Sensitive vs Non-Sensitive Yeast

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1 Introduction of the Problem

In our project, we were tasked with taking data that was acquired via chemostat of yeast growth and make some analysis based on that data. There were two different strains of yeast being grown, one that was susceptible to a certain toxin and one that secreted the toxin that the first was susceptible to. Before we worked with the given data however, we needed to understand the experiment first and create a model.

In the given description of the chemostat experiment, we started off with a culture chamber that had an inflow of nutrients from a stock of reservoir, along with an outflow from the culture chamber that is at the same rate as the inflow. It is good to note that the flow in the model needs to not be too high so that the culture doesn't get entirely flushed out of the chamber. As such let us go over the model for this first.

2 Pertinent Models

With regards to the chemostat experiments, it bears to mention the fact that there was no competition in place with these experiments. The two different strains of yeast were grown in seclusion. As such we first needed to analyze the experiments as they happened before doing theoretical combinations.

The first thing we did was derive a system of ODE's that described the situation which are given as follows:

$$\frac{\delta N}{\delta t} = KCN - \frac{F}{V}N \tag{1}$$

$$\frac{\delta C}{\delta t} = -\alpha KCN - \frac{F}{V}C + \frac{F}{V}C_0 \tag{2}$$

In this model, N is representative of the population level of the yeast, C is the amount of nutrient available, K is the growth coefficient, C_0 is the steady amount of nutrient being supplied, while $\frac{F}{V}$ is the rate of flow into and out of the chemostat.

What this model stats is that the rate of growth of the population is the rate of growth of the yeast, given by $KCN - \frac{F}{V}N$ which says that it grows based on the growth coefficient, amount of food, and the existing level of population minus the amount that flows out in the general cycling of the chemostat. The model of the rate of change of nutrient is modelled by $-\alpha KCN - \frac{F}{V}C + \frac{F}{V}C_0$ which states that the change in nutrients is represented by the amount of nutrients being consumed by the yeast, which is based off of how much of the yeast is growing naturally, and then how much is flowing out and how much is flowing in from the reservoir, which is a constant amount.

With these two models, we were able to look at their steady states and find formulas for α and K. The derivation for this are as follows:

$$
\begin{aligned}
0 &= KCN - \frac{F}{V}N && \text{(Steady state solution)} \\
0 &= -\alpha KCN - \frac{F}{V}C + \frac{F}{V}C_0 && \text{(Steady state solution)} \\
C &= \frac{F}{VK} && \text{(Solving the first equation for C)} \\
0 &= -\alpha K\frac{F}{VK}N - \frac{F}{V}\frac{F}{VK} + \frac{F}{V}C_0 && \text{(Substitute in C)} \\
0 &= -\alpha N - \frac{F}{VK} + C_0 && \text{(Simplify)} \\
N &= -\frac{1}{K\alpha}\frac{F}{V} + \frac{C_0}{\alpha} && \text{(Rearrangement)}
\end{aligned}
$$

With this we have a linear equation. With it we took the data we were given and ran a linear regression, only using the data values up to a certain point. This was because the data seems to follow a linear patten until it essentially bottomed out, so we decided to model the linear portion in order to find K and α.

3 Competition Model Experiments

Armed with derived K and α, we are able to expand our model to form a competition model. One note for this is that a competition system was never tested, so all that is being done here is mostly theoretical. The modified model is as follows:

$$
\begin{aligned}
\frac{\delta N_1}{\delta t} &= K_1CN_1 - \gamma N_1N_2 - \frac{F}{V}N_1 \\
\frac{\delta N_2}{\delta t} &= K_2CN_2 - \frac{F}{V}N_2 \\
\frac{\delta C}{\delta t} &= -\alpha_1K_1CN_1 - \alpha_2K_2CN_2 - \frac{F}{V}C + \frac{F}{V}C_0
\end{aligned}
$$

In this model we are simply relating the two populations, with N_1 being the yeast sensitive to toxin and N_2 being the toxin-producing yeast. The added portion in the first equation is the portion of the toxin-sensitive yeast that are dying out as a result of the toxins. In this cast the γ is the 'toxicity coefficient' that governs the behaviour of this model.

After figuring out our αs and Ks, all that we had left to do is to mess with the γ value. While manipulating the γ value, it became clear that as γ trended toward 1 the toxin-sensitive yeast died out more quickly, which has an intuitive logic to it. When the the toxin-producing the yeast creates a more 'potent' toxin the sensitive yeast are more affected by it.

The following figures demonstrate how our competition model predicts yeast population behavior if the two populations were to be grown in the same chemostat. These figures more specifically display the relationship between various possible toxin potencies (γ) and flow-rates ($\frac{F}{V}$) through the chemostat.

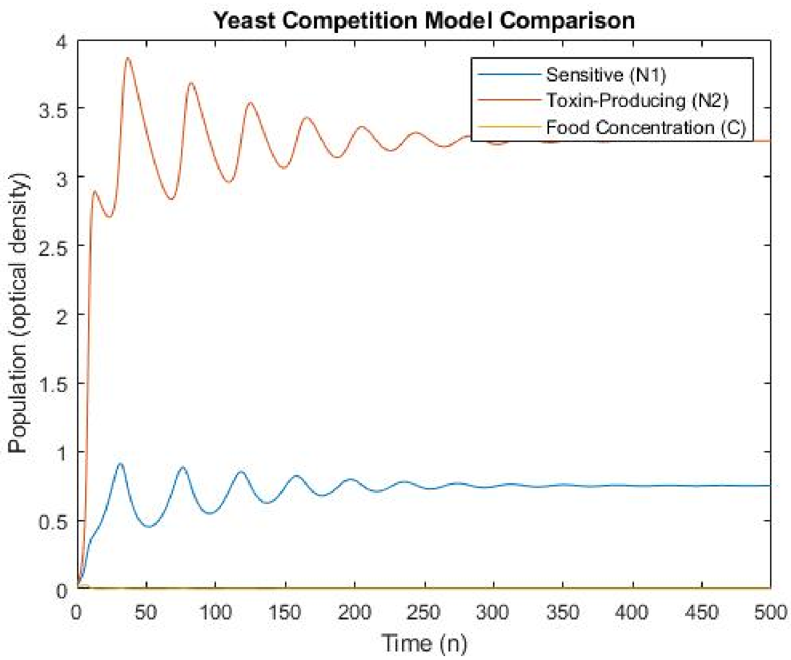

Figure 1: $\frac{F}{V} = 0.02$, $\gamma = 0.1$

When comparing these models in Fig.1, where only γ is varied, it becomes clear that as γ trends toward 1 the sensitive yeast struggles to compete with its the toxin-producing relative. Some specific phenomena that are observed when manipulating the γ and $\frac{F}{V}$ values include Fig.2, Fig.3, and Fig.4.

In Fig.2 we see an competitive model displayed in a less "violent" fashion. Where both populations grow at an rapid pace and then even out at the carrying capacity of those two specific yeast strains in that chemostat without the ebb-and-flow of your more traditional competition model, implying fierce competition with varying "battles" being won and lost.

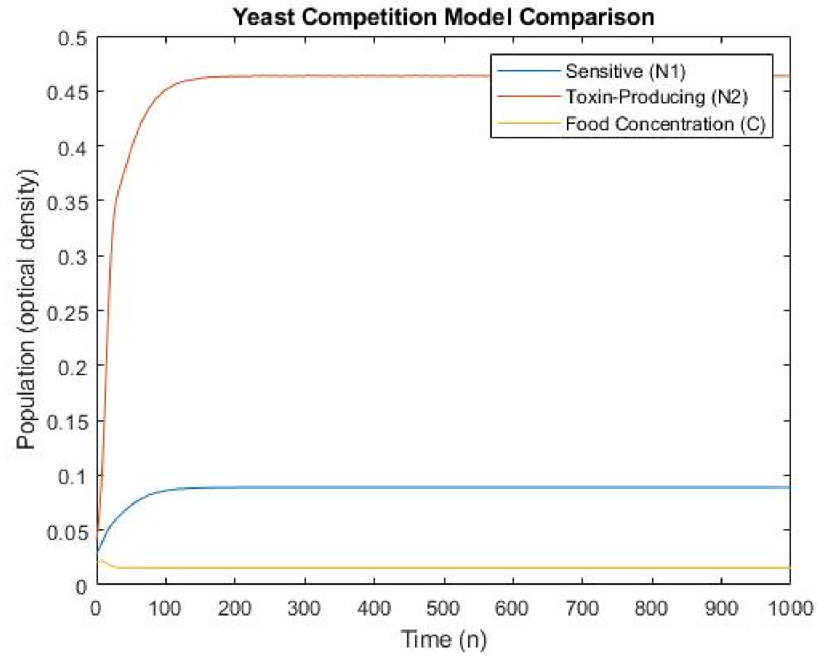

Figure 2: $\frac{F}{V} = 0.3$, $\gamma = 0.1$

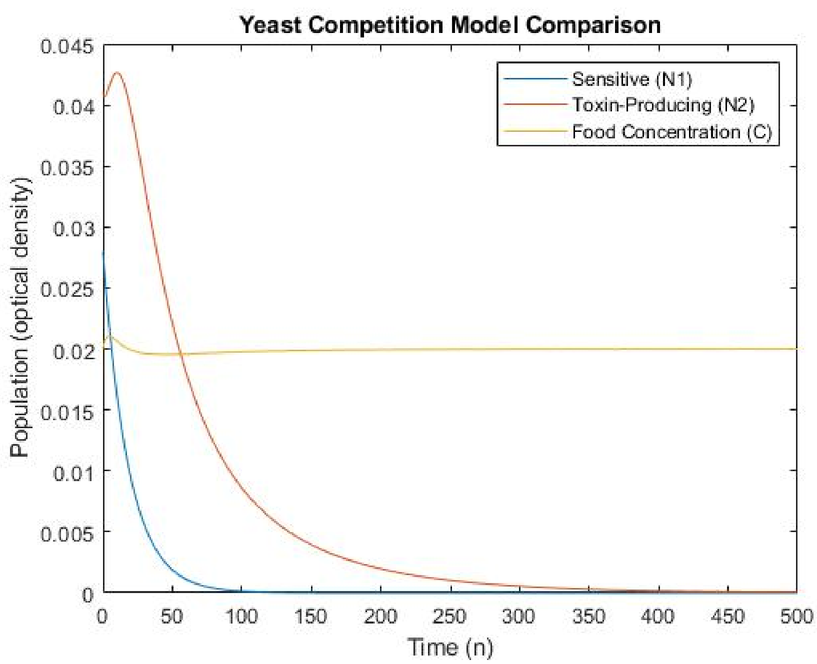

Figure 3: $\frac{F}{V} = 0.4$, $\gamma = 0.01$

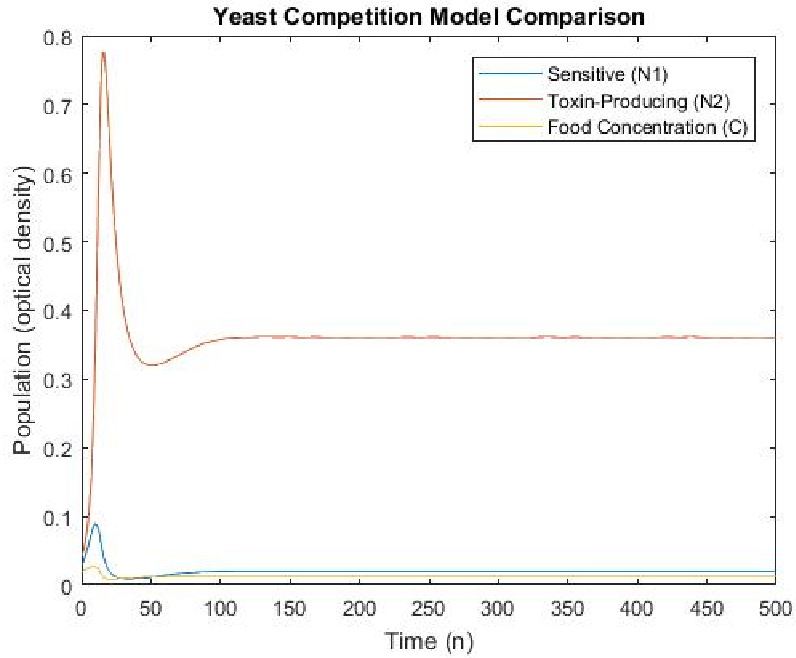

Figure 4: $\frac{F}{V} = 0.02$, $\gamma = 0.5$

Fig.3 likely demonstrates why the researchers who conducted this experiment stopped making observations above a flow-rate ($\frac{F}{V}$) of .4 or so. It would seem that is a $\frac{F}{V}$ enters this value range the environment becomes unsustainable for either yeast to survive it. We would assume that this may mean that the flow is too strong for the yeast populations stay in the chemostat, or something along that line of reasoning.

Fig.4 models what would happen if the killer yeast's toxin was just potent enough for the sensitive yeast to barely survive.

4 Conclusions and Discussion

The sensitive yeast's response to a stronger toxin (γ) varies from a general struggle to survive to being blatantly out-competed/killed by the killer-yeast when γ is greater than 0.4. Further data gathering may need to occur before a realistic competition model can be derived. One issue we encountered while modelling was a lack of data for certain vessel runs, making some of our linear-regressions not as statistically relevant as they could have been if more data had been available. It was interesting to see that there are more behaviors then just one species living and one dying. In particular, the model states in which both species simply reached an general carrying capacity without huge population spikes and declines was not something we had hypothesized as an potential outcome. Ultimately, it would be very interesting to compare and contrast the model we created and the behavior these two species exhibit when grown in the same chemostat.