Homework 4

MATH 591 Mathematics of Machine Learning
Fall 2019

due: 5pm Monday Nov 25th, submit on MyCourses

Homework based on

- Mohri, Ch 5, Support Vector Machines
- Mohri, Ch 6, Kernel Methods, Section 6.1, 6.2, 6.3, 6.6 (Bochner's Theorem)

Refer to 2nd edition for correct exercises.

1. *SVMs duality* Mohri 5.1
2. *SVMs duality* Mohri 5.6
3. *Kernels* Mohri 6.1
4. *Kernels* Mohri 6.2 (d) (e)
5. *Kernels* Mohri 6.21
6. *Kernels and Fourier Transform* Suppose $q(x)$ is a smooth probability density (a non-negative with integral one). Define $\Phi(x_0)$ to be the function which maps $x \mapsto q(x - x_0)$. The goal of this exercise is to show that we can define a Reproducing Kernel Hilbert space (RKKS) using this map. Let $\mathbb{H}$ be the Hilbert space of functions which can be written as the convolution $f = q * a$ and which have bounded norm (the norm will be defined below).

   (a) Define the inner product $\langle f, g \rangle = \int a(x)g(x)dx = \int f(x)b(x)dx$ where $f = q * a$, $g = q * b$. Show that the inner product is well defined (meaning that the inner product does not depend on whether it is written the first way or the second way).

   (b) Define $K(x, y) = q(x - y)$. Show that $K(x, y) = \langle \Phi(x, \cdot), \Phi(y, \cdot) \rangle$. Hint use the Dirac delta, to write $\Phi(x, \cdot)$ as a convolution.

   (c) Prove reproducing kernel property, $h(x) = \langle h, \Phi(x, \cdot) \rangle$ for all x and all $h \in \mathbb{H}$.