Econ 546 Assignment 3

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Problem 1. We will use backward induction, and start by analyzing day 2. The action-payoff space is as follows:

	Withdraw	Not Withdraw
Withdraw	($\underline{R}$,$\underline{R}$)	(2R-D,D)
Not Withdraw	(D,2R-D)	(R,R)

Note that $R>D \Rightarrow 2R-D>R$. Therefore (Withdraw, Withdraw) is the unique Nash Equilibrium of day 2. We can propagate this payoff back into action-payoff space for day 1 and analyze as follows:

	Withdraw	Not Withdraw
Withdraw	($\underline{r}$,$\underline{r}$)	(D,2r-D)
Not Withdraw	(2r-D,D)	($\underline{R}$,$\underline{R}$)

Note that $r<D \Rightarrow 2r-D<r$. Therefore (Withdraw, Withdraw) and (Not Withdraw, Not Withdraw) are the two Nash Equilibrium of day 1.

We conclude that there are 2 subgame perfect equilibrium. (Withdraw Withdraw, Withdraw Withdraw) yielding payoffs of (r, r) and (Not-Withdraw Withdraw, Not-Withdraw Withdraw) yielding payoffs of (R, R).

Problem 2. We will use backward induction.

Base Case: In the last round each player will play D regardless of the game history, since (D,D) is the Nash equilibrium of a single iteration of the Prisoner's dilemma.

Inductive Step: Assume that at iteration T, (D,D) is played. Now at iteration T-1 the actions and payoffs of player's 1 and 2 is independent of future iterations and therefore they will only consider the payoffs for a single iteration of the Prisoner's dilemma and thus (D,D) will be played at T-1.

We conclude that at any iteration, (D,D) will be played and therefore in any subgame perfect equilibrium both players choose D in each iteration.

Problem 3.

$$x + x\delta + x\delta^2 + ... \geq y + \delta + \delta^2 + ...$$

$$x(1 + \delta + \delta^2 + ...) \geq (y - 1) + (1 + \delta + \delta^2 + ...)$$

$$\frac{x}{1-\delta} \geq (y-1) + \frac{1}{1-\delta}$$

$$x \geq (y-1)(1-\delta)+1$$
$$\frac{x-1}{y-1} \geq 1-\delta$$
$$\delta \geq 1-\frac{x-1}{y-1}$$
$$\delta \geq \frac{y-x}{y-1}$$

Problem 4. Note: Game tree attached on the next page

From lecture 'Mixed Strategies 2' we have the following version of the 'Battle of the Sexes' game

	Football (F)	Opera (O)
Football (F)	(3,1)	(0,0)
Opera (O)	(0,0)	(1,3)

From class we have seen that this game has the following pure strategy Nash Equilibria, (F,F) and (O,O) and a mixed strategy Nash Equilibrium $((\frac{3}{4}, \frac{1}{4}), (\frac{1}{4}, \frac{3}{4}))$

Using Backward induction:

Player 1 at Stage 2:

- Play F if outcome from Stage 1 was (O,O)
- Play O if outcome from Stage 1 was (F,F)
- Play (F,O) $= (\frac{3}{4}, \frac{1}{4})$ otherwise

Player 2 at Stage 2:

- Play F if outcome from Stage 1 was (O,O)
- Play O if outcome from Stage 1 was (F,F)
- Play (F,O) $= (\frac{1}{4}, \frac{3}{4})$ otherwise

Adding these payoffs from the second stage to the first stage of the game we get:
(note: expected utility of mixed strategy $= \frac{3}{16} * 3 + \frac{3}{16} * 1 + \frac{9}{16} * 0 + \frac{1}{16} * 0 = \frac{3}{4}$)

	Football (F)	Opera (O)
Football (F)	(4,4)	$(\frac{3}{4}, \frac{3}{4})$
Opera (O)	$(\frac{3}{4}, \frac{3}{4})$	(4,4)

The Nash Equilibrium of this updated game are (F,F),(O,O) and $((\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}))$.

Therefore the subgame perfect Nash equilibrium of the 2 stage Battles of the Sexes game is:

- At stage 1, Player 1 plays F, O or (F,O) $= (\frac{1}{2}, \frac{1}{2})$. Player 2 plays F, O or (F,O) $= (\frac{1}{2}, \frac{1}{2})$.
- At stage 2, Player 1 plays:

- F if outcome from Stage 1 was (O,O)
- O if outcome from Stage 1 was (F,F)
- (F,O) = $(\frac{3}{4}, \frac{1}{4})$ otherwise

- At stage 2, Player 2 plays:
  - F if outcome from Stage 1 was (O,O)
  - O if outcome from Stage 1 was (F,F)
  - (F,O) = $(\frac{1}{4}, \frac{3}{4})$ otherwise