
Contents

Preface	xi
1 Introduction and Examples	1
1.1 The Science of Rule-Making	1
1.2 When Is Selfish Behavior Near-Optimal?	3
1.3 Can Strategic Players Learn an Equilibrium?	6
Notes, Problems, and Exercises	9
2 Mechanism Design Basics	11
2.1 Single-Item Auctions	11
2.2 Sealed-Bid Auctions	12
2.3 First-Price Auctions	12
2.4 Second-Price Auctions and Dominant Strategies	13
2.5 Ideal Auctions	15
2.6 Case Study: Sponsored Search Auctions	16
Notes, Problems, and Exercises	20
3 Myerson's Lemma	24
3.1 Single-Parameter Environments	24
3.2 Allocation and Payment Rules	26
3.3 Statement of Myerson's Lemma	26
*3.4 Proof of Myerson's Lemma	28
3.5 Applying the Payment Formula	31
Notes, Problems, and Exercises	34
4 Algorithmic Mechanism Design	39
4.1 Knapsack Auctions	39
4.2 Algorithmic Mechanism Design	42
4.3 The Revelation Principle	46
Notes, Problems, and Exercises	49

5	Revenue-Maximizing Auctions	55
5.1	The Challenge of Revenue Maximization	55
5.2	Characterization of Optimal DSIC Mechanisms	58
5.3	Case Study: Reserve Prices in Sponsored Search	65
*5.4	Proof of Lemma 5.1	66
	Notes, Problems, and Exercises	69
6	Simple Near-Optimal Auctions	74
6.1	Optimal Auctions Can Be Complex	74
6.2	The Prophet Inequality	75
6.3	Simple Single-Item Auctions	77
6.4	Prior-Independent Mechanisms	79
	Notes, Problems, and Exercises	82
7	Multi-Parameter Mechanism Design	87
7.1	General Mechanism Design Environments	87
7.2	The VCG Mechanism	88
7.3	Practical Considerations	91
	Notes, Problems, and Exercises	93
8	Spectrum Auctions	97
8.1	Indirect Mechanisms	97
8.2	Selling Items Separately	98
8.3	Case Study: Simultaneous Ascending Auctions	100
8.4	Package Bidding	105
8.5	Case Study: The 2016 FCC Incentive Auction	106
	Notes, Problems, and Exercises	110
9	Mechanism Design with Payment Constraints	113
9.1	Budget Constraints	113
9.2	The Uniform-Price Multi-Unit Auction	114
*9.3	The Clinching Auction	116
9.4	Mechanism Design without Money	119
	Notes, Problems, and Exercises	123
10	Kidney Exchange and Stable Matching	128
10.1	Case Study: Kidney Exchange	128
10.2	Stable Matching	136
*10.3	Further Properties	139

Notes, Problems, and Exercises	142
11 Selfish Routing and the Price of Anarchy	145
11.1 Selfish Routing: Examples	145
11.2 Main Result: Informal Statement	147
11.3 Main Result: Formal Statement	149
11.4 Technical Preliminaries	152
*11.5 Proof of Theorem 11.2	153
Notes, Problems, and Exercises	156
12 Over-Provisioning and Atomic Selfish Routing	159
12.1 Case Study: Network Over-Provisioning	159
12.2 A Resource Augmentation Bound	161
*12.3 Proof of Theorem 12.1	162
12.4 Atomic Selfish Routing	163
*12.5 Proof of Theorem 12.3	165
Notes, Problems, and Exercises	169
13 Equilibria: Definitions, Examples, and Existence	173
13.1 A Hierarchy of Equilibrium Concepts	173
13.2 Existence of Pure Nash Equilibria	179
13.3 Potential Games	181
Notes, Problems, and Exercises	183
14 Robust Price-of-Anarchy Bounds in Smooth Games	187
*14.1 A Recipe for POA Bounds	187
*14.2 A Location Game	188
*14.3 Smooth Games	194
*14.4 Robust POA Bounds in Smooth Games	195
Notes, Problems, and Exercises	199
15 Best-Case and Strong Nash Equilibria	202
15.1 Network Cost-Sharing Games	202
15.2 The Price of Stability	205
15.3 The POA of Strong Nash Equilibria	208
*15.4 Proof of Theorem 15.3	210
Notes, Problems, and Exercises	213
16 Best-Response Dynamics	216
16.1 Best-Response Dynamics in Potential Games	216

16.2	Approximate PNE in Selfish Routing Games	219
*16.3	Proof of Theorem 16.3	221
*16.4	Low-Cost Outcomes in Smooth Potential Games	223
	Notes, Problems, and Exercises	226
17	No-Regret Dynamics	230
17.1	Online Decision Making	230
17.2	The Multiplicative Weights Algorithm	234
*17.3	Proof of Theorem 17.6	236
17.4	No Regret and Coarse Correlated Equilibria	239
	Notes, Problems, and Exercises	242
18	Swap Regret and the Minimax Theorem	247
18.1	Swap Regret and Correlated Equilibria	247
*18.2	Proof of Theorem 18.5	249
18.3	The Minimax Theorem for Zero-Sum Games	253
*18.4	Proof of Theorem 18.7	255
	Notes, Problems, and Exercises	258
19	Pure Nash Equilibria and \mathcal{PLS}-Completeness	261
19.1	When Are Equilibrium Concepts Tractable?	261
19.2	Local Search Problems	264
19.3	Computing a PNE of a Congestion Game	271
	Notes, Problems, and Exercises	276
20	Mixed Nash Equilibria and \mathcal{PPAD}-Completeness	279
20.1	Computing a MNE of a Bimatrix Game	279
20.2	Total \mathcal{NP} Search Problems (\mathcal{TFNP})	280
*20.3	\mathcal{PPAD} : A Syntactic Subclass of \mathcal{TFNP}	285
*20.4	A Canonical \mathcal{PPAD} Problem: Sperner's Lemma	288
*20.5	MNE and \mathcal{PPAD}	290
20.6	Discussion	293
	Notes, Problems, and Exercises	294
	The Top 10 List	299
	Hints to Selected Exercises and Problems	301
	Bibliography	309
	Index	329