1. A discrete system can 1) linear or non linear 2) casual or non casual
   Examine the following systems with respect to the properties above.

① Causal: if output $y(n)$ of the system at any time n depends only in the present and past inputs, but doesn't depend on future inputs

② Linearity: if

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 \overbrace{T[x_1(n)]}^{y_1} + a_2 \overbrace{T[x_2(n)]}^{y_2}$$

a) Y(n) = cos(x[n])

① Causal

② $y_1(n) = \cos(x_1(n))$

$y_2(n) = \cos(x_2(n))$

$y_3(n) = T[a_1 x_1(n) + a_2 x_2(n)] = \cos[a_1 x_1(n) + a_2 x_2(n)] \rightarrow \underline{1}$

Right hand side $\rightarrow a_1 y_1 + a_2 y_2$

$\rightarrow a_1 \cos(x_1(n)) + a_2 \cos(x_2(n)) \rightarrow \underline{2}$

$\boxed{1 \neq 2}$ non-linear

b) Y(n) = x(-n + 2)

① non Causal because it depends on future samples

② $y_1(n) = x_1(-n+2)$

$y_2(n) = x_2(-n+2)$

$y_3(n) = T[a_1x_1(n) + a_2x_2(n)]$

$= a_1x_1(-n+2) + a_2x_2(-n+2) \rightarrow 1$

R.H.S $\rightarrow a_1y_1(n) + a_2y_2$

$\rightarrow a_1x(-n+2) + a_2x_2(-n+2) \rightarrow 2$

$1 = 2$ Linear

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c) y(n)=x(2n)

① non-Causal "depends on future samples"

② $y_1(n) = x_1(2n)$

$y_2(n) = x_2(2n)$

$y_3(n) = T[a_1x_1(n) + a_2x_2(n)]$

$= a_1x_1(2n) + a_2x_2(2n) \rightarrow 1$

R.H.S $\rightarrow a_1y_1 + a_2y_2$

$\rightarrow a_1x_1(2n) + a_2x_2(2n) \rightarrow 2$

$1 = 2$ Linear

d) y(n) = |x(n)|

① Causal "doesn't depend on future samples"

② $y_1(n) = |x_1(n)|$

$y_2(n) = |x_2(n)|$

$y_3(n) = T[a_1 x_1(n) + a_2 x_2(n)]$

$= |a_1 x_1(n) + a_2 x_2(n)| \rightarrow 1$

R.H.S $\rightarrow a_1 y_1 + a_2 y_2$

$\rightarrow a_1 |x_1(n)| + a_2 |x_2(n)| \rightarrow 2$

$\neq$ non-linear

e) y(n) = Round[x(n)] integer part of x(n)

① Causal "doesn't depend on future"

② $y_1(n) = R[x_1(n)]$

$y_2(n) = R[x_2(n)]$

$y_3(n) = T[a_1 x_1(n) + a_2 x_2(n)]$

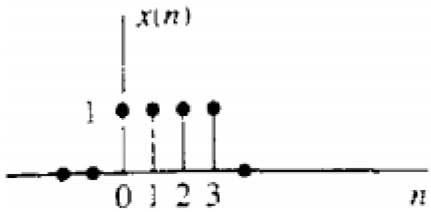
$= R[a_1 x_1(n) + a_2 x_2(n)] \rightarrow 1$

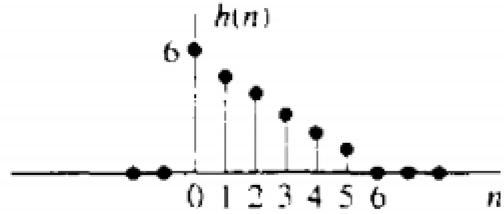
R.H.S $\rightarrow a_1 y_1 + a_2 y_2$

$\rightarrow a_1 R[x_1(n)] + a_2 R[x_2(n)] \rightarrow 2$

$\neq$ non-linear

2. Compute the convolution y(n) = x(n) * h(n)





$$x(n) = \{\overset{0}{1}, \overset{1}{1}, \overset{2}{1}, \overset{3}{1}\}$$

$$h(n) = \{\overset{0}{6}, \overset{1}{5}, \overset{2}{4}, \overset{3}{3}, \overset{4}{2}, \overset{5}{1}\}$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k)$$

$$\boxed{y(n) = \sum_{k=min\_i\_x}}$$

$$min\_i\_x + min\_i\_h \leq n \leq max\_i\_x + max\_i\_h$$

$$y(0) = x(0) \cdot h(0) = 6 * 1 = 6$$

$$0 \leq n \leq 8$$

$x(n\uparrow)$ $\quad$ $h(n\downarrow)$

$$y(1) = x(0) \cdot h(1) + x(1)h(0) = (1*5) + (1*6) = 11$$

$$y(2) = x(0)h(2) + x(1)h(1) + x(2)h(0)$$
$$= (1*4) + (1*5) + (1*6) = 15$$

$$y(3) = x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0)$$
$$= (1*3) + (1*4) + (1*5) + (1*6) = 18$$

$$y(4) = x(0)h(4) + x(1)h(3) + x(2)h(2) + x(3)h(1) + \cancel{x(4)h(0)}$$
$$= (1*2) + (1*3) + (1*4) + (1*5) = 14$$

$$y(5) = x(0)h(5) + x(1)h(4) + x(2)h(3) + x(3)h(2) + \cdots$$

$$= (1*1) + (1*2) + (1*3) + (1*4) = 10$$

$$y(6) = \cancel{x(0)h(6)} + x(1)h(5) + x(2)h(4) + x(3)h(3) + \cdots$$

$$= 0 + (1*1) + (1*2) + (1*3) = 6$$

$$y(7) = \cdots + x(2)h(5) + x(3)h(4) + \cdots = 1+2 = 3$$

$$y(8) = \cdots + x(3)h(5) + \cdots = (1*1) = 1$$

$$y(n) = \{\underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1\}$$

3. Compute the convolution y(n) = x(n) * h(n)

$$x(n) = \begin{cases} 1, & n = -2, 0, 1 \\ 2, & n = -1 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$$

$x(n) = \{1, 2, \underset{\uparrow}{1}, 1\}$

$h(n) = \{\underset{\uparrow}{1}, -1, 0, 0, 1, 1\}$

$$\left\{\begin{array}{l} h(0) = \delta(0) - \delta(-1) + \delta(-4) + \delta(-5) = 1 \\ h(1) = \delta(1) - \delta(0) + \delta(-3) + \delta(-4) = -1 \\ h(2) = \delta(2) - \delta(1) + \delta(-2) + \delta(-3) = 0 \\ h(3) = \delta(3) - \delta(2) + \delta(-1) + \delta(-2) = 0 \\ h(4) = \delta(4) - \delta(3) + \delta(0) + \delta(-1) = 1 \\ h(5) = \delta(5) - \delta(4) + \delta(1) + \delta(0) = 1 \end{array}\right.$$

$$y(n) = \sum_{k=\min_x = -2}^{\infty} x(k)h(n-k)$$

$(-2+0) \le n \le (1+5)$

$-2 \le n \le 6$

$y(-2) = x(-2)h(0) + x(-1)h(-1) + x(0)h(-2) + \cdots$
$= (1*1) = 1$

$y(-1) = x(-2)h(1) + x(-1)h(0) + x(0)h(-1)$
$= (1*-1) + (2*1) + (1*0) = 1$

$y(0) = x(-2)h(2) + x(-1)h(1) + x(0)h(0) + x(1)h(-1) + \cdots$
$= (1*0) + (2*-1) + (1*1) = -1$

$y(1) = x(-2)h(3) + x(-1)h(2) + x(0)h(1) + x(1)h(0) + \cdots$
$= (1*0) + (2*0) + (1*-1) + (1*1) = 0$

$$y(2) = x(-2)h(4) + x(-1)h(3) + x(0)h(2) + x(1)h(1) + x(2)\cancel{h(0) + \cdots}$$
$$= (1*1) + (2*0) + (1*0) + (1*-1) = 0$$

$$y(3) = x(-2)h(5) + x(-1)h(4) + x(0)h(3) + x(1)h(2)$$
$$= (1*1) + (2*1) + (1*0) + (1*0) = 3$$

$$y(4) = x(-2)h(6) + x(-1)h(5) + x(0)h(4) + x(1)h(3)$$
$$= 0 + 2 + 1 + 0 = 3$$

$$y(5) = x(-2)h(7) + x(-1)h(6) + x(0)h(5) + x(1)h(4)$$
$$= 0 + 0 + 1 + 1 = 2$$

$$y(6) = x(-2)h(8) + x(-1)h(7) + x(0)h(6) + x(1)h(5)$$
$$= 0 + 0 + 0 + 1 = 1$$

$$y(n) = [1, 1, -1, 0, 0, 3, 3, 2, 1]$$

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