4 Interest Rates

Interest rates are among the most closely watched variables in the market everyday. The news regularly reports changes in interest rates as they impact our everyday lives and have important consequences for the health of the economy. They influence personal decisions such as whether to buy a new car/house and how much to save. Businesses also make important investment decisions based on interest rate movements.

4.1 Measuring Interest Rates

Different debt instruments have very different streams of cash flows (payments to holder) with very different timing. To compare the value of two different debt instruments, we use the following:

Def^n: *Present Value* - a dollar paid to you one year from now is less valuable than a dollar paid to you today.

Example: 4.1. *Simple Loan*

Suppose your bff Jill asks to borrow \$100. You agree if she agrees to pay you \$110 next year. Thus, the interest rate:

$$i = \frac{\$10}{\$100} = 0.10 = 10\% \tag{4.1}$$

Equation 4.1 can be rewritten:

$$\$100 \times (1 + 0.10) = \$110$$

Suppose you make a \$110 loan at the end of the first year. At the end of the second year:

$$\big[\$100 \times (1 + 0.10)\big] \times (1 + 0.10) = \$100 \times (1 + 0.10)^2 = \$121$$

Continuing with another loan again, at the end of the third year:

$$\$100 \times (1 + 0.10)^3 = \$133$$

In general, if we make a loan for \$100 that capitalizes interest every year, your investment would turn into:

$$\$100 \times (1 + i)^n \tag{4.2}$$

Today 0	Year 1	Year 2	Year 3	Year n
\$100	\$110	\$121	\$133	$\$100 \times (1 + 0.1)^n$

This timeline tells us that you are just as happy have \$100 today as having \$110 a year from now. This also means that we can work backwards to put future cash payments in terms of today. In other words, we are discounting the future which leads to the following general formula:

$$PV = \frac{CF}{(1 + i)^n} \tag{4.3}$$

Present Value formula is extremely useful. We can calculate a price/value of a debt/credit market instrument at a give interest rate of i by adding up the individual present values of all future payments received.

Example: 4.2. *What is the present value of \$100 to be paid in 18 years if the interest rate is 5.2%? (\$40)*

Example: 4.3. *Assume you hit the lottery for \$20 million promising to pay \$1 million for the next 20 years. Have you really won \$20 million?*

No. Assuming the interest rate of 10%, the present value of the \$20 million in payments is:

$$\begin{aligned} PV =& \$1,000,000 + \frac{\$1,000,000}{1.10} + \frac{\$1,000,000}{(1.10)^2} + \frac{\$1,000,000}{(1.10)^3} + \cdots + \frac{\$1,000,000}{(1.10)^{19}} \\ =& \$1,000,000 + \$909,090 + \$826,446 + \$751,314 + \cdots + \$148,643 \approx \$9.4M \end{aligned}$$

4.1.1 Four Types of Credit Market Instruments

1. Simple loan
   - Lender provides funds which must be repaid at maturity date with interest
2. Fixed-payment loan (full amortized loan) (e.g. Mortgages)
   - Lender provides funds which must be repaid by making the same payment every period.
   - Payment consists of interest and principle.
3. Coupon Bond
   - Pays owner a fixed interest payment (coupon rate: c) every year until maturity date when the face value(FV) is repaid.

4. Discount Bond (zero-coupon bond)
   - Purchased at price below face value which is paid at maturity.

4.1.2 Yield to Matruity (IRR)

Def^n: *Yield to Maturity* - The interest rate that equates the PV of cash flow payments from a debt instrument with its value today

Example: 4.4. *Simple Loan & Yield to Maturity*

If you make \$100 loan to your bff and she pays you back \$110 next year, what is the yield to maturity?

$$PV = \frac{CF}{(1+i)^n} \quad \Rightarrow \quad \$100 = \frac{\$110}{(1+i)^1} \quad \Rightarrow \quad i = 10\%$$

Example: 4.5. *Fixed-Payment Loan & Yield to Maturity*

If take out a loan for \$15,000 agreeing to pay \$306.25/month for 5 years, what is the yield to maturity?

$$LV = \frac{FP}{(1+i)} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \cdots + \frac{FP}{(1+i)^n}$$

In this example, $n = 60$, $FP = \$306.25$, & $LV = \$15,000$. Because this formula assumes i is yearly, we have to adjust the formula such that we account for monthly payments. In other words i must be divided by 12 to get the correct yield to maturity (8.294%).

Example: 4.6. *Coupon Bond & Yield to Maturity*

If the price of a 2 year \$1,000 Coupon Bond with a coupon rate of 10% is \$1,150, what is the yield to maturity?

$$\$1,150 = \frac{10\% \times \$1,000}{(1+i)} + \frac{10\% \times \$1,000 + \$1,000}{(1+i)^2} \Rightarrow i = 2.246\%$$

Example: 4.7. *Perpetuity (Consol) & Yield to Maturity*

What is the yield to maturity on a bond that has a price of \$2,000 and pays \$100 forever?

$$P_c = \frac{C}{i_c} \quad \Rightarrow \quad i_c = \frac{C}{P_c} = \frac{\$100}{\$2,000} \quad \Rightarrow i_c = 5\%$$

where C is the yearly payment, P_c is the price of the consol, & i_c is the yield to maturity of the perpetuity or consol.

Example: 4.8. *Discount Bond & Yield to Maturity*

What is the yield to maturity on a one-year discount bond with a face value of \$1,000 and can be purchased for \$900 today?

$$\$900 = \frac{\$1,000}{1+i}$$

$$\$900 \times (1+i) = \$1,000$$

$$\Rightarrow i = \frac{\$1,000 - \$900}{\$900} = 0.111 = 11.1\%$$

An important takeaway regarding the relationship between bonds & interest rates is that the price of a bond today is inversely related to interest rates. When the interest rate rises, the price of the bond falls, and vice versa.

4.2 Distinction between Interest Rates & Returns

Def^n: *Rate of Return* - The payments to the owner plus the change in value expressed as a fraction of the purchase price:

$$R = \frac{C + P_{t+1} - P_t}{P_t} \quad = \quad \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t} \tag{4.4}$$

where R is the return from holding the bond from time t to time $t+1$, P_t & P_{t+1} is the price of the bond at time t & $t+1$ respectively, and C is the coupon payment.

If we let $i_c = C/P_t$ is the current yield, & $\frac{P_{t+1}-P_t}{P_t}$ is the rate of capital gain: g. Eq. 4.4 can be rewritten as:

$$R = i_c + g \tag{4.5}$$

Eq. 4.5 shows the return on a bond is the current yield i_c plus the rate of capital return g. This is a key point: **the return can differ substantially from the interest rate.**

Example: 4.9. *Returns vs. Yield to maturity on Coupon bond*

Suppose we purchase a 10% coupon bond for \$1,000. It has a face value of \$1,000 and matures in 10 years. Initially, the yield to maturity equals 10%. After one year, we collect one coupon payment, then sell it the bond for \$1,200. The one year holding period return for this bond:

$$R = \frac{\$100}{\$1,000} + \frac{\$1,200 - \$1,000}{\$1,000} = 0.30 = 30\%$$

Although the yield to maturing is 10%, the rate of return is 30%.

Key findings generally true for all bonds:

- The return equals the yield to maturity only if the holding period equals the time to maturity
- A rise in interest rates is associated with a fall in bond prices, resulting in capital losses on bonds whose terms to maturity are longer than that holding period.
- The longer the time to maturity, the greater the size of the percentage price change associated with an interest rate change
- The longer the time to maturity, the lower the rate of return that occurs as a result of the increase in the interest rate
- Even if a bond has a substantial initial interest rate, its return can be negative if interest rates rise.

TABLE 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year* ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return [col (2) + col (5)] (%)
30	10	1,000	503	−49.7	−39.7
20	10	1,000	516	−48.4	−38.4
10	10	1,000	597	−40.3	−30.3
5	10	1,000	741	−25.9	−15.9
2	10	1,000	917	−8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated with a financial calculator, using Equation 3.

Figure 1: Returns vs. Yield to Maturity Rates

Potentially, purchasing a bond can turn out to be a poor investment. To see this, remember that a rise in interest rates means the bond price has fallen which therefore implies a capital loss has occurred. If the loss is large enough, a bond can have a negative return. For example, Figure 6 shows that a bond with 30 years to maturing when purchased has a capital loss of 49.7% when interest rates rise from 10% to 20%. This loss exceeds the current yield of 10% resulting in a negative return.

4.2.1 Maturity & Volatility of Bond Returns

Because of the potential for a negative return from interest rate changes, prices and returns for long-term bonds are more volatile than shorter-term bonds.

Def^n: *Interest Rate Risk* - The riskiness of an assets return that results from interest rate changes

4.3 Real vs. Nominal Interest Rates

Def^n: *Nominal Interest* - Contract interest rate or Interest rate today. Does not acccount for inflation

Def^n: *Real Interest* - Adjusts for inflation

Fischer Equation:

$$i = r + \pi^e \quad \Rightarrow \quad r = i - \pi^e \tag{4.6}$$

Example: 4.10. *Calculating Real Interest Rates*

Suppose your bff Jill wants to borrow \$100. She agrees to pay 8% interest. If the expected rate of inflation is 10% over the course of the year, what is the real interest rate?

$$\begin{aligned} r &= i - \pi^e \\ &= 8\% - 10\% = -2\% \end{aligned}$$

When the real interest rate is low, there is a greater incentive to borrow and fewer to lend.

5 Behavior of Interest Rates

One reason why we study money & bankings is to provide some explanation to the fluctuations in interest rates. Previously, we saw that interest rates are negatively related to the price of bonds. Therefore, if we can explain why bond prices changes, we can also explain why interest rates change. To do this, we use portfolio theory then apply this to a supply and demand analysis for bond & money markets.

5.1 Asset Demand

First, before looking at the supply and demand for bond/money markets, we should understand what determines the quantity demanded of an asset. Recall that an asset is anything that stores value. Items such as art, land, houses, cocaine, money, bonds, and stocks are all considered assets. When determining whether to buy an asset, individuals should consider the following:

- Wealth
  - Not surprisingly, as our wealth increases ⇒ the quantity of assets we demand increases
- Expected Return
  - All else equal, an increase in the expected return of an asset relative to alternatives will raise the quantity demanded of the asset
- Risk
  - All else equal, an increase in the risk of an asset relative to alternatives will decrease the quantity demanded of the asset

  e.g. Consider two firms:

  Firm A has an expected return of 15% half the time & 5% half the time (10% average)

  Firm B has an expected return of 10% all the time

  Depending on how risky you are your choice will vary (*risk adverse/ risk neutral/ risk loving*)
- Liquidity
  - All else equal, an increase in the liquidity of an asset relative to alternatives will raise the quantity demanded of the asset

5.2 Supply & Demand in the Bond Market

Recall the existence of a negative relationship between bond prices and interest rates. In other words, as interest rates rise, bond prices fall, and vice versa.

5.2.1 Demand Curve

Consider the demand for one-year discount bonds with a face value of $1,000. Since the holding period is one year, then the rate of return (R^e) on the bond is exactly equal to the interest rate as measured by the yield to maturity.

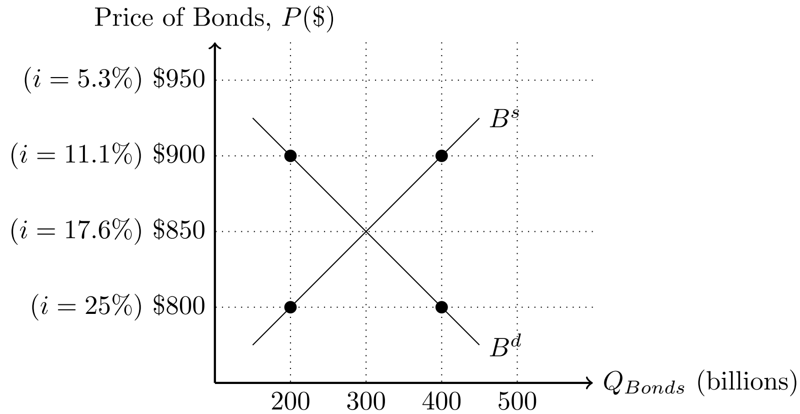
$$i = R^e = \frac{FV - PV}{PV} \tag{5.1}$$

If the bond sells for ($PV =$) $900, then the interest rate & expected rate of return:

$$\frac{\$1,000 - \$900}{\$900} = 0.0111 = 11.1\%$$

If the bond sells for ($PV =$) $800, then the interest rate & expected rate of return:

$$\frac{\$1,000 - \$800}{\$800} = 0.25 = 25\%$$



5.2.2 Supply Curve

Deriving the supply curve can be done using the same intuition from any micro market. If the price of bonds is \$800, the implied interest rate is 25%. If the price were to raise to \$900, firms are more likely to issue bonds. This occurs because they could offer a smaller discount and thus, pay a smaller interest rate making them more likely to borrow.

5.2.3 Market Equilibrium

Occurs when:

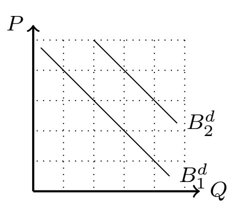
$$B^d = B^s \tag{5.2}$$

The market will tend to move to this equilibrium point. In the above figure, equilibrium price: $P = \$900$ & equilibrium quantity: $Q_{bonds} = 300$ billion.

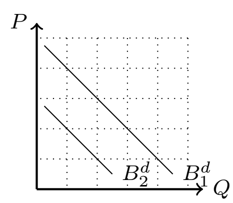
When price is above equilibrium ($P > \$850$), there is excess supply. Pice pushed downward toward equilb. When price is below equilibrium ($P < \$850$), there is excess demand. Price pushed upward toward equilb.

5.2.4 Shifting Bond Demand

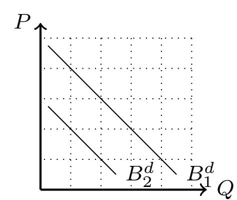
Remember from previous economic courses, a change in demand/supply is a shift of the curve. In this context, when something outside of price impacts the demand/supply, the curve is shifted implying a change in quantity demanded/supplied at each an every price. Do not confuse this with a change in quantity demanded/supplied that results from a price change.



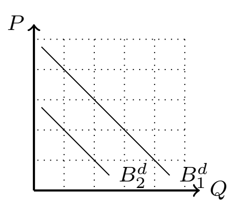
1. Increases in Wealth



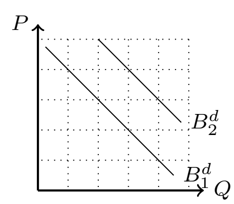
2. Higher Expected Interest Rate



3.Higher Expected Inflation

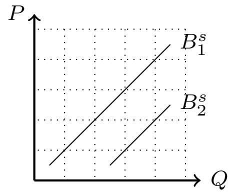


4. Higher Relative Riskiness

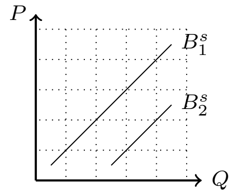


5. Higher Relative Liquidity

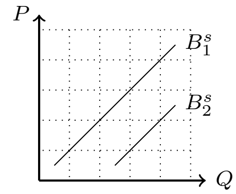
5.2.5 Shifting Bond Supply



1. Profitability of Investments

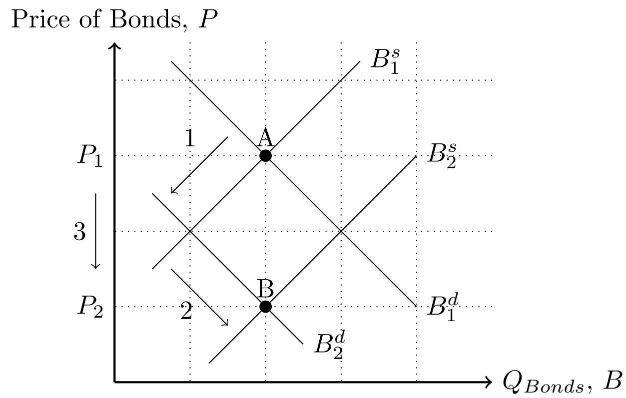


2. Higher Expected Inflation



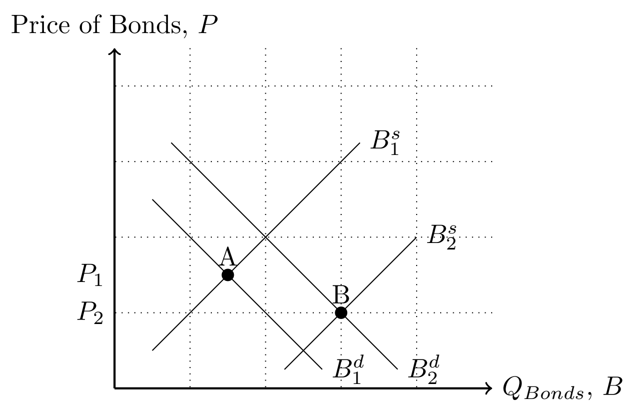
3. Govt. Deficit

Response to a Rise in Expected Inflation



1. A rise in expected inflation shifts the bond demand curve leftward
2. and also shifts the bond supply curve rightward
3. causing the price of bonds to fall and equilibrium interest rate to rise.

Response to a Business Cycle Expansion



1. A business cycle expansion shifts B^d outward because of the rising national income
2. Higher income means businesses will be more likely to borrow to fund investment shifting B^s right
3. Changes in the price depends on which curve shifts further. Typically, data shows the above outcome will occur.

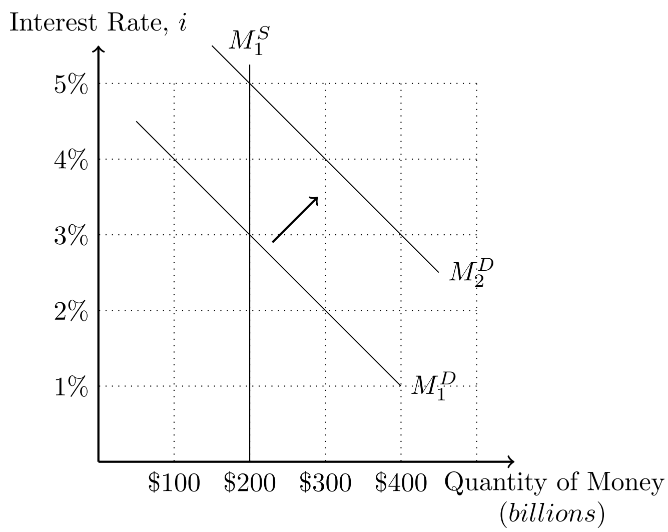
5.3 Money Market: Liquidity Preference Framework

An alternative method of determining equilibrium interest rates is the Market for Money developed by John Keynes. This is know as the *liquidity preference framework*.

Assumption: People can either store wealth by holding Money and/or Bonds.

$$B^s + M^s = B^d + M^d \text{ or} \tag{5.3}$$
$$B^s - B^d = M^d - M^s \tag{5.4}$$

Equation 5.4 tells us that if the maker for money is in equilibrium ($M^d = M^s$), the bond market must also equal zero implying it must too also be in equilibrium.



5.3.1 Shifting the Money Demand

1. Income Effect
   - Higher income causes demand for money at each interest rate to increase shifting M^d out.
2. Price Level Effect
   - Higher price level causes the demand for money at each interest rate to increase shifting M^d out.

5.4 Money Growth & Interest Rates?

6 Risk and Term Structure of Interest Rates

In the previous chapter, we learned the supply and demand analysis of interest rate behavior. Here, we will examine the relationship of various interest rates to one another.

Def^n: *Risk Structure* - The relationship among the interest rates of bonds with the same term to maturity

6.1 Risk Structure of Interest Rates

<u>Default Risk</u> is one attribute of a bond that will influence the interest rate. This occurs when the issuer of the bond is unable/unwilling to make interest payments when promised or unable to pay the face value at maturity.

In the Fall on 2013, debt ceiling negotiations in Congress halted as Republications refused to increase the debt ceiling. This lead to the govt. shutting down and credit rating agencies were considering lowering the U.S. bond rating as they were weary of politicians ability to reach a resolution.

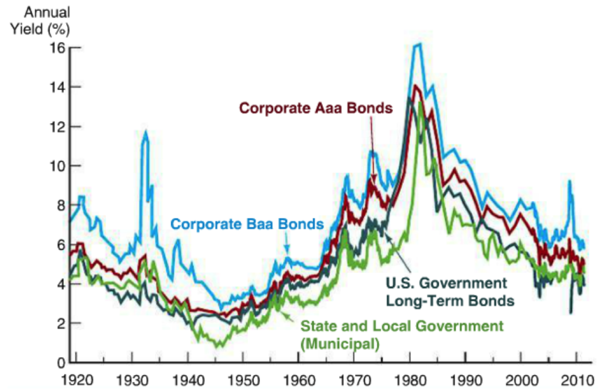


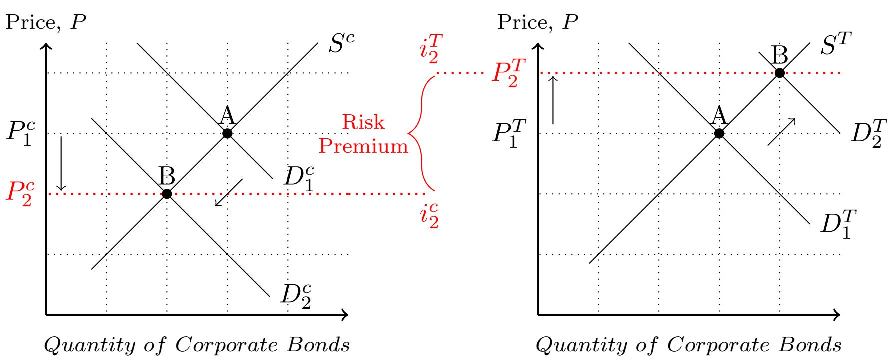
Figure 2: Long Term Bond Yields, 1919-2011

<u>Risk Premium</u> is the spread between interest rates on bonds with risk & default-free bonds (same maturity). It indicates how much additional interest people must earn to be wiling to hold that risky bond

Q: *Why do corporate bonds have higher yields than government bonds?*

A1: Government bonds are considered **default-free bonds** as they have almost zero default risk. The govt. can print more money/increase taxes to pay debt.

A2: Corporate bonds are more likely to suspend interest payments during bad economic times.



Assume initially both corporate bonds & Treasury bonds both have the same attributes (risk/maturity). This means $P_1^c = P_1^T$ & therefore $i_1^c = i_1^T$ implying a zero *risk premium:* ($i_1^c - i_1^T$=0).

- Suppose a corporation suffers large losses $\Rightarrow$ Default Risk $\uparrow$ causing D_1^c to decrease to D_2^c.
- Simultaneously, the expected relative return of Treasury Bonds $\uparrow$ making them more desirable causing D_1^T to increase to D_2^T.

$\Rightarrow$ Conclude: a bond with default risk will always have a risk premium > 0 &

$\Rightarrow$ the higher the default risk, the higher the risk premium

Bond Ratings by Moody's, Standard and Poor's, and Fitch

	Rating		
Moody's	**S&P**	**Fitch**	**Definitions**
Aaa	AAA	AAA	Prime Maximum Safety
Aa1	AA−	AA−	High Grade High Quality
Aa2	AA	AA	
Aa3	AA−	AA−	
A1	A+	A+	Upper Medium Grade
A2	A	A	
A3	A−	A−	
Baa1	BBB+	BBB+	Lower Medium Grade
Baa2	BBB	BBB	
Baa3	BBB−	BBB−	
Ba1	BB+	BB+	Noninvestment Grade
Ba2	BB	BB	Speculative
Ba3	BB−	BB−	
B1	B−	B−	Highly Speculative

Figure 3: Bond Rating Categories

Because default risk is so important to the size of the risk premium, bond buyers need to know the likelihood of default. This information is provided by:

Credit Rating Agencies assess quality of corporate & govt. bonds in terms of the probability of default.

Junk Bonds are bonds with a rating of *Baa* and below. They are dubbed speculative-grade due to their high risk of default.

Other factors influencing the risk premium:

- Default Risk *(see above)*
- Liquidity

  Q: If liquidity of a corporate bond decreases, what will happen to the risk premium?

  A: The risk premium will increase. As liquidity of corporate bonds relative to Treasury bills decrease, they become less desirable causing D_1^c to decrease to D_2^c just as before. Simultaneously, Treasury Bonds become more desirable causing D_1^T to increase to D_2^T just as before

- Income Tax Considerations

  Municipal (state & local) bonds are not risk free and are not as liquid as U.S. Treasury bonds. If so:

  Q: Why would you ever buy municipal bonds with lower interest rates, higher risk of default, and less liquid than U.S. Treasury bonds?

  A: Interest payments are exempt from federal income tax.

  Example: 6.1. *Income Tax Considerations*

  Suppose you make enough money to put you in the 35% tax bracket. This implies that every additional dollar you earn, you will pay $0.35 to the government. Consider two scenarios:

  1. Buy $1,000 U.S. Treasury Bond with 10% coupon rate
     - Each year, you'll make $100 and after taxes you will keep $65
  2. Buy $1,000 Municipal Bond with 8% coupon rate
     - Because these are tax free, each year, you'll make & keep $80

Clearly, you earn more on municipal bond after taxes, so you are willing to hold the risker/less liquid bond even though it has a lower interest rate.

6.2 Term Structure of Interest Rates

Def^n: *Term Structure* - The relationship among interest rates on bonds with different terms to maturity
Another factor that influences the interest rate is the bond's term to maturity.

Def^n: *Yield Curve* - A plot of the yields on bonds with different terms to maturity

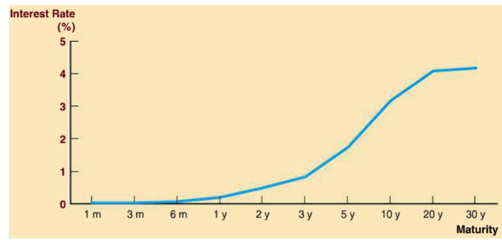


Figure 4: Yield Curve: Treasury Securities

Figure 5 shows the most usual case with the curve upward sloping. This is because as the maturity date is further out, the bond becomes more risky. However, this is not always the case. Yield curves can be upward sloping, flat, or inverted.

A good theory of the Term Structure must provide an explanation to three important empirical facts:

1. Interest rates on bonds of different maturities move together over time
2. When short-term interest rates are low, yield curves are more likely to have an upward slope <u>AND</u>

   when they are high, yield curves are more likely to be inverted
3. Yield curves almost always slope upward

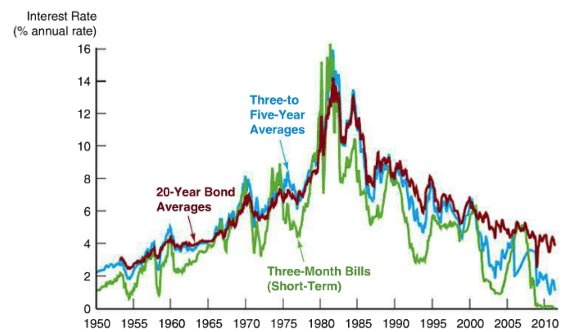


Figure 5: Interest Rate Movements

Three theories have been developed in attempt to explain the patterns of the term structure:

6.2.1 Expectations Theory - *(1 & 2)*

Expectations theory makes the assumption that all bonds, regardless of term to maturity, with the same expected return are *perfect substitutes.*

In order for this to be true, the interest rate on a long-term bond will equal the average of the short-term interest rates that people expect to occur over the life of the long-term bond.

Define the following:

- i_t = todays interest rate on a one-year bond
- i^e_{t+1} = expected interest rate next period on a one-year bond
- i_{nt} = todays interest rate on a n-year bond

$$i_{nt} = \frac{i_t + i^e_{t+1} + i^e_{t+2} + \cdots + i^e_{t+(n-1)}}{n} \tag{6.1}$$

Example: 6.2. *Suppose the one-year interest rate over the next five years is expected to be 5%, 6%, 7%, 8%, & 9%. Based on expectations theory, what is the interest rate today on a two-year bond?*

$$i_{2t} = \frac{5\% + 6\%}{2} = 5.5\%$$

This theory explains fact 1: interest rates move together & fact 2: upward sloped yield curve when short-term interest rates are low and inverted when they are high. Because it suggests the yield curve will be flat, it is unable to explain fact 3: yield curve typically upward sloping.

6.2.2 Segmented Markets Theory - *(3)*

As it's name suggests, this theory believes bonds with different maturities are completely separate and belong in segmented markets. This theory makes the opposite assumption as Expectations does: bonds with different maturity dates are not perfect substitutes at all.

Segmented markets theory explains the third fact very well. Since investors are typically risk adverse, they will only purchase a longer period bond if they are compensated with a higher expected return. Recall a longer maturity date is undesirable and risky as the future is uncertain.

6.2.3 Liquidity Preference Theory - *(1, 2, & 3)*

Liquidity premium theory states that the interest rate on a long term bond will equal an average of the short-term interest rates expected to occur over the life of the long-term bond plus a liquidity premium which responds to supply and demand conditions for that bond.

$$i_{nt} = \frac{i_t + i^e_{t+1} + i^e_{t+2} + \cdots + i^e_{t+(n-1)}}{n} + l_{nt} \tag{6.2}$$

This theory modifies the assumption make in expectations theory. It assumes that bonds can be substitutes (not perfect) but it is also possible for the investor to prefer one bond maturity over another. For this reason, investors must be compensated with a positive liquidity premium if they purchase a longer-term bond.

As an alternative to the liquidity preference theory, the **Preferred Habitat Theory** assumes investors will prefer a given maturity over the others available. Thus they must be compensated for breaking their preferred purchase. Because they are likely to prefer shorter term bonds, this assumption leads to the same conclusions above and results in the same equation.

Example: 6.3. *Suppose the one-year interest rate over the next five years is expected to be 5%, 6%, 7%, 8%, & 9% and the liquidity premium for one to five year bonds is 0%, 0.25%, 0.5%, 0.75%, & 1%. Based on Liquidity Preference Theory (Preferred Habitat Theory), what is the interest rate today on a two-year bond?*

$$i_{2t} = \frac{5\% + 6\%}{2} + 0.25\% = 5.75\%$$

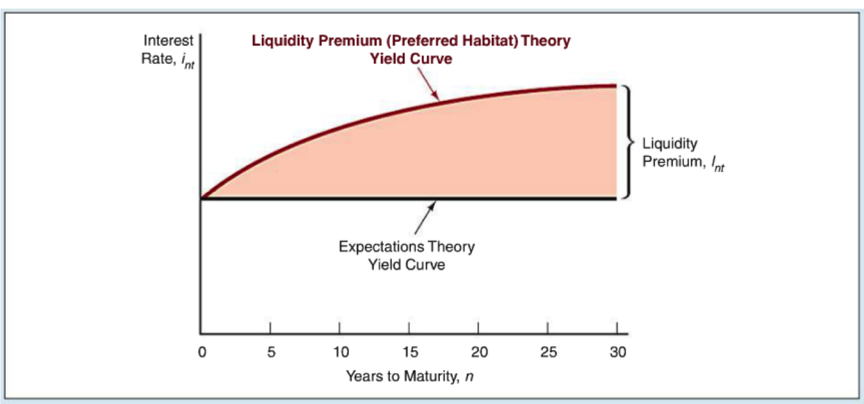


FIGURE 5 The Relationship Between the Liquidity Premium (Preferred Habitat) and Expectations Theory

Figure 6: Expectations Theory vs. Liquidity Premium Theory