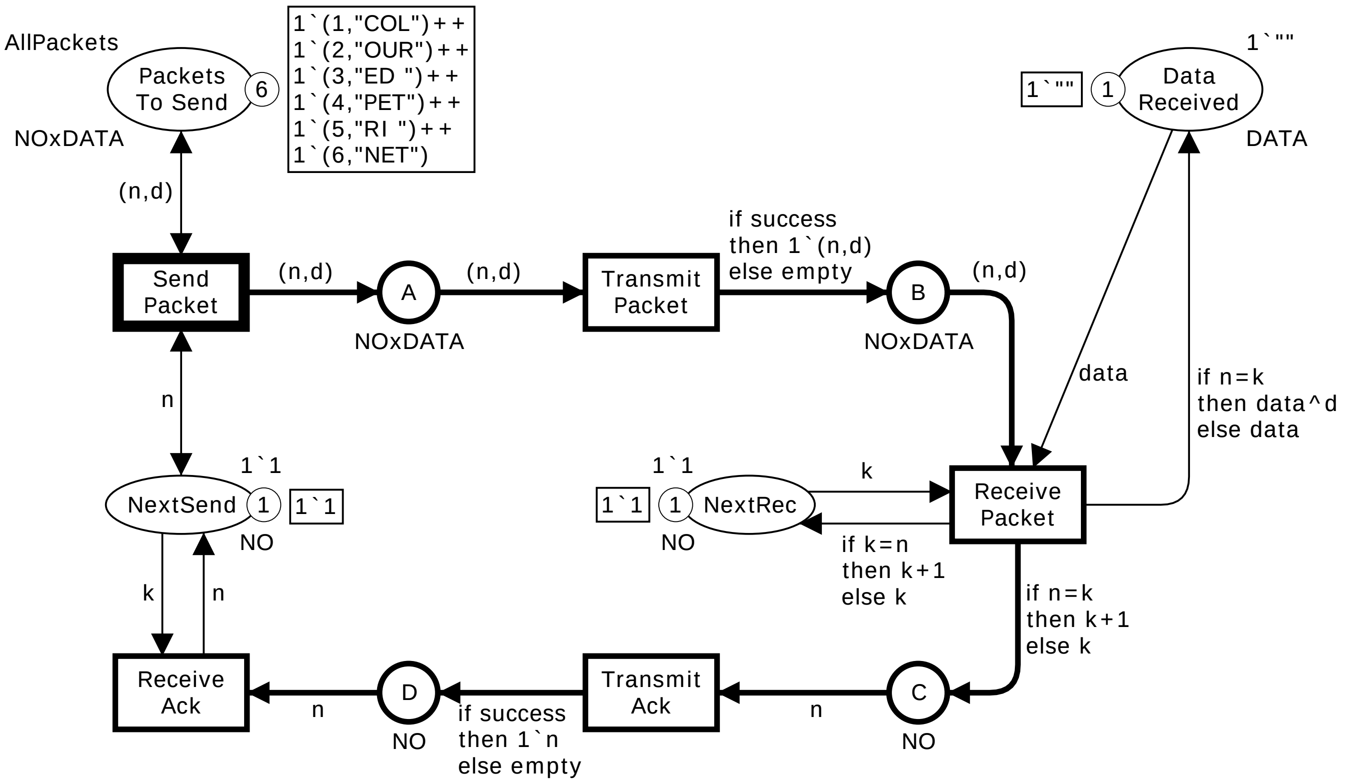
State spaces

- A state space is a directed graph with:
  - A node for each reachable marking (state).
  - An arc for each occurring binding element.
- State spaces can be used to investigate the behavioural properties of the CPN model.



Simple protocol

- SendPacket can occur an unlimited number of times producing an unlimited number of tokens on place A.
- This means that the state space becomes infinite.



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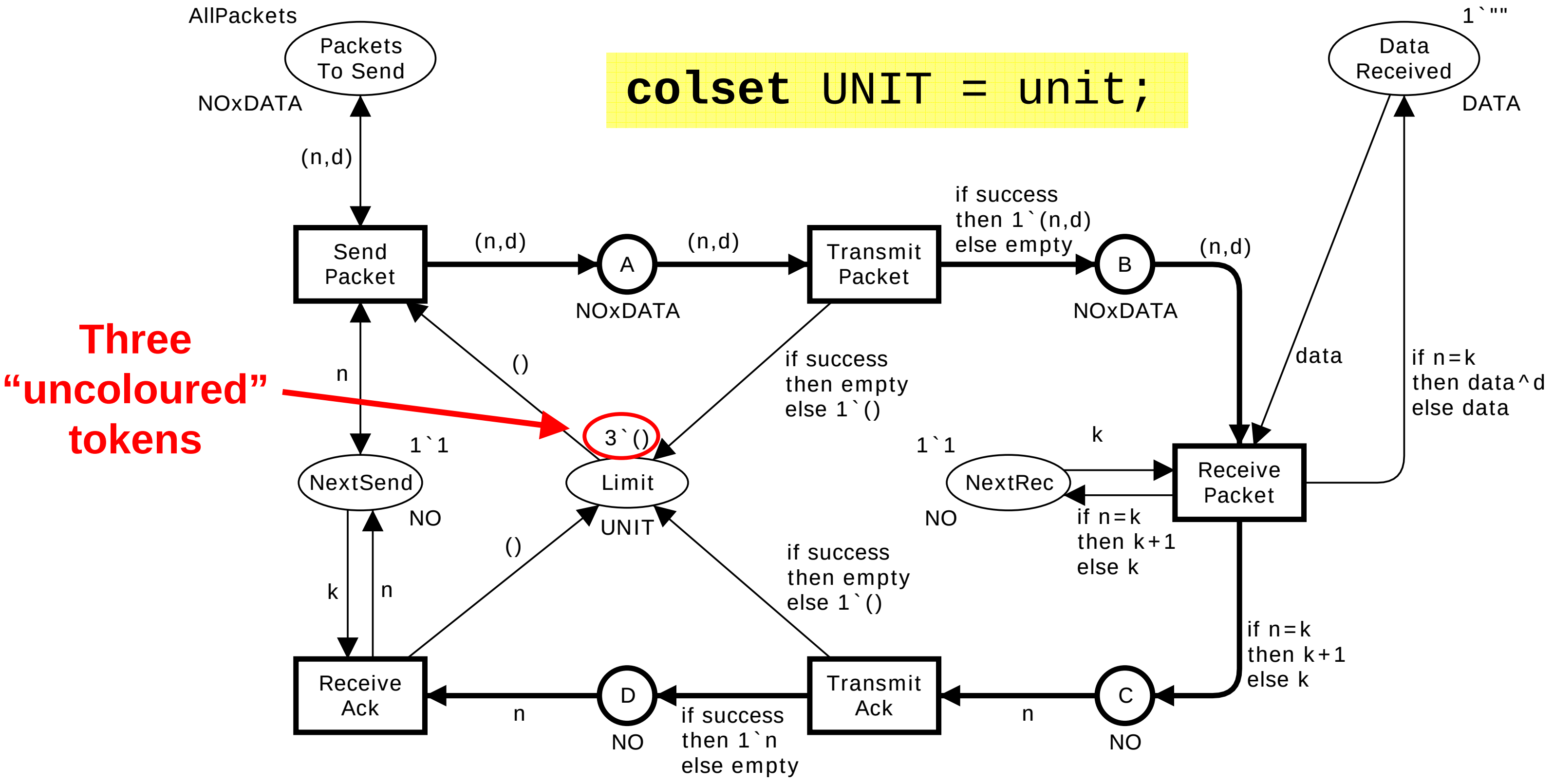
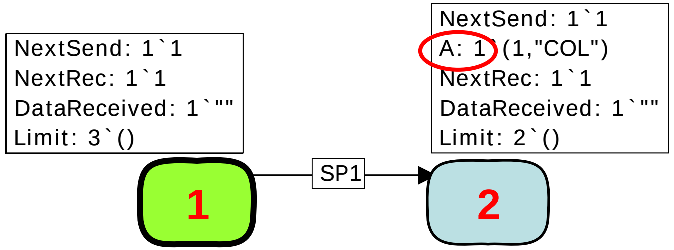
Simple protocol for state space analysis

- We add a new place Limit, which limits the total number of tokens on the buffer places A, B, C, and D.
- This makes the state space finite.



State space



- Construction of the state space starts with the processing of node 1 which represents the initial marking.

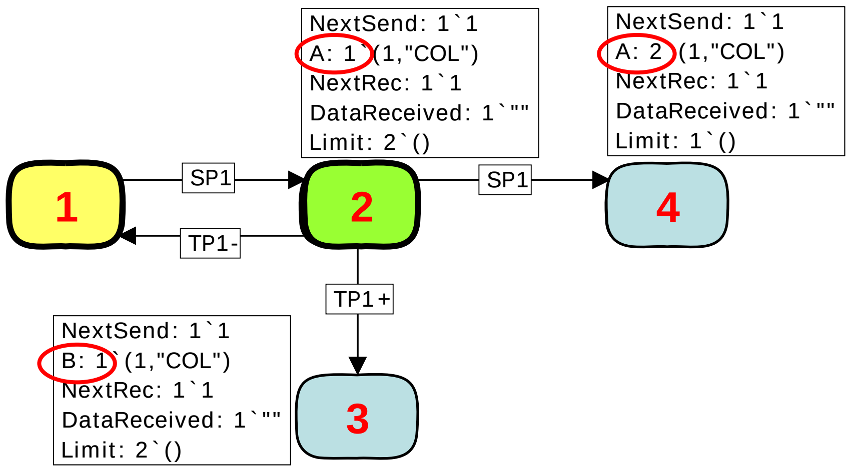
- Node 1 has one enabled binding element:

SP1 = (SendPacket, <n=1, d="COL">)

- This gives us one new arc and one new node 2.
- Node 2 has one copy of data packet 1 on place A.
- Node 1 is now marked as processed (thick border line).

State space

- Next we process node 2.
- It has three enabled binding elements:



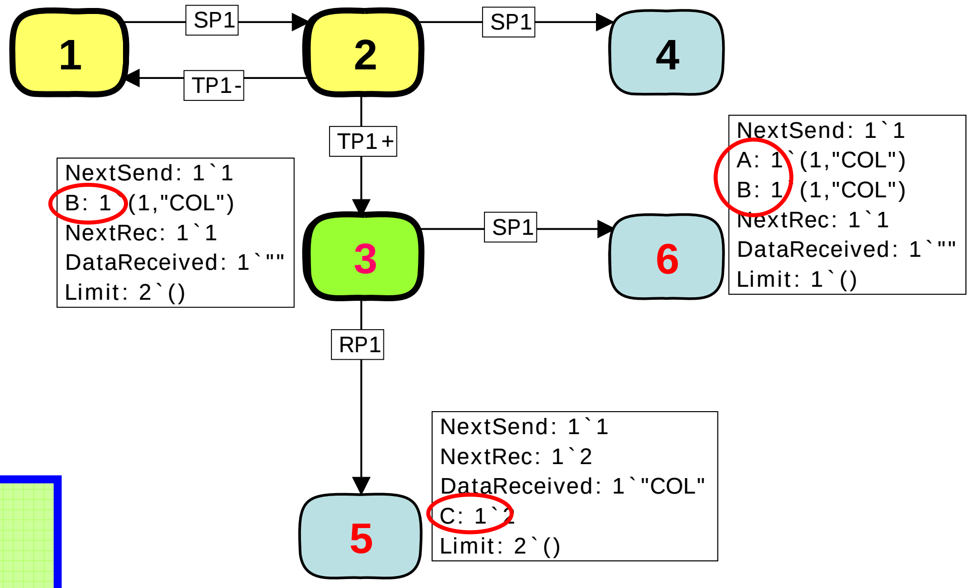
SP1 = (SendPacket, <n=1, d="COL">)

TP1+ = (TransmitPacket, <n=1, d="COL", success=true>)

TP1– = (TransmitPacket, <n=1, d="COL", success=false>)

- This gives us three new arcs and two new nodes 3 and 4.
- Node 3 has one copy of data packet 1 on place B.
- Node 4 has two copies of data packet 1 on place A.
- Node 2 is now marked as processed (thick border line).

State space



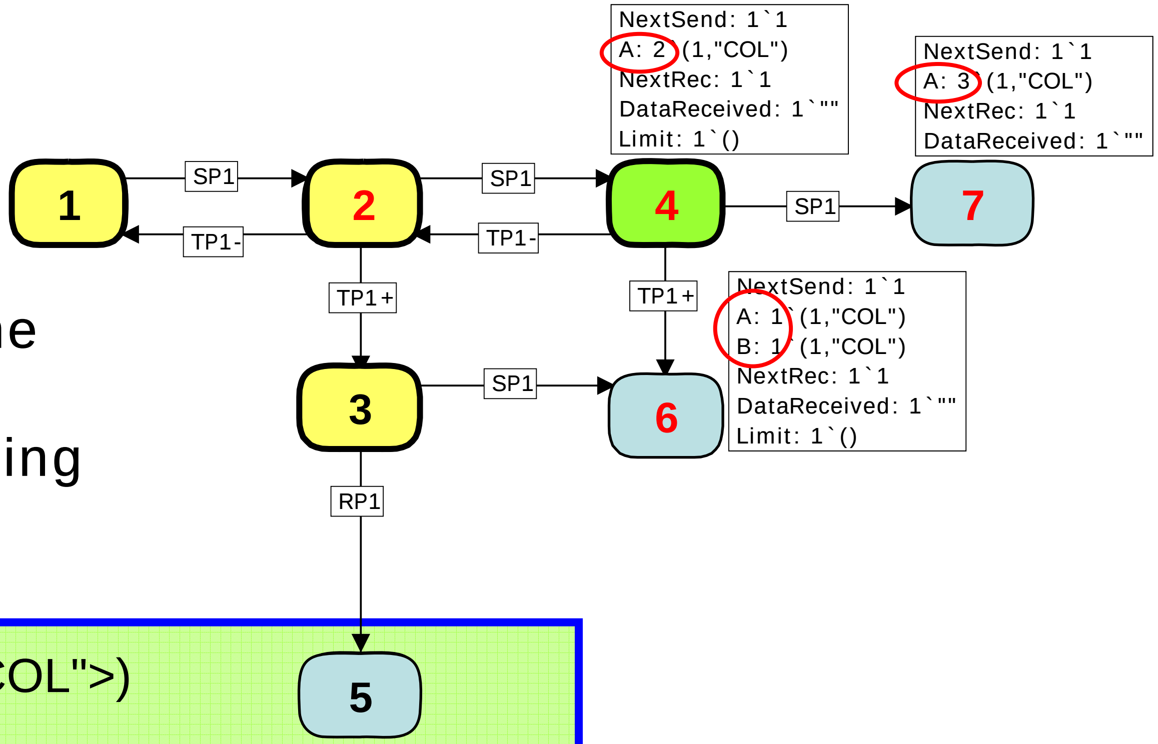
- Next we choose one of the unprocessed nodes: 3.
- It has two enabled binding elements:

SP1 = (SendPacket, <n=1, d="COL">)

RP1 = (ReceivePacket,
<n=1, d="COL", k=1, data = "">)

- This gives us two new arcs and two new nodes 5 and 6.
- Node 5 has one copy of acknowledgement 2 on place C.
- Node 6 has one copy of packet 1 on place A and another on place B.
- Node 3 is now marked as processed (thick border line).

State space



- Next we choose one of the unprocessed nodes: 4.
- It has three enabled binding elements:

SP1 = (SendPacket, <n=1, d="COL">)

TP1+ = (TransmitPacket, <n=1, d="COL", success=true>)

TP1– = (TransmitPacket, <n=1, d="COL", success=false>)

- This gives us a three new arcs and one new node 7.
- Node 7 has three copies of data packet 1 on place A.
- Node 4 is now marked as processed (thick border line).

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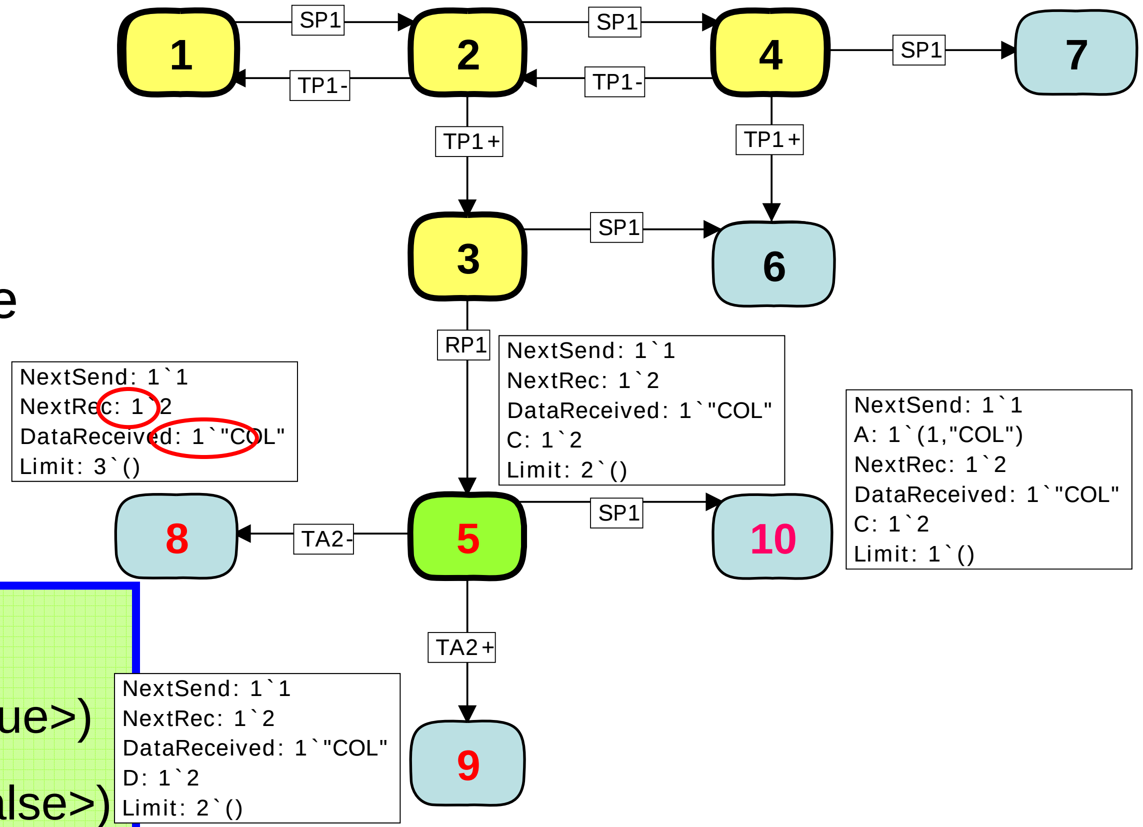
State space



- Next we choose one of the unprocessed nodes: 5.
- It has three enabled binding elements:

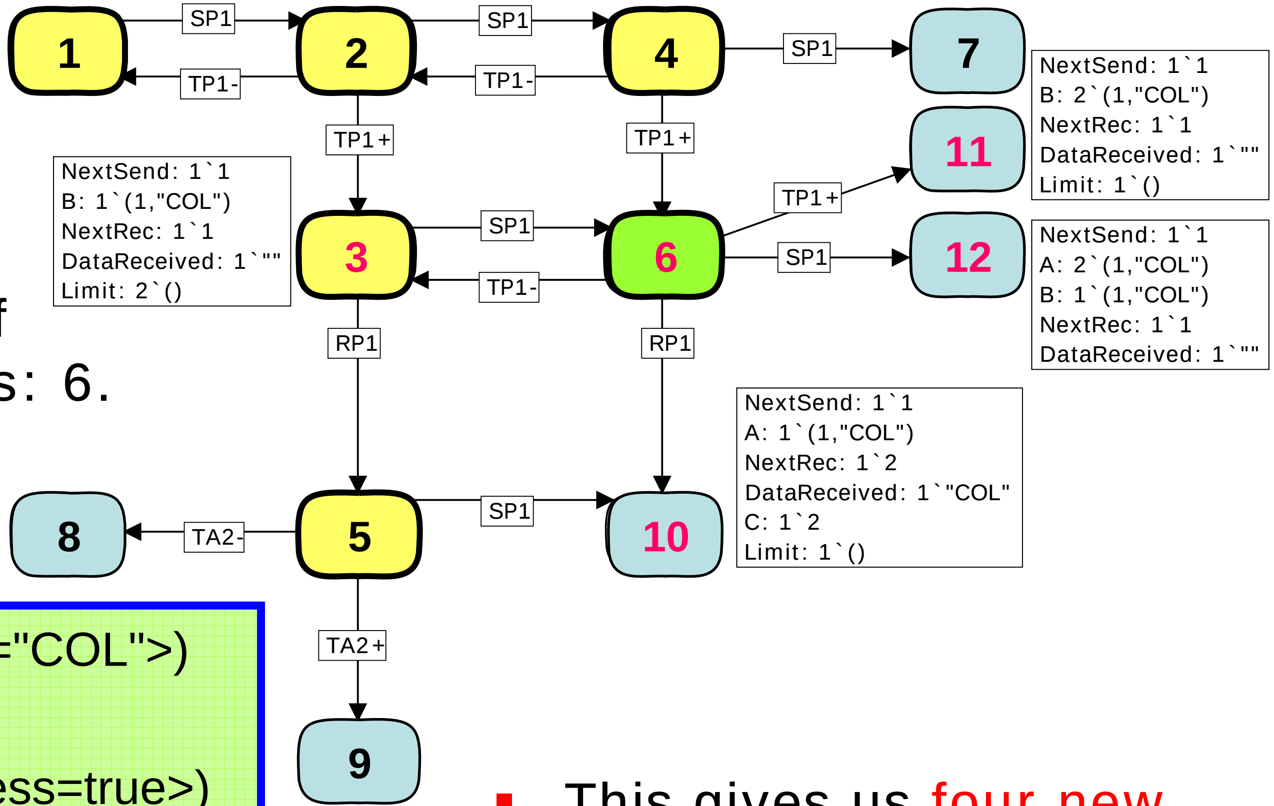
SP1 = (SendPacket, <n=1, d="COL">)

TA2+ = (TransmitAck, <n=2, success=true>)

TA2– = (TransmitAck, <n=2, success=false>)

- This gives us three new arcs and three new nodes 8, 9 and 10.
- Node 8 is identical to the initial marking except that NextRec and Data Received have been changed.
- Node 5 is now marked as processed (thick border line).

State space



- Next we choose one of the unprocessed nodes: 6.
- It has four enabled binding elements:

SP1 = (SendPacket, <n=1, d="COL">)

TP1+ = (TransmitPacket, <n=1, d="COL", success=true>)

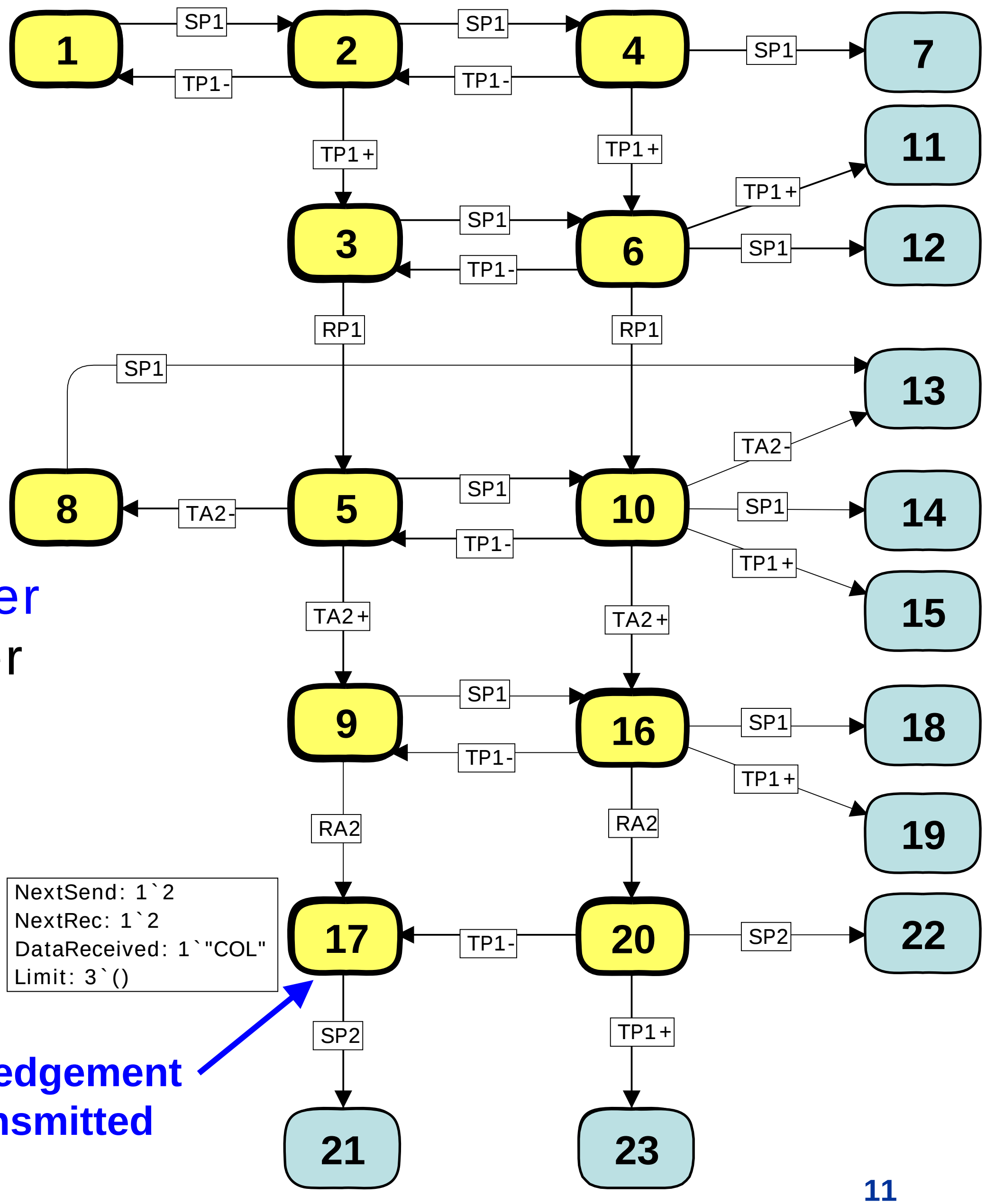
TP1– = (TransmitPacket, <n=1, d="COL", success=false>)

RP1 = (ReceivePacket, <n=1, d="COL", k=1, data = "">)

- This gives us four new arcs and two new nodes 11 and 12.
- Node 6 is now marked as processed.

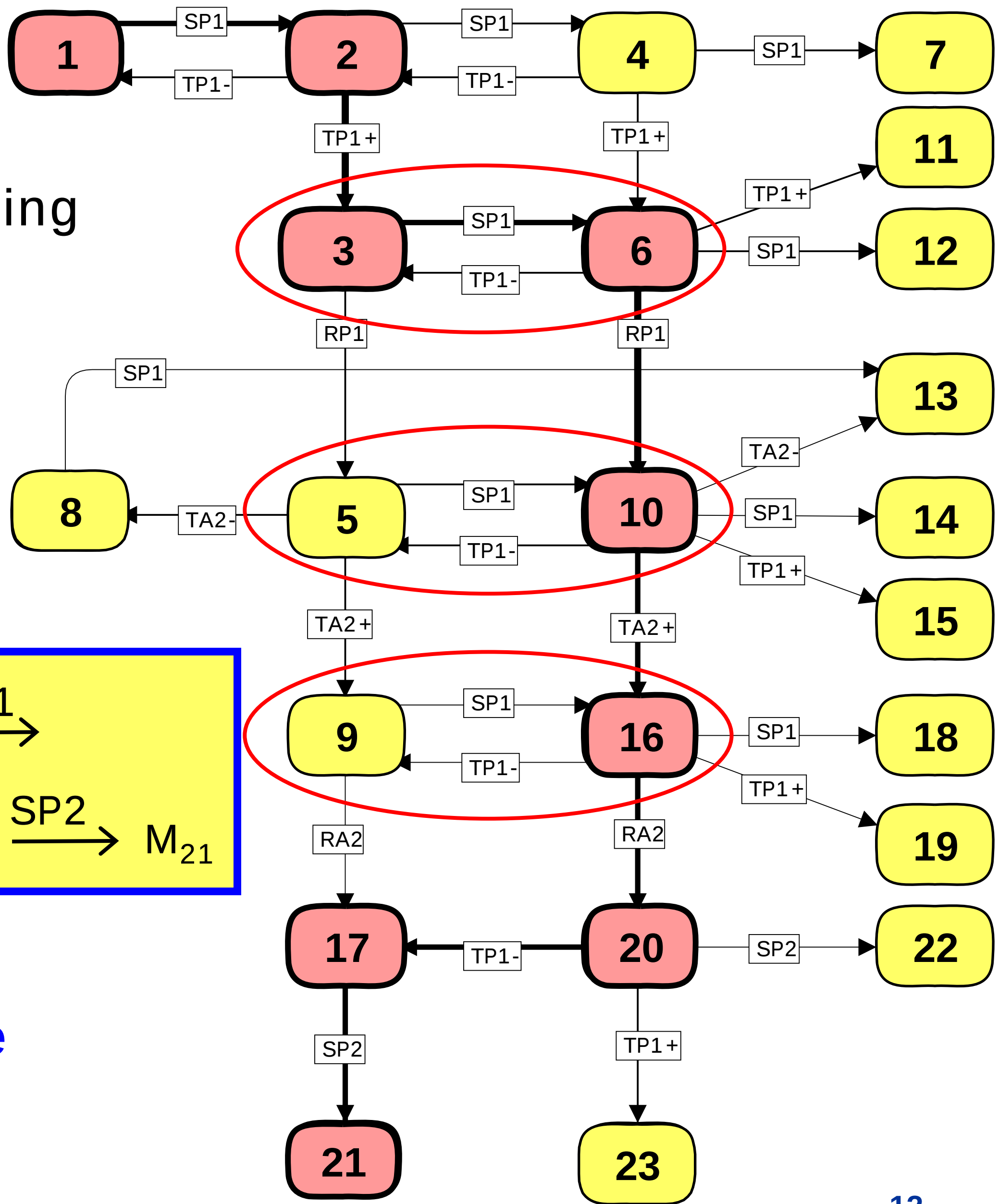
State space

- We continue to process the nodes one by one.
- If the state space is finite construction terminates when all reachable markings have been processed.
- Otherwise, we continue forever – obtaining a larger and larger part of the state space.
- This partial state space is visualised using the drawing facilities of the CPN state space tool.



Packet no. 1 and its acknowledgement have been successfully transmitted

Directed path



- A directed path is an alternating sequence of nodes and arcs.
- Each directed path in the state space corresponds to an occurrence sequence where all steps contain a single binding element.

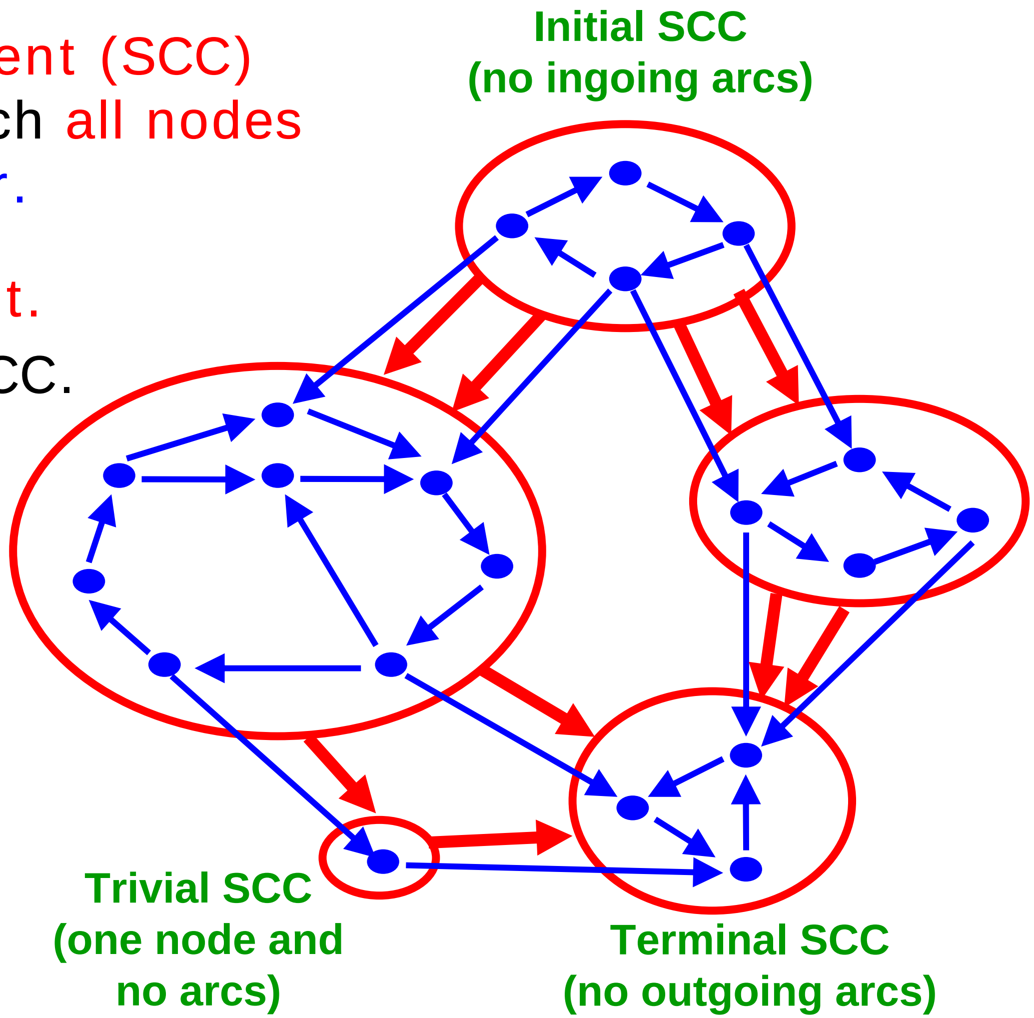
$$M_1 \xrightarrow{SP1} M_2 \xrightarrow{TP1+} M_3 \xrightarrow{SP1} M_6 \xrightarrow{RP1}$$

$$M_{10} \xrightarrow{TA2+} M_{16} \xrightarrow{RA2} M_{20} \xrightarrow{TP1-} M_{17} \xrightarrow{SP2} M_{21}$$

- Loops can be repeated.
- Infinite number of occurrence sequences.

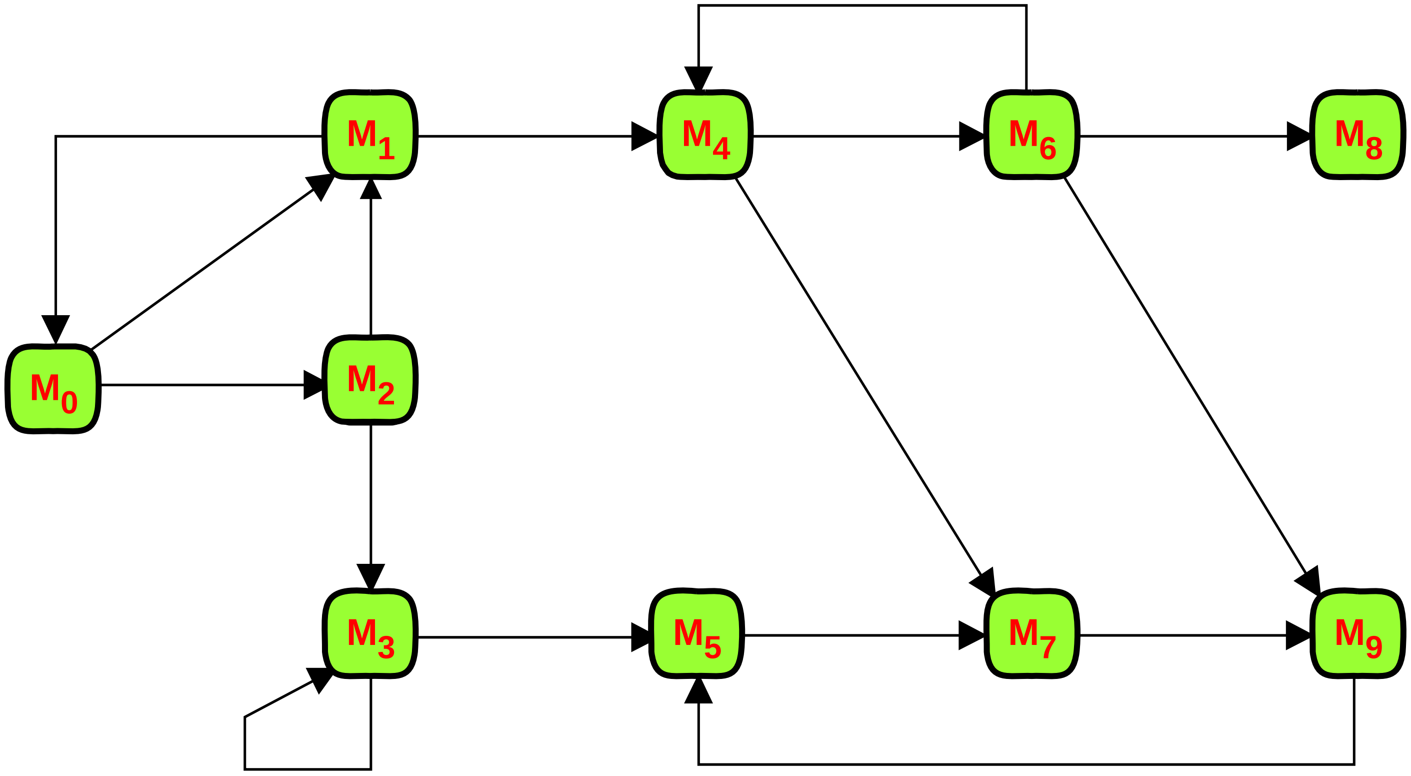
Strongly connected components

- A strongly connected component (SCC) is a maximal subgraph in which all nodes are reachable from each other.
- The SCCs are mutually disjoint.
- Each node is in exactly one SCC.
- SCC graph contains:
- A node for each SCC.
- An arc from S_i to S_j for each state space arc from a node $n_i \in S_i$ to a node $n_j \in S_j$ $(i \neq j)$.
- The SCC graph is acyclic.



State space (example)

- 10 nodes and 16 arcs.



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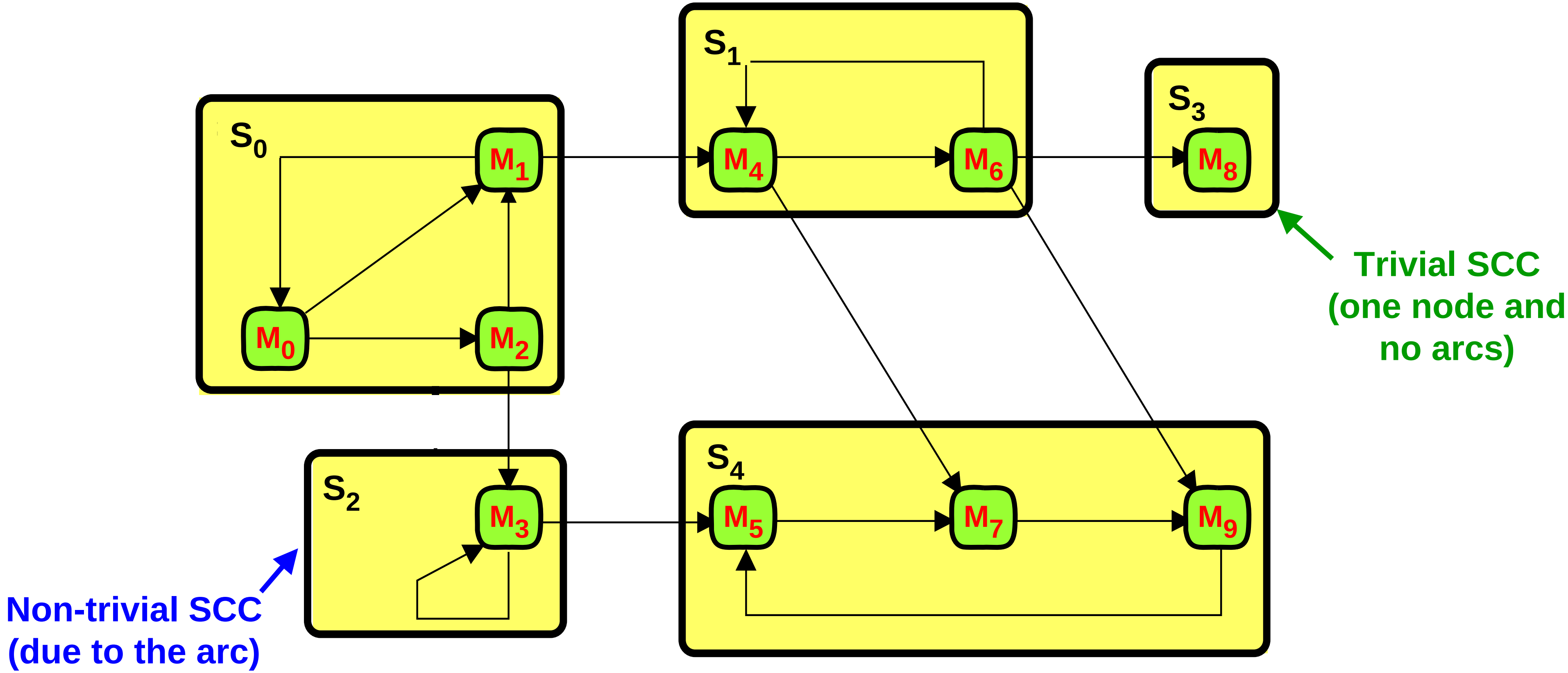
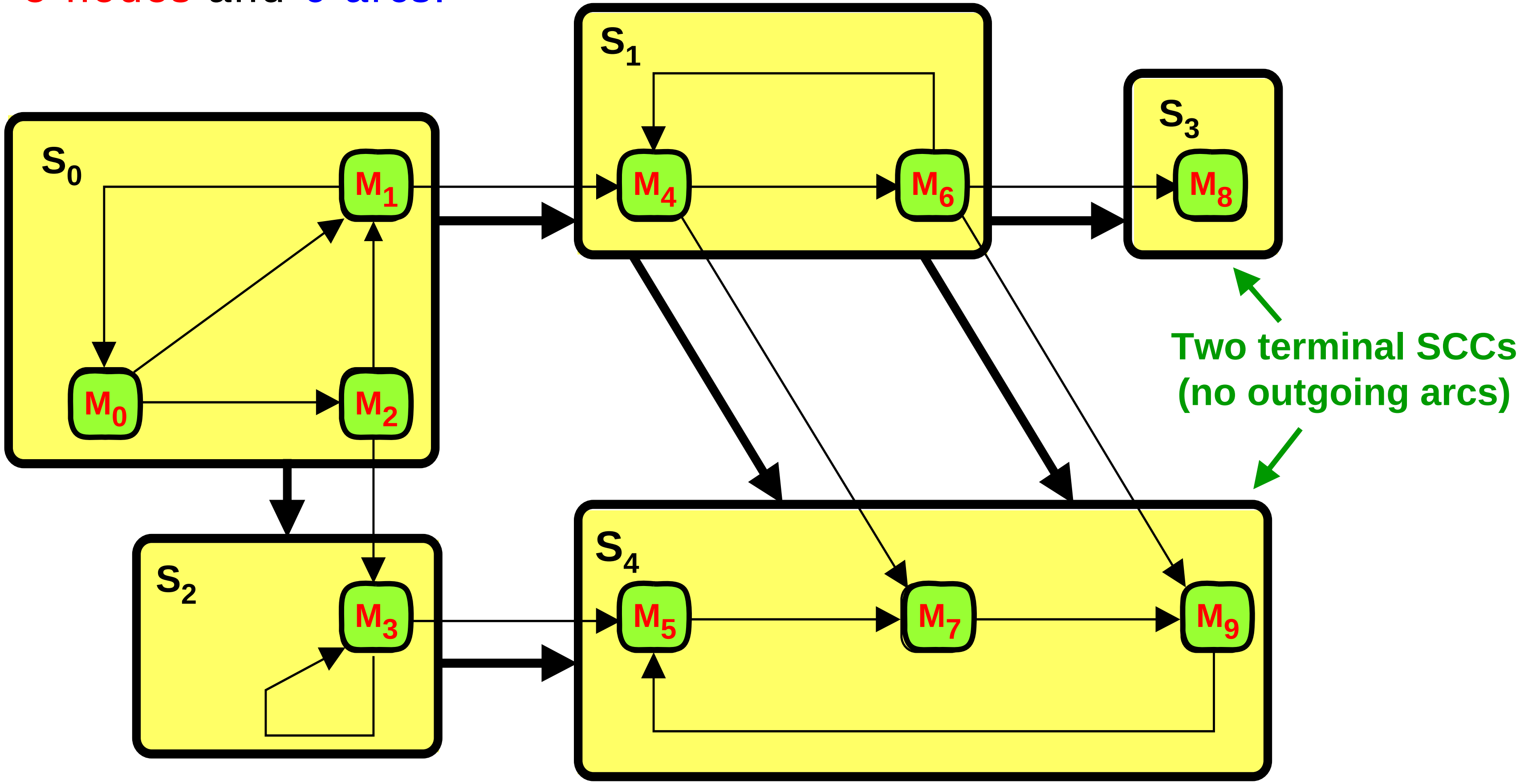
Strongly connected components

- 5 different SCCs.



SCC graph

- 5 nodes and 6 arcs.



State space construction and analysis

- State spaces may be very large and hence we need computer tools to construct and analyse them.

- Analysis of the state space starts with the generation of the state space report.
- This is done totally automatic.

- The report contains a lot of useful information about the behavioural properties of the CPN model.
- The report is excellent for locating errors or increase our confidence in the correctness of the system.

State space report

- The state space report contains information about standard behavioural properties which make sense for <u>all</u> CPN models:
  - Size of the state space and the time used to generate it.
  - Bounds for the number of tokens on each place and information about the possible token colours.
  - Home markings.
  - Dead markings.
  - Dead and live transitions.
  - Fairness properties for transitions.

State space report: size and time

State Space Statistics

	State Space		Scc Graph	
	Nodes:	13.215	Nodes:	5.013
	Arcs:	52.784	Arcs:	37.312
	Secs:	53	Secs:	2
	Status:	Full		

- State space contains more than 13.000 nodes and more than 52.000 arcs.
- The state space was constructed in less than one minute and it is full – i.e. contains all reachable markings.
- The SCC graph is smaller. Hence we have cycles.
- The SCC graph was constructed in 2 seconds.

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Reachability properties

- The standard query function below checks whether marking M_{17} is reachable from M_1 – i.e. whether there is a path from node 1 to node 17.

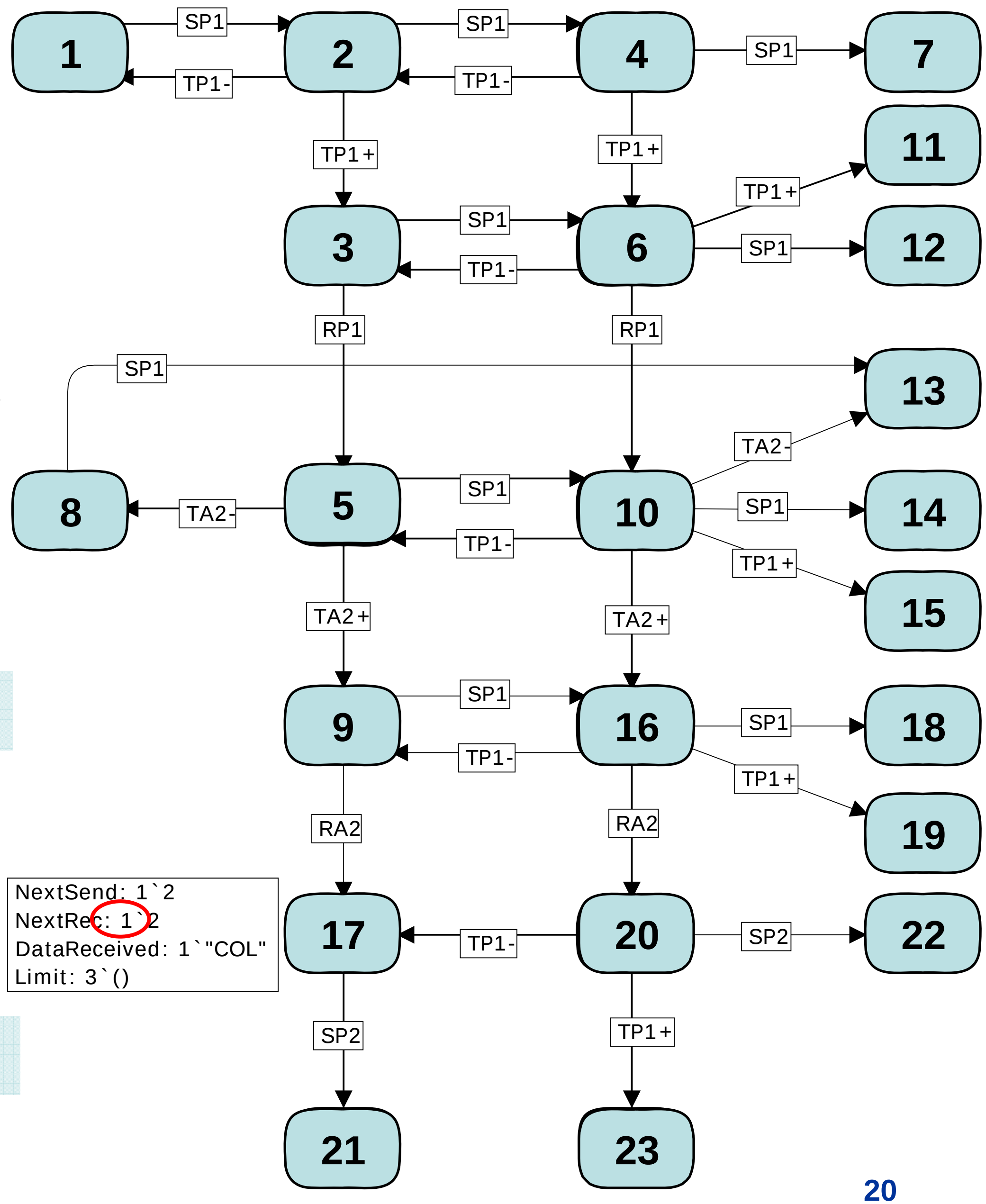
```
Reachable (1,17);
```

```
true
```

- We can also check whether M_1 is reachable from M_{17}:

```
Reachable (17,1);
```

```
false
```

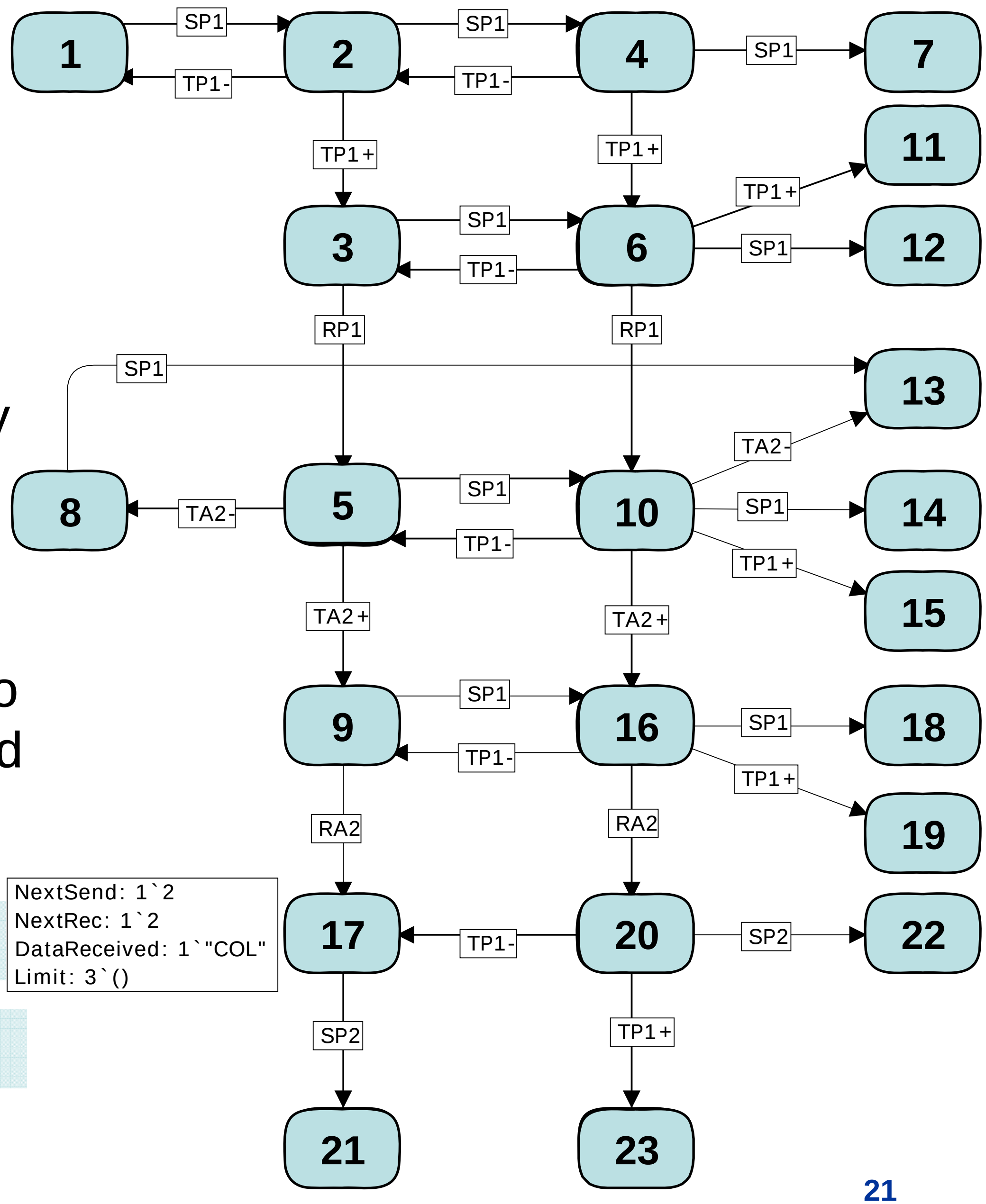


Reachability properties (SCC)

- It is also possible (and more efficient) to check reachability from the SCC graph.
- Then we check whether there exists a path from the SCC containing the first marking to the SCC containing the second marking.

`SccReachable (1,17);` `true`

`SccReachable (17,1);` `false`



Desired terminal marking

- The following predicate checks whether node n represents a marking in which all data packets have been successfully received.

```
fun DesiredTerminal n =
  ((Mark.Protocol’PacketsToSend 1 n) == AllPackets) andalso
  ((Mark.Protocol’NextSend 1 n) == 1‘7) andalso
  ((Mark.Protocol’A 1 n) == empty) andalso
  ((Mark.Protocol’B 1 n) == empty) andalso
  ((Mark.Protocol’NextRec 1 n) == 1‘7) andalso
  ((Mark.Protocol’C 1 n) == empty) andalso
  ((Mark.Protocol’D 1 n) == empty) andalso
  ((Mark.Protocol’DataReceived 1 n) == 1‘"COLOURED PETRI NET")
```

Structure **Module** **Place**

Equality of two multi-sets

State space node

Instance number

Predefined function:
- Returns the marking of DataReceived

Reachability of desired terminal marking

- The following query checks whether the desired terminal marking is reachable:

```
ReachablePred DesiredTerminal;
```

```
true
```

Standard query function:
- Searches through all nodes
- Determines whether some of these fulfil the predicate

- It is also possible to find the node(s) which represents the desired terminal marking:

```
PredAllNodes DesiredTerminal;
```

```
[4868]
```

Standard query function:
- Searches through all nodes
- Returns a list with those that fulfil the predicate

State space report: reachability properties

- The state space report does not contain information about reachability properties.
- The specific markings which it is of interest to investigate is highly model dependent – and there are too many to investigate all pairs.

- The statistics in the state space report for the protocol shows that there are more than one SCC.
- This implies that not all nodes in the state space are mutually reachable – as demonstrated above using standard query functions.

Integer bounds

- Integer bounds counts the number of tokens on a place.
- The best upper integer bound for a place is the maximal number of tokens on the place in a reachable marking.
- The best lower integer bound for a place is the minimal number of tokens on the place in a reachable marking.
- Places with an upper integer bound are bounded.
- Places with no upper integer bound are unbounded.
- 0 is always a lower integer bound, but it may not be the best.

State space report: integer bounds

Best Integers Bounds	Upper	Lower
PacketsToSend	6	6
DataReceived	1	1
NextSend, NextRec	1	1
A, B, C, D	3	0
Limit	3	0

- PacketsToSend has exactly 6 tokens in all reachable markings.
- DataReceived, NextSend and NextRec have exactly one token each in all reachable markings.
- The remaining five places have between 0 and 3 tokens each in all reachable markings.

More general integer bounds

- It is also possible to find integer bounds for a set of places.
- As an example, we might investigate how many tokens we have simultaneously on places A and B.

```
fun SumMarkings n =
    (Mark.StateSpaceProtocol'A 1 n) ++
    (Mark.StateSpaceProtocol'B 1 n);
```

`UpperInteger SumMarkings;`	3
`LowerInteger SumMarkings;`	0

Standard query functions

Argument must be a function mapping from a state space node into a multi-set type 'a ms

More general integer bounds

- It is also possible to investigate integer bounds which consider only certain token colours and places.
- As an example, we will investigate the minimal and maximal number of tokens with the colour (1,"COL") that can simultaneously reside on the places A and B:

Standard list function:
- Takes a predicate and a list as arguments
- Returns those elements that fulfil the predicate

```
fun SumFirstDataPacket n =
    (List.filter
      (fn p => p = (1,"COL"))
      (SumMarkings n));
```

Marking of places A and B

CPN tools represents multi-sets as lists

`UpperInteger SumFirstDataPacket;`	3
`LowerInteger SumFirstDataPacket;`	0

Multi-set bounds

- Integer bounds count the number of tokens ignoring the token colours.
- Multi-set bounds provide information about the possible token colours.

- The best upper multi-set bound for a place is a multi-set over the colour set of the place.
- The coefficient for a colour c is the maximal number of occurrences of tokens with colour c in a reachable marking.

- The best lower multi-set bound for a place is a multi-set over the colour set of the place.
- The coefficient for a colour c is the minimal number of occurrences of tokens with colour c in a reachable marking.

State space report: upper multi-set bounds

Best Upper Multi-set Bounds

PacketsToSend	1‘(1,"COL")++1‘(2,"OUR")++1‘(3,"ED ")++ 1‘(4,"PET")++1‘(5,"RI ")++1‘(6,"NET")
DataReceived	1‘""++1‘"COL"++1‘"COLOUR"++1‘"COLOURED "++ 1‘"COLOURED PET"++1‘"COLOURED PETRI "++ 1‘"COLOURED PETRI NET"
NextSend, NextRec	1‘1++1‘2++1‘3++1‘4++1‘5++1‘6++1‘7
A, B	3‘(1,"COL")++3‘(2,"OUR")++3‘(3,"ED ")++ 3‘(4,"PET")++3‘(5,"RI ")++3‘(6,"NET")
C, D	3‘2++3‘3++3‘4++3‘5++3‘6++3‘7
Limit	3‘()

- The upper bound for DataReceived is a multi-set with seven elements although the place always has exactly one token.

State space report: lower multi-set bounds

Best Lower Multi-set Bounds	
PacketsToSend	1‘(1,"COL")++1‘(2,"OUR")++1‘(3,"ED ")++ 1‘(4,"PET")++1‘(5,"RI ")++1‘(6,"NET")
DataReceived	empty
NextSend, NextRec	empty
A, B, C, D	empty
Limit	empty

- The lower bound for DataReceived is empty although the place always has exactly one token.

More general multi-set bounds

- Upper and lower multi-set bounds can be generalised to sets of places in a similar way as described for integer bounds.

```
UpperMultiSet SumMarkings;

LowerMultiSet SumMarkings;
```

Standard query functions

Argument must be a function mapping from a state space node into a multi-set type 'a ms

```
3`(1,"COL") ++ 3`(2,"OUR") ++ 3`(3,"ED ") ++
3`(4,"PET") ++ 3`(5,"RI ") ++ 3`(6,"NET")
```

```
empty
```

More general multi-set bounds

- Upper and lower multi-set bounds can also be generalised to specific token colours residing on a set of places in a similar way as described for integer bounds.

```
UpperMultiSet SumFirstDataPacket;
LowerMultiSet SumFirstDataPacket;
```

```
3‘(1,"COL")
```

```
empty
```

Standard query functions

Argument must be a function mapping from a state space node into a multi-set type ’a ms

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Integer and multi-set bounds

- The two kinds of bounds supplement each other and provides different kinds of information.

DataReceived	1

Tells us that DataReceived has at most one token, but gives us no information about the token colours.

DataReceived	1`""++1`"COL"++1`"COLOUR"++1`"COLOURED "++ 1`"COLOURED PET"++1`"COLOURED PETRI "++ 1`"COLOURED PETRI NET"

Tells us that DataReceived can have seven different token colours, but not whether they can be present simultaneously.

Home marking

- A home marking is a marking M_{home} which can be reached from any reachable marking.

- This means that it is impossible to have an occurrence sequence which cannot be extended to reach M_{home}.

- The home property tells that it is possible to reach M_{home}.
- However, there is no guarantee that this will happen.

State space report: home markings

Home Properties Home Markings: [4868]

- There is a single home marking represented by node number 4868.
- The marking of this node can be shown in the CPN simulator.

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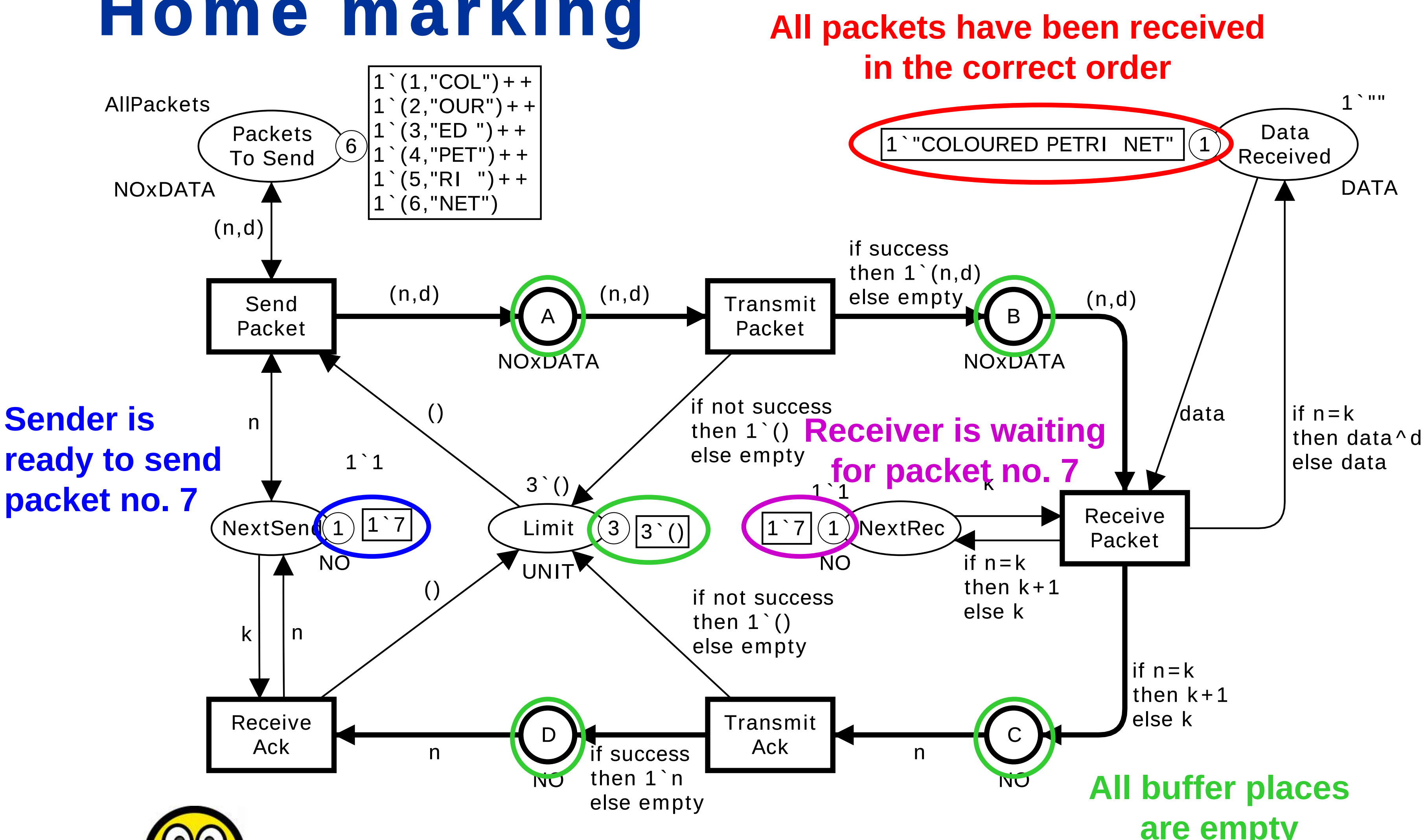
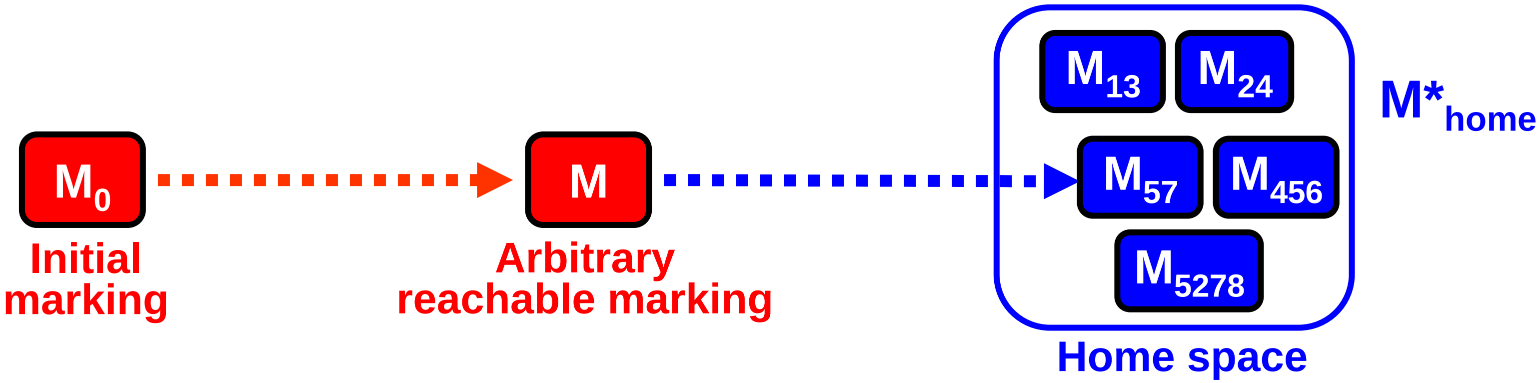
Home marking



Successful completion of transmission.

Home space

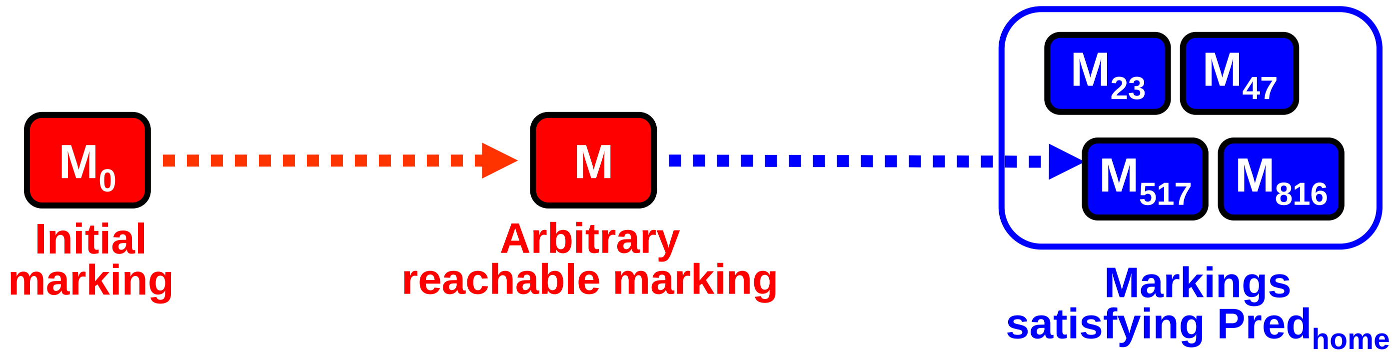
- A home space is a set of markings M^*_{home} such that at least one marking in M^*_{home} can be reached from any reachable marking.



- This means that it is impossible to have an occurrence sequence which cannot be extended to reach a marking in M^*_{home}.
- The home property tells that it is possible to reach a marking in M^*_{home}.
- However, there is no guarantee that this will happen.

Home predicate

- A home predicate is a predicate on markings $Pred_{home}$ such that at least one marking satisfying $Pred_{home}$ can be reached from any reachable marking.



- This means that it is impossible to have an occurrence sequence which cannot be extended to reach a marking satisfying $Pred_{home}$.
- The home property tells that it is possible to reach a marking satisfying $Pred_{home}$.
- However, there is no guarantee that this will happen.

Use of home predicate

- Instead of inspecting node 4868 in the CPN simulator we can check whether DesiredTerminal is a home predicate:

```
fun DesiredTerminal n =
  ((Mark.Protocol'PacketsToSend 1 n) == AllPackets) andalso
  ((Mark.Protocol'NextSend 1 n) == 1`7) andalso
  ((Mark.Protocol'A 1 n) == empty) andalso
  ((Mark.Protocol'B 1 n) == empty) andalso
  ((Mark.Protocol'NextRec 1 n) == 1`7) andalso
  ((Mark.Protocol'C 1 n) == empty) andalso
  ((Mark.Protocol'D 1 n) == empty) andalso
  ((Mark.Protocol'Data_Received 1 n) == 1`"COLOURED PETRI NET")
```

```
HomePredicate DesiredTerminal;
```

```
true
```

Standard query function (HomePredicate)

Argument must be a predicate on markings (DesiredTerminal)

Use of home properties to locate errors

- Home properties are excellent to locate certain kinds of errors.
- As an example, consider a CPN model of a telephone system.
- If all users stop calling and terminate all ongoing calls, the system is expected to reach an idle system state in which all lines and all equipment are unused and no calls are in progress.
- The idle system state will be represented:
  - by a home marking (if the system is without memory)
  - by a home space / home predicate (if information is stored about prior activities).
- If one or more reachable markings exist from which we cannot reach the idle system state, we may have made a modelling error or a design error – e.g., forgotten to return some resources.

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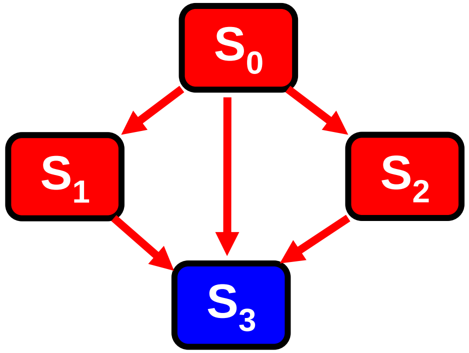
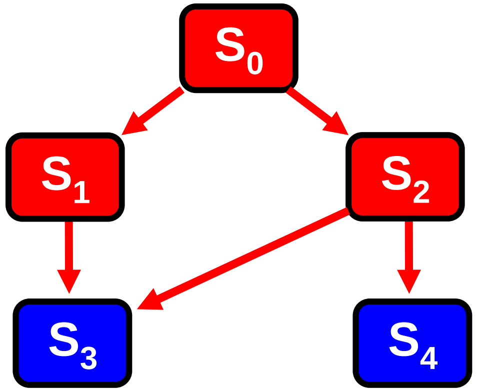
Home markings and SCCs

- The existence of home markings can be determined from the number of terminal SCCs.

- Only one terminal SCC:
  - All markings in the terminal SCC are home markings.
  - No other markings are home markings.

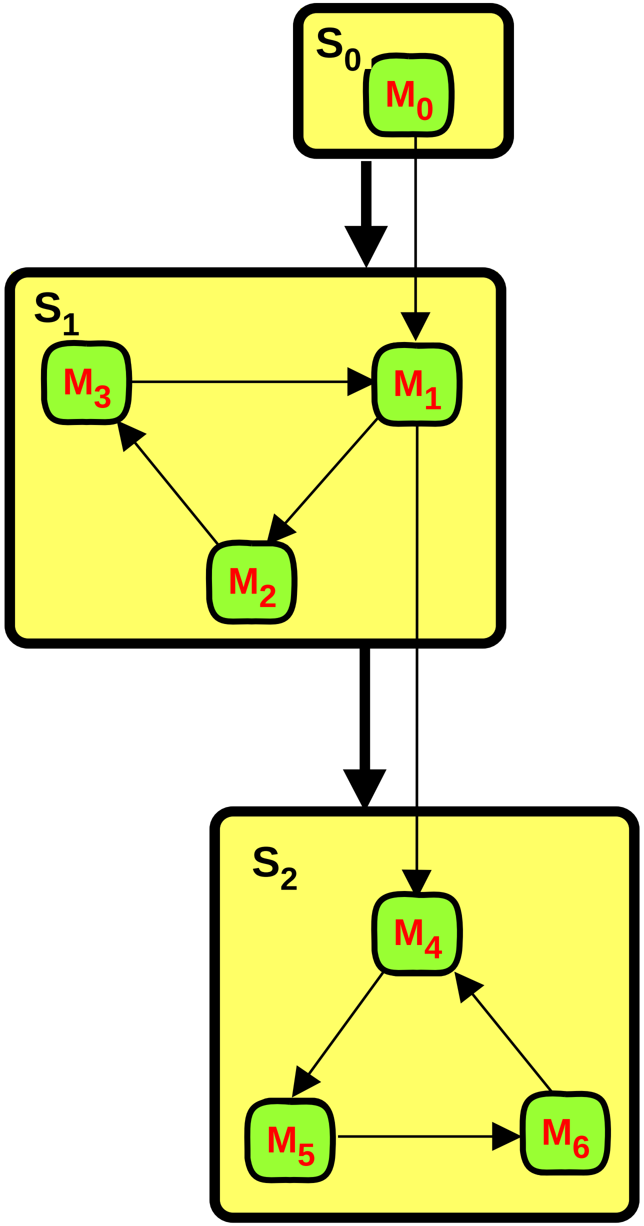


- More than one terminal SCC:
  - No home markings.



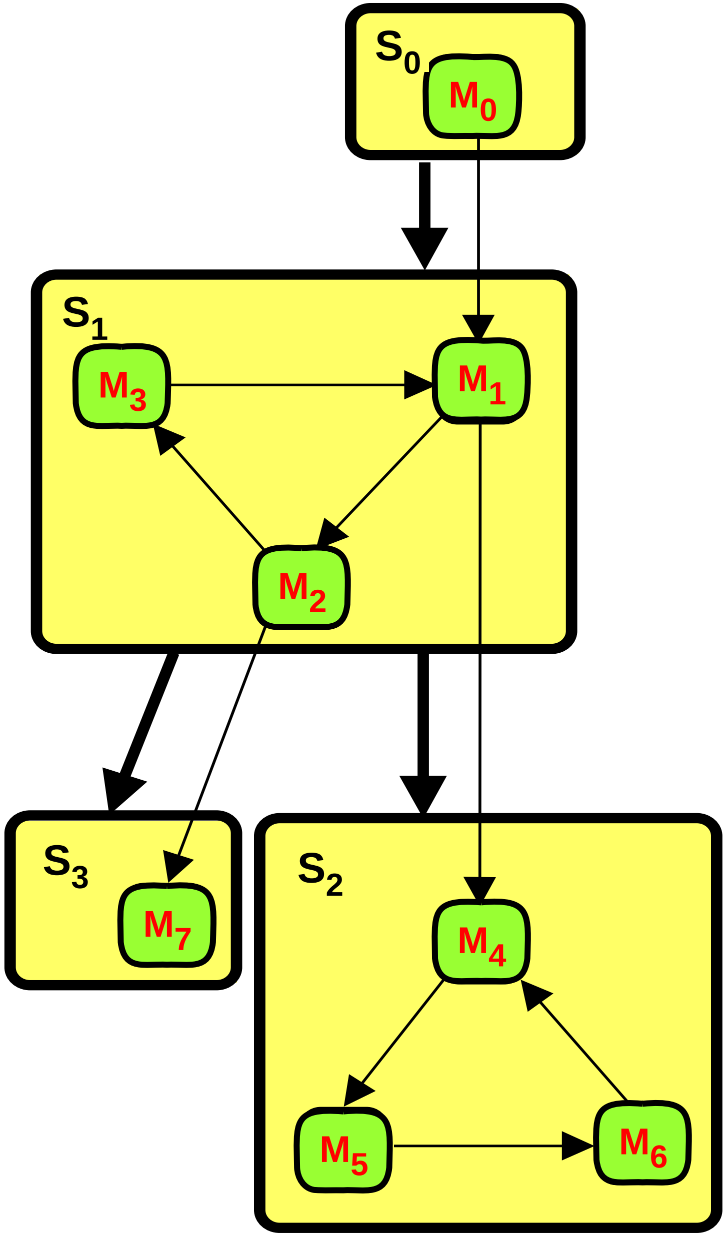
Single terminal SCC

- All markings in the terminal SCC S_2 are home markings.
- No other markings are home markings.



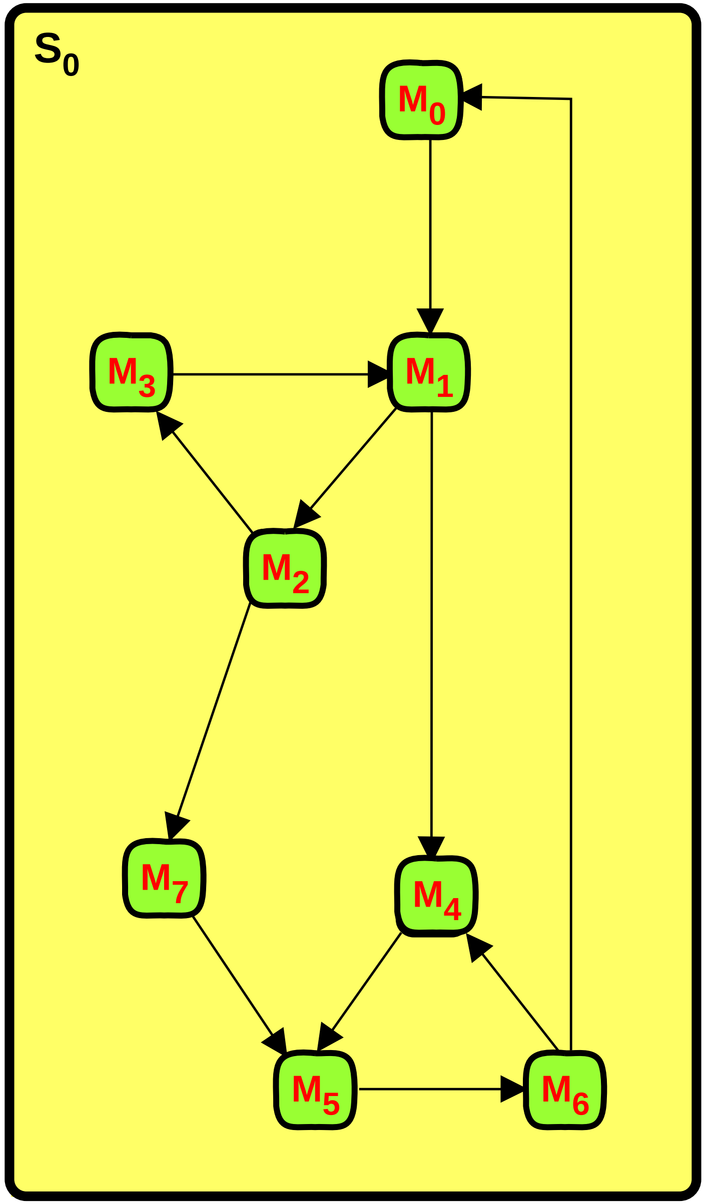
More than one terminal SCC

- No home markings.
- When one of the terminal SCCs S_2 and S_3 has been reached, it is impossible to leave it again.



Single SCC

- All reachable markings are home markings.
- They are mutually reachable from each other.



Calculation of home markings

- The CPN state space tool uses the following query to calculate the set of all home markings:

```
fun ListHomeMarkings () =
   let
      val Terminal_Sccs = PredAllSccs SccTerminal;
   in
     case Terminal_Sccs of
       [scc] => SccToNodes scc
      | _ => []
end;
```

Checks whether an SCC is terminal

Standard query function:
- Searches through all nodes in the SCC graph
- Returns those which fulfil the predicate

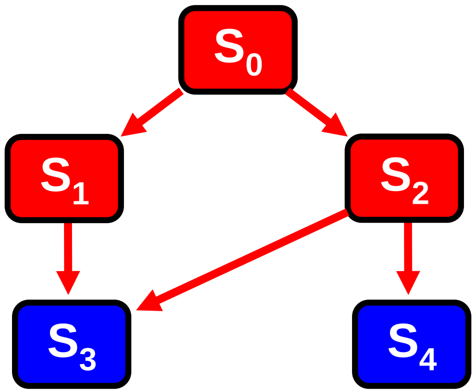
Exactly one terminal SCC

Returns the state space nodes in the strongly connected component scc

Home spaces and SCCs

- The size of home spaces can be determined from the number of terminal components in the SCC graph.

- A set of markings is a home space if and only if it contains a node from each terminal SCC.
- Home spaces must have at least as many elements as there are terminal SCCs.



- Each home marking is a home space with only one element.
- A system may have home spaces without having home markings.

Liveness properties – being dead

- A marking M is dead if M has no enabled transitions.
- A transition t is dead if t never can occur – i.e. is disabled in all reachable markings.

- Generalisations:
- A binding element is dead if it can never become enabled.
- A set of binding elements is dead if none of the binding elements can become enabled.
- A set of transitions is dead if the union of their binding elements is dead.

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State space report: being dead

Liveness Properties	
Dead Markings:	[4868]
Dead Transitions:	None
Live Transitions:	None

- There is a single dead marking represented by node number 4868.
  - Same marking as home marking.
- There are no dead transitions.

Marking no 4868

- We have seen that marking M_{4868} represents the state in which we have achieved successful completion of the transmission.

- M_{4868} is the only dead marking.
- Tells us that the system is partially correct. If execution terminates we will have the correct result.

- M_{4868} is a home marking.
- Tells us that it always is possible to reach the correct result – independently of the number of losses and overtakings.

Being dead

- It is straightforward to check whether markings, transitions and binding elements are dead.
- A marking is dead if the corresponding state space node has no outgoing arcs.
- A transition is dead if it does not appear on an arc in the state space.
- A binding element is dead if it does not appear on an arc in the state space.
- A set of binding elements is dead if no binding element in the set appears on an arc in the state space.
- A set of transitions is dead if none of their binding elements appear on an arc in the state space.

Calculation of dead markings

- The CPN state space tool uses the following query to calculate the set of all dead markings:

Maps a state space node into its outgoing arcs

```
fun ListDeadMarkings () =
    PredAllNodes (fn n => (OutArcs n) = []);
```

Standard query function:
- Searches through all nodes in the state space
- Returns a list with those that fulfil the predicate

Checks whether the set of output arcs is empty

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Coloured Petri Nets

Calculation of dead transitions

- The CPN state space tool uses the following query to check whether a transition instance is dead:

Maps a state space arc into its transition instance

```
fun TransitionInstanceDead ti =
    (PredAllArcs (fn a => ArcToTI a = ti)) = [];
```

Standard query function:
- Searches through all arcs in the state space
- Returns a list with those that fulfil the predicate

Checks whether the arc a has the transition instance ti in its label

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Calculation of dead binding elements

- We want to check whether the Sender can receive an acknowledgement with sequence number 1.

```
BEsDead ([Bind.Protocol'ReceiveAck (1,{k=1,n=1}),
          Bind.Protocol'ReceiveAck (1,{k=2,n=1}),
          Bind.Protocol'ReceiveAck (1,{k=3,n=1}),
          Bind.Protocol'ReceiveAck (1,{k=4,n=1}),
          Bind.Protocol'ReceiveAck (1,{k=5,n=1}),
          Bind.Protocol'ReceiveAck (1,{k=6,n=1}),
          Bind.Protocol'ReceiveAck (1,{k=7,n=1})],1);
```

Standard query function

List of binding elements

Structure Module Transition

Constructor

Instance

Binding

Returns the specified binding element

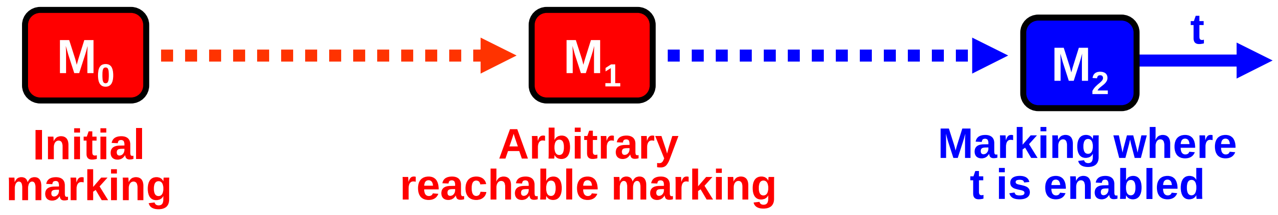
Initial Marking

```
true
```

Not possible to receive such acknowledgments.

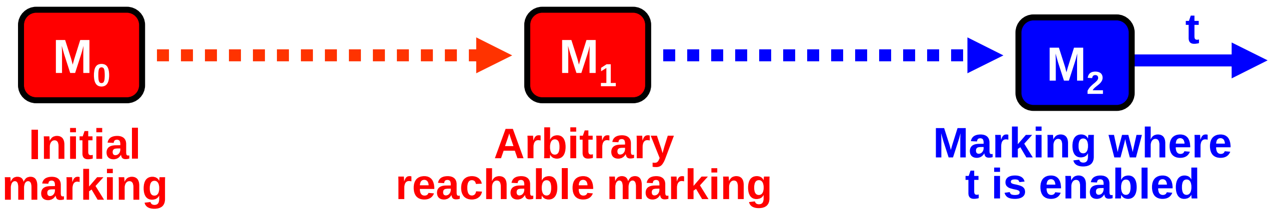
Liveness properties – being live

- A transition t is live if we from all reachable markings can find an occurrence sequence containing t.



- Liveness tells that it is possible for t to occur.
- However, there is no guarantee that this will happen.

Liveness is a strong property



- If the live transition t occurs in the marking M_2 we reach another reachable marking.
- We can use the new marking as M_1 and hence t is able to occur once more, and so on.
- This means that there exists infinite occurrence sequences from M_1 in which t occurs infinitely many times.

- It is possible to be non-dead without being live.

State space report: being live

Liveness Properties	
Dead Markings:	[4868]
Dead Transitions:	None
Live Transitions:	None

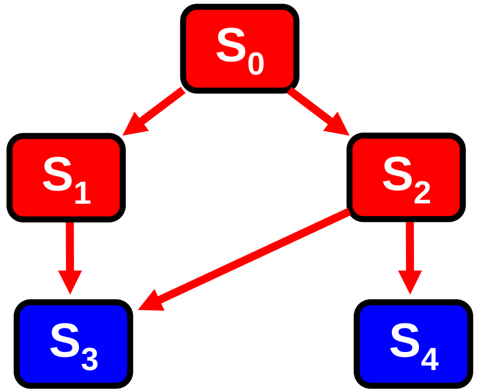
- There are no live transitions
- Trivial consequence of the existence of a dead marking.

Generalisations of liveness

- A binding element is live if it can always become enabled.
- A set of binding elements is live if it is always possible to enable at least one binding element in the set.
- A set of transitions is live if the union of their binding elements is live.

Liveness properties and SCCs

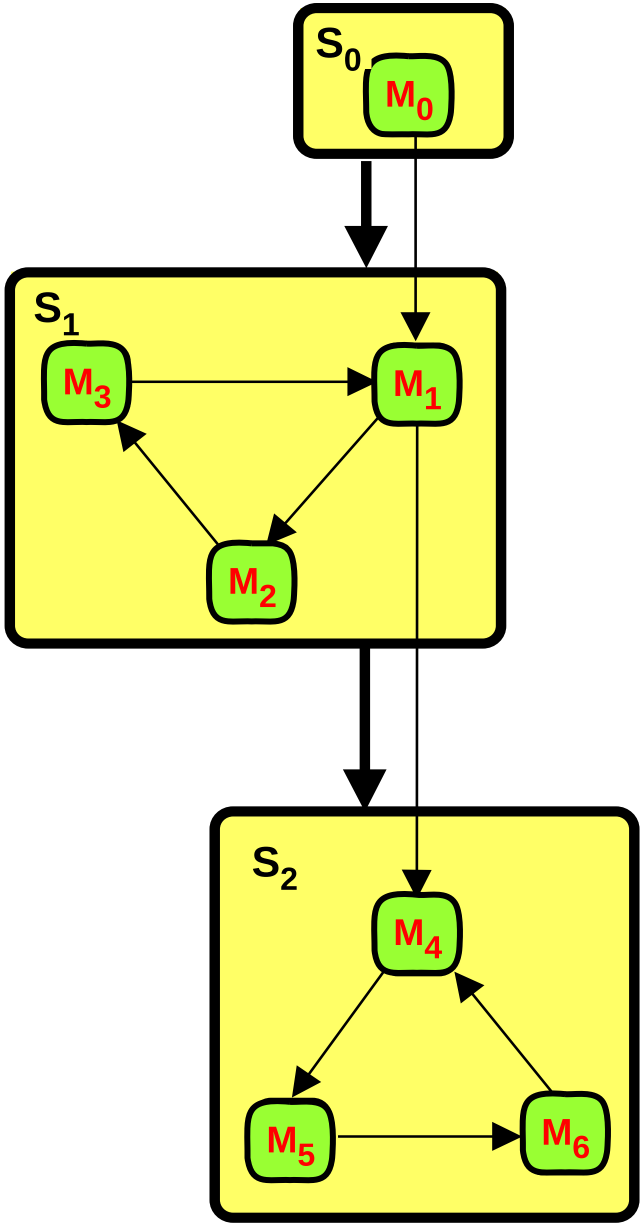
- Liveness can be determined from the SCC graph.



- A transition/binding element is live if and only if it appears on at least one arc in each terminal SCC.
- A set of transitions/binding elements is live if and only if each of the terminal SCCs contains at least one arc with a transition/binding element from the set.

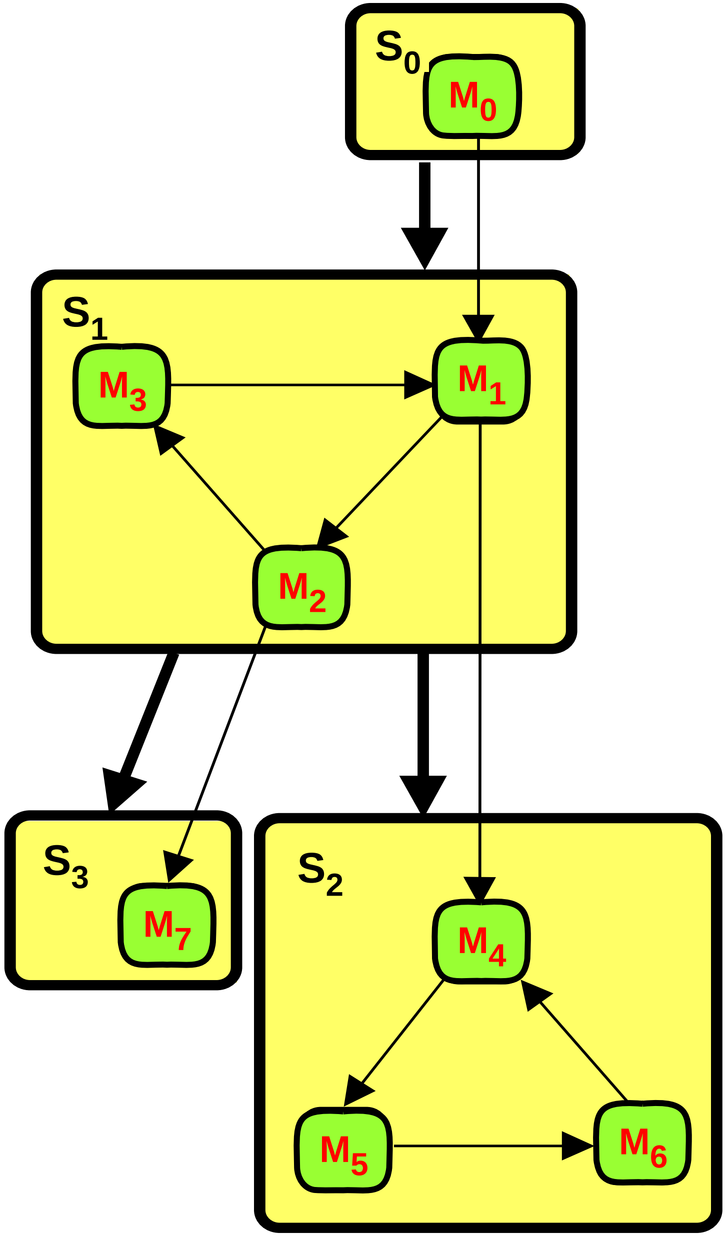
Single terminal SCC

- A transition is live if it appears on an arc in the terminal SCC S_2.



More than one terminal SCC

- No live transitions.
- S_3 is terminal and trivial.
- M_7 is a dead marking.



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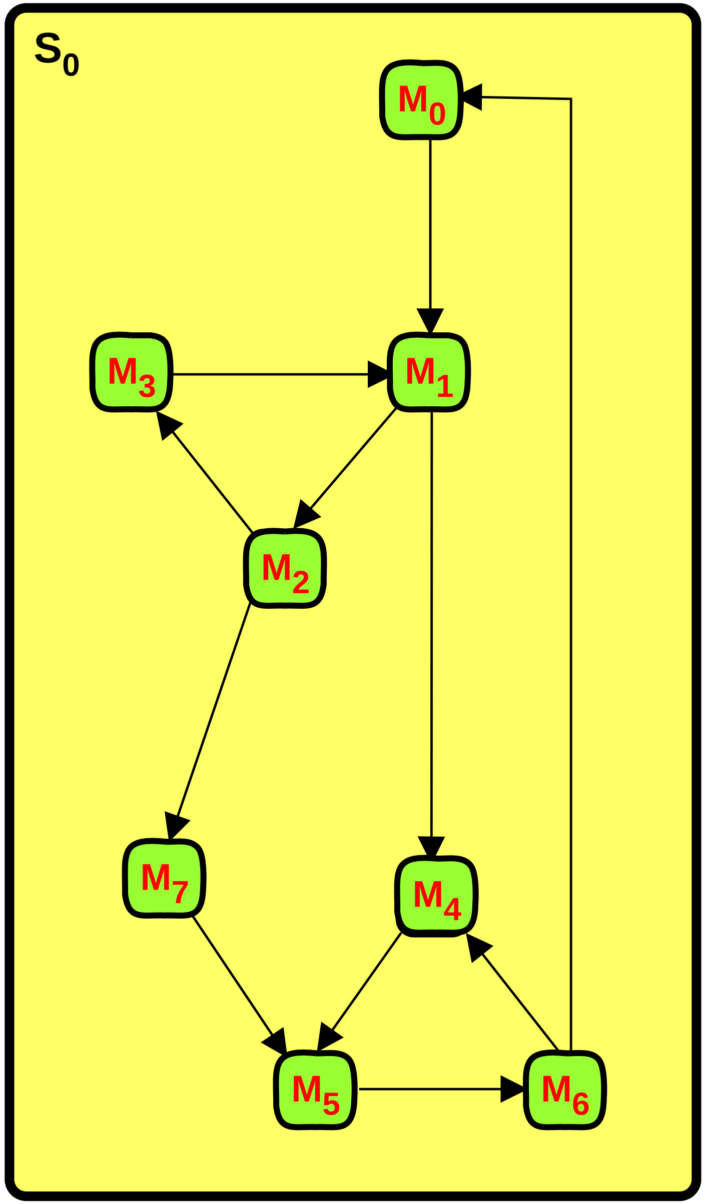
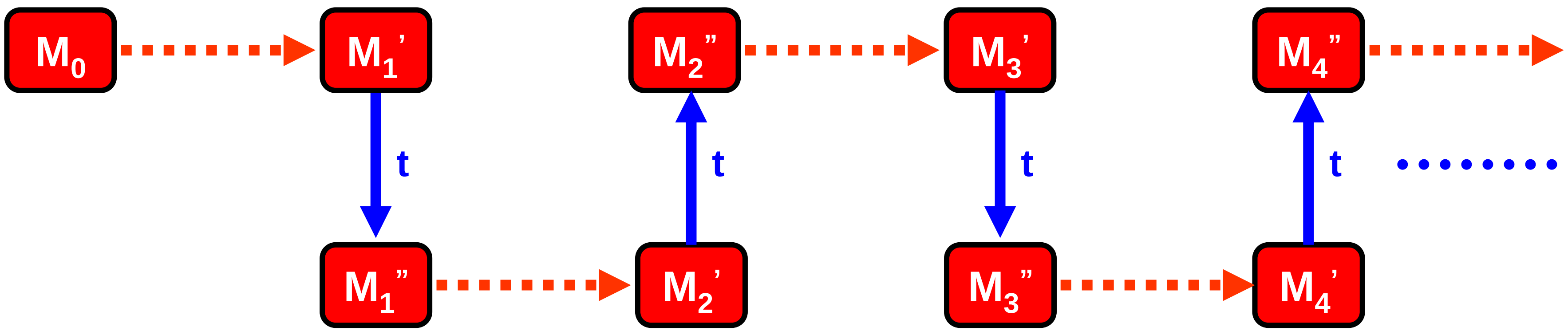
Single SCC

- A transition is live if it appears on an arc in the SCC.
- A transition is live if and only is it is non-dead.



Fairness properties

- A transition t is impartial if t occurs infinitely often in all infinite occurrence sequences.



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State space report: fairness properties

```
Fairness Properties

   Impartial Transitions: [SendPacket 1,TransmitPacket 1]
```

Instance Instance

- SendPacket and TransmitPacket are impartial.
- If one of these are removed (or blocked by the guard false) the protocol will have no infinite occurrence sequences.
- The other three transitions are not impartial.
- If we remove the Limit place only SendPacket will be impartial.
- Adding the Limit place has changed the behavioural properties.

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Generalisations of impartial

- A binding element is impartial if it occurs infinitely often in all infinite occurrence sequences.
- A set of binding elements is impartial if binding elements from the set occurs infinitely often in all infinite occurrence sequences.
- A set of transitions is impartial if the union of their binding elements is impartial.

Fairness properties and SCCs

- Impartiality of a transition/binding element can be checked by means of an SCC graph:

  1. Construct the pruned state space in which all arcs with appearances of the transition/binding element are removed.
  2. Construct the SCC graph of the pruned state space.
  3. Check whether the SCC graph for the pruned state space has the same number of nodes and arcs as the state space for the pruned state space.
  4. If this is the case the pruned state space has no cycles.
  5. This implies that all cycles in the original state space contain an arc with an appearance of the transition/binding element.
  6. Hence the transition/binding element is impartial.

- Impartiality of a set of transitions/binding elements is checked in a similar way.

Use of fairness properties

- As an example, we will investigate whether the set of binding elements corresponding to loss of data packets and acknowledgements is impartial.
- If the protocol does not terminate we expect this to be because the network keeps losing packets, and we therefore expect this set of binding elements to be impartial.

Use of fairness properties

Binding elements which lose a data packet

```
BEsImpartial
      (List.map
        (fn (n,d) =>
          Bind.StateSpaceProtocol'TransmitPacket
                         (1,{n=n,d=d,success=false}))
       AllPackets)
      ^^
      (List.map
        (fn (n,_) =>
          Bind.StateSpaceProtocol'TransmitAck
                         (1,{n=n+1,success=false}))
       AllPackets);
```

Standard query function

List concatenation

```
true
```

Binding elements which lose an acknowledgement

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Query functions – summary

- The state space report contains information about standard behavioural properties which make sense for all CPN models.
- Non-standard behavioural properties can be investigated by means of queries.
- For some purposes it is sufficient to provide arguments to a predefined query function – e.g. to check whether a set of markings constitute a home space.
- For other more special purposes it is necessary to write your own query functions using the CPN ML programming language.

Example of user-defined query function

- We want to check whether the protocol obeys the stop-and-wait strategy – i.e. that the sender always sends the data packet expected by the receiver (or the previous one)

Converts a multi-set 1`x with one element to the colour x

```
fun StopWait n =
  let
    val NextSend = ms_to_col (Mark.Protocol'NextSend 1 n);
    val NextRec  = ms_to_col (Mark.Protocol'NextRec 1 n);
  in
    (NextSend = NextRec) orelse (NextSend = NextRec - 1)
  end;
```

```
val SWviolate = PredAllNodes (fn n => not(StopWait));
```

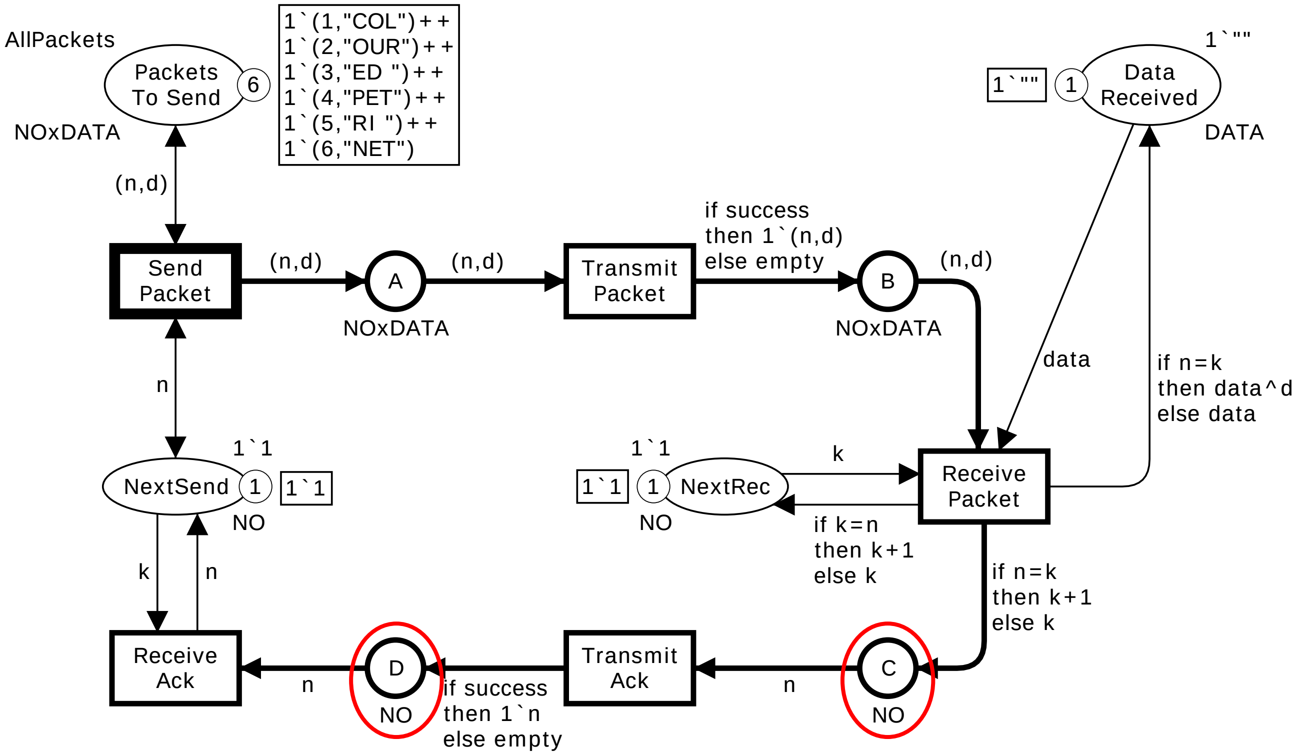
Negation

- The stop-and-wait strategy is not satisfied (7020 violations).

We check whether some states violate the property.
This is easier than checking that all states fulfil the property.

Violation of stop-and-wait strategy

- Acknowledgements may overtake each other on C and D.
- This means that it is possible for the sender to receive an old acknowledgement which decrements NextSend.



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Shortest counter example

- We want to construct a shortest counter example – i.e. to find one of the shortest occurrence sequences leading from the initial marking to a marking where the predicate does not hold.
- The state space is generated in breadth-first order.
- Hence, we search for the lowest numbered node in the list SWviolate.

Lowest node in SWviolate

```
List.foldr
        Int.min
        (List.hd SWviolate)
        (List.tl SWviolate);
```

Predefined function:
- Takes 3 arguments

Combination function
Initial value
List

```
557
```

- The function iterates over the list.
- In each iteration the combination function is applied to the pair consisting of the current element in the list and the value returned by the previous application of the combination function.
- In the first iteration, the initial value plays the role of the result from the previous application.

Shortest counter example

```
ArcsInPath(1,557);
```

```
[1,3,9,16,27,46,71,104,
 142,201,265,362,489,652,
 854,1085,1354,1648]
```

Predefined function:
- Returns the arcs in one of the shortest paths from 1 to 557

18 arcs

- The path can be visualised using the drawing facilities in the CPN state space tool.
- This is the same drawing facilities that were used to visualise the initial fragment of the state space (at the beginning of this lecture).

Bindings elements in counter example

- The binding elements in the shortest path can be obtained by the following query:

```
List.map (ArcToBE (ArcsInPath(1,557)));
```

Maps a state space arc into its binding element

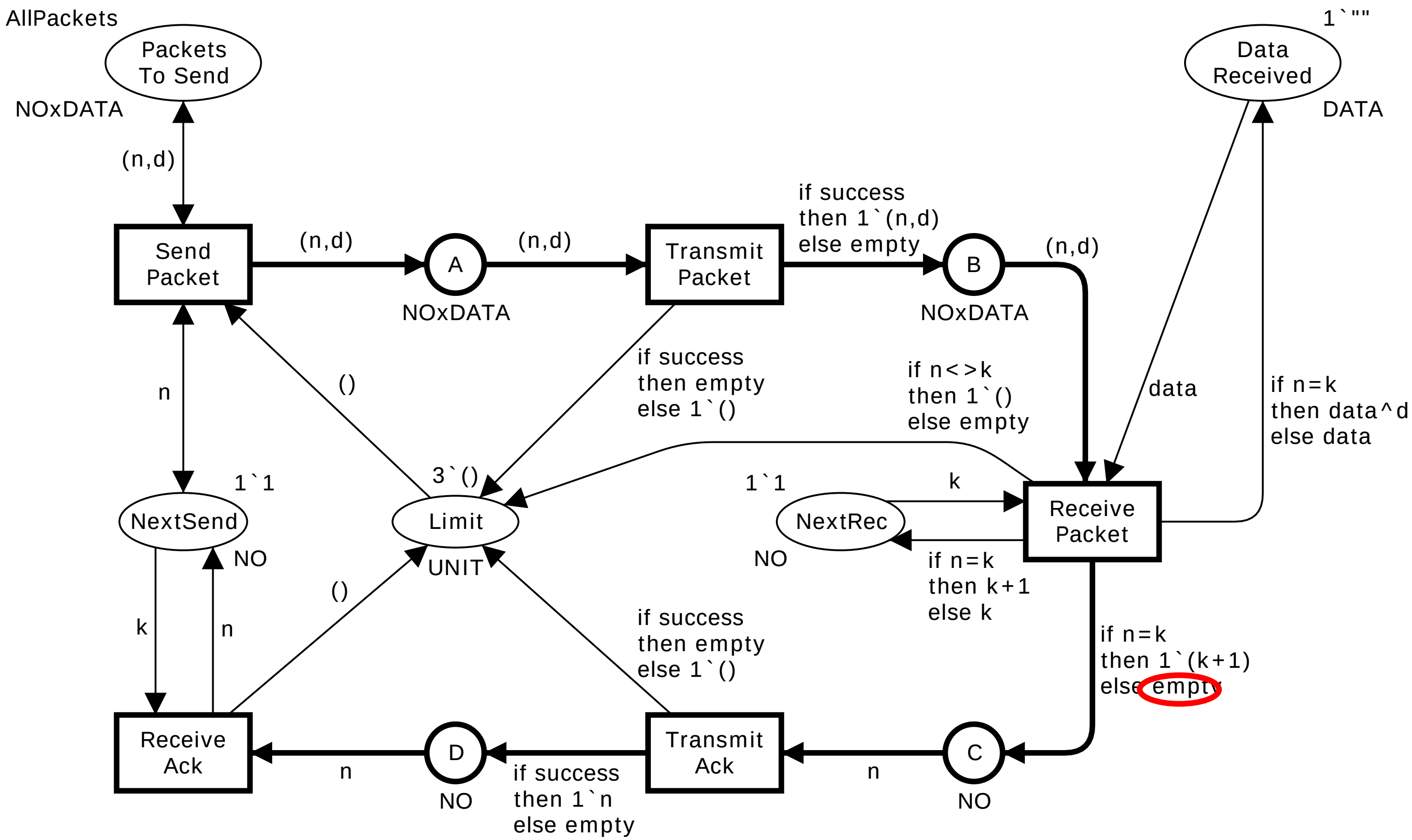
Shortest path with counter example

Shortest counter example

	1	(SendPacket, <d="COL",n=1>
	2	(TransmitPacket, <n=1,d="COL",success=true>
Packet no 1	3	(ReceivePacket, <k=1,data="",n=1,d="COL“>
and its ack	4	(SendPacket, <d="COL",n=1>
	5	(TransmitAck, <n=2,success=true>
	6	(ReceiveAck, <k=1,n=2>
	7	(SendPacket, <d="OUR",n=2>
	8	(TransmitPacket, <n=1,d="COL",success=true>
Packet no 2	9	(TransmitPacket, <n=2,d="OUR",success=true>
and its ack	10	(ReceivePacket, <k=2,data="COL",n=1,d="COL“>
	11	(ReceivePacket, <k=2,data="COL",n=2,d="OUR“>
	12	(TransmitAck, <n=3,success=true>
	13	(ReceiveAck, <k=2,n=3>
Packet no 3	14	(SendPacket, <d="ED ",n=3>
NextRec = 4	15	(TransmitPacket, <n=3,d="ED ",success=true>
Retrans-	16	(ReceivePacket, <k=3,data="COLOUR",n=3,d="ED “>
mission	17	(TransmitAck, <n=2,success=true>
NextSend = 2	18	(ReceiveAck, <k=3,n=2>

Revised protocol

- Now we only send an acknowledgement when an expected packet is received.
- Is the new protocol correct?
- Are the behavioural properties the same as before?



State space for revised protocol

- The state space contains 1,823 nodes and 6,829 arcs.
- Before we had 13,215 nodes and 52,874 arcs.

- As before there is a single dead marking which corresponds to the desired terminal marking, where all packets have been successfully transmitted.
- The new protocol is partially correct.

- Now there are no home markings.
- We can reach situations from which it is impossible to reach the desired terminal marking.
- The new protocol may enter a live-lock.

Analysis of revised protocol

- The dead marking is no longer a home marking and hence we must have one or more terminal SCCs from which we cannot reach the dead marking.
- These terminal SCCs can be found by the following query which returns all SCCs that are terminal but not trivial:

```
PredAllSccs (fn scc => SccTerminal scc andalso
                       not (SccTrivial scc));
```

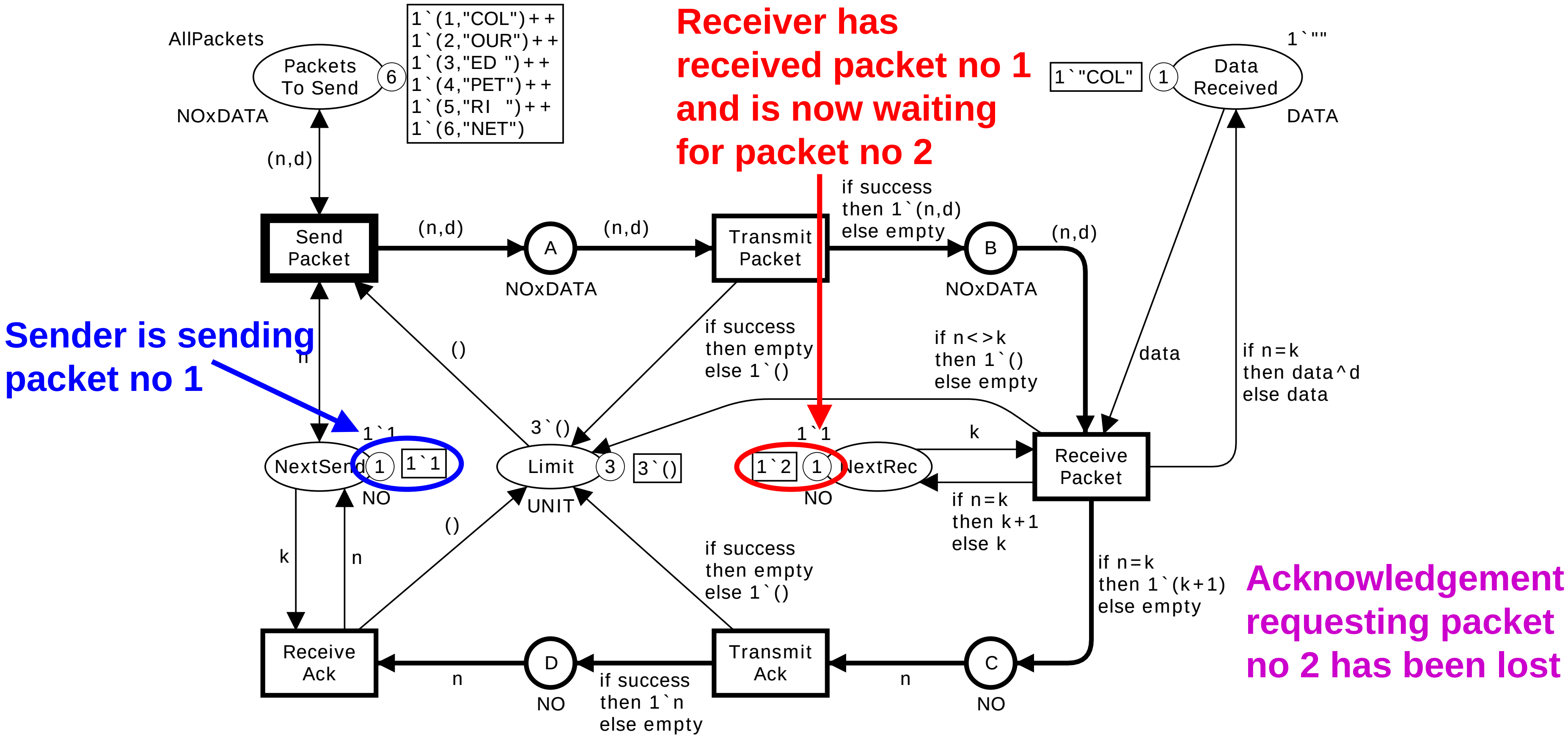
Standard query function:

- Searches through all nodes in the SCC graph

- Returns those which fulfil the predicate

- The result of the query is a list with six SCCs.
- The state space nodes in the six SCCs can be obtained using the function SccToNodes.
- To get a shortest counter example, we choose the lowest numbered node which is node 12.

Marking no 12



All data packets will we “wrong”.

No acknowledgements will be sent – no progress.

What is wrong

- The analysis of marking no 12 has told us what the problem is.
- The sender continues to send wrong packets and the receiver never sends an acknowledgement which can correct the problem.

- We might also want to know how we arrived at this unfortunate situation.
- This is done by constructing an error trace / counter example.

Counter example

- The query below returns a list with all the arcs in one of the shortest paths from node 1 (initial marking) to node number 12:

```
ArcsInPath(1,12);
```

- The binding elements in the shortest path can be obtained by the following query:

```
List.map (ArcToBE (ArcsInPath(1,12)));
```

Maps a state space arc into its binding element

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Counter example

- The result of the query is the following list of binding elements:

```
1   (SendPacket, <d="COL", n=1>)
2   (TransmitPacket, <d="COL",n=1,success=true>)
3   (ReceivePacket, <d="COL",n=1,k=1, data="">)
4   (TransmitAck, <n=2,success=false>)
```

- We see that data packet no 1 was sent, successfully transmitted, and received.
- However, the acknowledgment requesting data packet no 2 was lost on the network.

System configurations

- With state space analysis we always investigate a system for a particular configuration of the system parameters.
- In practice it is often sufficient to consider a few rather small configurations – although we cannot be totally sure that larger configurations will have the same properties.

- As system parameters increase the size of the state space increases – often in an exponential way.
- This is called the state space explosion, and it is one of the most severe limitations of the state space method.

Different system configurations

Limit	Packets	Nodes	Arcs	Limit	Packets	Nodes	Arcs
1	1	9	11	5	1	217	760
1	2	17	22	5	3	17,952	97,963
1	200	1,601	2,200	5	5	269,680	1,655,021
1	600	4,801	6,600	7	1	576	2,338
2	1	26	53	7	2	11,280	64,297
2	5	716	1,917	7	3	148,690	1,015,188
2	50	93,371	258,822	10	1	1,782	8,195
2	140	746,456	2,072,682	10	2	76,571	523,105
3	1	60	159	12	1	3,276	15,873
3	5	7,156	28,201	12	2	221,117	1,636,921
3	10	70,131	286,746	13	1	4,305	21,294
3	20	622,481	2,583,361	13	2	357,957	2,737,878

Is it worthwhile?

- State space analysis can be a time consuming process where it takes many hours to generate the state spaces and verify the desired properties.
- However, it is fully automatic and hence requires much less human work than lengthy simulations and tests.

- It may take days to verify the properties of a system by means of state spaces.
- However, this is still a relatively small investment:
  - compared to the total number of resources used in a system development project.
  - compared to the cost of implementing, deploying and correcting a system with errors that could have been detected in the design phase.

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Partial state spaces

- It is sometimes impossible to generate the full state space for a given system configuration – either because it is too big or takes too long time.
- This means that only a partial state space – i.e. a fragment of the state space is generated.

- Partial state spaces cannot in general be used to verify properties, but they may identify errors.
- As an example, an undesirable dead marking in a partial state space will also be present in the full state space.

- Partial state spaces can in that sense be viewed as being positioned between simulation and state spaces.
- The CPN state space tool has a number of parameters to control the generation of partial state spaces.

State spaces - summary

- State spaces are powerful and easy to use.
  - Construction and analysis can be automated.
  - The user do not need to know the mathematics behind the analysis methods.
- The main drawback is the state explosion – i.e. the size of the state space.
  - The present CPN state space tool handles state spaces with up to one million states.
  - For many systems this is not sufficient.
  - A much more efficient state space tool is under development.

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Reduced state spaces

- Fortunately, it is often possible to construct reduced state spaces – without losing analytic power.
- This is done by exploiting:
  - Progress measure.
  - Symmetries in the modelled system.
  - Other kinds of equivalent behaviour.
  - Concurrency between events.
- The reduction methods rely on complex mathematics.
- An overview of the advanced state space methods is given in Chapter 8.