Math 256 Winter 2017

Due Friday, January 20

1. (Ax 2.A.1) Suppose v_1, v_2, v_3, v_4 spans V. Prove that the list
$$v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4$$
also spans V.

2. Find a number t such that
$$(3,1,4),(1,5,9),(2,6,t)$$
is not linearly independent in $\mathbb{R}^3$. Explain why the set is not linearly independent.

3. (Ax 2.A.5)
   (a) Show that if we consider $\mathbb{C}$ as a vector space over $\mathbb{R}$, then $(1+i, 1-i)$ is linearly independent.
   (b) Show that if we consider $\mathbb{C}$ as a vector space over $\mathbb{C}$, then $(1+i, 1-i)$ is not linearly independent.

4. (Ax 2.A.6) Suppose v_1, v_2, v_3, v_4 is linearly independent in V. Prove that the list
$$v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4$$
is also linearly independent in V.

5. (Ax 2.A.8) Prove or give a counterexample: If $v_1, v_2, \ldots, v_m$ is a linearly independent set of vectors in V and $\lambda \in \mathbb{F}$ with $\lambda \neq 0$, then $\lambda v_1, \lambda v_2, \ldots, \lambda v_m$ is linearly independent.

6. (Ax 2.A.13) Explain why no list of four polynomials spans $\mathcal{P}_4(\mathbb{F}) = \{a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 \mid a_i \in \mathbb{F}\}$.

7. (Ax 2.B.3) Let U be the subspace of $\mathbb{R}^5$ defined by
$$U = \{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5 \mid x_1 = 3x_2 \text{ and } x_3 = 7x_4\}.$$
   (a) Find a basis for U.
   (b) Extend the basis in part (a) to a basis for $\mathbb{R}^5$.
   (c) Find a subspace W of $\mathbb{R}^5$ such that $\mathbb{R}^5 = U \oplus W$.