Econ 546 Assignment 2

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Problem 1. This game is symmetric, so we will only consider Player 1's moves. Let A represent the aggressive action and P represent the passive action. From the question we can formulate a system of equations:

$$\begin{cases} U_1(P,P) = \frac{1}{2}U_1(A,A) + \frac{1}{2}U_1(A,P) \\ U_1(P,A) = \frac{2}{3}U_1(A,A) + \frac{1}{3}U_1(P,P) \end{cases}$$

The question also states $U_1(A,A) = 0$ and $U_1(P,A) = 1$. Using these we can solve the system and obtain the solution:

$$U_1(A,A) = 0, U_1(P,A) = 1, U_1(P,P) = 3, U_1(A,P) = 6$$

Using the symmetry of the game to get the utility for Player 2, we establish the game as follows:

	Aggressive	Passive
Aggressive	(0,0)	(6,1)
Passive	(1,6)	(3,3)

We know that (P,A) and (A,P) are the pure strategy Nash Equilibrium (Assignment 1). These can be expressed as mixed strategy Nash Equilibrium; $\{(1,0),(0,1)\}$ and $\{(0,1),(1,0)\}$. We can now solve for the final mixed strategy Nash Equilibrium. Denote p the probability Player 1 is aggressive and denote q the probability Player 2 is Aggressive.

$$\begin{cases} U_1(A) = 0 * q + 6(1-q) = 6 - 6q \\ U_1(P) = 1 * q + 3(1-q) = 3 - 2q \end{cases}$$

Setting $U_1(A) = U_1(P)$ we get $q = \frac{3}{4}$. By symmetry $p = \frac{3}{4}$.

We conclude the 3 mixed strategy Nash equilibrium are $\{(1,0),(0,1)\}$, $\{(0,1),(1,0)\}$ and $\{(\frac{3}{4},\frac{1}{4}),(\frac{3}{4},\frac{1}{4})\}$

Problem 2. This is a symmetric game so we will only need to analyse the utility from Player 1's perspective. Examining Player 1's utilities:

$$U_1(day1 = \text{swim}) = \pi(-c+0) + (1-\pi)(1+1) = -\pi c + 2(1-\pi)$$

$$U_1(day1 = \text{no swim and friend swims}) = 0 + (1-\pi)(1) = 1 - \pi$$

$$U_1(day1 = \text{no swim and friend doesn't swim}) = \pi(-c) + (1-\pi)(1) = -\pi c + 1 - \pi$$

However recall that from the question we only swim on day 2 if $-\pi c + 1 - \pi > 0$. Therefore we can update our 3rd utility equation.

$$U_1(day1 = \text{no swim and friend doesn't swim}) = max(0, -\pi c + 1 - \pi)$$

Therefore we can represent the game as follows,

	Swim	No Swim
Swim	$(-\pi c+2(1-\pi),-\pi c+2(1-\pi))$	$(-\pi c+2(1-\pi),1-\pi)$
No Swim	$(1-\pi,-\pi c+2(1-\pi))$	$(max(0,-\pi c+1-\pi),max(0,-\pi c+1-\pi))$

We will find the mixed strategy Nash Equilibria through different cases.

Case 1: suppose $\alpha=-\pi c+2(1-\pi)<0$. The game becomes:

	Swim	No Swim
Swim	(α,α)	$(\alpha,1-\pi)$
No Swim	$(1-\pi,\alpha)$	$(0,0)$

Therefore the mixed strategy Nash Equilibrium in this case is $\{(0,1),(0,1)\}$, both players will choose to not swim on the first day.

Case 2: suppose $\beta=-\pi c+1-\pi>0$. The game becomes:

	Swim	No Swim
Swim	$(\beta+(1-pi),\beta+(1-pi))$	$(\beta+(1-pi),1-\pi)$
No Swim	$(1-\pi,\beta+(1-pi))$	(β,β)

Therefore the mixed strategy Nash Equilibrium in this case is $\{(1,0),(1,0)\}$, both players will choose to swim on the first day.

Case 3: suppose $-\pi c+1-\pi<0<-\pi c+2(1-\pi)$. Therefore $(max(0,-\pi c+1-\pi)=0$ and the game can be simplified to the following (this will help with solving for the mixed strategy):

	Swim	No Swim
Swim	$(-\pi c+2(1-\pi),-\pi c+2(1-\pi))$	$(-\pi c+2(1-\pi),1-\pi)$
No Swim	$(1-\pi,-\pi c+2(1-\pi))$	$(0,0)$

Denote p the probability Player 1 swims on the first day and denote q the probability Player 2 swims on the first day.

$$\begin{cases} U_1(Swim)=q(-\pi c+2(1-\pi))+(1-q)(-\pi c+2(1-\pi))=-\pi c+2(1-\pi) \\ U_1(NoSwim)=q(1-\pi)+(1-q)(0)=q(1-\pi) \end{cases}$$

Setting $U_1(Swim)=U_1(NoSwim)$ we get $q=\frac{-\pi c+2(1-\pi)}{1-\pi}$. Therefore the mixed strategy Nash equilibrium in this case is $\{(\frac{-\pi c+2(1-\pi)}{1-\pi},\frac{\pi c+\pi-1)}{1-\pi}),(\frac{-\pi c+2(1-\pi)}{1-\pi},\frac{\pi c+\pi-1)}{1-\pi}))\}$. Note, the inequality at the beginning of Case 3 ensures that $q\in[0,1]$.

The presence of a friend decreases the likelihood of someone swimming on the first day, since we can get our friend to check the water safety while we wait for day 2.

Problem 3. Let p the denote probability that Player 2 plays L. We will now examine the expected utility for Player 2 given Player 1's move:

$$\begin{cases} U_2(Player1=T)=p(-2)+(1-p)(1)=1-3p \\ U_2(Player1=B)=p(1)+(1-p)(-1)=2p-1 \end{cases}$$

Setting $U_2(Player1=T)=U_2(Player1=B)$ we get that the security strategy for Player 2 is $p^*=\frac{2}{5}$ and the maxmin payoff is $\frac{-1}{5}$.

Problem 4. Consider the following game. Let $\alpha, \beta \in \mathbb{R}$ and $\alpha > \beta > 0$

	a	b
A	(α ,α)	(β,0)
B	(0,β)	(0,0)

Player 1 and Player 2's maxminimized payoff is β. $\{A, a\}$ is the unique Nash equilibrium, therefore each Player's payoff is α and recall $\alpha > \beta$.

Problem 5a.

- Given Player 1 chooses C, Player 2's unique optimal action is F
- Given Player 1 chooses D, Player 2's unique optimal action is G

Propagating these backwards through the game tree we can see that Player 1's unique optimal action for every combination of the optimal actions of player 2 is C. Therefore the unique subgame perfect equilibrium is (C, FG).

Problem 5b.

- Given Player K chooses R and Player R chooses B, Player E's unique optimal action is B
- Given Player K chooses R and Player R chooses H, Player E's unique optimal action is H
- Given Player K chooses E and Player E chooses B, Player R's unique optimal action is B
- Given Player K chooses E and Player E chooses H, Player R's unique optimal action is H

Propagating these backwards through the game tree.

- Given Player K chooses R, Player R's unique optimal action is B
- Given Player K chooses E, Player E's unique optimal action is H

Propagating these backwards through the game tree we can see that Player K's unique optimal action for every combination of the optimal actions at the second and third stage is E. Therefore there is a unique subgame perfect equilibrium. Writing this subgame perfect equilibrium in brackets as we did in 5a. is difficult because Rosa and Ernesto's actions are all labelled either B and H.

Problem 5c.

- Given Player 1 chooses C, Player 2's optimal actions are E and F
- Given Player 1 chooses D, Player 2's optimal actions are G and H

Thus there are four combinations of player 2s optimal actions (given Player 1's actions): EG, EH, FG and FH. Propagating these backwards through the game tree.

- For the combinations EH, player 1's optimal action is C
- For the combinations FG, player 1's optimal action is D
- For the combinations EG and FH, player 1's optimal action are C and D

Therefore the subgame perfect equilibria are (C, EH), (D, FG), (C,EG), (D,EG), (C, FH) and (D, FH).

Problem 6.

- Given Army 1 chooses to Attack, Army 2's unique optimal action is Retreat

Propagating this backwards through the game tree we can see that Army 1's unique optimal action for the optimal action of Army 2 is Attack. Therefore the unique subgame perfect equilibrium is (Attack, Retreat).

Suppose now that Army 2 burns the bridge to its mainland. The subgame perfect equilibrium is now the action set in which army 1 does not attack. Therefore army 2 can increase its subgame perfect equilibrium payoff (and reduce army 1's payoff) by burning the bridge to its mainland, eliminating its option to retreat if attacked.