# Comp 424 Assignment 2

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**Problem 1.1.** There are 26 cells in the tunnel system. Ignoring G, there are 25 cells. Therefore the total number of possible beliefs in this domain is $2^{25} - 1$

**Problem 1.2.** There are 7 distinct percepts possible in this domain:

$$(N, S, E), (N, S, W), (N, E, W), (S, E, W), (N, S), (E, W), ()$$

**Problem 1.3.** There are 2 unique percepts. The first one is at (1,3), the $(N, S, W)$ percept. The second one is at (11,3), the $(N, S, E)$ percept

**Problem 1.4.** Conformant Plan: $N \Rightarrow E \Rightarrow N(\Rightarrow E)^2 \Rightarrow S \Rightarrow E(\Rightarrow S)^2$ This plan guarantees that whoever teleports to portal 1 and 2 will reach Grasshopper. The plan is 9 moves long and therefore takes 45 minutes.

**Problem 2.1.** Graph attached on second last page. A win was assigned a value of 1, a loss was assigned a value of -1.

**Problem 2.2.** Minimax scores are found on graph on the second last page. The root node has value 1, therefore we conclude that we can save Grasshopper. There are multiple game paths that will save grasshopper, I assume that my algorithm would work left to right on my game state tree. Therefore the game would play out as follows,

| | | |
|---|---|---|
| B | R | B |
| R | | |
| B | | |

$\Rightarrow$

| | | |
|---|---|---|
| B | R | B |
| R | | R |
| B | | |

$\Rightarrow$

| | | |
|---|---|---|
| B | R | B |
| R | B | R |
| B | | |

$\Rightarrow$

| | | |
|---|---|---|
| B | R | B |
| R | B | R |
| B | | R |

$\Rightarrow$

| | | |
|---|---|---|
| B | R | B |
| R | B | R |
| B | B | R |

**Problem 2.3.A.** Alpha Beta Pruning graph is on the last page. I Modified the order of the branches to make the best case scenario readable from left to right in the tree. Note I assumed that if we ever got a value of 1 at a max node we would prune the remaining branches since we can do no better (i.e. we have found a guaranteed win).

**Problem 2.3.B.** In this branch I was able to explore 4 less nodes with alpha-beta pruning. If I expanded the algorithm to the full tree I would be able to prune the other two initial moves immediately which would mean I would save an additional 16 nodes, for 20 total.

**Problem 2.3.C.** If I were to go down the right branch (from the full game state tree) first (i.e. the first move of Red is on the tile on the third row, third column), then we would prune 23 nodes in this scenario, which is the best case.

**Problem 3.1.a.** $2^2 = 4$ total models. Statement is satisfied unless both $A$ and $B$ are both false. Therefore **3** models satisfy $A \lor B$.

**Problem 3.1.b.** $2^5 = 32$ total models. Statement is satisfied unless $A, B, C, D$ and $E$ are all false. Therefore **31** models satisfy $A \lor B \lor C \lor D \lor E$.

**Problem 3.1.c.** $2^3 = 8$ total models. Statement is satisfied if $C$ is true, regardless of $A$ and $B$. If $C$ is false, then $A \land B$ must be true, this only occurs when both $A$ and $B$ are true. Therefore **5** models satisfy $(A \land B) \lor C$

**Problem 3.1.d.** $2^2 = 4$ total models. $A$ and $\neg B$ must be true, therefore $A$ must be true and $B$ must be false. However true $\Rightarrow$ false is false. Therefore **0** models satisfy $A \land (A \Rightarrow B) \land \neg B$

**Problem 3.1.e.** $2^6 = 64$ total models. $A \land B \land C \land D \land E \land F$ is false unless every element is true. Therefore $\neg(A \land B \land C \land D \land E \land F)$ is true in 63 models. In the 1 remaining model, where every element is true, $B \land C$ is true. Therefore all **64** models satisfy $\neg(A \land B \land C \land D \land E \land F) \lor (B \land C)$

**Problem 3.2.a.**

| $A$ | $\neg A$ | $A \lor \neg A$ |
|---|---|---|
| True | False | True |
| False | True | True |

Sentence is valid.

**Problem 3.2.b.**

| $A$ | $B$ | $\neg A$ | $A \land \neg A$ | $(A \land \neg A) \lor B$ |
|---|---|---|---|---|
| True | True | False | False | True |
| True | False | False | False | False |
| False | True | True | False | True |
| False | False | True | False | False |

Sentence is satisfiable.

**Problem 3.2.c.**

| $A$ | $B$ | $A \Rightarrow B$ | $(A \Rightarrow B) \land A$ | $((A \Rightarrow B) \land A) \Rightarrow B$ | $\neg B$ | $B \lor \neg B$ |
|---|---|---|---|---|---|---|
| True | True | True | True | True | False | True |
| True | False | False | False | True | True | True |
| False | True | True | False | True | False | True |
| False | False | True | False | True | True | True |

| $(((A \Rightarrow B) \wedge A) \Rightarrow B) \Leftrightarrow (B \vee \neg B)$ |
|---|
| True |
| True |
| True |
| True |

Sentence is valid.

**Problem 3.2.d.**
False is always False, therefore it does not matter what $A \vee B \vee C \vee D \vee \neg A$ evaluates to. It follows that the sentence is valid

**Problem 3.2.e.**

| $A$ | $B$ | $\neg A$ | $\neg B$ | $A \vee B$ | $(A \vee B) \wedge \neg A$ | $(A \vee B) \wedge \neg A \wedge \neg B$ |
|---|---|---|---|---|---|---|
| True | True | False | False | True | False | False |
| True | False | False | True | True | False | False |
| False | True | True | False | True | True | False |
| False | False | True | True | False | False | False |

Sentence is unsatisfiable.

**Problem 4.1.**
Constant: $X = \{Dustey, Elody, Michael, William\}$
Variable: person $x$
Constant: $Y = \{eggo, chocolate\ pudding, 3 - musketeers\}$
Variable: snack $y$

1. $\forall x \in X\ \exists y \in Y\ \ Bought(x, y)$
2. $\forall x \in X\ \ Bought(x, chocolate\ pudding) \Rightarrow \neg Bought(x, eggo)$
3. $\forall x \in X\ \ Bought(x, 3 - musketeers) \Rightarrow Bought(x, chocolate\ pudding)$
4. $\forall y \in Y\ \ Bought(Michael, y) \Leftrightarrow \neg Bought(Elody, y)$
5. $Bought(Michael, 3 - musketeers)$
6. $Bought(Dustey, 3 - musketeers)$

**Problem 4.2.**

1. $Bought(x, eggo) \vee Bought(x, chocolate\ pudding) \vee Bought(x, 3 - musketeers)$
2. $\neg Bought(y, chocolate\ pudding) \vee \neg Bought(y, eggo)$

3. $\neg Bought(z, 3 - musketeers) \lor Bought(z, chocolate\, pudding)$
4. $\neg Bought(Michael, w) \lor \neg Bought(Elody, w)$
5. $Bought(Michael, w) \lor Bought(Elody, w)$
6. $Bought(Michael, 3 - musketeers)$
7. $Bought(Dustey, 3 - musketeers)$

**Problem 4.3.** We will prove that Elody only buys eggos

query: $\alpha = Bought(Elody, eggo) \land \neg Bought(Elody, chocolatepudding) \land \neg Bought(Elody, 3-musketeers)$

$\neg\alpha = \neg(Bought(Elody, eggo) \land \neg Bought(Elody, chocolate\, pudding) \land \neg Bought(Elody, 3 - musketeers))$

By De Morgan's Law
$\neg\alpha = \neg Bought(Elody, eggo) \lor Bought(Elody, chocolatepudding) \lor Bought(Elody, 3-musketeers)$

We will show that $KB \land \neg\alpha$ is unsatisfiable using proof by resolution:

(i) use 6. and 3. with $\sigma = \{z/michael\}$ : $Bought(Michael, chocolate\, pudding)$

(ii) use (i) and 2. with $\sigma = \{y/michael\}$ : $\neg Bought(Michael, eggo)$

(iii) use 6. and 4. with $\sigma = \{w/3 - musketeers\}$ : $\neg Bought(Elody, 3 - musketeers)$

(iv) use (i) and 4. with $\sigma = \{w/chocolate\, pudding\}$ : $\neg Bought(Elody, chocolate\, pudding)$

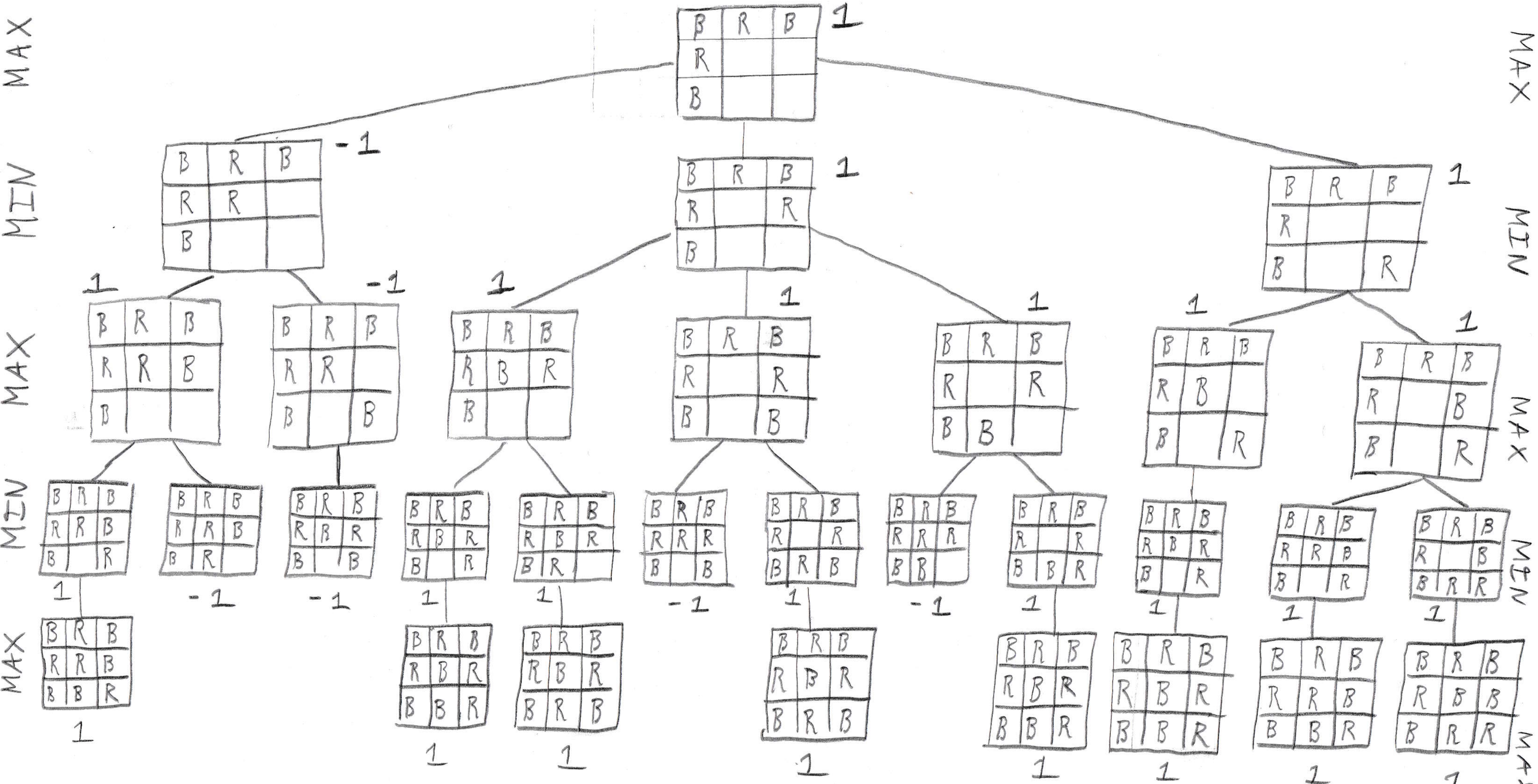
(v) use (ii) and 5. with $\sigma = \{w/eggo\}$ : $Bought(Elody, eggo)$

(vi) use (v) and $\neg\alpha$ : $Bought(Elody, chocolate\, pudding) \lor Bought(Elody, 3 - musketeers)$

(vii) use (iv) and (vi) : $Bought(Elody, 3 - musketeers)$

(viii) use (iii) and (vii) : $NIL$

Therefore we conclude that $KB \land \neg\alpha$ is unsatisfiable. It follows that $\alpha$ is true and Elody only purchased eggos. Therefore the question of 'Did any of them only buy eggos?' is true.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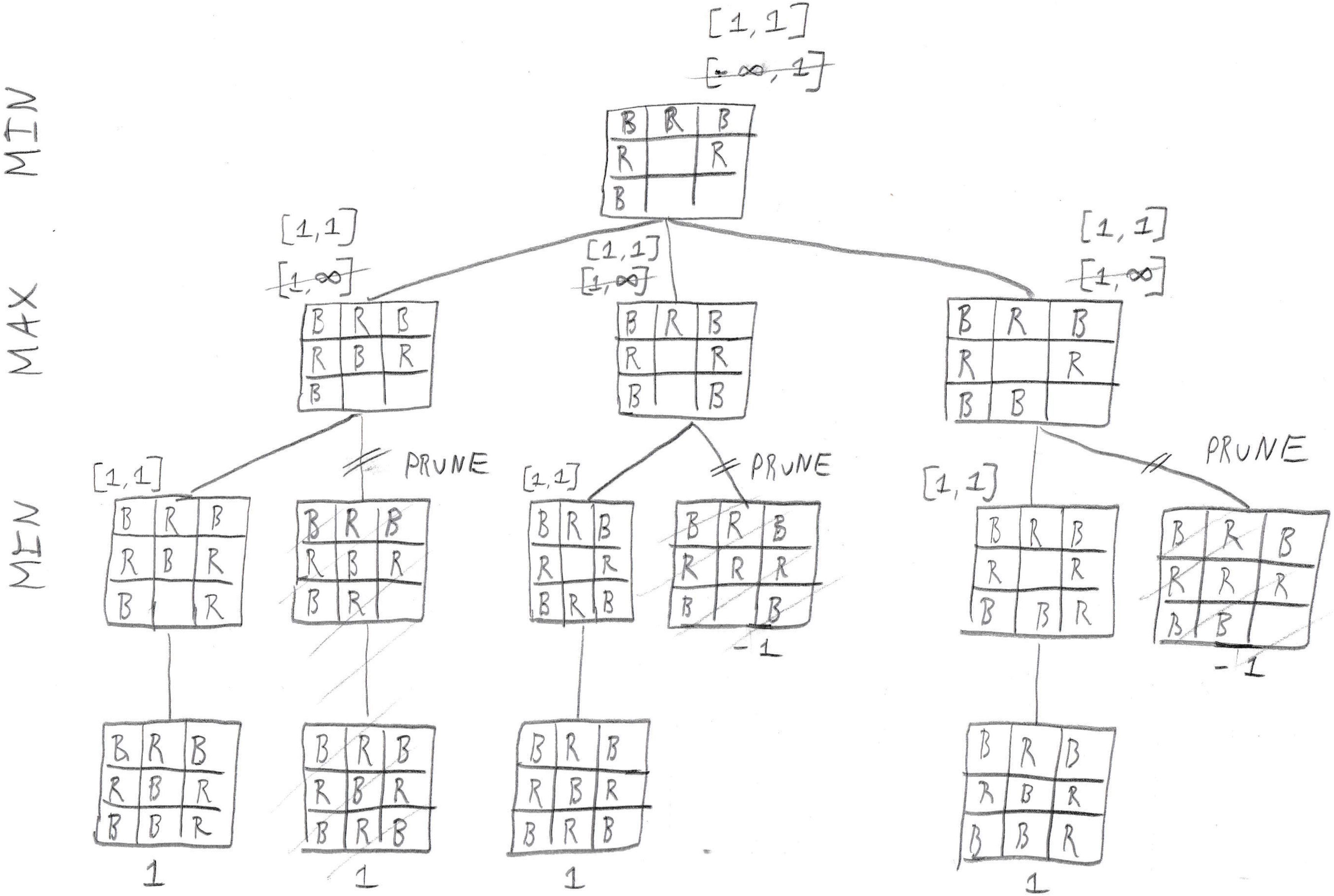
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