2ab) Explicit Version

Editor - D:\Users\jpearc6\Documents\MATLAB\HouseHolder.m

HouseHolder.m

```
1   function [Q,R] = HouseHolder(A)
2
3 -  [m,n] = size(A);
4
5 -  R = A;
6 -  Q = eye(m);
7 -  for k = 1:n
8 -      u = R(k:m,k);
9 -      e = zeros(length(u),1); e(1) = 1;
10 -     v = u - norm(u)*e;
11
12 -     H = eye(m);
13 -     H(k:m,k:m) = H(k:m,k:m) - (2/(v'*v))*(v*v');
14
15 -     R = H'*R;
16 -     Q = Q*H;
17 -  end
```

Command Window

New to MATLAB? See resources for Getting Started.

```
>> [Q,R] = HouseHolder(A)

Q =

    0.7071   -0.5883    0.1903    0.3430
         0    0.3922    0.9037    0.1715
    0.7071    0.5883   -0.1903   -0.3430
         0    0.3922   -0.3330    0.8575


R =

    1.4142   -0.7071    0.7071
   -0.0000    2.5495    0.1961
    0.0000    0.0000    3.2344
    0.0000    0.0000         0

fx >>
```

2ab) Implicit Version

Editor - D:\Users\jpearc6\Documents\MATLAB\HouseHolderIm.m

HouseHolderIm.m

```
function [Q,R] = HouseHolderIm(A)

[m,n] = size(A);
V = zeros(m,n);
R = A;
for k = 1:n
    u = R(k:m,k);
    e = zeros(length(u),1); e(1) = 1;
    v = u - norm(u)*e;
    v = v/norm(v);

    R(k:m,k:n) = (R(k:m,k:n)-(2*v*(v'*R(k:m,k:n))));
    V(k:m,k) = v; %keep vectors to recover Q
end

Q = eye(m);

for j=n:-1:1
    v = V(j:m,j);
    Q(j:m,j:m) = Q(j:m,j:m) - 2*v*(v' * Q(j:m,j:m));
end
```

Command Window

New to MATLAB? See resources for Getting Started.

```
>> [Q,R] = HouseHolderIm(A)

Q =

    0.7071   -0.5883    0.1903    0.3430
         0    0.3922    0.9037    0.1715
    0.7071    0.5883   -0.1903   -0.3430
         0    0.3922   -0.3330    0.8575


R =

    1.4142   -0.7071    0.7071
         0    2.5495    0.1961
    0.0000         0    3.2344
         0         0   -0.0000

>>
```

2c) Code and Output

Editor - D:\Users\jpearc6\Documents\MATLAB\compareHouseholderV1V2.m

HouseHolderIm.m | compareHouseholderV1V2.m | HouseHolder.m

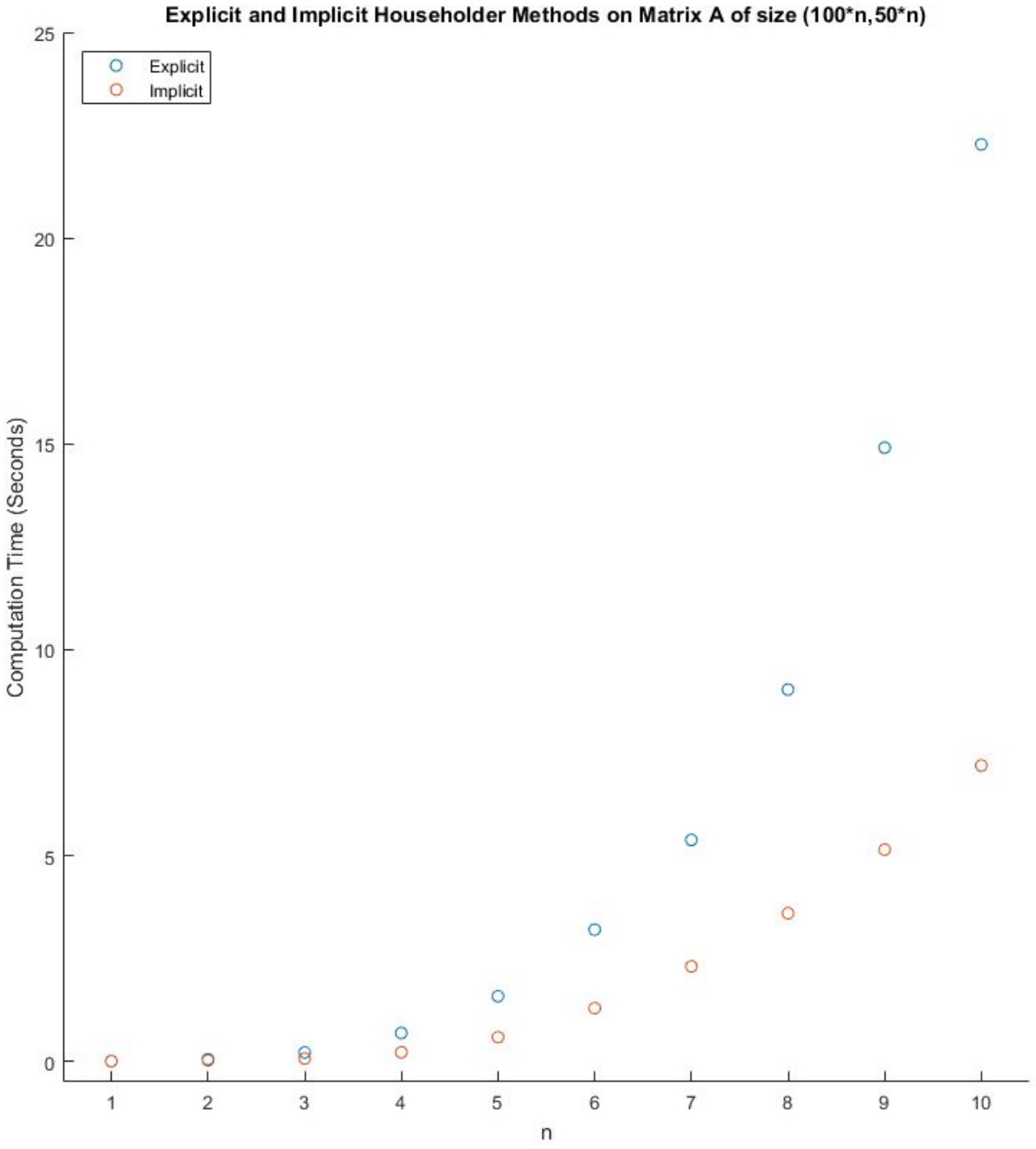
```
1   function [expTimes, imTimes] = compareHouseholderV1V2(n)
2
3   expTimes = zeros(1,10);
4   imTimes = zeros(1,10);
5
6   for k = 1:n
7       A =randn (100 * k, 50 * k);
8       %Explicit Method
9       tic;
10      HouseHolder(A);
11      expTimes(k) = toc;
12      %Implicit Method
13      tic;
14      HouseHolderIm(A);
15      imTimes(k) = toc;
16  end
```

Command Window

New to MATLAB? See resources for Getting Started.

```
>> [Explicit,Implicit] = compareHouseholderV1V2(10)

Explicit =

    0.0072    0.0501    0.2181    0.6892    1.5830    3.1996    5.3804    9.0319   14.9093   22.2760


Implicit =

    0.0037    0.0250    0.0681    0.2191    0.5875    1.2932    2.3080    3.5981    5.1426    7.1873

fx >>
```

2c) Computation Time Chart



3)a)

As in class the least squares problem can be represented as follows:

$$min_{\alpha} ||y - A\alpha||_2$$

Where,

$$y = (0.0131,\ 0.0542,\ 0.1652,\ 0.5614,\ 1.6412,\ 2.9546,\ 4.9099,\ 8.3626,\ 10.9059,\ 15.4361)^T$$

$$n = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)^T$$

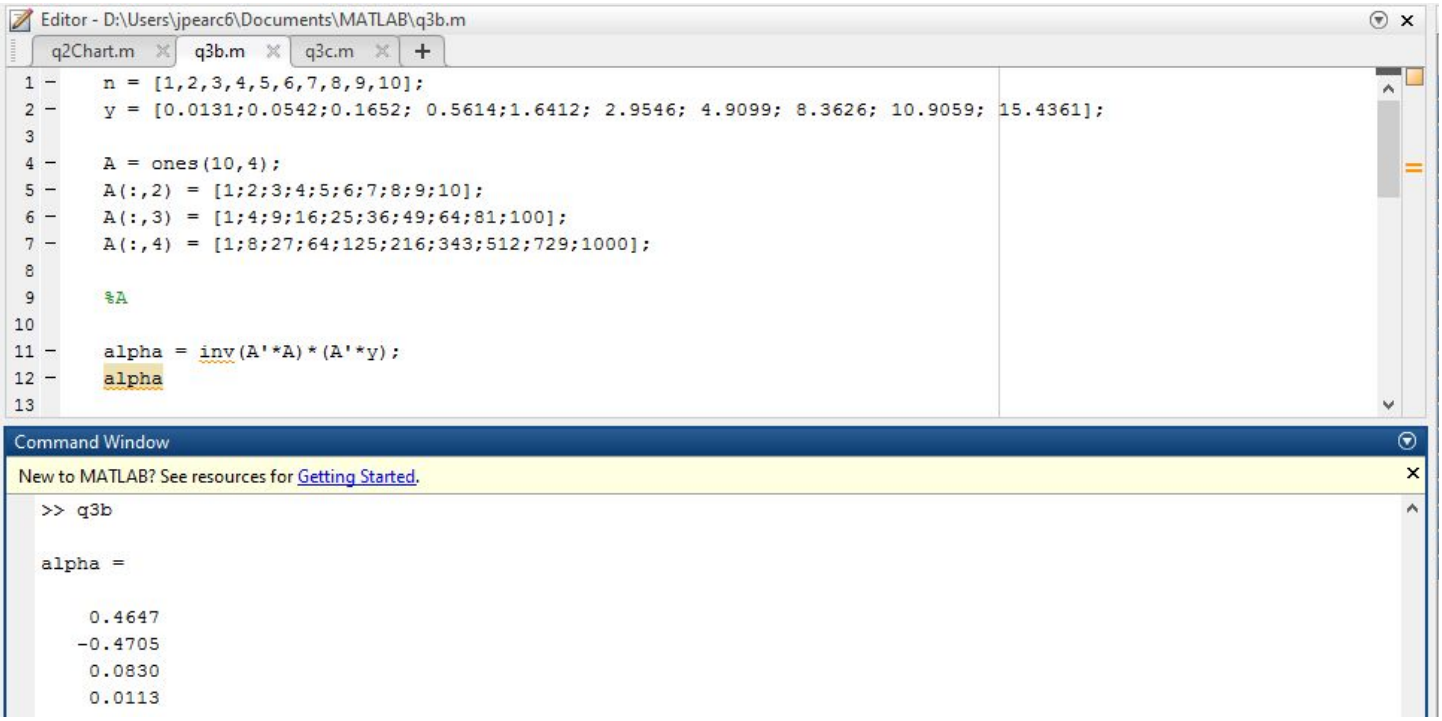
$$A = (n^0, n^1, n^2, n^3)$$

$$\alpha = (w_0, w_1, w_2, w_3)^T$$

3)b)

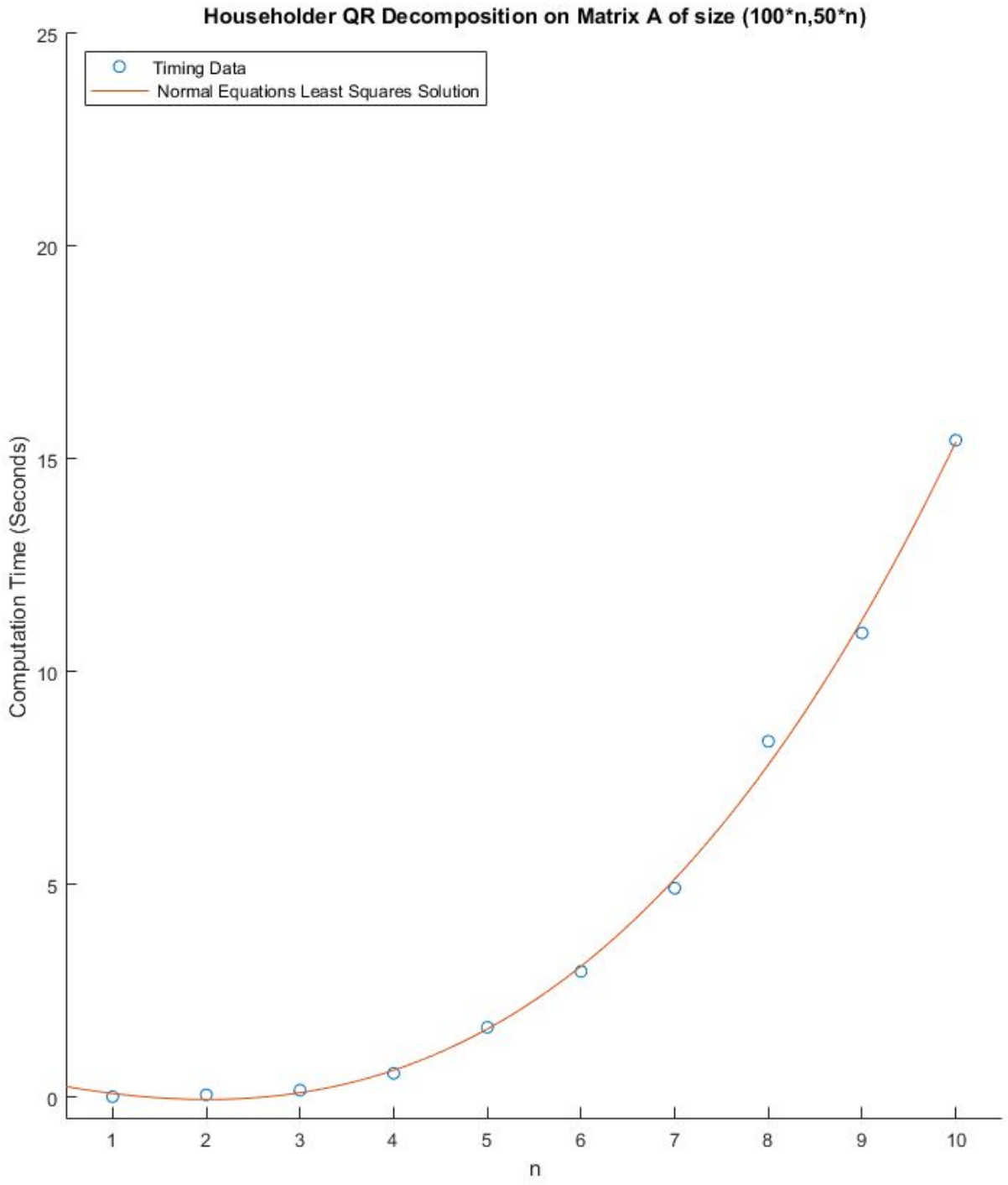
$$\alpha = (A^T A)^{-1} (A^T y)$$

Using Matlab to solve for α:



Normal Equations Least Squares Cubic Polynomial Solution:

$$time(x) = 0.4647 - 0.4705x + 0.0830x^2 + 0.0113x^3$$

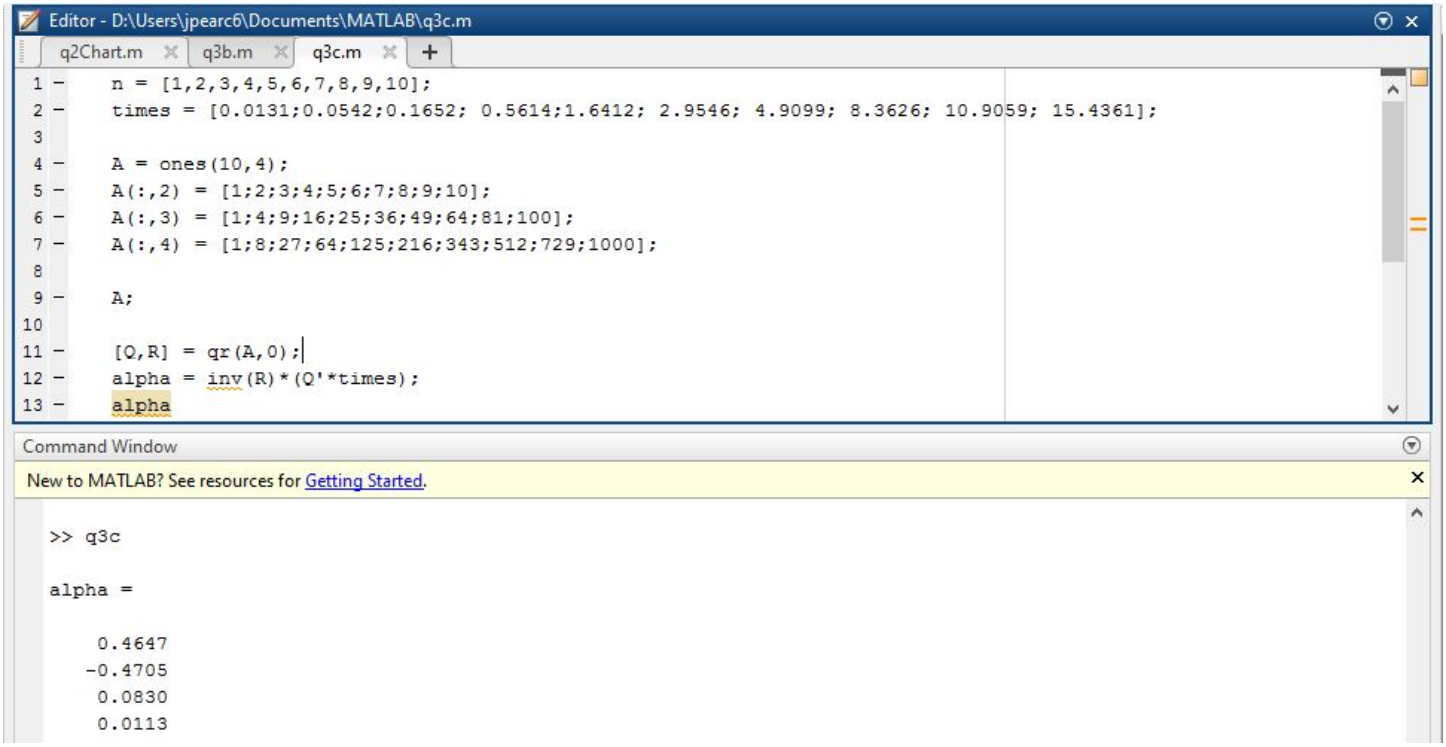

Householder QR Decomposition on Matrix A of size (100*n,50*n)
Timing Data
Normal Equations Least Squares Solution
Computation Time (Seconds)
n
0
5
10
15
20
25
1
2
3
4
5
6
7
8
9
10

3)c)

Given the reduced QR decomposition of A,

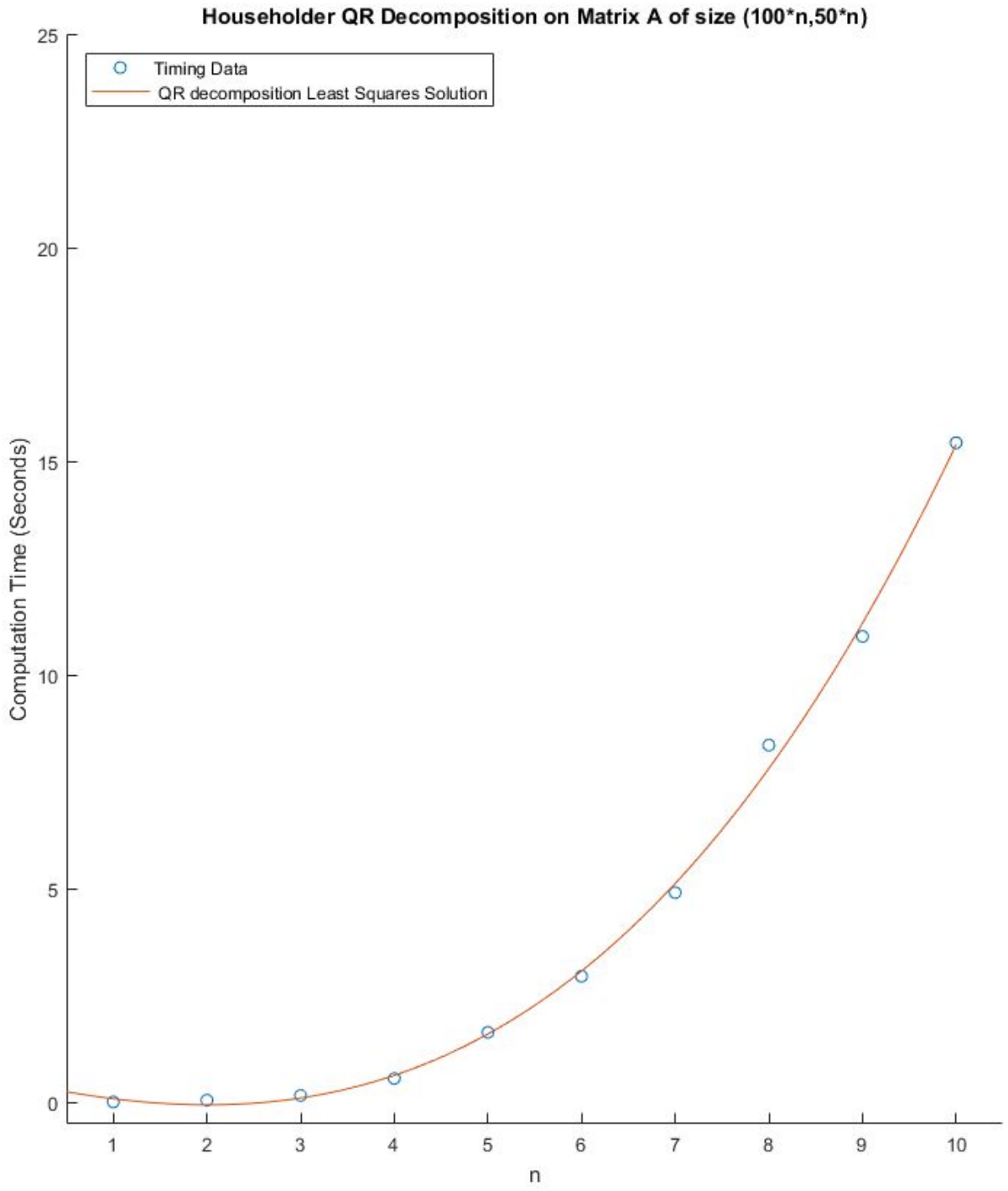
$$\alpha = R^{-1}(Q^T y)$$

Using Matlab to solve for α:



QR Decomposition Least Squares Cubic Polynomial Solution:

$$time(x) = 0.4647 - 0.4705x + 0.0830x^2 + 0.0113x^3$$


Householder QR Decomposition on Matrix A of size (100*n,50*n)
Timing Data
QR decomposition Least Squares Solution
Computation Time (Seconds)
n
0
5
10
15
20
25
1
2
3
4
5
6
7
8
9
10

3d)i)

750 x 375 Matrix. Therefore $x = \frac{750}{100} = 7.5$

$$time\,(7.5) = 0.4647 - 0.4705\,(7.5) + 0.0830(7.5)^2 + 0.0113(7.5)^3 = 6.372$$

According to my model, I expect this Householder algorithm to run for 6.372 seconds for a 750 x 375 size matrix

3d)ii)

10000 x 5000 Matrix. Therefore $x = \frac{10000}{100} = 100$

$$time\,(100) = 0.4647 - 0.4705\,(100) + 0.0830(100)^2 + 0.0113(100)^3 = 12083.415$$

According to my model, I expect this Householder algorithm to run for 12083.415 seconds for a 10000 x 5000 size matrix