Sheet 6

1) Obtain the coefficients of an FIR lowpass filter to meet the specifications given below using the window method

Passband edge frequency = 1.5 KHz
transition width = 0.5 KHz
Stopband attenuation > 50 dB
sampling frequency = 8 KHz

* Window method steps Summary:
1. Specify the "ideal" frequency response of filter "$H_D(\omega)$"
2. obtain the impulse response, $h_D(n)$ of the desired filter by evaluating inverse of Fourier transform "see table"
3. Select a window function
4. obtain values of $w(n)$, and values of the actual FIR coefficients
$h(n) = h_D(n) \cdot w(n)$

* From table 2 in the slides:

low pass filter: $h_D(0) = 2f_c$
$h_D(n), n \neq 0 = 2f_c \frac{\sin(n \cdot \omega_c)}{n \cdot \omega_c}$

* From table 3 in the slides:
Hamming and blackman can be used according to stopband attenuation

* So, we will use "Hamming Window" for the Simplicity.

→ Transition width = $3.3/N$

→ Window function = $0.54 + 0.46 \cdot \cos\left(\frac{2\pi n}{N}\right) \rightarrow |n| \leq \frac{N-1}{2}$

Now, Δf "transition width" = $0.5/8 = 0.0625$

From the table: $\Delta f = \frac{3.3}{N} \rightarrow$ So, $N = \frac{3.3}{\Delta f} = \frac{3.3}{0.0625} = 52.8$

So, we can say "$N = 53$"

_ Filter Coefficients are obtained from:

$h_D(n) \cdot w(n) \rightarrow -26 \leq n \leq 26$

$h_D(n) = 2f_c \rightarrow n = 0$

$h_D(n) = 2f_c \cdot \sin(n\omega_c) / n \cdot \omega_c \rightarrow n \neq 0$

$w(n) = 0.54 + 0.46 \cdot \cos(2\pi n/53) \rightarrow -26 \leq n \leq 26$

* Because, the Smearing effect of the window on the filter response, the Cutoff frequency of the resulting filter will be different from that given in the specifications.

_ Using f_c that is centered on the transition

$f_c' = f_c + (\Delta f/2)$

$= 1.5 + (0.5/2) = 1.75$ KHz → Normalized = $\frac{1.75}{8} = 0.21875$

* $h(n)$ is symmetrical, we need only compute values for $h(0), h(1), \ldots, h(26)$ and then use the symmetry property to obtain the other coeff.

$n=0 \rightarrow h_D(0) = 2f_c = 2 * 0.21875 = 0.4375$

$w(0) = 0.54 + 0.46 . \cos(0) = 1$

$\downarrow$

$h(0) = h_D(0) . w(0) = 0.4375$

$n=1 \rightarrow h_D(1) = \dfrac{2 * 0.21875}{2\pi * 0.21875} . \sin(2\pi * 0.21875) = 0.31219$

$w(1) = 0.54 + 0.46 . \cos(\frac{2\pi}{53}) = 0.98713$

$\downarrow$

$h(1) = h_D(1) . w(1) = 0.3119 = h(-1)$

Continue.

2) A requirement exists for an FIR digital filter to meet the following specifications

Passband → 150 - 250 Hz
Transition width → 50 Hz
Passband ripple → 0.1 dB
Stopband attenuation → 60 dB
Sampling frequency → 1 KHz

obtain the filter coefficients and spectrum using the window method

* According to "stopband attenuation" → Blackman will be suitable

* From "Table 2" in the slides

Band Pass $h_D(n) = \begin{cases} h_D(0) = 2(f_2 - f_1) \\ h_D(n) = 2f_2 \cdot \frac{\sin(n\omega_2)}{n\omega_2} - 2f_1 \cdot \frac{\sin(n\omega_1)}{n \cdot \omega_1} \end{cases}$

* From "Table 3" in the slides

→ Blackman method:

1. transition width = $5.5/N$

2. window func. = $0.42 + 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08 \cdot \cos\left(\frac{4\pi n}{N-1}\right)$

* Transition width = $50/1000 = 0.05$ Hz (1000 → Sampling freq)

* From the table : $\Delta f = \frac{5.5}{N} \rightarrow N = \frac{5.5}{0.05} = 110$

* Because of the smearing effect of the window on the filter response

$f_{c_1}' = f_{c_1} - \frac{\Delta f}{2}$ and $f_{c_2}' = f_{c_2} + \frac{\Delta f}{2}$

So, $f_{c_1}' = 150 - (50/2) = 125 \rightarrow$ Normalized : $f_{c_1}' = \frac{125}{1000} = 0.125$

$f_{c_2}' = 250 + (50/2) = 225 \rightarrow$ Normalized : $f_{c_2}' = \frac{225}{1000} = 0.225$

* To get the coefficients:

$n = 0 \rightarrow h_D(0) = 2(f_{c_2}' - f_{c_1}') = 2(0.225 - 0.125) = 2(0.1) = 0.2$

$w(0) = 0.42 + 0.5\cos(0) + 0.08\cos(0)$
$= 0.42 + 0.5 + 0.08 = 1$

$\downarrow$

$h(0) = h_D(0) \cdot w(0) = (0.2)(1) = 0.2$

$n = 1 \rightarrow h_D(1) = 2(0.225) \cdot \frac{\sin(2 \cdot \pi \cdot 0.225)}{2 \cdot \pi \cdot 0.225} - 2(0.125) \cdot \frac{\sin(2\pi \cdot 0.125)}{2\pi \cdot 0.125}$

$w(1) = 0.42 + 0.5 \cdot \cos\left(\frac{2\pi}{109}\right) + 0.08 \cos\left(\frac{4\pi}{109}\right)$

$\downarrow$

$h(1) = h_D(1) \cdot w(1)$