APPENDIX C

SUMMARY OF COMMON PROBABILITY DISTRIBUTIONS

DISCRETE DISTRIBUTIONS	
Distribution	**PMF, Expectation, Variance**
Uniform $(1, \ldots, n)$	$P(X = k) = \frac{1}{n}, \quad k = 1, \ldots, n$ $E(X) = \frac{n+1}{2} \quad Var(X) = \frac{n^2 - 1}{12}$
Binomial	$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \ldots, n$ $E(X) = np \quad Var(X) = np(1-p)$
Poisson	$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad k = 0, 1, 2, \ldots$ $E(X) = \lambda \quad Var(X) = \lambda$
Geometric	$P(X = k) = (1-p)^{k-1}p, \quad k = 1, 2, \ldots$ $E(X) = \frac{1}{p} \quad Var(X) = \frac{1-p}{p^2}$ $P(X > t) = (1-p)^t$
Negative binomial	$P(X = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r},$ $k = r, r+1, \ldots, \quad r = 1, 2, \ldots$ $E(X) = \frac{r}{p} \quad Var(X) = \frac{r(1-p)}{p^2}$
Hypergeometric	$P(X = k) = \frac{\binom{D}{k}\binom{N-D}{n-k}}{\binom{N}{n}}, k = 0, 1, \ldots, n$ $E(X) = \frac{nD}{N} \quad Var(X) = \frac{nD(N-D)}{N^2}\left(1 - \frac{n-1}{N-1}\right)$

Introduction to Stochastic Processes with R, First Edition. Robert P. Dobrow.

CONTINUOUS DISTRIBUTIONS	
Distribution	**Density, CDF, Expectation, Variance**
Uniform (a, b)	$f(x) = \frac{1}{b-a}, \quad a < x < b$ $F(x) = \frac{x-a}{b-a}, \quad a < x < b$ $E(X) = \frac{a+b}{2} \quad Var(X) = \frac{(b-a)^2}{12}$
Exponential	$f(x) = \lambda e^{-\lambda x}, \quad x > 0$ $F(x) = 1 - e^{-\lambda x}, \quad x > 0$ $E(X) = \frac{1}{\lambda} \quad Var(X) = \frac{1}{\lambda^2}$
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right), \quad -\infty < x < \infty$ $E(X) = \mu \quad Var(X) = \sigma^2$
Gamma	$f(x) = \lambda e^{-\lambda x} \frac{(\lambda x)^{r-1}}{\Gamma(r)}, \quad x > 0,$ where $\Gamma(r) = \int_0^\infty t^{r-1} e^{-t}\, dt$ $E(X) = \frac{r}{\lambda} \quad Var(X) = \frac{r}{\lambda^2}$
Beta	$f(x) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1}, \quad 0 < x < 1,$ where $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}.$ $E(X) = \frac{\alpha}{\alpha+\beta} \quad Var(X) = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$