Lecture 6

# *Simple Near-Optimal Auctions*

The preceding lecture identified the expected-revenue-maximizing auction for a wide range of Bayesian single-parameter environments. When agents' valuations are not identically distributed, the optimal mechanism is relatively complex, requires detailed information about the valuation distributions, and does not resemble the auction formats used in practice. This lecture pursues approximately optimal mechanisms that are simpler, more practical, and more robust than the theoretically optimal mechanism.

Section 6.1 motivates the pursuit of simple near-optimal auctions. Section 6.2 covers a fun result from optimal stopping theory, the "prophet inequality," and Section 6.3 uses it to design a simple and provably near-optimal single-item auction. Section 6.4 introduces prior-independent mechanisms, which are mechanisms whose description makes no reference to any valuation distributions, and proves the Bulow-Klemperer theorem, which explains why competition is more valuable than information.

## 6.1 Optimal Auctions Can Be Complex

Theorem 5.4 states that, for every single-parameter environment in which agents' valuations are drawn independently from regular distributions, the corresponding virtual welfare maximizer maximizes the expected revenue over all DSIC mechanisms. The allocation rule of this mechanism sets

$$\mathbf{x}(\mathbf{v}) = \mathrm{argmax}_X \sum_{i=1}^{n} \varphi_i(v_i) x_i(\mathbf{v})$$

for each valuation profile $\mathbf{v}$, where

$$\varphi_i(v_i) = v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}$$

is the virtual valuation corresponding to the distribution $F_i$.[1]

Section 5.2.6 noted that the optimal single-item auction with i.i.d. bidders and a regular distribution is shockingly simple: it is simply a second-price auction, augmented with the reserve price $\varphi^{-1}(0)$. This is a true "killer application" of auction theory—it gives crisp and practically useful guidance to auction design.

The plot thickens if the problem is a bit more complex. Consider again a single-item auction, but with bidders' valuations drawn independently from *different* regular distributions. The optimal auction can get weird, and it does not generally resemble any auctions used in practice (Exercise 6.1). Someone other than the highest bidder might win, and the payment made by the winner seems impossible to explain without referencing virtual valuations. This weirdness is inevitable if you really want every last cent of the maximum-possible expected revenue, with respect to the exact valuation distributions $F_1, \ldots, F_n$.

Are there simpler and more practical single-item auction formats that are at least *approximately* optimal?[2]

## 6.2 The Prophet Inequality

Consider the following game with $n$ stages. In stage $i$, you are offered a nonnegative prize $\pi_i$, drawn from a distribution $G_i$. You are told the distributions $G_1, \ldots, G_n$ in advance, and these distributions are independent. You are told the realization $\pi_i$ only at stage $i$. After seeing $\pi_i$, you can either accept the prize and end the game or discard the prize and proceed to the next stage. The decision's difficulty stems from the trade-off between the risk of accepting a reasonable prize early, and then missing out later on a great one, and the risk of having to settle for a lousy prize in one of the final stages.

The amazing "prophet inequality" offers a simple strategy that performs almost as well as a fully clairvoyant prophet.

[1] Since we only consider DSIC mechanisms, we assume truthful reports (i.e., $\mathbf{b} = \mathbf{v}$) throughout the lecture.

[2] This lecture leaves terms like "simple," "practical," and "robust" largely undefined. This contrasts with our use of approximation in algorithmic mechanism design (Lecture 4) to escape a different kind of complexity imposed by full optimality; there, we identified "practical" with "runs in polynomial time."

**Theorem 6.1 (Prophet Inequality)** *For every sequence $G_1, \ldots, G_n$ of independent distributions, there is a strategy that guarantees expected reward at least $\frac{1}{2}\mathbf{E}_{\pi\sim\mathbf{G}}[\max_i \pi_i]$. Moreover, there is such a* threshold strategy, *which accepts prize $i$ if and only if $\pi_i$ is at least some threshold $t$.*

*Proof:* Let $z^+$ denote $\max\{z, 0\}$. Consider a threshold strategy with threshold $t$. It is difficult to compare directly the expected payoff of this strategy with the expected payoff of a prophet. Instead, we derive lower and upper bounds, respectively, on these two quantities that are easy to compare.

Let $q(t)$ denote the probability that the threshold strategy accepts no prize at all.[3] As $t$ increases, the risk $q(t)$ increases but the expected value of an accepted prize goes up.

What payoff does the $t$-threshold strategy obtain? With probability $q(t)$, zero, and with probability $1 - q(t)$, at least $t$. Let's improve our lower bound in the second case. If exactly one prize $i$ satisfies $\pi_i \geq t$, then we should get "extra credit" of $\pi_i - t$ above and beyond our baseline payoff of $t$. If at least two prizes exceed the threshold, say $i$ and $j$, then things are more complicated: our "extra credit" is either $\pi_i - t$ or $\pi_j - t$, according to which corresponds to the earlier stage. We'll be lazy and punt on this complication: when two or more prizes exceed the threshold, we'll only credit the baseline $t$ to the strategy's payoff.

Formally, we can bound

$$\mathbf{E}_{\pi\sim\mathbf{G}}[\text{payoff of the } t\text{-threshold strategy}]$$

from below by

$$\begin{aligned}
&(1 - q(t))t + \\
&\sum_{i=1}^{n} \mathbf{E}_\pi[\pi_i - t \mid \pi_i \geq t, \pi_j < t \ \forall j \neq i] \, \mathbf{Pr}[\pi_i \geq t] \, \mathbf{Pr}[\pi_j < t \ \forall j \neq i] \\
&= (1 - q(t))t + \sum_{i=1}^{n} \underbrace{\mathbf{E}_\pi[\pi_i - t \mid \pi_i \geq t] \, \mathbf{Pr}[\pi_i \geq t]}_{=\mathbf{E}[(\pi_i - t)^+]} \underbrace{\mathbf{Pr}[\pi_j < t \ \forall j \neq i]}_{\geq q(t)} \\
&\geq (1 - q(t))t + q(t) \sum_{i=1}^{n} \mathbf{E}_\pi\left[(\pi_i - t)^+\right], \qquad (6.1)
\end{aligned}$$

[3]Note that discarding the final stage's prize is clearly suboptimal!

where we use the independence of the $G_i$'s to factor the two probability terms and drop the conditioning on the event that $\pi_j < t$ for every $j \neq i$. In (6.1), we use that $q(t) = \mathbf{Pr}[\pi_j < t \ \forall j] \leq \mathbf{Pr}[\pi_j < t \ \forall j \neq i]$.

Now we produce an upper bound on the prophet's expected payoff $\mathbf{E}_\pi[\max_i \pi_i]$ that is easy to compare to (6.1). The expression $\mathbf{E}_\pi[\max_i \pi_i]$ doesn't reference the strategy's threshold $t$, so we add and subtract it to derive

$$\begin{aligned} \mathbf{E}_\pi\left[\max_{i=1}^{n} \pi_i\right] &= \mathbf{E}_\pi\left[t + \max_{i=1}^{n}(\pi_i - t)\right] \\ &\leq t + \mathbf{E}_\pi\left[\max_{i=1}^{n}(\pi_i - t)^+\right] \\ &\leq t + \sum_{i=1}^{n} \mathbf{E}_\pi\left[(\pi_i - t)^+\right]. \end{aligned} \tag{6.2}$$

Comparing (6.1) and (6.2), we can set $t$ so that $q(t) = \frac{1}{2}$, with a 50/50 chance of accepting a prize, and complete the proof.[4] ■

**Remark 6.2 (Guarantee with Adversarial Tie-Breaking)** The proof of Theorem 6.1 implies a stronger statement that is useful in the next section. Our lower bound (6.1) on the revenue of the $t$-threshold strategy only credits $t$ units of value when two or more prizes exceed the threshold $t$. Only the realizations in which exactly one prize exceeds the threshold contribute to the second, "extra credit" term in (6.1). For this reason, the guarantee of $\frac{1}{2}\mathbf{E}_\pi[\max_i \pi_i]$ holds for the strategy even if, whenever there are multiple prizes above the threshold, it somehow always picks the smallest of these.

## 6.3 Simple Single-Item Auctions

We now return to our motivating example of a single-item auction with $n$ bidders with valuations drawn independently from regular distributions $F_1, \ldots, F_n$ that need not be identical. We use the prophet inequality (Theorem 6.1) to design a relatively simple and near-optimal auction.

[4] If there is no such $t$ because of point masses in the $G_i$'s, then a minor extension of the argument yields the same result (Exercise 6.2).

The key idea is to define the $i$th prize as the positive part $\varphi_i(v_i)^+$ of bidder $i$'s virtual valuation. $G_i$ is then the corresponding distribution induced by $F_i$; since the $F_i$'s are independent, so are the $G_i$'s. To see an initial connection to the prophet inequality, we can use Theorem 5.2 to note that the expected revenue of the optimal auction is

$$\mathbf{E}_{\mathbf{v}\sim\mathbf{F}}\left[\sum_{i=1}^{n}\varphi_i(v_i)x_i(\mathbf{v})\right] = \mathbf{E}_{\mathbf{v}\sim\mathbf{F}}\left[\max_{i=1}^{n}\varphi_i(v_i)^+\right],$$

precisely the expected value obtained by a prophet with prizes $\varphi_1(v_1)^+, \ldots, \varphi_n(v_n)^+$.

Now consider any allocation rule that has the following form.

**Virtual Threshold Allocation Rule**

1. Choose $t$ such that $\mathbf{Pr}[\max_i \varphi_i(v_i)^+ \geq t] = \frac{1}{2}$.[5]
2. Give the item to a bidder $i$ with $\varphi_i(v_i) \geq t$, if any, breaking ties among multiple candidate winners arbitrarily.

The prophet inequality, strengthened as in Remark 6.2, immediately implies the following guarantee for such allocation rules.

**Corollary 6.3 (Virtual Threshold Rules Are Near-Optimal)** *If* $\mathbf{x}$ *is a virtual threshold allocation rule, then*

$$\mathbf{E}_{\mathbf{v}}\left[\sum_{i=1}^{n}\varphi_i(v_i)^+x_i(\mathbf{v})\right] \geq \frac{1}{2}\mathbf{E}_{\mathbf{v}}\left[\max_{i=1}^{n}\varphi_i(v_i)^+\right]. \tag{6.3}$$

Because a virtual threshold allocation rule never awards the item to a bidder with negative virtual valuation, the left-hand side of (6.3) also equals $\mathbf{E}_{\mathbf{v}}[\sum_i \varphi_i(v_i)x_i(\mathbf{v})]$.

Here is a specific virtual threshold allocation rule.

**Second-Price with Bidder-Specific Reserves**

1. Set a reserve price $r_i = \varphi_i{}^{-1}(t)$ for each bidder $i$, with $t$ defined as for virtual threshold allocation rules.

[5]See Exercise 6.2 for the case where no such $t$ exists.

2. Give the item to the highest bidder that meets her reserve, if any.

This auction first filters bidders using bidder-specific reserve prices, and then awards the item to the highest bidder remaining. With regular valuation distributions, this allocation rule is monotone (Exercise 6.3) and hence can be extended to a DSIC auction using Myerson's lemma. The winner's payment is then the maximum of her reserve price and the highest bid by another bidder that meets her reserve price. By Theorem 5.2 and Corollary 6.3, this auction approximately maximizes the expected revenue over all DSIC auctions.

**Theorem 6.4 (Simple Versus Optimal Auctions)** *For all $n \geq 1$ and regular distributions $F_1, \ldots, F_n$, the expected revenue of a second-price auction with suitable reserve prices is at least 50% of that of the optimal auction.*

The guarantee of 50% can be improved for many distributions, but it is tight in the worst case, even with only two bidders (see Problem 6.1).

The second-price auction with bidder-specific reserve prices is simpler than the optimal auction in two senses. First, virtual valuation functions are only used to set reserve prices. Second, the highest bidder wins, as long as she clears her reserve price.

An even simpler auction would use a common, or "anonymous," reserve price for all bidders. For example, the opening bid in eBay is anonymous.[6] See the Notes for approximation guarantees for single-item auctions with anonymous reserve prices.

## 6.4 Prior-Independent Mechanisms

This section explores a different critique of the theory of optimal mechanisms developed in Lecture 5: the valuation distributions $F_1, \ldots, F_n$ were assumed to be known to the mechanism designer in advance. In some applications, where there is lots of data and

[6] Some real-world auctions do use bidder-specific reserve prices. For example, in some sponsored search auctions, "higher-quality" advertisers (as estimated by the search company) face lower reserve prices than "lower-quality" advertisers.

bidders' preferences are not changing too rapidly, this is a reasonable assumption. But what if the mechanism designer does not know, or is not confident about, the valuation distributions? This problem is especially relevant in thin markets where there is not much data, including sponsored search auctions for rarely used but potentially valuable keywords (as in Exercise 5.10).

Removing advance knowledge of the valuation distributions might seem to return us to the single-bidder single-item quandary that motivated the Bayesian approach (Section 5.1.2). The difference is that we continue to assume that bidders' valuations are drawn from distributions; it's just that these distributions are unknown to the mechanism designer. That is, we continue to use distributions in the *analysis* of mechanisms, but not in their *design*. The goal is to design a good *prior-independent* mechanism, meaning one whose description makes no reference to a valuation distribution. Examples of prior-independent mechanisms include second-price single-item auctions, and more generally welfare-maximizing DSIC mechanisms (as in Exercise 4.1). Non-examples include monopoly prices, which are a function of the underlying distribution, and more generally virtual welfare maximizers.

Next is a beautiful result from auction theory: the expected revenue of an optimal single-item auction is at most that of a second-price auction (with no reserve price) with one extra bidder.

**Theorem 6.5 (Bulow-Klemperer Theorem)** *Let $F$ be a regular distribution and $n$ a positive integer. Let $\mathbf{p}$ and $\mathbf{p}^*$ denote the payment rules of the second-price auction with $n+1$ bidders and the optimal auction (for $F$) with $n$ bidders, respectively.*[7] *Then*

$$\mathbf{E}_{\mathbf{v}\sim F^{n+1}}\left[\sum_{i=1}^{n+1} p_i(\mathbf{v})\right] \geq \mathbf{E}_{\mathbf{v}\sim F^{n}}\left[\sum_{i=1}^{n} p_i^*(\mathbf{v})\right]. \tag{6.4}$$

The usual interpretation of the Bulow-Klemperer theorem, which also has anecdotal support in practice, is that extra competition is more important than getting the auction format just right. It is better to invest your resources to recruit more serious participants

[7] The latter auction is a second-price auction with reserve price $\varphi^{-1}(0)$, where $\varphi$ is the virtual valuation function of $F$ (Section 5.2.6).

than to sharpen your knowledge of their preferences. (Of course, do both if you can!)

The Bulow-Klemperer theorem gives a sense in which the (prior-independent) second-price auction is simultaneously competitive with an infinite number of different optimal auctions, ranging over all single-item environments with bidders' valuations drawn i.i.d. from a regular distribution. Exercise 6.4 shows another consequence of the theorem: for every such environment and $n \geq 2$, the expected revenue of the second-price auction with $n$ bidders is at least $\frac{n-1}{n}$ times that of an optimal auction (again with $n$ bidders). Problem 6.4 outlines some further extensions and variations of the Bulow-Klemperer theorem.

*Proof of Theorem 6.5:* The two sides of (6.4) are tricky to compare directly, so for the analysis we define a fictitious auction $\mathcal{A}$ to facilitate the comparison. This $(n+1)$-bidder single-item DSIC auction works as follows.

**The Fictitious Auction $\mathcal{A}$**

1. Simulate an optimal $n$-bidder auction for $F$ on the first $n$ bidders $1, 2, \ldots, n$.

2. If the item was not awarded in the first step, then give the item to bidder $n+1$ for free.

We defined $\mathcal{A}$ to possess two properties useful for the analysis. First, its expected revenue equals that of an optimal auction with $n$ bidders, the right-hand side of (6.4). Second, it always allocates the item.

We can finish the proof by arguing that the expected revenue of a second-price auction (with $n+1$ bidders) is at least that of $\mathcal{A}$. We show the stronger statement that, when bidders' valuations are drawn i.i.d. from a regular distribution, the second-price auction maximizes the expected revenue over all DSIC auctions that always allocate the item.

We can identify the optimal such auction using the tools developed in Section 5.2. By the equivalence of expected revenue and expected virtual welfare (Theorem 5.2), it suffices to maximize the latter. The allocation rule with maximum possible expected virtual welfare subject to always allocating the item always awards the item to a bidder with the highest virtual valuation, even if this is negative.

A second-price auction always awards the item to a bidder with the highest valuation. Since bidders' valuations are drawn i.i.d. from a regular distribution, all bidders share the same nondecreasing virtual valuation function $\varphi$. Thus, a bidder with the highest valuation also has the highest virtual valuation. We conclude that the second-price auction maximizes expected revenue subject to always awarding the item, and the proof is complete. ■

**The Upshot**

☆ When bidders' valuations are drawn from different distributions, the optimal single-item auction is complex, requires detailed information about the distributions, and does not resemble the auction formats used in practice.

☆ The prophet inequality states that, given a sequence of prizes drawn from known and independent distributions, there is a threshold strategy with expected value at least 50% of the expected value of the biggest prize.

☆ The prophet inequality implies that a second-price auction with suitably chosen bidder-specific reserve prices has expected revenue at least 50% of the maximum possible.

☆ A prior-independent mechanism is one whose description makes no reference to any valuation distributions. Welfare-maximizing mechanisms are prior-independent; virtual welfare-maximizing mechanisms are not.

☆ The Bulow-Klemperer theorem states that the expected revenue of an optimal single-item auction is at most that of a second-price auction with one extra bidder.

## Notes

The prophet inequality (Theorem 6.1) is due to Samuel-Cahn (1984). Theorem 6.4 is from Chawla et al. (2007). Approximation guarantees for second-price auctions with anonymous reserve prices are considered by Hartline and Roughgarden (2009), and a recent result of Alaei et al. (2015) shows that such an auction can always extract at least a $1/e \approx 37\%$ fraction of the optimal expected revenue. Problem 6.2 also appears in Hartline and Roughgarden (2009). The result in Problem 6.3 is due to Chawla et al. (2010).

The Bulow-Klemperer theorem (Theorem 6.5) and its extension in Problem 6.4(a) are from Bulow and Klemperer (1996). Our proof follows Kirkegaard (2006). The consequent approximation guarantee (Exercise 6.4) is observed in Roughgarden and Sundararajan (2007). The general agenda of designing good prior-independent mechanisms is articulated in Dhangwatnotai et al. (2015), and Problem 6.4(b) is a special case of their "single sample" mechanism. Prior-independent mechanism design can be considered a relaxation of "prior-free" mechanism design, as developed by Goldberg et al. (2006).

In contrast to the classical optimal auction theory developed in Lecture 5, the theories of simple near-optimal and prior-independent mechanisms emerged only over the past 10 years, primarily in the computer science literature. See Hartline (2016) for a survey of the latest developments.

## Exercises

**Exercise 6.1** Consider an $n$-bidder single-item auction, with bidders' valuations drawn independently from regular distributions $F_1, \ldots, F_n$.

(a) Give a formula for the winner's payment in an optimal auction, in terms of the bidders' virtual valuation functions.

(b) *(H)* Show by example that, in an optimal auction, the highest bidder need not win, even if it has a positive virtual valuation.

(c) Give an intuitive explanation of why the property in (b) might be beneficial to the expected revenue of an auction.

**Exercise 6.2** *(H)* Extend the prophet inequality (Theorem 6.1) to the case where there is no threshold $t$ with $q(t) = \frac{1}{2}$, where $q(t)$ is the probability that no prize meets the threshold.

**Exercise 6.3** Prove that with regular valuation distributions $F_1, \ldots, F_n$, the allocation rule of a second-price auction with bidder-specific reserve prices (Section 6.3) is monotone.

**Exercise 6.4** *(H)* Consider an $n$-bidder single-item auction, with bidders' valuations drawn i.i.d. from a regular distribution $F$. Prove that the expected revenue of a second-price auction (with no reserve price) is at least $\frac{n-1}{n}$ times that of an optimal auction.

## Problems

**Problem 6.1** This problem investigates improvements to the prophet inequality (Theorem 6.1) and its consequences for simple near-optimal auctions (Theorem 6.4).

(a) *(H)* Show that the factor of $\frac{1}{2}$ in the prophet inequality cannot be improved: for every constant $c > \frac{1}{2}$, there are distributions $G_1, \ldots, G_n$ such that *every* strategy, threshold or otherwise, has expected value less than $c \cdot \mathbf{E}_{\pi \sim \mathbf{G}}[\max_i \pi_i]$.

(b) Prove that Theorem 6.4 does not hold with 50% replaced by any larger constant factor.

(c) Can the factor of $\frac{1}{2}$ in the prophet inequality be improved for the special case of i.i.d. distributions, with $G_1 = G_2 = \cdots = G_n$?

**Problem 6.2** This problem steps through a reasonably general result about simple and near-optimal mechanisms. Consider a single-parameter environment in which every feasible outcome is a 0-1 vector, indicating the winning agents (cf., Exercise 4.2). Assume that the feasible set is *downward-closed*, meaning that if $S$ is a feasible set of winning agents and $T \subseteq S$, then $T$ is also a feasible set of winning agents. Finally, assume that the valuation distribution $F_i$

of every agent $i$ satisfies the monotone hazard rate (MHR) condition (Exercise 5.4), meaning that $\frac{f_i(v_i)}{1-F_i(v_i)}$ is nondecreasing in $v_i$.

Let $\mathcal{M}^*$ denote the expected revenue-maximizing DSIC mechanism. Our protagonist is the following DSIC mechanism $\mathcal{M}$.

**Welfare Maximization with Monopoly Reserves**

1. Let $r_i$ be a monopoly price (i.e., in $\text{argmax}_{r \geq 0}\{r \cdot (1 - F_i(r))\}$) for the distribution $F_i$.
2. Let $S$ denote the agents $i$ that satisfy $v_i \geq r_i$.
3. Choose winners $W \subseteq S$ to maximize the social welfare:
$$W = \underset{T \subseteq S \,:\, T \text{ feasible}}{\text{argmax}} \sum_{i \in T} v_i.$$
4. Define payments according to Myerson's payment formula (3.5).

(a) Let $\varphi_i$ denote the virtual valuation function of $F_i$. Use the MHR condition to prove that, for every $v_i \geq r_i$, $r_i + \varphi_i(v_i) \geq v_i$.

(b) *(H)* Prove that the expected social welfare of $\mathcal{M}$ is at least that of $\mathcal{M}^*$.

(c) *(H)* Prove that the expected revenue of $\mathcal{M}$ is at least half of its expected social welfare.

(d) Conclude that the expected revenue of $\mathcal{M}$ is at least half the expected revenue of the optimal mechanism $\mathcal{M}^*$.

**Problem 6.3** Consider a single consumer who is interested in purchasing at most one of $n$ non-identical items. Assume that the consumer's private valuations $v_1, \ldots, v_n$ for the $n$ items are drawn from known independent regular distributions $F_1, \ldots, F_n$. The design space is the set of *posted prices*, with one price per item. Faced with prices $p_1, \ldots, p_n$, the consumer selects no item if $p_j > v_j$ for every item $j$. Otherwise, she selects the item that maximizes $v_j - p_j$, breaking ties arbitrarily, providing revenue $p_j$ to the seller.

(a) Explain why this setting does not correspond to a single-parameter environment.

(b) *(H)* Prove that for every $F_1, \ldots, F_n$, the maximum expected revenue achievable by posted prices is at most that of an optimal single-item auction with $n$ bidders with valuations drawn independently from the distributions $F_1, \ldots, F_n$.

(c) *(H)* Prove that, for every $F_1, \ldots, F_n$, there are posted prices that achieve expected revenue at least half that of the upper bound identified in (b).

**Problem 6.4** This problem considers some variations of the Bulow-Klemperer theorem (Theorem 6.5). Consider an $n$-bidder $k$-unit auction (Example 3.2) with $n \geq k \geq 1$ and with bidders' valuations drawn i.i.d. from a regular distribution $F$.

(a) Prove that the expected revenue of the optimal auction for $F$ (Exercise 5.7) is at most that of the DSIC welfare-maximizing auction (Exercise 2.3) with $k$ extra bidders.

(b) *(H)* Assume that $n \geq k+1$. Prove that the following randomized auction is DSIC and has expected revenue at least $\frac{n-1}{2n}$ times that of the optimal auction.[8]

**A Prior-Independent Auction**

1. Choose one bidder $j$ uniformly at random.
2. Let $S$ denote the $k$ highest bidders other than $j$, and $\ell$ the next-highest such bidder. (If $n = k = 1$, interpret $v_\ell$ as 0.)
3. Award an item to every bidder $i$ of $S$ with $v_i \geq v_j$, at a price of $\max\{v_j, v_\ell\}$.

[8] A randomized mechanism is DSIC if for every agent $i$ and reports $\mathbf{v}_{-i}$ by the others, truthful reporting maximizes the expected utility of $i$. The expectation is over the coin flips of the mechanism.