
Index

- acknowledgments, xii
- agent, 25, 87, 173
- allocation curve, 29
 - area under, 31
- allocation rule, 26
 - approximately welfare-maximizing, 43
 - cycle monotonicity, 89, 93
 - deferred, 107, 111–112
 - greedy, 44, 106, 112
 - implementable, 27
 - monotone, 27
 - non-monotone, 45
 - scoring function, 108
 - virtual threshold, 78
 - virtual welfare-maximizing, 62
 - welfare-maximizing, 19, 22, 42, 50, 89
- approximation algorithm, 43, 44
- auction
 - approximately optimal, 75, 84
 - ascending, 21, 97–98, 112, 115
 - clinching, 116–119, 126
 - combinatorial, *see* combinatorial auction
 - competition, 80
 - English, *see* auction, ascending
 - first-price, 12, 64, 72–73
 - ideal, 15–16
 - Japanese, 98
 - k -unit, 20, 25, 37, 71
 - knapsack, 39–40, 43–46
 - multi-item, *see* combinatorial auction
 - multi-unit, 114, 126
 - online, 22
 - optimal, 57, 64
 - prior-free, 83
 - prior-independent, 79–82, 86
 - procurement, 21
 - revenue-maximizing, 57
 - reverse, 21, 106
 - sealed-bid, 12
 - second-price, 13, 32
 - separate single-item, 98
 - sequential, 100
 - simple, 75, 77–79, 84
 - simultaneous sealed-bid, 101, 201
 - single-item, 11–16, 24, 31, 88
 - sniping, 102
 - spectrum, *see* spectrum auction

- sponsored search, *see* sponsored search
 - third-price, 20
 - two-step design approach, 18
 - uniform-price, 114–116
 - versus negotiation, 80
 - weird, 75, 83
 - welfare-maximizing, 15
 - with budgets, 113
 - with one bidder, 56, 57
- badminton, 1–3
- Bayesian analysis, 56, 63, 72–73
- Bayesian incentive compatible (BIC), 63
- best response, 175
- best-response dynamics, 216
- as a graph walk, 217
 - as local search, 274
 - convergence time, 218
 - ϵ -, 219, 275
 - fast convergence, 220–223, 227, 228, 228
 - in atomic selfish routing, 220–223, 228
 - in congestion games, 274
 - in potential games, 217
 - in scheduling, 227–229
 - in smooth games, 223–226
 - in symmetric congestion games, 275
 - lower bounds, 274, 275
 - maximum-gain, 219, 224
 - maximum-relative-gain, 228
 - maximum-weight, 227
 - non-convergence, 227, 229
 - vs. better-response dynamics, 217
- bid, 26
- critical, 41, 63
 - false-name, 92, 94
- bimatrix game, 7, 279
- bounded rationality, 8
- Braess's paradox, 4, 145, 158
- Brouwer's fixed-point theorem, 290, 295
- and Nash's theorem, 291, 293, 296
 - in higher dimensions, 291
- budget, 114
- private, 119, 124
- budget-balance, 21, 95–96
- Bulow-Klemperer theorem, 80–82, 86
- bundle, 88
- case studies, xii, 16, 65–66, 99–109, 128–136, 159–161
- CCE, *see* coarse correlated equilibrium
- CE, *see* correlated equilibrium
- Chernoff-Hoeffding bounds, 308
- Christie's, 98
- coalition
- blocking (core allocation), 121
 - deviation (strong Nash equilibrium), 209
- coarse correlated equilibrium, 177–178
- and no-regret dynamics, 240–241, 243, 245
 - approximate, 241
 - as a linear program, 260

- interpretation, 241
 - tractability, 178, 241, 260, 263, 293
- coffee, 290
- collusion, 23, 38, 92, 94
- combinatorial auction, 88, 93–95, 97
 - applications, 88
 - approximation, 96–97
 - complements, 99
 - exposure problem, 104
 - package bidding, 105
 - price of anarchy, 201
 - substitutes, 99
- common prior, 48, 57, 72
- communication network, 159, 169
- computational efficiency, 16
- congestion game, 181, 271
 - as a potential game, 186, 271
 - computing a correlated equilibrium, 277
 - intractability of pure Nash equilibria, 272–275
 - lower bound for best-response dynamics, 274, 275
 - symmetric, 274–275
- $\text{co}\mathcal{NP}$, 281
- core allocation, 122
- correlated equilibrium, 176–177
 - and no-swap-regret dynamics, 248, 257
 - as a linear program, 260, 277
 - in congestion games, 277
 - interpretation, 176, 241
 - swapping function, 183, 247
 - tractability, 177, 249, 260, 263, 277, 293
 - traffic light example, 177
- cost function
 - in cost-minimization games, 174
 - in selfish routing, 148
- cryptography, 293, 295
- CTR, *see* sponsored search, click-through rate
- deferred acceptance algorithm, 137–141
 - applicant-optimality, 140
 - convergence, 139
 - incentive properties, 141, 144
- deferred allocation rule, *see* allocation rule, deferred
- demand
 - aggregate, 115
 - of a bidder, 114
 - reduction, 103, 115
 - residual, 116
- density function, 57
- dependencies, xii
- direct-revelation, *see* mechanism, direct-revelation
- distribution function, 57
- dominant strategy, 13
- dominant-strategy incentive compatible, *see* DSIC
- DSIC, 15, 16, 46
 - for randomized mechanisms, 86, 126
- dynamics, 216

- best-response, *see* best-response dynamics
- no-regret, *see* no-regret dynamics
- no-swap-regret, *see* no-swap-regret dynamics
- vs. algorithms, 263
- eBay, 13, 21, 58, 64
- economics and computation, xi
- environment, *see* single-parameter environment
- envy-free, *see* sponsored search, envy-free
- equilibrium
 - Bayes-Nash, 48, 57, 63, 72–73
 - coarse correlated, *see* coarse correlated equilibrium
 - competitive, *see* Walrasian equilibrium
 - computation, 263
 - correlated, *see* correlated equilibrium
 - dominant-strategy, 46
 - flow, 152, 164
 - hierarchy, 173, 208
 - Nash, *see* Nash equilibrium
 - refinement, 208
 - sponsored search, 36
 - Walrasian, *see* Walrasian equilibrium
- experts, *see* online decision making
- externality, 50, 90, 195, 202
- negative vs. positive, 202
- feasible set (of outcomes), 24
- Federal Communications Commission (FCC), 106
- first welfare theorem, 111
- $FLXP$, 295
- flow network, 152–153, 156
 - multicommodity, 157
- \mathcal{FNP} (functional \mathcal{NP}), 280
 - and mixed Nash equilibria, 281
 - decision vs. search problems, 280
- \mathcal{FNP} -completeness, 281
- Gale-Shapley algorithm, *see* deferred acceptance algorithm
- game
 - compact, 263, 277
 - congestion, *see* congestion game
 - constant-sum, 258
 - cost-minimization, 173
 - description length, 263
 - location, 188, 199
 - network cost-sharing, *see* network cost-sharing game
 - payoff-maximization, 174, 190
 - potential, *see* potential game
 - smooth, *see* smooth game
 - symmetric, 297
 - two-player, *see* bimatrix game
 - zero-sum, *see* zero-sum game
- Google, 17

- greedy, *see* allocation rule, greedy
- group-strategyproof, 38
weakly, 112
- GSP, *see* sponsored search, generalized second-price auction
- hints, xii, 301–308
- \mathcal{H}_k , 205
- house allocation, 119–122, 125
and kidney exchange, 129
- implementable, *see* allocation rule, implementable
- incentive guarantee, 16
- individual rationality, 15, 26, 46
- information rent, 60
- intended audience, xii
- intractability
of approximate mixed Nash equilibria, 295
of local search, 267–271
of mixed Nash equilibria, 8, 284, 292–293
of pure Nash equilibria, 272–275
of welfare maximization, 41, 92
- Iran, 129
- item, 11, 88
- kidney exchange, 128–136
altruistic donor, 131
chain, 131, 143
for money, 129
incentives for hospitals, 134
incompatibility, 128
pairwise, 131–134
patient-donor pair, 128
priorities, 132
simultaneous vs. sequential, 130
- knapsack problem, 40
auction, *see* auction, knapsack
fully polynomial-time approximation scheme (FPTAS), 45, 52
greedy heuristic, 43
- learning, *see* dynamics
online, *see* online decision making
- lecture videos, xii
- Lemke-Howson algorithm, 292, 295
- linear programming, 259–260, 277, 292
- local optimum, 265
- local search, 265
abstract, 267
and best-response dynamics, 274
as a walk in a graph, 265, 269
generic procedure, 268
improving move, 265
intractability, 267–271
polynomial, *see* \mathcal{PLS}
unconditional lower bounds, 271
- Markov chain, 252, 257
- matching
in kidney exchange, 131
stable, *see* stable matching

- maximum cut problem, 264
 - and congestion games, 273
 - is \mathcal{PLS} -complete, 270, 277
 - unconditional lower bounds, 271
 - with unit edge weights, 265
- mechanism, 25
 - anonymous, 127
 - approximately optimal, 75, 84, 96
 - direct-revelation, 26, 46
 - ideal, 51, 89
 - indirect, 21, 26, 97–98
 - onto, 127
 - optimal, 57
 - prior-free, 83
 - prior-independent, 79–82, 86
 - priority, 125
 - randomized, 86, 126
 - serial dictatorship, 125
 - simple, 84
 - single-sample, 86
 - VCG, *see* VCG mechanism
- mechanism design, 3
 - algorithmic, 42–43, 45
 - applications, 3, 16, 65, 106
 - Bayesian, xi, 48
 - direct-revelation, 91
 - multi-parameter, 87
 - preference elicitation, 91
 - single-parameter, 24
 - two-step design approach, 18, 89
 - with budgets, 113
- Minimax theorem, 254–256
 - and linear programming duality, 258
 - and no-regret dynamics, 255, 259
 - equivalent to equilibrium existence, 258
 - for constant-sum games, 258
 - history, 257
 - interpretation, 254
 - mix and match, 258
- mixed Nash equilibrium, 7, 175
 - brute-force search algorithm, 296
 - existence, 175, 290
 - in bimatrix games, 279
 - intractability, 8, 176, 284, 292–293
 - intractability of approximate equilibria, 295
 - intractability with three or more players, 295
 - quasitractability of approximate equilibria, 295–297
- mixed strategy, 7, 175
- MNE, *see* mixed Nash equilibrium
- monopoly price, 58, 85
- monotone, *see* allocation rule, monotone
- monotone hazard rate (MHR), 70, 84
- multiplicative weights (MW)
 - algorithm, *see* no-regret algorithm, multiplicative weights
- Myerson's lemma, 28–31

- in multi-parameter environments, 89
- Myerson's payment formula, 30
- Nash equilibrium, 7
 - approximate, 197, 219, 295–297
 - as a predictor of behavior, 8, 293
 - best-case, *see* price of stability
 - existence (mixed), 290
 - existence (pure), 179–182
 - existence (strong), 210, 214
 - in zero-sum games, 255, 258, 260
 - ℓ -strong, 215
 - mixed, *see* mixed Nash equilibrium
 - non-uniqueness, 293
 - pure, *see* pure Nash equilibrium
 - strong, 209
 - worst-case, *see* price of anarchy
- Nash's theorem, 7, 290
- network cost-sharing game, 203
 - examples, 203–205
 - opting out, 205
 - price of anarchy, 204, 214
 - price of stability, 205–208
 - strong Nash equilibria, 208–212
 - undirected networks, 213, 214
 - VHS or Betamax, 204
 - with concave cost functions, 214
- network formation, 202, 213
- network over-provisioning, 159, 169
- no-regret algorithm, 232
 - design principles, 234
 - deterministic, 233
 - existence, 234
 - follow-the-leader (FTL), 233
 - follow-the-perturbed-leader (FTPL), 245–246
 - learning rate, 235, 239
 - multiplicative weights, 234–239
 - with unknown time horizon, 239, 243
- no-regret dynamics, 239
 - converges to coarse correlated equilibria, 240–241, 245
 - in payoff-maximization games, 239
 - in smooth games, 241, 243
 - in zero-sum games, 255, 259
 - non-convergence to mixed Nash equilibria, 259
- no-swap-regret algorithm, 248
 - existence, 249
 - reduction to a no-regret algorithm, 249–252
- no-swap-regret dynamics, 248
 - converges to correlated equilibria, 248, 257
- \mathcal{NP} , 42
- \mathcal{NP} search problem, *see* \mathcal{FNP}

- \mathcal{NP} -completeness, 264, 276
 - unsuitability for equilibrium computation, 281–283
- \mathcal{NP} -hard, 8, 42, 107
- Olympic scandal, 1–3
- online decision making, 230
 - adversary (adaptive), 231, 241
 - adversary (oblivious), 236
 - algorithm, 231
 - bandit model, 231, 243
 - examples, 231–234
 - history, 243
 - in games, *see* no-regret dynamics
 - mistake bound, 244
 - regret-minimization, *see* no-regret algorithm
 - with an omniscient expert, 244
 - with large costs, 243
 - with payoffs, 231, 255
- organization of book, xi
- outcome, 87, 174
- \mathcal{P} , 42
- Pareto optimal, 124
- payment rule, 26
 - computation, 42, 51
 - explicit formula, 30, 33, 41
 - nonnegative, 26
 - uniqueness, 28
- payoff, 7
- performance guarantee, 16
 - ex post, 56
- Pigou's example, *see* selfish routing, Pigou's example
- player, 253
- \mathcal{PLS} , 269, 276
 - as a subset of \mathcal{FNP} , 281
 - as a subset of \mathcal{TFNP} , 283
 - as a syntactic class, 286
 - as a walk in a graph, 269
- \mathcal{PLS} -completeness, 270
 - analogy with \mathcal{NP} -completeness, 267
 - justification, 283
 - of computing a locally maximum cut, 270
 - of computing a pure Nash equilibrium of a congestion game, 272–275
 - vs. unconditional results, 266
- PNE, *see* pure Nash equilibrium
- POA, *see* price of anarchy
- polynomial time, 41, 263
- posted price, 56, 85
- potential function, 181, 206
 - generalized ordinal, 227
 - minimizer, 207, 213, 215
- potential game, 181, 185–186, 199
 - as a congestion game, 186
 - convergence of best-response dynamics, 217
 - equilibrium existence, 182, 217
 - generalized ordinal, 227
 - smooth, 223
- \mathcal{PPAD} , 286, 294
 - and mixed Nash equilibria, 292–293

- and Sperner's lemma, 289
- as a subset of \mathcal{TFNP} , 288
- as a syntactic class, 287
- as a walk in a graph, 286
- evidence of intractability, 293
- PPAD*-completeness
 - of computing a mixed Nash equilibrium of a bimatrix game, 284
 - of computing a trichromatic triangle, 289
- prerequisites, xii
- price of anarchy, 5
 - four-step recipe, 187
 - in location games, 188–194
 - in network cost-sharing games, 204, 214
 - in scheduling, 171–172, 185, 200
 - in selfish routing, 146–155
 - in selfish routing (atomic), 165–168
 - in smooth games, 195–198, 200
 - in sponsored search, 200
 - of correlated equilibria, 177
 - of mixed Nash equilibria, 176
 - of no-regret dynamics, 241, 243
 - of strong Nash equilibria, 208–212
 - optimistic, *see* price of stability
- price of stability, 205
 - in atomic selfish routing networks, 215
 - in network cost-sharing games, 205–208
 - interpretation, 208
- private, 12
- prophet inequality, 75–77, 82–84
- public project, 25
- pure Nash equilibrium, 174
 - existence, 179–182, 184
 - intractability, 272–275, 277
 - tractability, 278
- pure strategy, 174
- quasilinear utility, 12, 23, 26
- quasipolynomial time, 297
- reduction (between search problems), 269, 280, 297
- regret, 232
 - minimization algorithm, *see* no-regret algorithm
 - external, 232
 - external vs. swap, 258
 - internal, 248
 - lower bound, 233, 244
 - swap, 248
 - vs. stronger benchmarks, 231
- regular distribution, *see* valuation distribution, regular
- report, 25
- reserve price, 58, 65–66
 - anonymous, 79, 83
 - bidder-specific, 79, 83, 85
- resource augmentation, 161

- revelation principle, 46–48, 64, 89
- revenue, 59
 - curve, 60, 72
 - equals virtual welfare, 60
 - equivalence, 70, 72
 - monotonicity, 93
 - of a mechanism, 59, 61
 - target, 37
- revenue maximization, *see*
 - auction, revenue-maximizing
- Rock-Paper-Scissors, 7, 253
- routing, *see* selfish routing

- SAA, *see* spectrum auction, simultaneous ascending
- satisfiability (SAT), 107
 - functional version, 281
- scheduling, 171–172, 185, 200, 227–229
- science of rule-making, *see* mechanism design
- secretary problem, 22
- selfish routing, 4, 148
 - affine cost functions, 151, 156, 171
 - affine cost functions (atomic), 165
 - α -bounded jump condition, 220
 - atomic, 163–168, 170, 184, 198
 - atomic splittable, 169
 - best-response dynamics (atomic), 220–223, 228, 277
 - Braess's paradox, 145, 158
 - computing an equilibrium flow (atomic), 277, 278
 - concave cost functions, 151, 156
 - cost function, 148
 - cost of a flow, 153
 - equilibrium existence, 146, 165, 179
 - equilibrium flow, 152
 - equilibrium flow (atomic), 164
 - equilibrium uniqueness, 146, 165, 180
 - examples, 145–147
 - flow, 152
 - history, 156
 - M/M/1 cost function, 162
 - maximum travel time, 157
 - multicommodity, 157, 169
 - nonlinear Pigou's example, 147
 - over-provisioned network, 160–161, 169
 - Pigou bound, 151, 157
 - Pigou's example, 146–147
 - Pigou's example (atomic), 164
 - Pigou-like network, 150
 - polynomial cost functions, 151, 156
 - potential function, 179, 180, 221
 - price of anarchy, 146, 148–152
 - price of anarchy (atomic), 165, 170, 171
 - resource augmentation bound, 161–163, 170,

- 171
- road traffic, 149, 156
- total travel time, 153
- weighted atomic, 169, 184
- with a common origin and destination (atomic), 220, 277–278
- semantic complexity class, 284, 295
- single-item auction, *see* auction, single-item
- single-parameter environment, 24
 - downward-closed, 84
- single-peaked preferences, 126
- sink vertex, 265
- smooth game, 194–198
 - best-response dynamics, 223–226
 - examples, 195, 200–201
 - interpretation, 194
 - potential, 223
 - price of anarchy of approximate equilibria, 198, 200
 - price of anarchy of coarse correlated equilibria, 196
 - price of anarchy of pure Nash equilibria, 196
 - with respect to a strategy profile, 195
- social welfare, 15, 55, 87
 - approximation, 43, 92
 - with budgets, 124, 126
- Sotheby's, 98
- source vertex, 286
- spectrum auction, 97–109
 - activity rule, 102, 105
 - bid signaling, 104
 - deferred allocation rule, 107
 - demand reduction, 103
 - descending implementation, 109
 - exposure problem, 104
 - hierarchical packages, 105
 - in New Zealand, 101
 - in Switzerland, 100
 - opening bids, 109
 - package bidding, 105
 - price discovery, 102
 - repacking, 106
 - rookie mistake, 100, 101
 - sanity checks, 102
 - scoring function, 108
 - simultaneous ascending, 102
 - substitutes vs. complements, 99, 104
- Sperner's lemma, 288
 - and Brouwer's fixed-point theorem, 290
 - and Nash's theorem, 290–292
 - as a *PPAD* problem, 289
 - as a walk in a graph, 289
 - legal coloring, 288
 - trichromatic triangle, 288
- sponsored search, 16–19, 25, 27, 32–33, 35, 65–66
- click-through rate, 17
- DSIC payment formula, 33
- envy-free, 36
- equilibrium, 36
- equivalence of DSIC and GSP auctions, 35

- generalized second-price auction, 33, 35
- locally envy-free, 37
- price of anarchy, 200
- revenue maximization, 71
- slot, 17
- welfare maximization, 22
- stable matching, 136–141
 - applicant-optimality, 140
 - blocking pair, 137
 - deferred acceptance, 137–141
 - DSIC mechanism, 141
 - existence, 139
 - in practice, 143
- starred sections, xii
- strategy profile, 174
- strategy set, 7, 174
- strings and springs, 5
- submodular function, 192, 199
- syntactic complexity class, 283
- \mathcal{TFNP} (total functional \mathcal{NP}), 283, 294
 - lack of complete problems, 283
 - reasons for membership, 284
- Top Trading Cycle algorithm, 119–122
 - in kidney exchange, 129–131
- truthful mechanism, *see* mechanism, direct-revelation
- TTC, *see* Top Trading Cycle algorithm
- unit-demand, *see* valuation, unit-demand
- utility, *see* quasilinear utility
- valuation, 12, 24, 87
 - downward-sloping, 126
 - marginal, 126
 - single-minded, 53
 - subadditive, 96
 - unit-demand, 94, 112, 201
- valuation distribution, 57
 - correlated, 69
 - irregular, 63, 69
 - regular, 62, 70
 - strictly regular, 64
 - unknown, 80
- VCG mechanism, 90, 103
 - allocation rule, 89
 - and affine maximizers, 96
 - ascending implementation, 123
 - flaws, 91–92, 97, 105
 - non-monotonicity, 92, 93
 - payment rule, 90, 93
 - pivot term, 93, 95–96
 - revenue, 92
 - with unit-demand valuations, 94
- Vickrey auction, *see* auction, second-price
- Vickrey-Clarke-Groves, *see* VCG mechanism
- virtual valuation, 59
 - ironing, 69
 - nondecreasing, 62
- virtual welfare, 61
- Walrasian equilibrium, 110–112
- welfare, *see* social welfare

wireless spectrum, *see* spectrum auction

Yahoo, 65–66

zero-sum game, 7, 253–254
 convergence of no-regret
 dynamics, 255, 259
 minimax pair, 258
 Minimax theorem, *see*
 Minimax theorem
 mixed Nash equilibrium,
 254, 263
 value, 255