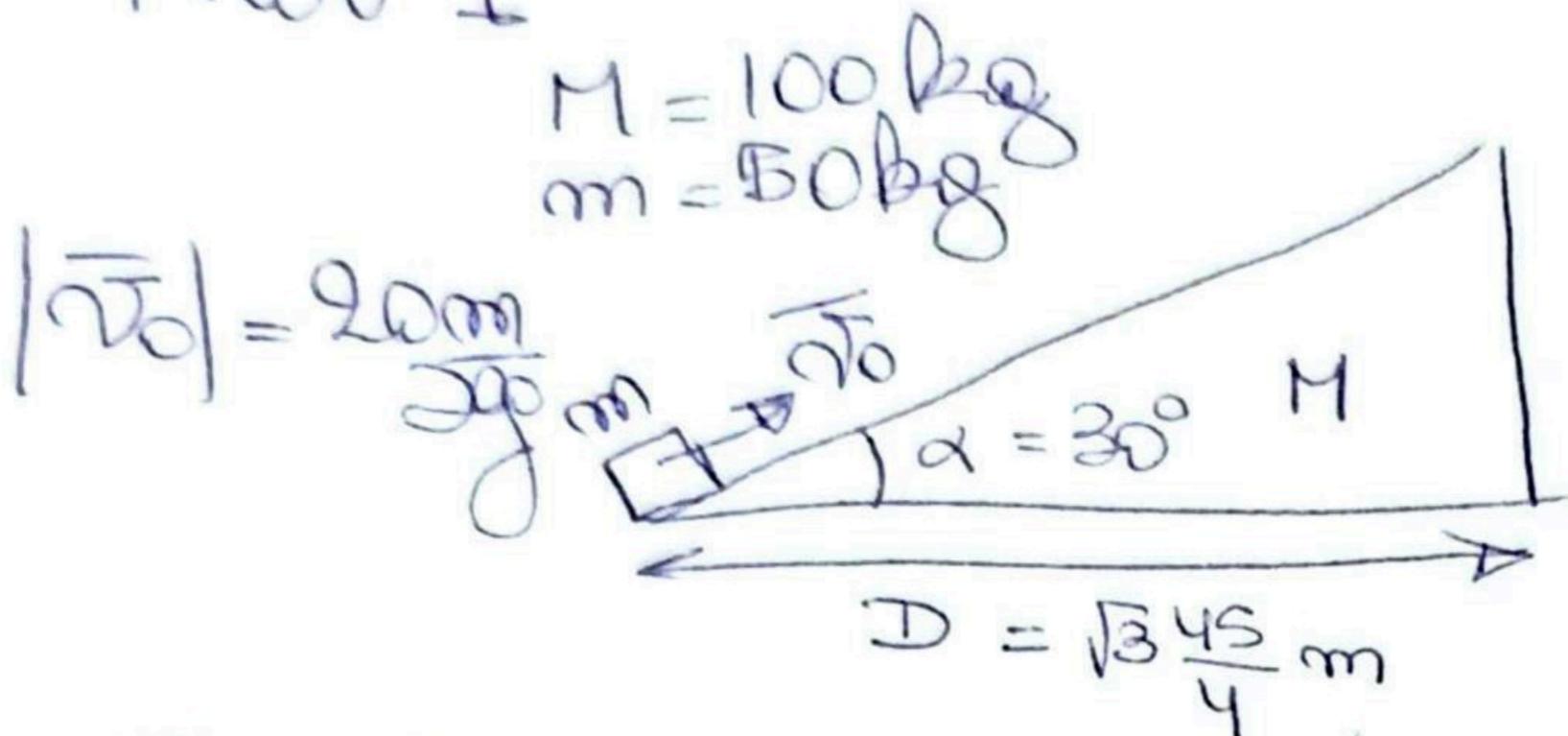


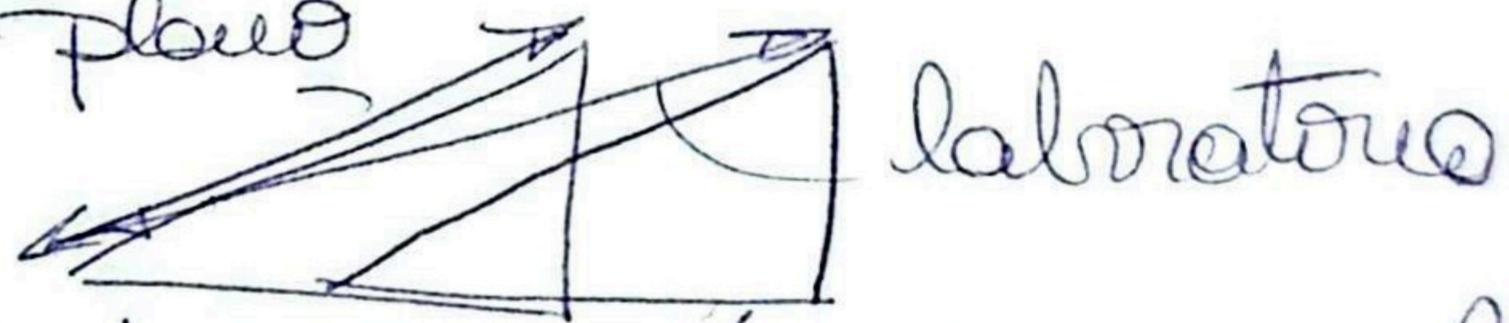
Prob 1



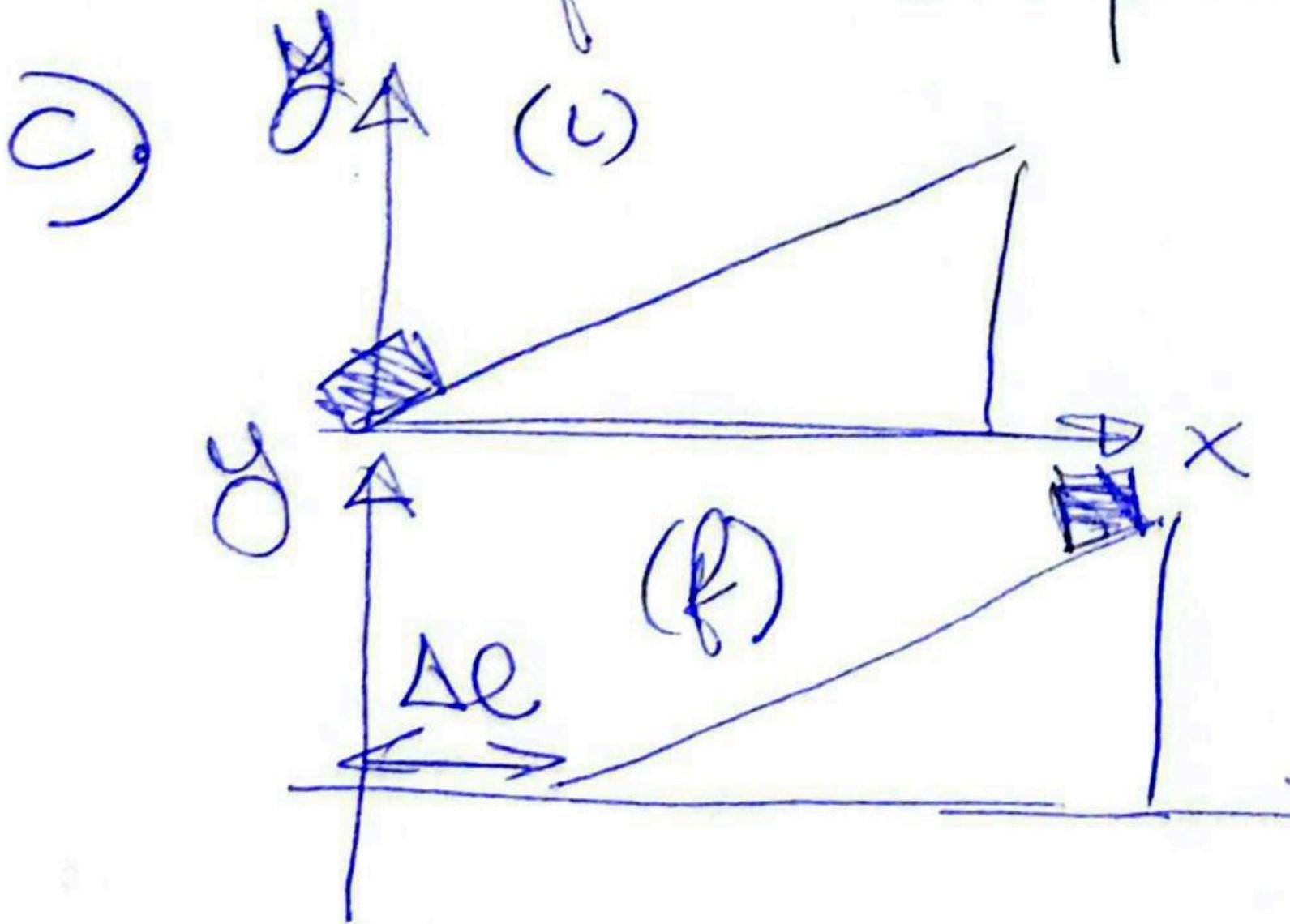
El plato se mueve horizontalmente - no hay rotaciones que se lo impida.

a) Se conservan la Eme y la componente horizontal de \bar{P}

b) \bar{v}_{CM} se mueve porque $\bar{V}_{CM} = \text{cte} \neq \bar{V}_{CM} + 0$
y la trayectoria cambia desde el sistema de laboratorio



la partícula pierde energía porque le transfiere al plato para moverse



$$x_{CM}^{(l)} = \frac{m \cdot 0 + M \cdot x_P}{m+M}$$

$$= \frac{M}{m+M} x_P$$

$$x_{CM}^{(f)} = \frac{m(D + \Delta l) + M(\Delta l + x_f)}{m+M}$$

$$= \Delta l + \frac{mD + Mx_P}{m+M}$$

$$x_{CM}^{(l)} - x_{CM}^{(i)} = V_{CM_x} \Delta t$$

$$V_{CM_x} = \frac{m \omega_0 \cos \alpha}{m+M}$$

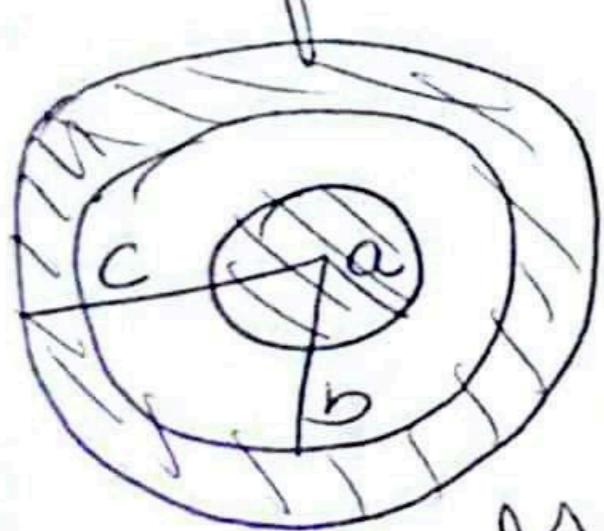
$$\Rightarrow \Delta l + \frac{mD}{m+M} + \cancel{\frac{M \times p}{m+M}} - \cancel{\frac{M \times p}{m+M}} = \frac{m \omega_0 \cos \alpha \Delta t}{m+M}$$

$$\Delta l = \frac{m}{m+M} (\omega_0 \cos \alpha \Delta t - D)$$

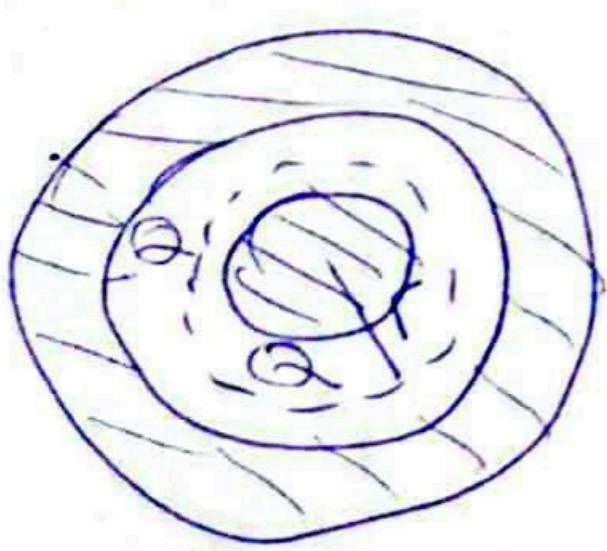
$$\Delta l = 10.83 \text{ m}$$

Prob 2

Tenemos dos capacitores en paralelo uno esférico



Tenemos que calcular la capacidad. Para ello colocamos cargas iguales y contrarias en cada conductor y calcularemos la diferencia de potencial entre los conductores



Sólo hay campo en $a < r < b$

Por Gauss (hay simetría radial) $\bar{E}(r) = E(r) \hat{r}$

$$\oint \bar{E} \cdot d\bar{s} = 4\pi r^2 E(r) = \frac{Q}{\epsilon_0} \Rightarrow \bar{E}(r) = \frac{Q}{4\pi \epsilon_0 r^2} \hat{r}$$

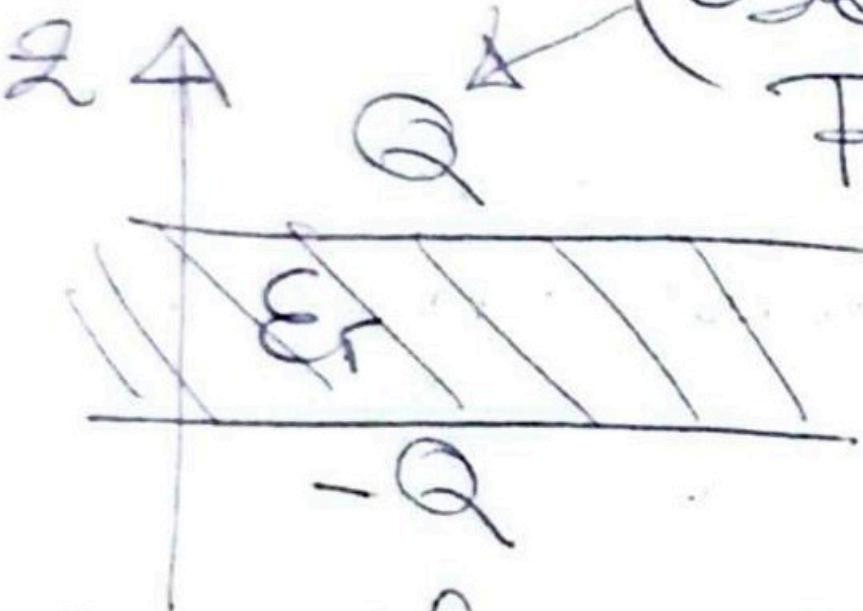
$$\Delta V = V(r=a) - V(r=b) = - \int_a^b \bar{E} \cdot dr = - \frac{Q}{4\pi \epsilon_0} \int_b^a \frac{dr}{r^2}$$

$$\frac{Q}{4\pi \epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right) \hat{r} \quad \boxed{\frac{Q}{8\pi \epsilon_0} \frac{1}{a} = \Delta V}$$

$$\Rightarrow C_{\text{pl}} = \frac{Q}{\Delta V} = \frac{8\pi\epsilon_0 a}{2}$$

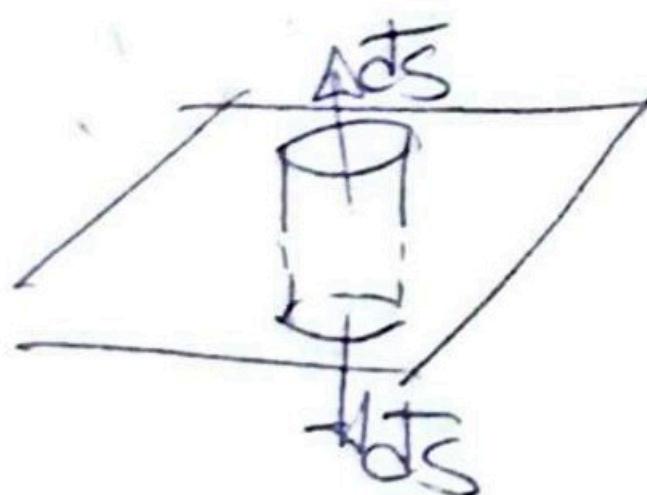
Hacemos lo mismo para el capacitor

plano



(Este Q es una carga genérica
Para calcular la capacidad)

Cada plato se calcula por Gauss
de modo independiente



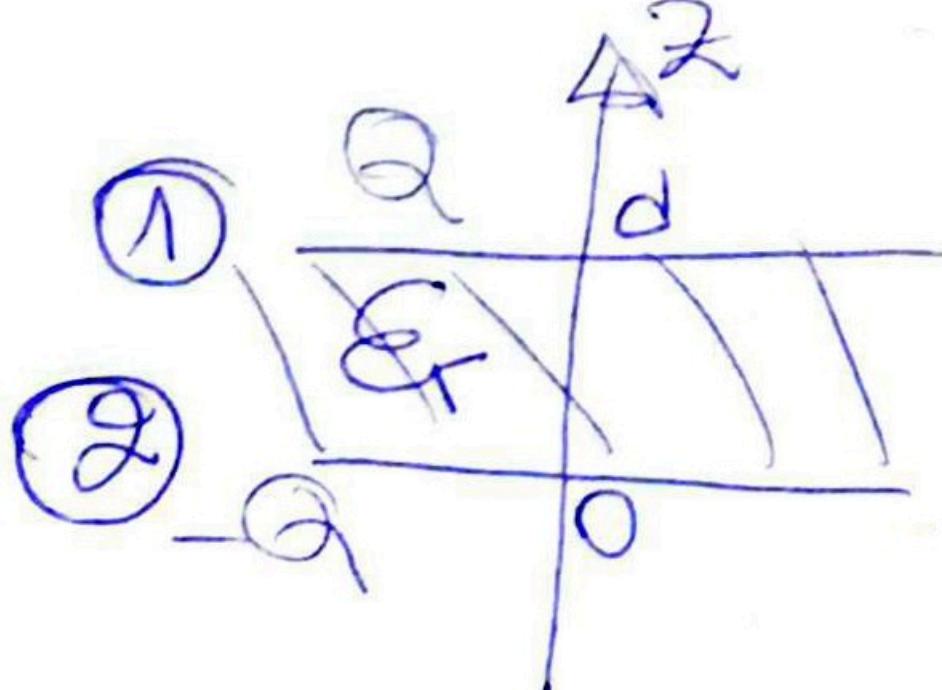
$$\oint \vec{E} \cdot d\vec{s} = 2E(z)AS = \frac{Q_{\text{enc}}}{\epsilon}$$

1 por cada
tapa

$$C = \frac{Q}{A^2}$$

$$N_A f_n c \Rightarrow D_m = \frac{C}{A}$$

$$\Rightarrow E(z) = \frac{C}{2\epsilon} \quad \begin{array}{l} \text{en cada dirección,} \\ \text{con carga permitida} \end{array}$$



$$E_1 = \begin{cases} -\frac{\sigma}{2\epsilon_0} & z < 0 \\ -\frac{\sigma}{2\epsilon_r \epsilon_0} & 0 < z < d \\ \frac{\sigma}{2\epsilon_0} & z > d \end{cases}$$

$$E_2 = \begin{cases} \frac{\sigma}{2\epsilon_0} & z < 0 \\ -\frac{\sigma}{2\epsilon_0 \epsilon_r} & 0 < z < d \\ -\frac{\sigma}{2\epsilon_0} & z > d \end{cases}$$

$\Rightarrow E_T =$

$$E_T = \begin{cases} 0 & z < 0 \\ -\frac{\sigma}{\epsilon_0 \epsilon_r} & 0 < z < d \\ 0 & z > d \end{cases}$$

$$\Delta V = - \int_0^d \bar{E} dz \stackrel{\bar{E} \text{ cte}}{=} \left(-\frac{\sigma}{\epsilon_0 \epsilon_r} \right) (-d) = \frac{\sigma d}{\epsilon_0 \epsilon_r A^2}$$

$$\Rightarrow C_{\text{plano}} = \frac{\epsilon_0 \epsilon_r A^2}{d} = \frac{32 \pi \epsilon_0 a}{d}$$

$$\epsilon_r = 2\pi$$

$$A = 2a$$

$$d = a/4$$

$$\therefore C_{\text{plano}} = 4C_{\text{esf}}$$

a)

Cuando C_1 y C_2 se ponen en contacto la carga Q de C_2 se reparte entre ambos capacitores

$$Q = Q_1 + Q_2 \quad (1) \quad C_1 \parallel \frac{Q_1}{C_1} \parallel \frac{Q_2}{C_2} \parallel C_2$$

$$\Delta V = \frac{Q_2}{C_2} = \frac{Q_1}{C_1} \quad (2)$$

de (1) y (2)

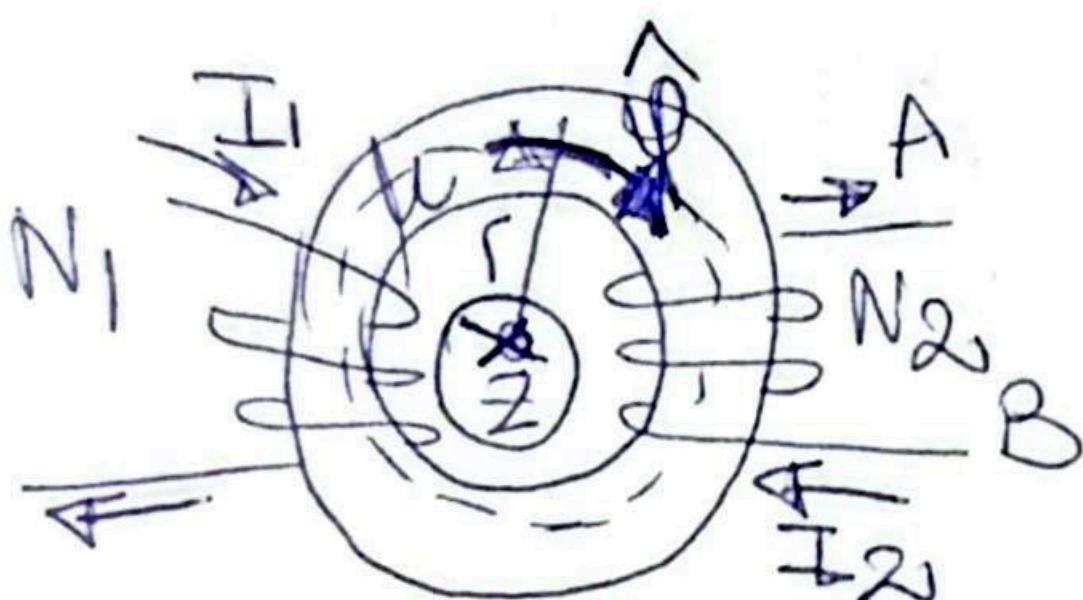
$$Q_1 = \frac{C_1}{C_1 + C_2} Q = \frac{Q}{5}$$

$$Q_2 = \frac{C_2}{C_1 + C_2} Q = \frac{4}{5} Q$$

b) Si llamamos $V_0 = \frac{Q}{C_2}$ (potencial inicial)

$$\Rightarrow \Delta V_{\text{final}} = \frac{Q_2}{C_2} = \frac{Q}{C_1 + C_2} = \frac{C_2}{C_1 + C_2} V_0 = \frac{4}{5} V_0$$

Problema 3



Para calcular las autounductancias y la reductancia magnetica debemos calcular qué campo hay en el toroide cuando circula corriente por alguno de los bobinados, o por los dos.

Si hay I_1 en el bobinado 1 y I_2 en el bobinado 2, entonces:

$$\bar{B}(r) = B(r) \hat{\phi} \quad (\text{toroide ancho})$$

$$\approx B(\text{radio medio}) \hat{\phi}$$

$$\Rightarrow \text{por Ampere} \quad \oint \bar{B} \cdot d\bar{l} = \mu I_c$$

$$B(r) 2\pi r = \mu \left(N_1 \frac{I_1}{2\pi r} + N_2 I_2 \right)$$

$$\bar{B}(r) = \frac{\mu}{2\pi r} (N_1 I_1 + N_2 I_2)$$

② Para calcular L , supongo $I_1 \neq 0, I_2 = 0$

$$\Rightarrow B(r) = \frac{\mu N_1 I_1}{2\pi r}$$

$$\text{El flujo} \quad \Phi_b = N_1 \iint \bar{B} \cdot d\bar{S} = \frac{\mu N_1^2 S}{2\pi r} I_1$$

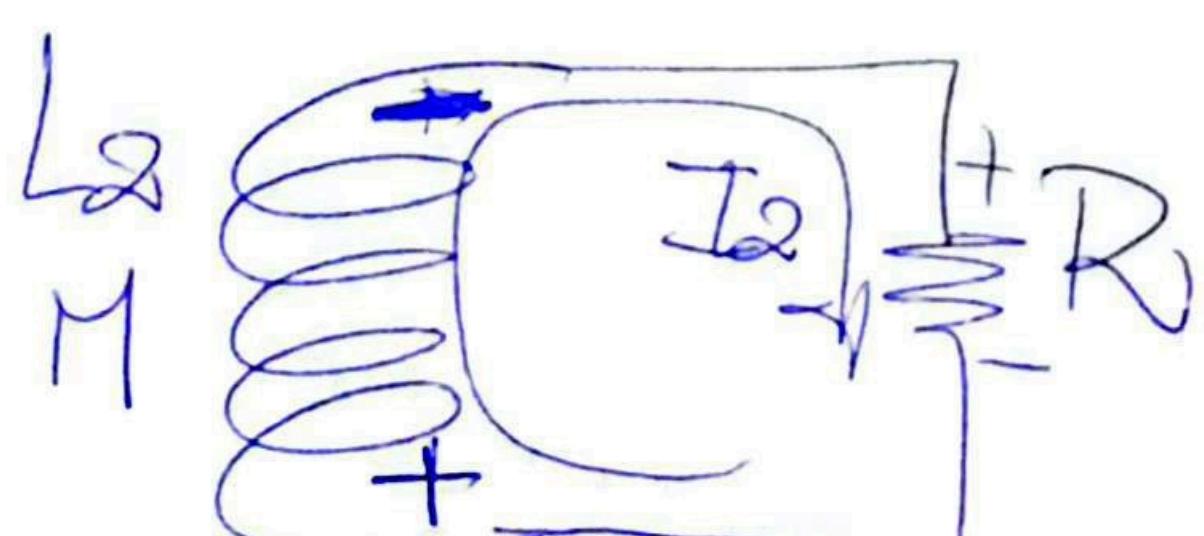
$$\Rightarrow L_1 = \frac{\phi_{11}}{I_1} = \frac{\mu N_1^2 S}{2\pi r}$$

b) Si $I_1 = I_1(t) = I_0 e^{-t/\tau}$ aparece una fuente independiente en el bobinado 2,

$$fue_2 = -\frac{d\phi_{21}}{dt} = -\frac{d}{dt}(\phi_{21}) = -\frac{d}{dt}(M I_1)$$

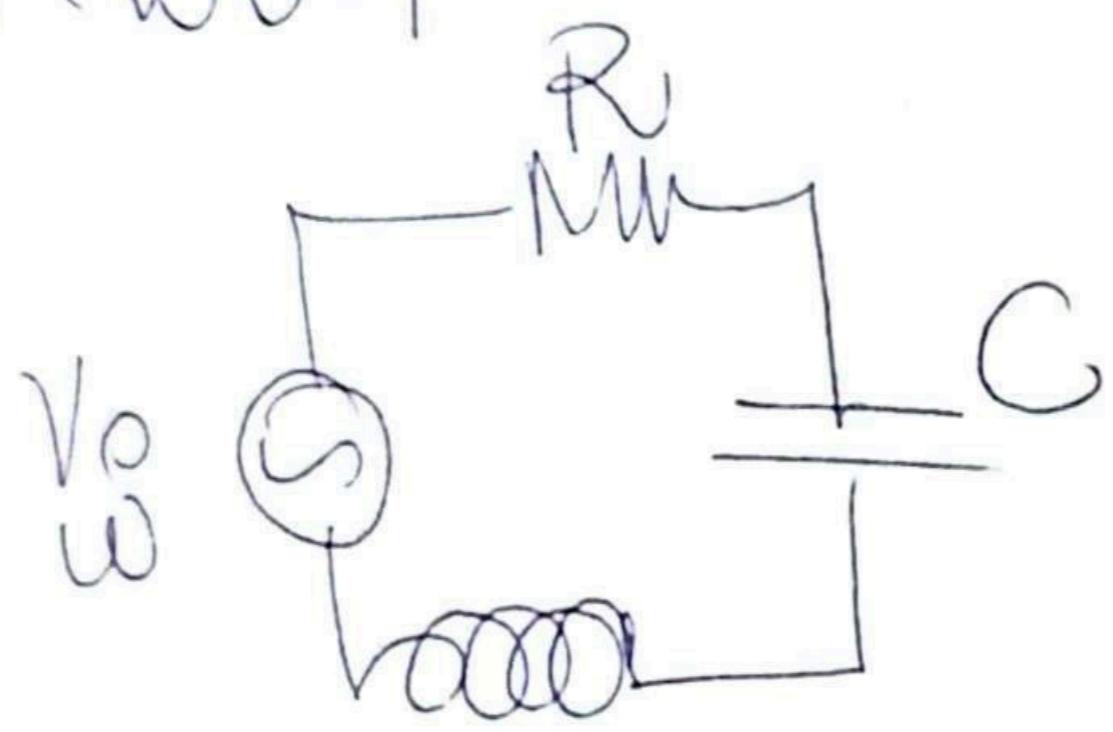
$$= -M \frac{dI_1}{dt} = -M \left(-\frac{I_0}{\tau} e^{-t/\tau} \right) = +\frac{M I_0 e^{-t/\tau}}{\tau}$$

c) Si $L_2 \neq 0 \Rightarrow$ el circuito en el segundo bobinado será



$$-\frac{M dI_1}{dt} - L_2 \frac{dI_2}{dt} - I_2 R = 0$$

Prob 4



$$\omega = 2 \cdot 10^6 \text{ rad/s}$$

$$C = 10^{-9} \text{ F}$$

$$L, R = ?$$

Sabewus $Z = R + j(\omega L - 1/\omega C)$

$$\varphi_I = \frac{\pi}{3} \rightarrow \varphi_Z = -\varphi_I = -\arctg \frac{\omega L - 1/\omega C}{R}$$

$$\Rightarrow \tan\left(-\frac{\pi}{3}\right) = -\frac{1}{\sqrt{3}} = \frac{\omega L - 1/\omega C}{R}$$

$$\therefore R = \sqrt{3} \left(\frac{1}{\omega C} - \omega L \right)$$

$$V_C = |I_{ef}^*| \left| -j \frac{1}{\omega C} \right| = \frac{V_{ef}}{|Z_{eq}|} \frac{1}{\omega C}$$

$$\frac{5}{6} = \frac{V_C}{V_{ef}} = \frac{1}{\omega C} \Rightarrow |Z_{eq}| = \frac{6}{5} \frac{1}{\omega C}$$

$$|Z_{eq}| = \frac{6}{5} \frac{1}{2 \cdot 10^6 \frac{1}{\text{rad/s}} \cdot 10^{-9} \text{ F}} = 600 \Omega$$

$$V_L = |I_{ef}^*| \omega L = \frac{V_{ef}}{|Z_{eq}|} \omega L$$

$$\frac{1}{3} = \frac{V_L}{V_{ef}} = \frac{\omega L}{600 \Omega} \Rightarrow \omega L = 200 \Omega = 2 \cdot 10^6 \frac{1}{\text{rad/s}} L$$

$$\Rightarrow L = 10^{-4} \text{ H} \quad \boxed{R = \sqrt{3} \cdot 300 \Omega}$$