Math 256 Winter 2017

Due Friday, January 6

1. (Ax 1.A.7) Show that for every $\alpha \in \mathbb{C}$, there exists a unique $\beta \in \mathbb{C}$ such that $\alpha + \beta = 0$.

2. (Ax 1.A.11) Explain why there does not exist $\lambda \in \mathbb{C}$ such that

$$\lambda(2 - 3i, 5 + 4i, -6 + 7i) = (12 - 5i, 7 + 22i, -32 - 9i).$$

3. (Ax 1.B.4) The empty set is not a vector space. The empty set fails to satisfy only one of the requirements listed in 1.19. Which one?

4. Let V be the set of positive real numbers. Prove that V is a vector space over $\mathbb{R}$ with "addition" defined by

$$x \boxplus y = xy, \text{ for all } x, y \in V$$

and scalar multiplication by

$$\lambda * x = x^{\lambda}, \text{ for all } x \in V, \lambda \in \mathbb{R}.$$

What is the zero vector?