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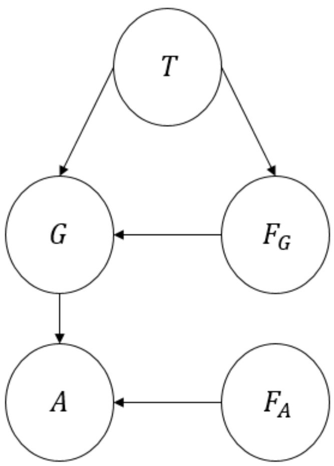
Comp 424 Assignment 3

1a.



b.

Our network is not a polytree. If we remove the edge directions, there is a cycle (G – T – F_G – G).

c.

$$P(G|T, F_G)$$

		G	
T	**F_G**	Normal	High
Normal	F_G	y	1-y
Normal	Not F_G	x	1-x
High	F_G	1-y	y
High	Not F_G	1-x	x

d.

$$P(A|G, F_A)$$

		A	
G	**F_A**	A	Not A
Normal	F_A	0	1
Normal	Not F_A	0	1
High	F_A	0	1
High	Not F_A	1	0

e.

Notation: Let T, G denote 'high temperature' and $\neg T, \neg G$ denote 'normal temperature

$$P(T|A, \neg F_G, \neg F_A) = \frac{P(T, A, \neg F_G, \neg F_A)}{P(A, \neg F_G, \neg F_A)}$$

$$= \frac{\sum_{g \in \{G, \neg G\}} P(T, A, \neg F_G, \neg F_A, g)}{\sum_{g \in \{G, \neg G\}, t \in \{T, \neg T\}} P(A, \neg F_G, \neg F_A, t, g)}$$

$$= \frac{\sum_{g \in \{G, \neg G\}} P(T) * P(\neg F_A) * P(\neg F_G|T) * P(g|\neg F_G, T) * P(A|g, \neg F_A)}{\sum_{g \in \{G, \neg G\}, t \in \{T, \neg T\}} P(t) * P(\neg F_A) * P(\neg F_G|t) * P(g|\neg F_G, t) * P(A|g, \neg F_A)}$$

We can factor and divide out $P(\neg F_A)$

$$= \frac{\sum_{g \in \{G, \neg G\}} P(T) * P(\neg F_G|T) * P(g|\neg F_G, T) * P(A|g, \neg F_A)}{\sum_{g \in \{G, \neg G\}, t \in \{T, \neg T\}} P(t) * P(\neg F_G|t) * P(g|\neg F_G, t) * P(A|g, \neg F_A)}$$

2a.

$$P(s, r) = \sum_{a,t,b} P(r, s, B = b, A = a, T = t)$$

$$= \sum_{a,t,b} P(r) P(B = b) P(A = a|B = b) P(s|A = a) P(T = t|r, A = a, B = b)$$

a	t	b	P(r)	P(B=b)	P(A=a | B=b)	P(S | A = a)	P(T = t | r, B=b, A=a)	
a	t	b	0.15	0.2	0.6	0.8	0.95	0.01368
a	t	not b	0.15	0.8	0.4	0.8	0.92	0.035328
a	not t	b	0.15	0.2	0.6	0.8	0.05	0.00072
a	not t	not b	0.15	0.8	0.4	0.8	0.08	0.003072
not a	t	b	0.15	0.2	0.4	0.2	0.9	0.00216
not a	t	not b	0.15	0.8	0.6	0.2	0.85	0.01224
not a	not t	b	0.15	0.2	0.4	0.2	0.1	0.00024
not a	not t	not b	0.15	0.8	0.6	0.2	0.15	0.00216
								0.0696

$$P(s, r) = 0.0696$$

2b.

$$P(a, \neg t) = \sum_{b,r,s} P(a, \neg t, B = b, R = r, S = s)$$

$$= \sum_{b,r,s} P(R = r) P(B = b) P(a|B = b) P(S = s|a) P(\neg t|a, R = r, B = b)$$

r	s	b	P(R=r)	P(B=b)	P(a | B=b)	P(S=s | a)	P(not t |R=r, B=b, a)	
r	s	b	0.15	0.2	0.6	0.8	0.05	0.00072
r	s	not b	0.15	0.8	0.4	0.8	0.08	0.003072
r	not s	b	0.15	0.2	0.6	0.2	0.05	0.00018
r	not s	not b	0.15	0.8	0.4	0.2	0.08	0.000768
not r	s	b	0.85	0.2	0.6	0.8	0.65	0.05304
not r	s	not b	0.85	0.8	0.4	0.8	0.4	0.08704
not r	not s	b	0.85	0.2	0.6	0.2	0.65	0.01326
not r	not s	not b	0.85	0.8	0.4	0.2	0.4	0.02176
								0.17984

$$P(a, \neg t) = 0.17984$$

2c.

$$P(t \mid s) = \frac{P(t, s)}{P(s)}$$

$$P(t, s) = \sum_{a,r,b} P(t, s, B = b, A = a, R = r)$$

$$= \sum_{a,r,b} P(R = r)P(B = b)P(A = a|B = b)P(s|A = a)P(t|R = r, A = a, B = b)$$

r	a	b	P(R=r)	P(B=b)	P(A=a | B=b)	P(s | A=a)	P(t |R=r, B=b, A=a)	
r	a	b	0.15	0.2	0.6	0.8	0.95	0.01368
r	a	not b	0.15	0.8	0.4	0.8	0.92	0.035328
r	not a	b	0.15	0.2	0.4	0.2	0.9	0.00216
r	not a	not b	0.15	0.8	0.6	0.2	0.85	0.01224
not r	a	b	0.85	0.2	0.6	0.8	0.35	0.02856
not r	a	not b	0.85	0.8	0.4	0.8	0.6	0.13056
not r	not a	b	0.85	0.2	0.4	0.2	0.4	0.00544
not r	not a	not b	0.85	0.8	0.6	0.2	0.05	0.00408
								0.232048

$$P(t, s) = 0.232048$$

$$P(s) = \sum_{a,r,b,t} P(s, T = t, B = b, A = a, R = r)$$

$$= \sum_{a,r,b,t} P(R = r)P(B = b)P(A = a|B = b)P(s|A = a)P(T = t|R = r, A = a, B = b)$$

t	r	a	b	P(R=r)	P(B=b)	P(A=a | B=b)	P(s | A=a)	P(T=t |R=r, B=b, A=a)	
t	r	a	b	0.15	0.2	0.6	0.8	0.95	0.01368
t	r	a	not b	0.15	0.8	0.4	0.8	0.92	0.035328
t	r	not a	b	0.15	0.2	0.4	0.2	0.9	0.00216
t	r	not a	not b	0.15	0.8	0.6	0.2	0.85	0.01224

t	not r	a	b	0.85	0.2	0.6	0.8	0.35	0.02856
t	not r	a	not b	0.85	0.8	0.4	0.8	0.6	0.13056
t	not r	not a	b	0.85	0.2	0.4	0.2	0.4	0.00544
t	not r	not a	not b	0.85	0.8	0.6	0.2	0.05	0.00408
not t	r	a	b	0.15	0.2	0.6	0.8	0.05	0.00072
not t	r	a	not b	0.15	0.8	0.4	0.8	0.08	0.003072
not t	r	not a	b	0.15	0.2	0.4	0.2	0.1	0.00024
not t	r	not a	not b	0.15	0.8	0.6	0.2	0.15	0.00216
not t	not r	a	b	0.85	0.2	0.6	0.8	0.65	0.05304
not t	not r	a	not b	0.85	0.8	0.4	0.8	0.4	0.08704
not t	not r	not a	b	0.85	0.2	0.4	0.2	0.6	0.00816
not t	not r	not a	not b	0.85	0.8	0.6	0.2	0.95	0.07752
									0.464

$$P(s) = 0.464$$

$$P(t \mid s) = \frac{P(t,s)}{P(s)} = \frac{0.232048}{0.464} = 0.5001$$

3.

Eliminate R:

Factors: $P(R)$, $P(B)$, $P(A|B)$, $P(T|A,B,R)$, $P(S|A)$

$$m_R(T,A,B) = \sum_{\{r \in R\}} P(r)P(T|A,B,r)$$

$$= P(r)P(T|A,B,r) + P(\neg r)P(T|A,B,\neg r)$$

t	a	b	P(T = t | A = a, B = b, r)	P(T = t | A = a, B = b, not r)	m_R(T,A,B)
t	a	b	0.95	0.35	0.44
t	a	not b	0.92	0.6	0.648
t	not a	b	0.9	0.4	0.475
t	not a	not b	0.85	0.05	0.17
not t	a	b	0.05	0.65	0.56
not t	a	not b	0.08	0.4	0.352
not t	not a	b	0.1	0.6	0.525
not t	not a	not b	0.15	0.95	0.83

$m_R(T,A,B)$ represents the joint probability distribution of T,A,B independent of R.

Eliminate S:

Factors: $P(B)$, $P(A|B)$, $P(S|A)$, $m_R(T,A,B)$

$$m_S(A) = \sum_{\{s \in S\}} P(s|A)$$

$$= P(s \mid A) + P(\neg s \mid A)$$

$$= 1$$

$m_S(A)$ represents the probability distribution of A independent of Independent of R and S.

Eliminate B:

Factors: $P(B)$, $P(A|B)$, $m_R(T, A, B)$, $m_S(A)$

$$m_B(T, A) = \sum_{\{b \in B\}} P(b)P(A|b)m_R(T, A, b)$$

$$= P(b)P(A|b)m_R(T, A, b) + P(\neg b)P(A|\neg b)m_R(T, A, \neg b)$$

t	a	P(A=a | b)	P(A=a | not b)	m_R(T=t,A=a,b)	m_R(T=t,A=a,not b)	m_B(T,A)
t	a	0.6	0.4	0.44	0.648	0.26016
t	not a	0.4	0.6	0.475	0.17	0.1196
not t	a	0.6	0.4	0.56	0.352	0.17984
not t	not a	0.4	0.6	0.525	0.83	0.4404

$m_B(T, A)$ represents the joint probability distribution of T,A independent of Independent of R,S and B.

Eliminate A:

Factors: $m_S(A)$, $m_B(T, A)$

$$\delta(A, a) = \begin{cases} 1 \; if \; A = a \\ 0 \; if \; A \neq a \end{cases}$$

$$m_A(T) = \sum_{\{a \in A\}} m_B(T, a)m_S(a)\delta(A, a)$$

$$= m_B(T, a)m_s(a)(1) + m_B(T, a)m_S(a)(0)$$

$$= m_B(T, a)m_S(a)(1)$$

t	m_B(T=t,a)	m_S(a)	m_A(T)
t	0.26016	1	0.26016
not t	0.17984	1	0.17984

$m_A(T)$ represents the probability distribution of T given A=a (provided by the indicator function) independent of R,S.B.

$$m_A(t) > m_A(\neg t)$$

$\Rightarrow T = t$ is the MAP result of the query.

4a.

i.

$$\theta_A = P(A=1)$$

$$\theta_{B|0} = P(B=1|A=0)$$

$$\theta_{B|1} = P(B=1|A=1)$$

$$\theta_{C|0} = P(C=1|A=0)$$

$$\theta_{C|1} = P(C=1|A=1)$$

$$\theta_{D|0,0} = P(D=1|B=0, C=0)$$

$$\theta_{D|1,0} = P(D=1|B=1, C=0)$$

$$\theta_{D|0,1} = P(D=1|B=0, C=1)$$

$$\theta_{D|1,1} = P(D=1|B=1, C=1)$$

ii.

$$\theta_A^{MLE} = P(A=1) = \frac{49}{146}$$

$$\theta_{B|0}^{MLE} = P(B=1|A=0) = \frac{68}{68+29} = \frac{68}{97}$$

$$\theta_{B|1}^{MLE} = P(B=1|A=1) = \frac{43}{43+6} = \frac{43}{49}$$

$$\theta_{C|0}^{MLE} = P(C=1|A=0) = \frac{56}{56+41} = \frac{56}{97}$$

$$\theta_{C|1}^{MLE} = P(C=1|A=1) = \frac{19}{19+30} = \frac{19}{49}$$

$$\theta_{D|0,0}^{MLE} = P(D=1|B=0, C=0) = \frac{4}{4+2} = \frac{4}{6}$$

$$\theta_{D|1,0}^{MLE} = P(D=1|B=1, C=0) = \frac{21}{21+44} = \frac{21}{65}$$

$$\theta_{D|0,1}^{MLE} = P(D=1|B=0, C=1) = \frac{8}{8+21} = \frac{8}{29}$$

$$\theta_{D|1,1}^{MLE} = P(D=1|B=1, C=1) = \frac{0}{0+46} = \frac{0}{46}$$

iii.

$$\theta_A^{LS} = P(A=1) = \frac{49+1}{146+2} = \frac{50}{148}$$

$$\theta_{B|0}^{LS} = P(B = 1|A = 0) = \frac{68+1}{97+2} = \frac{69}{99}$$

$$\theta_{B|1}^{LS} = P(B = 1|A = 1) = \frac{43+1}{49+2} = \frac{44}{51}$$

$$\theta_{C|0}^{LS} = P(C = 1|A = 0) = \frac{56+1}{97+2} = \frac{57}{99}$$

$$\theta_{C|1}^{LS} = P(C = 1|A = 1) = \frac{19+1}{49+2} = \frac{20}{51}$$

$$\theta_{D|0,0}^{LS} = P(D = 1|B = 0, C = 0) = \frac{4+1}{6+2} = \frac{5}{8}$$

$$\theta_{D|1,0}^{LS} = P(D = 1|B = 1, C = 0) = \frac{21+1}{65+2} = \frac{22}{67}$$

$$\theta_{D|0,1}^{LS} = P(D = 1|B = 0, C = 1) = \frac{8+1}{29+2} = \frac{9}{31}$$

$$\theta_{D|1,1}^{LS} = P(D = 1|B = 1, C = 1) = \frac{0+1}{46+2} = \frac{1}{48}$$

b.

i.

$$w_{D_1=1} = P(D_1 = 1|A_1, B_1, C_1)$$

$$= P(D = 1\,|A = 1, B = 0, C = 1)$$

$$= \frac{P(D = 1, A = 1, B = 0, C = 1)}{P(A = 1, B = 0, C = 1)}$$

$$= \frac{P(A = 1, B = 0, C = 1, D = 1)}{\sum_{\{d \in D\}} P(A = 1, B = 0, C = 1, D = d)}$$

$$= \frac{\theta_A(1-\theta_{B|1})\theta_{C|1}\theta_{D|0,1}}{\theta_A\left(1-\theta_{B|1}\right)\theta_{C|1}\theta_{D|0,1} + \ \theta_A(1-\theta_{B|1})\theta_{C|1}(1-\theta_{D|0,1})}$$

$$= \frac{\theta_{D|0,1}}{\theta_{D|0,1} + \ (1-\theta_{D|0,1})}$$

$$= \theta_{D|0,1}$$

$$= \frac{8}{29}$$

$$\Rightarrow w_{D_1=0} = \frac{21}{29}$$

$$w_{C_2=1} = P(C_2 = 1|A_2, B_2, D_2)$$

$$= P(C = 1 \,|A = 1, B = 1, D = 0)$$

$$= \frac{P(C = 1, A = 1, B = 1, D = 0)}{P(A = 1, B = 1, D = 0)}$$

$$= \frac{P(A = 1, B = 1, C = 1, D = 0)}{\sum_{\{c \in C\}} P(A = 1, B = 1, C = c, D = 0)}$$

$$= \frac{\theta_A \theta_{B|1} \theta_{C|1} (1 - \theta_{D|1,1})}{\theta_A \theta_{B|1} \theta_{C|1} (1 - \theta_{D|1,1}) + \theta_A \theta_{B|1} (1 - \theta_{C|1})(1 - \theta_{D|1,0})}$$

$$= \frac{\theta_{C|1} (1 - \theta_{D|1,1})}{\theta_{C|1} (1 - \theta_{D|1,1}) + (1 - \theta_{C|1})(1 - \theta_{D|1,0})}$$

$$= \frac{\frac{19}{49} * (1 - \frac{0}{46})}{\frac{19}{49} * \left(1 - \frac{0}{46}\right) + (1 - \frac{19}{49}) * (1 - \frac{21}{65})}$$

$$= \frac{247}{511}$$

$$\Rightarrow w_{C_2=0} = \frac{264}{511}$$

ii.

$$\theta_A^{MLE} = P(A = 1) = \frac{49 + 2}{146 + 2} = \frac{51}{148}$$

$$\theta_{B|0}^{MLE} = P(B = 1|A = 0) = \frac{68}{97}$$

$$\theta_{B|1}^{MLE} = P(B = 1|A = 1) = \frac{43 + 1}{49 + 2} = \frac{44}{51}$$

$$\theta_{C|0}^{MLE} = P(C = 1|A = 0) = \frac{56}{97}$$

$$\theta_{C|1}^{MLE} = P(C = 1|A = 1) = \frac{19 + 1 + w_{C_2=1}}{49 + 2} \approx 0.4016$$

$$\theta_{D|0,0}^{MLE} = P(D = 1|B = 0, C = 0) = \frac{4}{6}$$

$$\theta_{D|1,0}^{MLE} = P(D = 1|B = 1, C = 0) = \frac{21}{65 + w_{C_{2=0}}} \approx 0.3205$$

$$\theta_{D|0,1}^{MLE} = P(D = 1|B = 0, C = 1) = \frac{8 + w_{D_1=1}}{29 + 1} \approx 0.2759$$

$$\theta_{D|1,1}^{MLE} = P(D = 1|B = 1, C = 1) = \frac{0}{46 + w_{C_2=1}} = 0$$

iii.

We use the simplified formulas derived in part i.

$$w_{D_1=1} = \theta_{D|0,1}$$

$$= 0.2759$$

$$\Rightarrow w_{D_1=0} = 0.7241$$

$$w_{C_2=1} = \frac{\theta_{C|1}(1-\theta_{D|1,1})}{\theta_{C|1}(1-\theta_{D|1,1}) + (1-\theta_{C|1})(1-\theta_{D|1,0})}$$

$$= \frac{0.4016 * (1-0)}{0.4016 * (1-0) + (1-0.4016)(1-0.3205)}$$

$$= 0.4969$$

$$\Rightarrow w_{C_2=0} = 0.5031$$