Math 236 Algebra 2 Assignment 1

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Problem 1. Show that for every $\alpha \in \mathbb{C}$, there exists a unique $\beta \in \mathbb{C}$ such that $\alpha + \beta = 0$

Proof. Suppose $\alpha = a + bi$ such that $a, b \in \mathbb{R}$ and i is the imaginary unit.
Let $\beta = -a - bi$. Then $\alpha + \beta = a + bi + (-a - bi) = 0$ $\square$

Problem 2. Explain why there does not exist $\lambda \in \mathbb{C}$ such that

$$\lambda(2 - 3i, 5 + 4i, -6 + 7i) = (12 - 5i, 7 + 22i, -32 - 9i).$$

Proof. We can rewrite the vector equation as a system of equations containing the x,y and z component of each vector

$$\begin{cases} \lambda(2 - 3i) = 12 - 5i \\ \lambda(5 + 4i) = 7 + 22i \\ \lambda(-6 + 7i) = (-32 - 9i) \end{cases}$$

From the first equation we get $\lambda = \frac{12-5i}{2-3i}$
Substituting this λ into the second equation of the system of equations we get

$$\frac{12 - 5i}{2 - 3i}(5 + 4i) = (7 + 22i)$$
$$80 + 23i = 80 + 23i$$

This λ satisfies the second equation
Substituting this λ into the third equation of the system of equations we get

$$\frac{12 - 5i}{2 - 3i}(-6 + 7i) = (-32 - 9i)$$
$$-37 + 114i = -91 + 78i$$
$$54 + 36i \neq 0$$

This λ does not satisfy the third equation. Therefore there does not exist a λ that will satisfy the equation defined in the problem statement

$\square$

Problem 3. The empty set is not a vector space. The empty set fails to satisfy only one of the requirements listed in 1.19. Which one?

Proof. A vector space must contain the zero vector. Since the empty set does not contain any elements (vectors) it does not contain the zero vector. Therefore the empty set is not a vector space. □

Problem 4. Let V be the set of positive real numbers. Prove that V is a vector space over $\mathbb{R}$ with addition defined by

$$x \boxplus y = xy,\ for\ all\ x, y \in V$$

and scalar multiplication by

$$\lambda * x = x^{\lambda},\ for\ all\ x \in V, \lambda \in \mathbb{R}$$

What is the zero vector?

Proof. Let x, y, z $\in$ V and a, b $\in \mathbb{R}$

- x + y = xy = yx (since x and y are both positive real numbers) = y + x
- (x + y) + z = (xy) + z = (xy)z = xyz = x(yz) = x + (yz) = x + (y + z)
  (ab)x = $x^{(ab)}$ = $(x^b)^a$ = a(x^b) = a(bx)
- Note: The zero vector is 1
- x + 1 = x1 = x
- x + (1/x) = x(1/x) = 1
- 1*x = x^1 = x
- ax + ay = x^a + y^a = $x^a y^a$ = $(xy)^a$ = a(xy) = a(x + y)
  ax + bx = x^a + x^b = $x^a x^b$ = x^{a+b} = (a + b)x

  Therefore V is a vector space

□