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Modelling and Validation of Concurrent Systems

Chapter 8: Advanced State Space Methods

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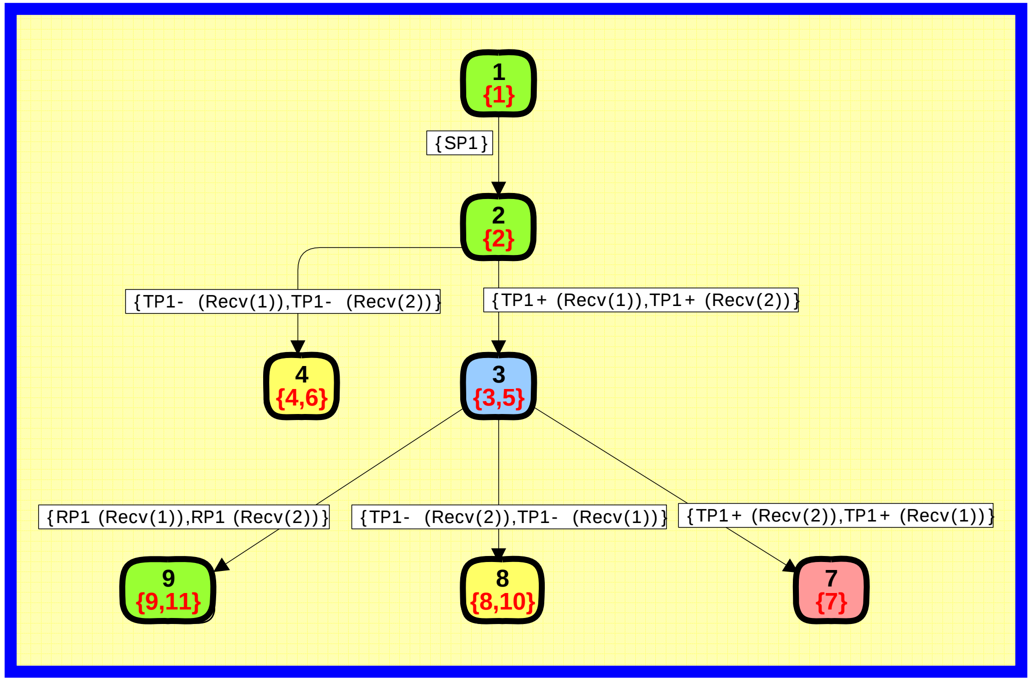

State space methods

- The main limitation of using state spaces to verify behavioural properties of systems is the state explosion problem.

- State spaces of many systems have an astronomical number of reachable states.

- This means that they are too large to be handled with the available computing power:
  - Memory.
  - CPU speed.

State space reduction methods

- Methods for alleviating the state explosion problem is an active area of research. They allow:
  - faster construction,
  - more compact representation (less memory).

- A large collection of state space reduction methods exists.

- The reduction methods have significantly increased the class of systems that can be verified in practice.

- State spaces can now be used to verify systems of industrial size.

Independent of modelling language

- Most state space reduction methods are independent of the concrete modelling language and hence applicable for a large class of such languages (e.g. all transition systems).

- Some of the reduction methods have been developed within the context of the CPN modelling language:
  - Sweep-line method.
  - Symmetry method.
  - Equivalence method.

- Other reduction methods have been developed outside the context of the CPN modelling language.

Why different reduction methods?

- State space reduction methods typically exploit certain characteristics of the system under analysis.
- No single reduction method works well for all kind of systems.

- Furthermore, the methods often limit the verification questions that can be answered.

- When verifying a concrete system one must therefore choose a method that:
  - exploits characteristics present in the system.
  - preserves the behavioural properties to be verified.

On-the-fly verification

- Many reduction methods are based on the paradigm of on-the-fly verification.

- The verification question is stated before the exploration of the state space starts.
- The state space exploration is done relative to the provided verification question.

- This makes it possible to terminate the state space exploration as soon as the answer to the verification question has been obtained – ignoring irrelevant parts.

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Model checking

- Many advanced state space reduction methods use temporal logic for stating the verification questions :
  - Linear-time temporal logic (LTL).
  - Computation tree temporal logic (CTL).
- The use of temporal logic for stating and checking verification questions is referred to as model checking.

State spaces are kept in main memory

- The amount of available main memory is often the limiting factor in the practical use of state spaces.

- During construction of the state space, the set of markings encountered are kept in main memory.

- This allows us to recognise already visited markings and thereby ensure that the state space exploration terminates.

Method 1: Sweep-line method

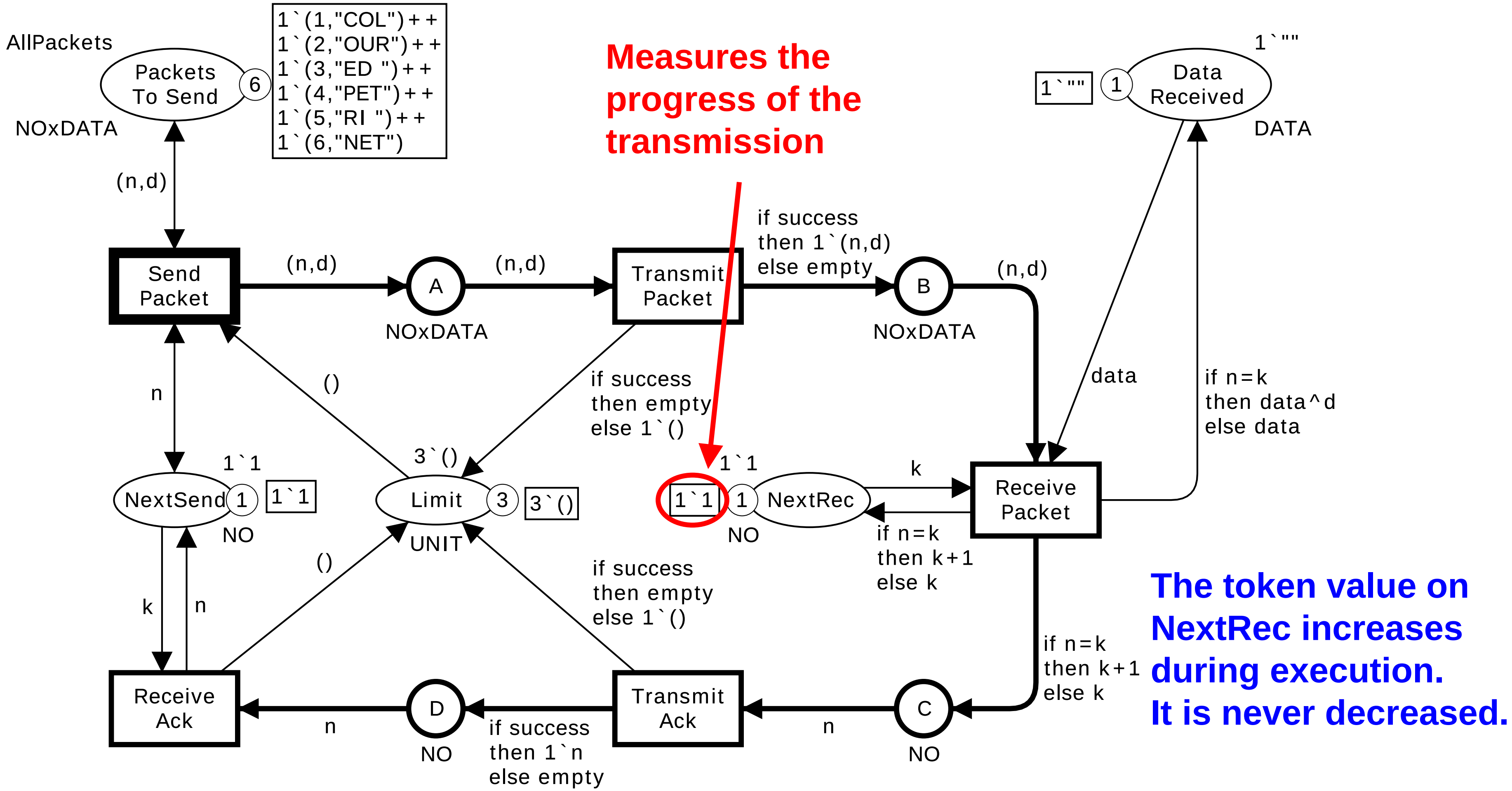
- The basic idea of the sweep-line method is to exploit a certain kind of progress exhibited by many systems.

- Exploiting progress makes it possible to explore all the reachable markings of a CPN model, while only storing small fragments of the state space in main memory at a time.

- This means that the peak memory usage is significantly reduced.

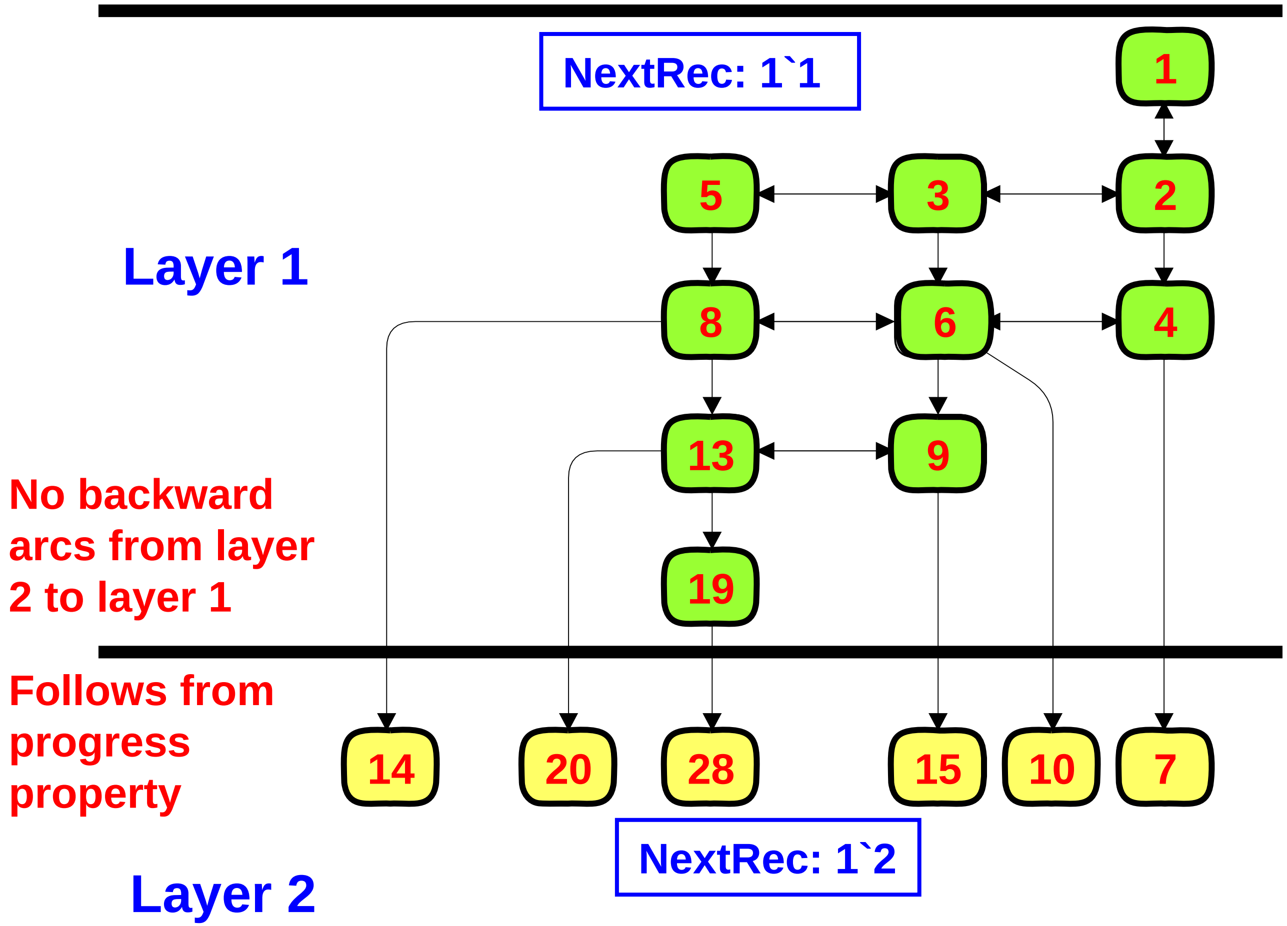
- The sweep-line method is aimed at on-the-fly verification of safety properties (e.g., determining whether a reachable marking exists satisfying a given predicate).

Simple protocol



Initial fragment of state space

- Each marking has successor markings either in the same layer or in higher layers – never in lower layers.



We process markings layer by layer

- We process the markings (i.e., calculate successor markings) one layer at a time.
- We only move from one layer to the next when all markings in the first layer have been processed.

- We can think of this as a sweep-line moving through the state space (layer by layer).

- At any time during state space exploration, the sweep-line corresponds to a single layer.
  - All markings in the layer are "on" the sweep-line.
  - All new markings calculated are either on the sweep-line or in front of the sweep-line (i.e. in a higher layer).

Progress measure

- The progress in the protocol system is captured by a progress measure which is a function mapping each marking into a progress value.

Converts a multi-set 1`x with one element to the colour x

```
fun ProtocolPM n = ms_to_col (Mark.Protocol'NextRec 1 n);
```

- Monotonic progress measure:

$$M' \in \Re(M) \Rightarrow \text{ProtocolPM } M \leq \text{ProtocolPM } M'$$

Statistics for sweep-line method

Limit	Packets	Nodes	Arcs	Nodes (peak)	Nodes	Time
1	4	33	44	33	1.00	1.00
2	4	293	764	134	2.19	1.00
3	4	1,829	6,860	758	2.41	1.00
4	4	9,025	43,124	4,449	2.03	1.78
5	4	37,477	213,902	20,826	1.80	1.65
6	4	136,107	891,830	82,586	1.65	1.51
4	5	20,016	99,355	8,521	2.35	1.95
4	6	38,885	198,150	14,545	2.67	2.19
4	7	68,720	356,965	22,905	3.00	2.27
4	8	113,121	596,264	33,985	3.33	2.41

Configuration — Standard method — Sweep-line — Gain

Summary for sweep-line method

- From the statistics on the previous slide, it can be seen that the sweep-line method yields a reduction in both space and time.
- The space reduction was expected since markings are deleted during state space exploration.
- The time reduction is because the deletion of states implies that there are fewer markings to compare with when determining whether a marking has been seen before.

Generalised sweep-line method

- Above we have used a monotonic progress measure:

$$M' \in \Re(M) \Rightarrow \text{ProtocolPM } M \leq \text{ProtocolPM } M'$$

- It is also possible to use a generalised sweep-line method where the monotonicity property only is satisifed by most steps.
- The generalised sweep-line method performs multiple sweeps of the state space, and it makes certain markings persistent which means that they cannot be deleted from memory.
- The sweep-line method has also been generalised to use external storage such that counter examples and diagnostic information can be obtained.
- This is not possible in the basic method since it deletes the markings from memory.

Method 2: Symmetry method

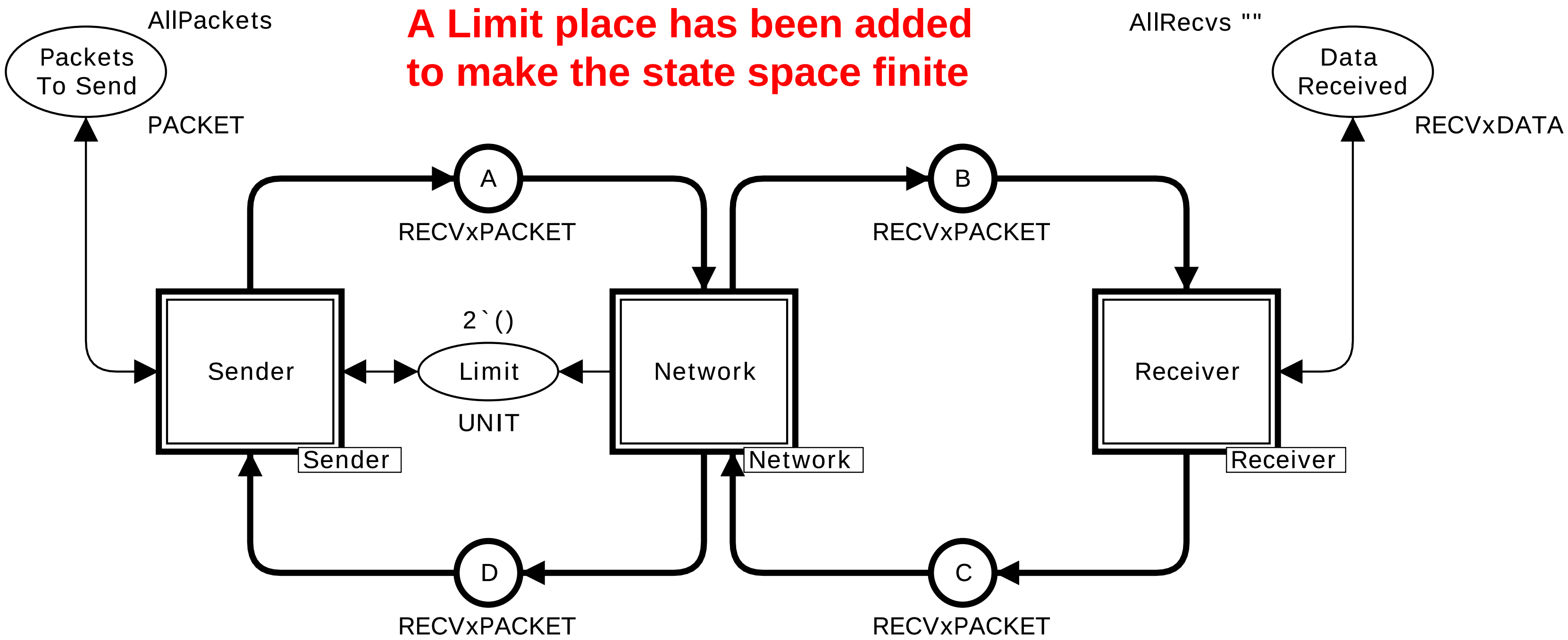
- Many concurrent systems possess a certain degree of symmetry.
- They may e.g. have similar components whose identities are interchangeable from a verification point of view.

- The basic idea in the symmetry method is to represent symmetric markings and symmetric binding elements using equivalence classes.
  - Each node represents a class of equivalent markings (instead of a single marking).
  - Each arc represents a class of equivalent binding elements (instead of a single binding element).

Construction and analysis

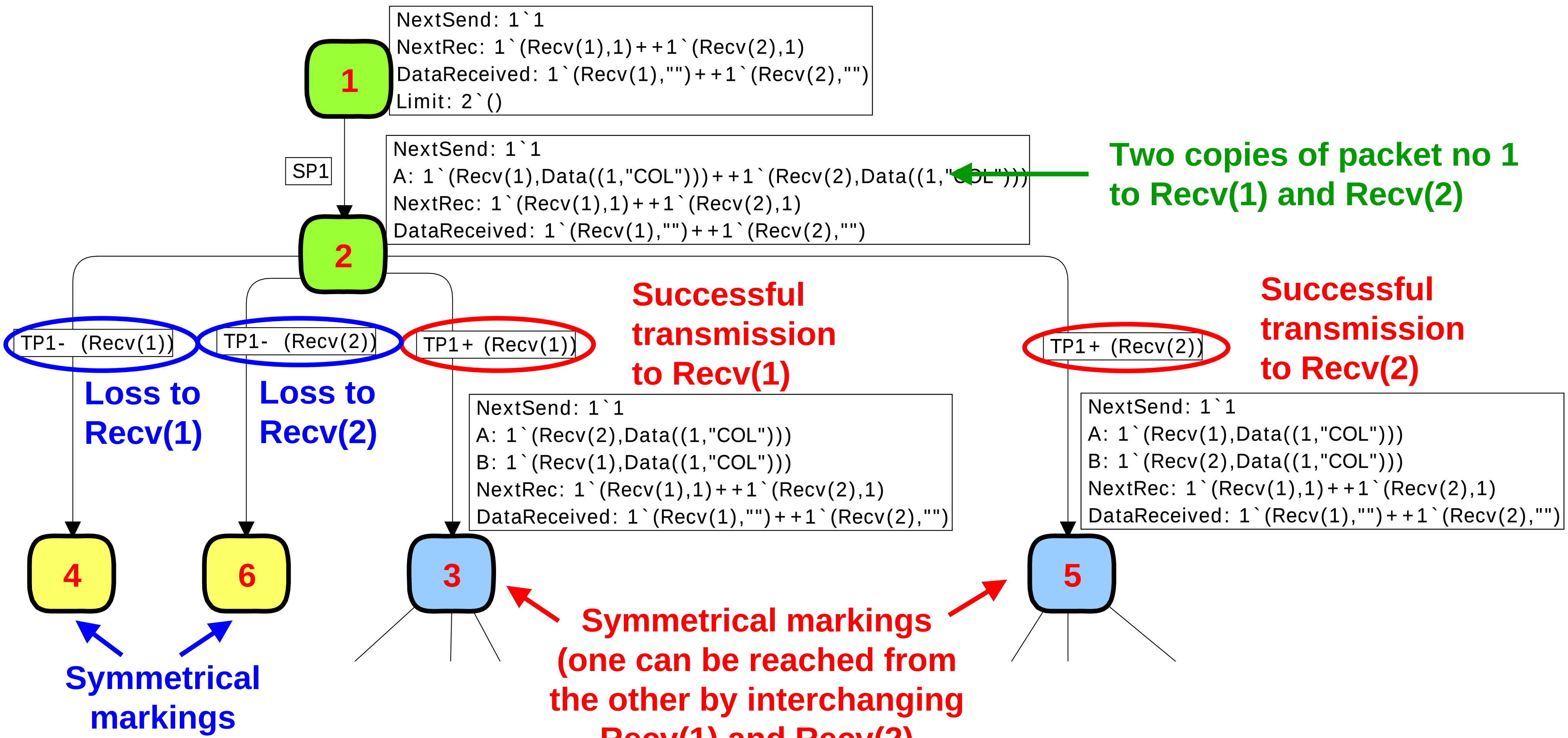
- Symmetry condensed state spaces are typically orders of magnitude smaller than the corresponding full state spaces.
- They can be constructed directly without first constructing the full state space and then grouping nodes and arcs into equivalence classes.
- Furthermore, behavioural properties can be verified directly on the symmetry condensed state space without unfolding to the full state space.

Protocol with multiple receivers

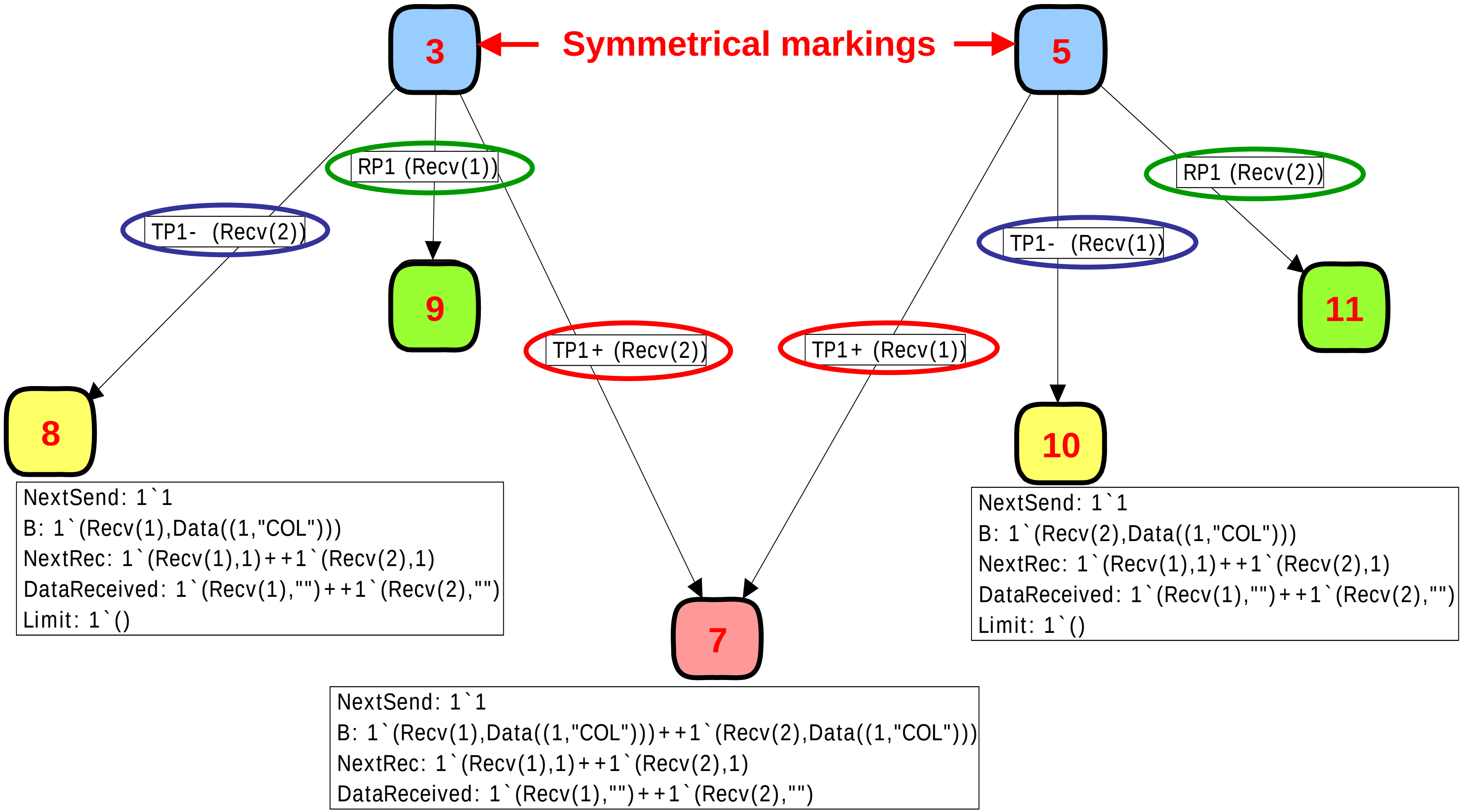


- The receivers in the protocol system are symmetric, in the sense that they all behave in the same way.
- They are only distinguishable by their identity.

State space (ordinary)



Symmetrical successors

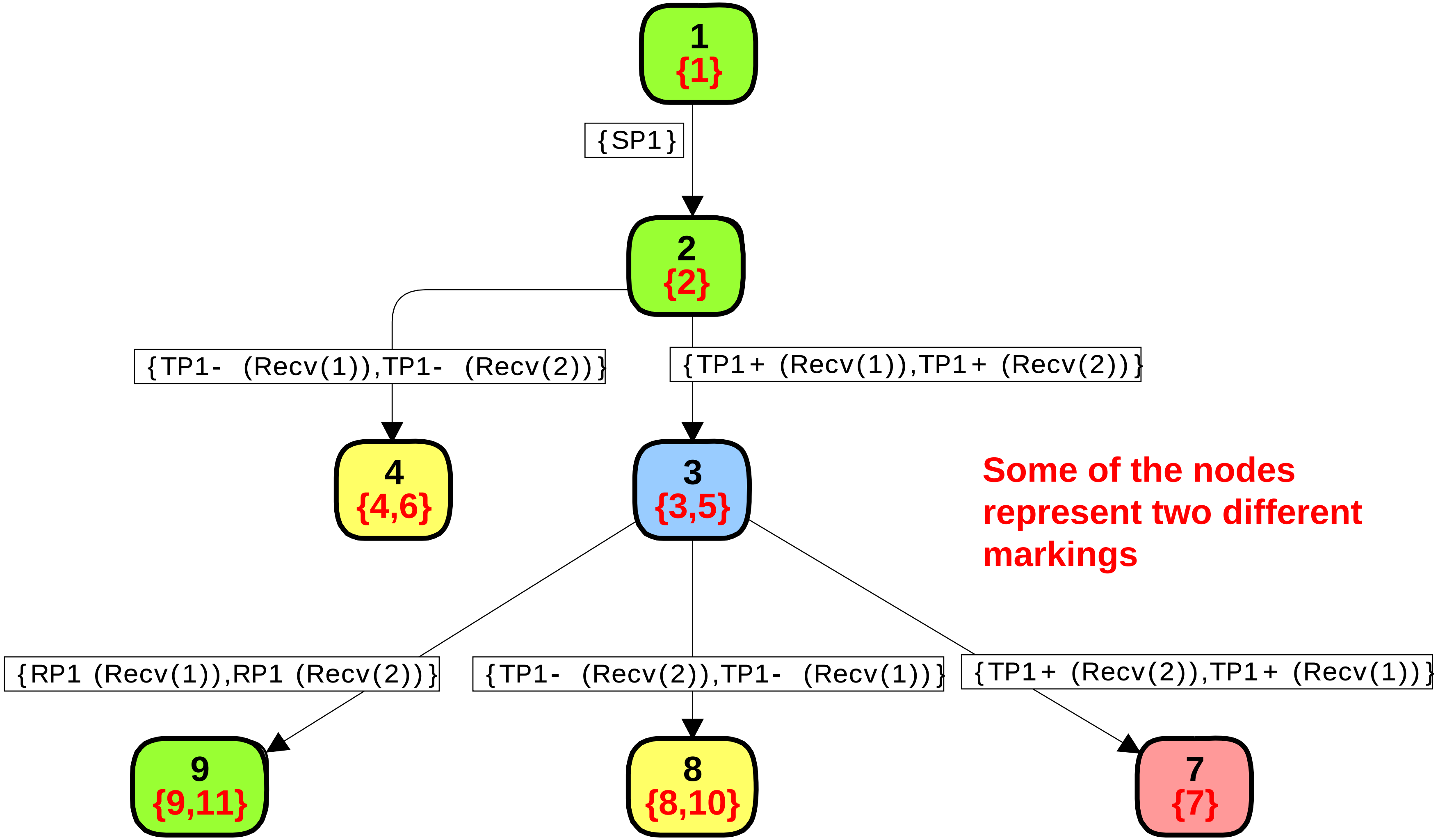


Symmetrical markings

- On the previous slide we saw that the two symmetric markings M_3 and M_5 have:
  - symmetric sets of enabled binding elements,
  - symmetric sets of direct successor markings.

- By induction this property can be extended to finite and infinite occurrence sequences:
  - For any occurrence sequence starting in a marking M and all markings M′ symmetric with M there exists a symmetric occurrence sequence starting in M′.
  - The things which can happen from M can also happen from M′ (up to symmetry).

Symmetry condensed state space



Some of the nodes represent two different markings

Soundness criteria

- The symmetries used to reduce the state space are required to be symmetries actually present in the CPN model:

- All initial marking inscriptions must be symmetric (applying a permutation to the initial marking does not change the initial marking).
- All guard expressions must be symmetric (evaluating the guard in a binding must give the same result as first permuting the binding element and then evaluating the guard).
- All arc expressions must be symmetric (evaluating the arc expression in a binding and then applying a permutation must give the same result as first permuting the binding element and then evaluating the arc expression).

Static checks by local examination of net inscriptions

Specification of symmetries

- Colour sets are divided into:
  - Atomic (Int, Bool, String, Unit, enumerations, indexed).
  - Structured (products, records, unions, lists, subsets).

- Each atomic colour set is associated with an algebraic group of allowed permutations.
- The structured colours sets inherits their permutations from the colour sets from which they are constructed.

- Examples of permutation groups are:
  - all permutations in the colour set,
  - all rotations in an ordered colour set,
  - identity element alone (no permutation allowed).

Protocol with multiple receivers

- Atomic colour sets:

```
colset NO   = int;
colset DATA = string;
colset RECV = index Recv with 1..NoRecv;
```

No permutations

All permutations

- Structured colour sets:

```
colset NOxDATA = product NO * DATA;
colset PACKET  = union Data : NoxDATA + Ack : NO;
colset RECVxDATA   = product RECV * DATA;
colset RECVxPACKET = product RECV * PACKET;
colset RECVxNO     = product RECV * NO;
```

No permutations

All permutations of Recv-component

Statistics for symmetry method

L P R	Nodes	Arcs	Nodes	Arcs	Nodes	Arcs	Time	R!
2 3 2	921	1,832	477	924	1.93	1.98	0.7	2
3 3 3	22,371	64,684	4,195	11,280	5.33	5.73	2.0	6
4 3 4	172,581	671,948	9,888	32,963	17.45	20.38	23.9	24
5 2 5	486,767	2,392,458	8,387	31,110	58.04	76.90	—	120
6 2 6	5,917,145	35,068,448	24,122	101,240	245.30	346.39	—	720

Configuration Standard method Symmetry Gain

L = Limit
P = Packets
R = Receivers

Number of possible permutations

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Summary for symmetry method

- Significant reductions can be obtained as illustrated on the protocol with multiple receivers.
- The method can be sued to check all behavioural properties that are invariant under symmetry.

- Computation of the canonical representations of markings and binding elements is computational expensive.
  - At least as hard as the graph isomorphism problem for which no polynomial time algorithm is known.
  - The present algorithms exploits a number of advanced algebraic techniques and can efficiently deal with systems where the number of permutation symmetries are below 10!
  - This is usually sufficient in practice.

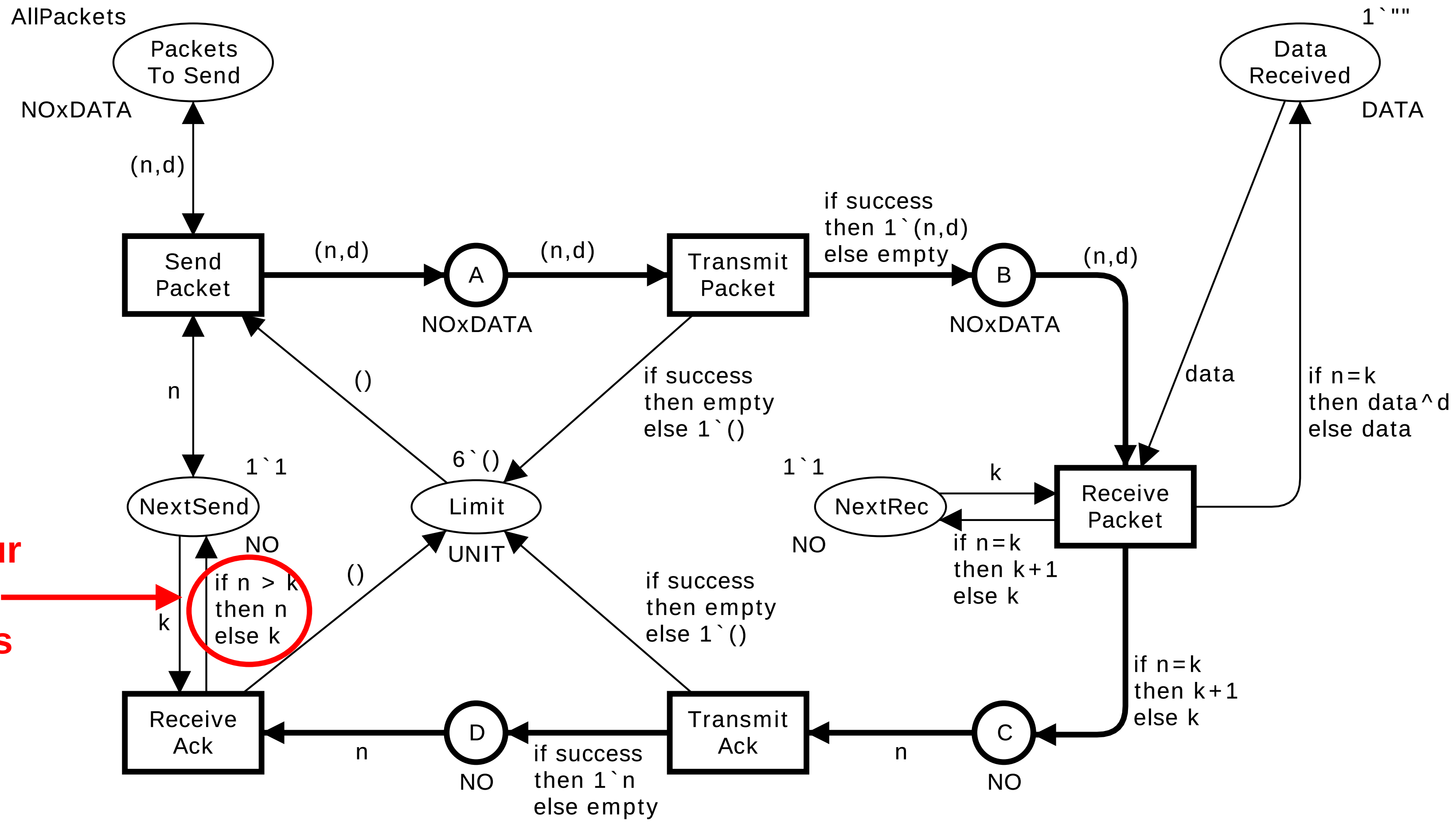
Method 3: Equivalence method

- The equivalence method is a generalisation of the symmetry method.

- In the symmetry method we have equivalence relations on the markings and on the binding elements.
- The equivalence relations are induced by the permutation symmetries.

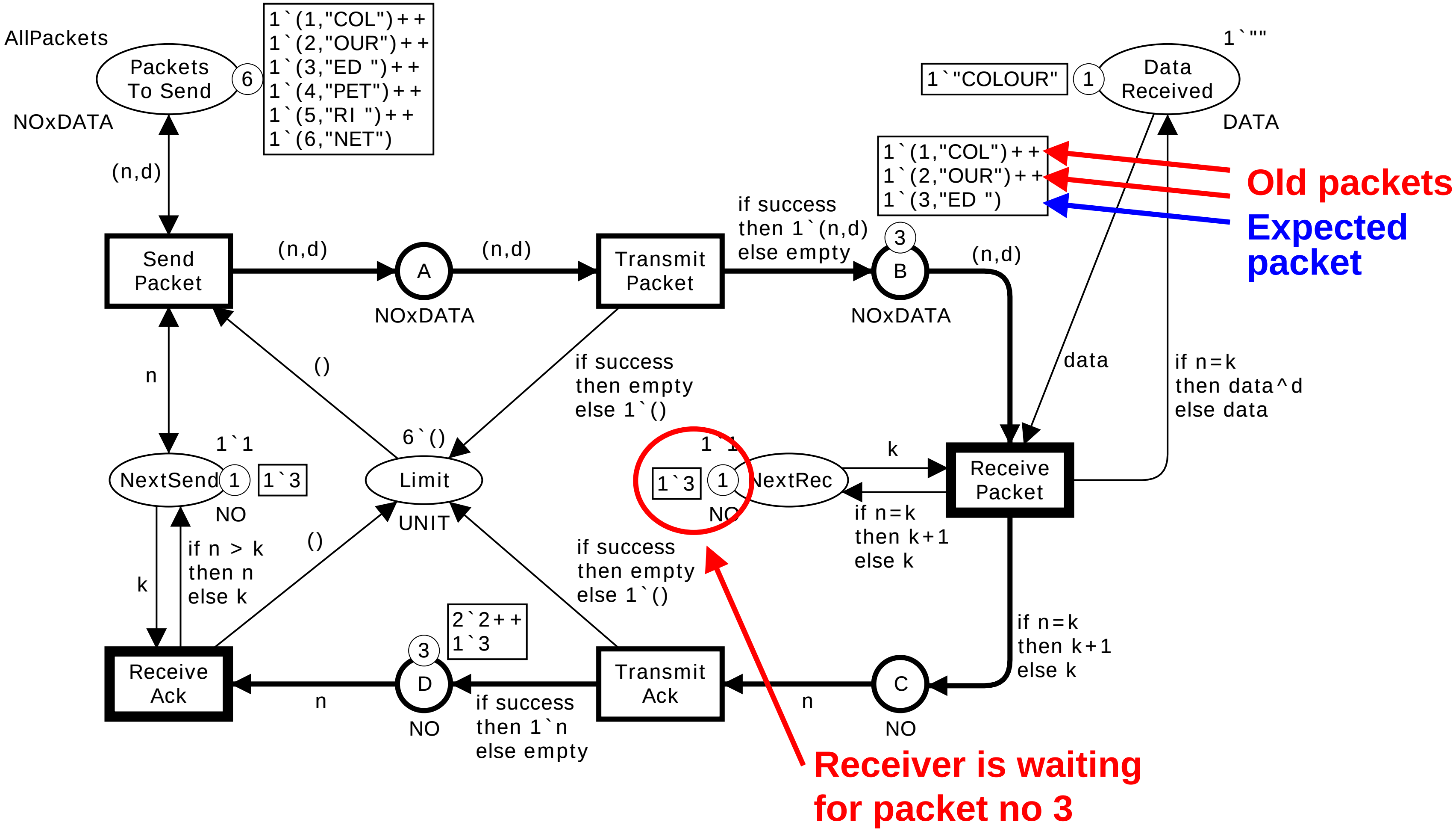
- In the equivalence method the equivalence relations are specified directly (without the use of symmetries).
- Soundness criteria: Equivalent markings must have equivalent sets of enabled binding elements and equivalent sets of successor markings.

Simple protocol (slightly modified)



The token colour on NextSend never decreases

Old packets



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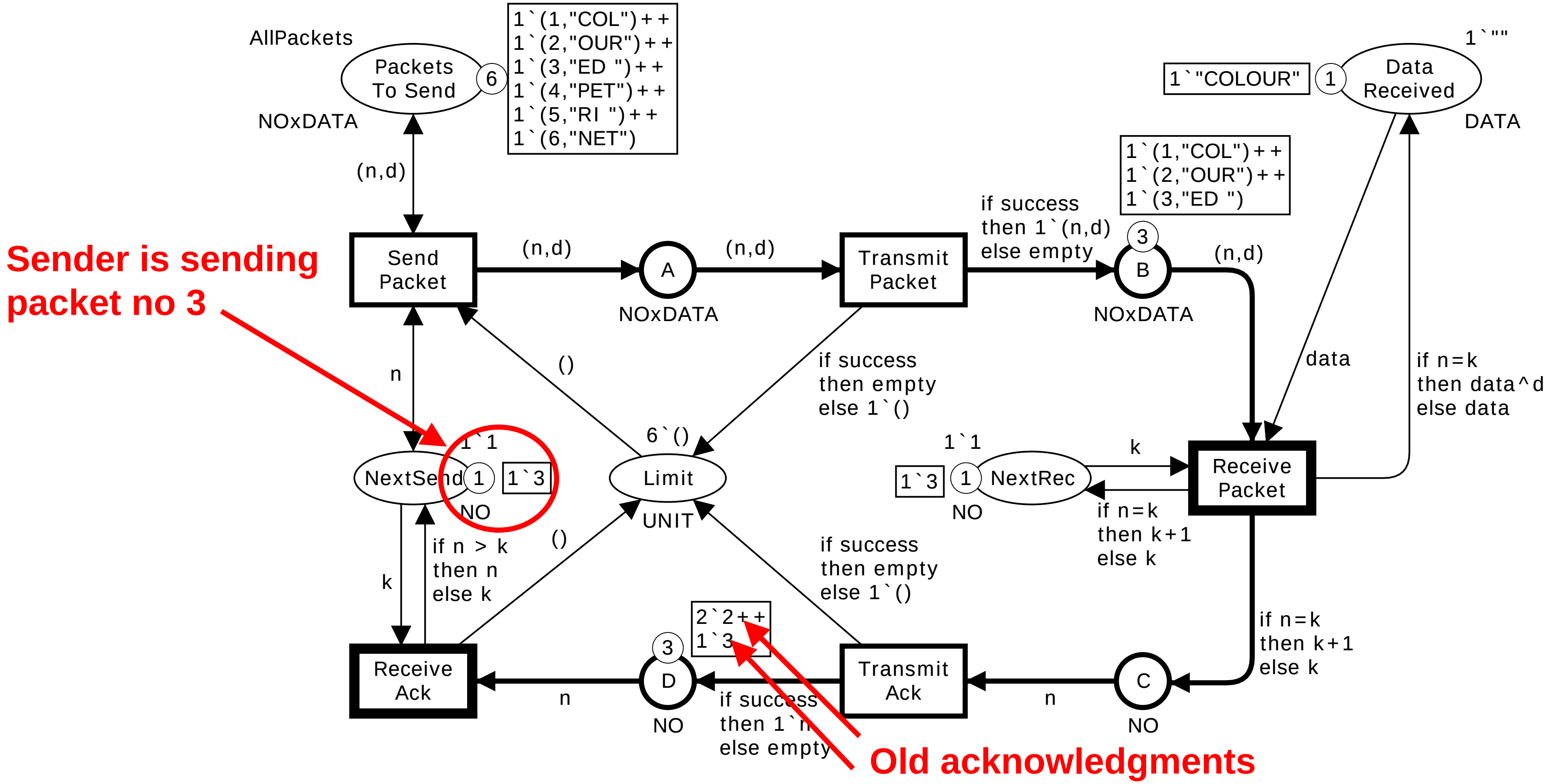
Old acknowledgments



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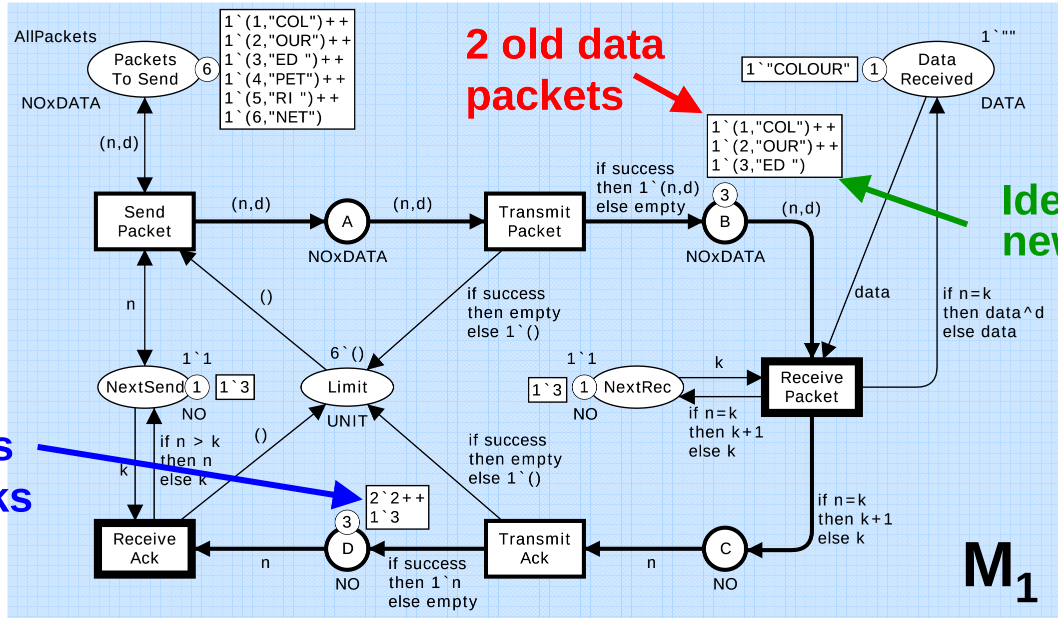
Equivalence relation for markings

- Basic idea:
  - Old data packets can be replaced by other old data packets.
  - Old acknowledgements can be replaced by other old acknowledgements.

- Two markings are equivalent if the following conditions hold:
  - Markings of A, B, C, and D: Identical non-old packets and the same number of old packets.
  - All other places must have identical markings.

Two equivalent markings

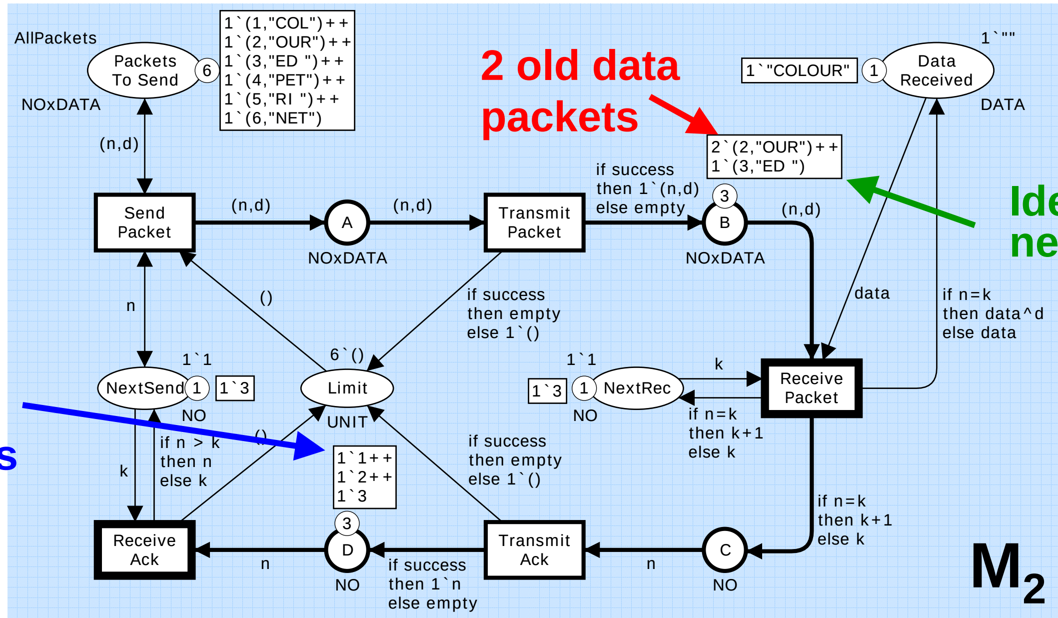


Identical new packet

3 old acks
0 new acks

All other places have identical markings

The two markings are equivalent to each other



Identical new packet

3 old acks
0 new acks

Equivalence relation for binding elements

- Two bindings of the same transition are equivalent to each other if they both involve old data packets or both involve old acknowledgements.
- All other binding elements are non-equivalent.

Statistics for equivalence method

Configuration		Standard method		Equivalence		Gain		
L	P	Nodes	Arcs	Nodes	Arcs	Nodes	Arcs	Time
1	4	33	44	33	44	1.00	1.00	1.00
2	4	293	764	155	383	1.89	1.99	1.00
3	4	1,829	6,860	492	1,632	3.72	4.20	0,90
4	4	9,025	43,124	1,260	5,019	7.16	8.59	1.56
5	4	37,477	213,902	2,803	12,685	13.37	18.86	4.09
6	4	136,107	891,830	5,635	28,044	24.15	31.80	13.58

L = Limit
P = Packets

Summary for equivalence method

- The equivalence method allows a more dynamic/general notion of equivalence than the symmetry method.
- Hence it can be used in situations where the symmetry method are of no use.

- The consistency proof must be done manually.
- The equivalence relations must be implemented manually (as ML functions).

- Later we shall see that the equivalence method can be used to reduce state spaces for timed CPN models (without manual consistency proof and with automatic implementation).

Multiple reduction methods

- It is often possible to simultaneously use two or more state space reduction methods.

- This leads to more reduction:
  - in CPU, and
  - memory usage

  than each method used in isolation.

- The sweep-line, symmetry, and equivalence methods can be used simultaneously with each other.

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