McGill ECON546

Winter 2019

Due Date: Jan 30th, 2019 9:50am

Prof. Zhang

Assignment 1

Question 1: Provide your own definition of Game Theory.

Question 2: Consider the Game of Nim. There are two piles of straws (Pile A and Pile B) in front of two players (Player 1 and Player 2), and players take turns to remove straws. Players can remove straws from one pile at a time. Whoever removes the last straw wins the game.

Now, consider the two piles are unbalanced, in that Pile A has two straws and Pile B has only one.

(a) (5 points) Does Player 1 have a winning strategy? If so, clearly describe that winning strategy. (Note: Player 1 is the first mover.)
(b) (5 points) Suppose the rules of the game remain the same except that the person who removes the last straw loses the game. Does Player 1 have a winning strategy? If so, clearly describe that winning strategy.

Question 3: Six pirates were sailing one day and stumbled upon a treasure chest with 20 gold coins. The captain and pirate 2,3,4,5,6 had to decide how to share the coins. The captain gives the seniority ranking: himself > 2 > 3 > 4 > 5 > 6. The rules and conditions are the same as the pirates game we have discussed in class. Please show your analysis and write down the equilibrium properly.

Question 4: Compute all the Nash equilibria in the following game.

	L	M	R
T	3,2	4,0	0,0
M	2,0	3,3	0,0
B	0,0	0,0	3,3

Question 5: (Exercise 34.1 in Osborne) Guessing two-third of the average: Each of the three people announces an integer from 1 to K. If the three integers are different, the person whose integer is closet to 2/3 of the average of the three integers wins $100. If there is a tie, $100 will be split equally among the winners.

1. Is there any integer K such that the strategy profile (K, K, K) is a Nash equilibrium?
2. Is any other strategy profile a Nash equilibrium?
3. For the guessing game we played in class where there were 60 players. Players were asked to pick an integer from (o,100) inclusive. Find all the Nash equilibria. Show your analysis. Explain from your understanding why the outcome in the in-class game was not a NE.

Question 6 (exercise 31.2) Hawk-Dove: Two animals are fighting over some prey. Each can be passive or aggressive. Each prefers to be aggressive if its opponent is passive, and passive if its opponent is aggressive; given its own stance, it prefers the outcome in which its opponent is passive to that in which its opponent is aggressive. Formulate this situation as a strategic game and find its Nash equilibrium.

Question 7 (exercise 42.1) Find the Nash equilibrium of the two-player strategic game in which each player's set of actions is the set of nonnegative numbers and the players' payoff functions are

$$u_1(a1, a2) = a1(a2 - a1)$$

$$u_2(a1, a2) = a2(1 - a2 - a1)$$

Question 8 (exercise 49.1) Suppose there are three candidates, A, B and C, and no citizen is indifferent between any two of them. A tie for first place is possible; assume that a citizen who prefers a win by x to a win by y ranks a tie between x and y between an outright win for x and an outright win for y. Show that a citizen's only weakly dominated action is a vote for her least preferred candidate. Find a Nash equilibrium in which some citizen does not vote for her favorite candidate, but the action she takes is not weakly dominated.