COMP 766: Assignment 1
Available: Wednesday, October 2nd, 2019
Due Date: Friday, October 18th, 2019

Notes: You are welcome (and even encouraged) to use the literature to help you answer questions. However, whenever you base your answer upon e.g. an article or a textbook, please provide appropriate references. Points may be deducted if references are needed but missing.

Also, if a figure/drawing can help clarify your answer (e.g. for Question 2!), feel free to include hand-drawn sketches and/or actual figures.

Question 1:

a) Provide the equation(s) for constructing the average of two images in the diffeomorphic setting.

b) The equation(s) provided in part (a) should involve all the pixels/voxels of each image. This is the general setting for diffeomorphic registration. However, if we work in a simpler domain, these equations should simplify accordingly. For instance, in class we saw that the average of two binary images of a circle should simply be an image with a circle of average radius (as opposed to a pixelwise averaging of image values). How would you simplify the equation(s) provided in part (a) for this particular case? Assume that the circle centers are aligned, and that the only parameter that changes is the circle radius.

c) What happens if the circles' centers are not aligned and you need to estimate both the average radius as well as the average location of the circle center?

Question 2:

We saw in class that in the diffeomorphic registration setting for 3D images, we seek to minimize an energy function which involves a smoothness term of the form

$$\int_0^1 \|Lv(t)\|^2 dt$$

with a typical choice for L as follows:

$$L = \begin{pmatrix} -\nabla^2 + c & 0 & 0 \\ 0 & -\nabla^2 + c & 0 \\ 0 & 0 & -\nabla^2 + c \end{pmatrix} \tag{1}$$

Remember that ∇^2 denotes the Laplacian. For simplicity, let's say that $c = 0$ and let's multiply L by -1 to drop the negative sign.

Justify the use of this operator. What does L compute? How does (1) change as the vector field $v(t)$ varies from a smooth one to a perturbed one?

Question 3:

a) List three examples each of possible unary and pairwise terms for the energy function of a Markov Random Field, specify their formulation and explain in words what their intended behavior is. Be creative! (but also accurate)

b) In class we saw that diffeomorphic registration can be formulated as an energy minimization problem where we seek a displacement u' minimizing an energy function of the following general form:

$$u' = \arg\min_u E_D(I_T, I_S(\psi)) + \lambda E_R(u), \tag{2}$$

where I_T and I_S are the target and source images, respectively, ψ is the deformation, E_D is the data similarity energy term and E_R is the regularization energy term. Describe how you would formulate **diffeomorphic** registration using a Markov Random Field. Describe the structure of the graph and the potentials involved, and the meaning of graph nodes and the values they can take on. Please be as specific as possible.

Question 4:

For this question, you will need to read the following paper, which is provided on the MyCourses page:

BH Menze et al (2016) A generative probabilistic model and discriminative extensions for brain lesion segmentation – with application to tumor and stroke. IEEE Trans Med Imaging 35(4): 933-946.

You will also need the HMRF-EM toolbox for Matlab provided on the MyCourses website in a zip file. It comes with documentation in the form of a paper (included in the zip file).

The course website also provides two input images. One image shows a cropped axial slice through an abdominal CT image with a liver tumor. FYI the non-cropped image is

provided as well, though you will not need it for this assignment. The other image shows a cropped axial slice through an abdominal CT image of a healthy liver, to be used as an atlas prior.

Your task is to implement the Expectation-Maximization model described in Section II of the reference paper as a Markov Random Field, using the HMRF-EM toolbox which you will need to modify to accomodate for the model of the paper.

You are asked to implement this model for the case of 2D images, with a single input channel. Please show results for (a) three tissue classes: tumor, healthy liver tissue and background, and (b) four tissue classes: tumor core, tumor border, healthy liver tissue and background.

When submitting your assignment, please include your source code with the modifications you have implemented clearly indicated with appropriate comments. Please also include a discussion of your results, the model, the decisions you have made and the challenges you have encountered, and anything else that you deem important/relevant.