$\cdot\, e^{\frac{jkn2\pi}{N}}$

3) Compute the auto_correlation of the following signal using direct and fast method

$x = \{1,0,0,1\}$ and FFT of x is $\{2, 1+j, 0, 1-j\}$

Fast Method:

$r_{12}(j) = \frac{1}{N} FD^{-1}[X_1^*(K) \,.\, X_2(K)]$

$X_1(K) = \{2, 1+j, 0, 1-j\} = X_2(K)$ "Auto_Correlation"

$X_1^*(K) = \{2, 1-j, 0, 1+j\}$

$X_1^*(0) . X_1(0) = 2 * 2 = 4$

$X_1^*(1) . X_1(1) = (1-j)(1+j) = 1 - j^2 = 2$

$X_1^*(2) . X_1(2) = 0 * 0 = 0$

$X_1^*(3) . X_1(3) = (1+j).(1-j) = 1 - j^2 = 2$

So, $FD^{-1} = \{4,2,0,2\}$

$r_{12}(j) = \frac{1}{4} . FD^{-1}\{X_1^*(K) . X_2(K)\}$

$r_{12}(0) = \frac{1}{4}[(4*1) + (2*1) + (0*1) + (2*1)] = \frac{1}{4}[8] = \boxed{2}$

$r_{12}(1) = \frac{1}{4}[4 + (2*e^{\pi/2 j}) + 0 + (2*e^{3/2\pi j})]$

$= \frac{1}{4}[4 + \frac{1}{2}[\cos\frac{\pi}{2} + j\overset{1}{\sin}\frac{\pi}{2}] + 0 + \frac{1}{2}[\overset{0}{\cos}\frac{3\pi}{2} + j\sin\frac{3\pi}{2}]$

$= \frac{1}{4}[4 + \frac{1}{2}j + 0 - \frac{1}{2}j] = \boxed{1}$

$$r_{12}(2) = \frac{1}{4}[4 + 2.e^{\pi j} + 0 + 2e^{3\pi j}]$$

$$= \frac{1}{4}[4 + 2[\cos\pi + j\sin\pi] + 0 + 2[\cos 3\pi + j\sin 3\pi]]$$

$$= \frac{1}{4}[4 - 2 - 2] = \boxed{0}$$

$$r_{12}(3) = \frac{1}{4}[4 + 2e^{3/2\pi j} + 0 + 2e^{9/2\pi j}]$$

$$= \frac{1}{4}[4 + 2[\cos\frac{3\pi}{2} + j\sin\frac{3\pi}{2}] + 0 + 2[\cos\frac{9\pi}{2} + j\sin\frac{9\pi}{2}]]$$

$$= \frac{1}{4}[4 - 2 + 0 + 2] = \boxed{1}$$

So, $r_{12}(j) = \frac{1}{4}\{2, 1, 0, 1\} = \{\frac{1}{2}, \frac{1}{4}, 0, \frac{1}{4}\}$

* Direct Method: $(1, 0, 0, 1)$

$$r_{11}(j) = \frac{1}{N}\sum_{n=0}^{N-1} x_1(n).x_2(n+j)$$

$$r_{11}(0) = \frac{1}{4}[(1)(1) + (0)(0) + (0)(0) + (1)(1)] = \frac{2}{4} = \frac{1}{2}$$

$X_2 : (0, 0, 1, 1)$ → "Shifting Left"

$$r_{11}(1) = \frac{1}{4}[(1)(0) + (0)(0) + (0)(1) + (1)(1)] = \frac{1}{4}[1] = \frac{1}{4}$$

$X_2 : (0, 1, 1, 0)$

$$r_{11}(2) = \frac{1}{4}[(1)(0) + (0)(1) + (0)(1) + (1)(0)] = 0$$

$X_2 : (1, 1, 0, 0)$

$$r_{11}(3) = \frac{1}{4}[(1)(1) + (0)(1) + (0)(0) + (1)(0)] = \frac{1}{4}$$

4) Compute the normalized cross-correlation r12 of these two periodic signals

$$X_1 = \{2, 1, 0, 0, 3\}$$
$$X_2 = \{3, 2, 1, 1, 5\}$$

$$P_{12}(n) = \frac{r_{12}(n)}{\frac{1}{N}\left[\sum_{j=0}^{N-1} X_1^2(j) \cdot \sum_{j=0}^{N-1} X_2^2(n)\right]^{\frac{1}{2}}}$$

⊗ using direct Method $r_{12}(j) = \frac{1}{N} \sum_{n=0}^{N-1} X_1(n) \cdot X_2(n+j)$ to solve numerator

2	1	0	0	3	
3	2	1	1	5	→ 0
2	1	1	5	3	→ 1
1	1	5	3	2	→ 2
1	5	3	2	1	→ 3
5	3	2	1	1	→ 4
3	2	1	1	5	→ 5

$$r_{12}(0) = \frac{1}{5}\left[(2*3) + (1*2) + (3*5)\right] = \frac{1}{5} * 23 = \boxed{4.6}$$

$$P_{12}(0) = \frac{4.6}{\frac{1}{5}\left[\underbrace{(2^2+1^2+3^2)}_{14} \cdot \underbrace{(3^2+2^2+1^2+1^2+5^2)}_{40}\right]^{\frac{1}{2}}} = \boxed{\frac{4.6}{4.7}}$$

$$r_{12}(1) = \frac{1}{5}\left[(2*2) + (1*1) + (3*3)\right] = \frac{1}{5} * 14 = \boxed{2.8}$$

$$P_{12}(1) = \frac{2.8}{\frac{1}{5}\left[(2^2+1^2+3^2) \cdot (2^2+1^2+1^2+5^2+3^2)\right]^{\frac{1}{2}}} = \boxed{\frac{2.8}{4.7}}$$

$$r_{12}(2) = \frac{1}{5}\left[(2*1) + (1*1) + (2*3)\right] = \frac{9}{5} = \boxed{1.8}$$

$$P_{12}(2) = \frac{1.8}{\frac{1}{5}\left[(14) \cdot (40)\right]^{\frac{1}{2}}} = \boxed{\frac{1.8}{4.7}}$$

$r_{12}(3) = \frac{1}{5}[(1*2) + (5*1) + (1*3)] = \boxed{2}$

$P_{12}(3) = \frac{2}{\frac{1}{5}[(14)\cdot(40)]^{\frac{1}{2}}} = \boxed{\frac{2}{4.7}}$

$r_{12}(4) = \frac{1}{5}[(2*5) + (1*3) + (3*1)] = \frac{16}{5} = \boxed{3.2}$

$P_{12}(4) = \frac{3.2}{4.7}$

$r_{12}(5) = \frac{1}{5}[(2*3) + (1*2) + (3*5)] = \boxed{4.6}$

$P_{12}(5) = \boxed{\frac{4.6}{4.7}}$ and repeat.

Time delay:

1) Calculate correlation
2) find the max absolute value
3) save its lag (j)
4) Time delay= j * Ts