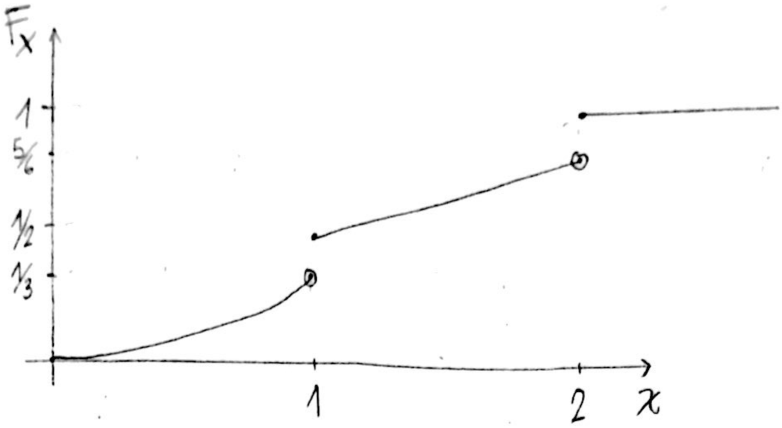
3.1) $F_X(x) = \frac{x^3}{3} \mathbb{1}\{0 \leq x < 1\} + \frac{2x+1}{6} \mathbb{1}\{1 \leq x < 2\} + \mathbb{1}\{x \geq 2\}$



a) $E[X] = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \int_0^1 x^3 \, dx + \int_1^2 \frac{2x}{6} \, dx = \frac{5}{4}$

b) $E[X \mid X<1] = \frac{E[X \mathbb{1}\{X<1\}]}{P(X<1)} = \frac{\int_0^1 x^3 dx}{\int_0^1 x^2 dx} = \frac{3}{4}$

$E[X \mid X \leq 1] = \frac{\int_0^1 x^3 dx + 1 \cdot 1/6}{1/3} = \frac{5}{4}$

3.2) $E[X] = (-2) \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} + \int_{-1}^{1} \frac{1}{6} x \, dx = 0$

a)

b) $E[X \mid |X| = 2] = \frac{-2 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3}}{\frac{1}{3} + \frac{1}{3}} = 0$

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3.3) a) $E[N_k] = \frac{k}{p} = \frac{8}{5}k$

b) $E[N_1] = \frac{8}{5}$ e) $E[N_2] = \frac{16}{5}$

3.4) $\Omega = \{1,2,3,4,5,6\}$

$\mathcal{A} = \{\emptyset, \{1,2\}, \{3,4\}, \{5,6\}, \{1,2,3,4\}, \{1,2,5,6\}, \{3,4,5,6\}, \Omega\}$

$P(\{1,2\}) = \frac{1}{6}$ $P(\{3,4\}) = \frac{1}{3}$ $P(\{5,6\}) = \frac{1}{2}$

a) Definir $X: \Omega \to \mathbb{R} / \{-1, 1/2\} \subset X(\Omega)$ y $E[X] = 0$

$P(\{1,2,3,4\}) = \frac{1}{2}$ $P(\{1,2,5,6\}) = \frac{2}{3}$ $P(\{3,4,5,6\}) = \frac{5}{6}$

<u>OPCIÓN 1</u>

$X(1) = X(2) = -1$

$X(3) = X(4) = 1/2$

$X(5) = X(6) = 0$

$\Rightarrow E[X] = 0$

y hay más opciones...

$X(3) = X(4) = -1$

$X(1) = X(2) = X(5) = X(6) = \frac{1}{2}$

$\Rightarrow E[X] = 0$

) $E[X|X>-1] = \frac{E[X \mathbb{1}\{X>-1\}]}{P(X>-1)} = \frac{1/2}{P(X=0,1/2)}$

opción 1

$$= \frac{1/2}{1/2 + 1/3} = \frac{3}{5}$$

c) $\{-1,1\} \subset X(\Omega)$

OPCIÓN 1

NO ES UNICA

$X(5) = X(6) = 1$

$X(1) = X(2) = X(3) = X(4)$
$= -1$

$\Rightarrow E[X] = 0$

$$E[X|X>-1] = \frac{1/2}{1/2} = 1$$

opción 1

3.5) $E[N] = 10$ $\quad p_N(n) = \frac{10^n \ldots}{n!}$

$$E[N \mid N \leq 4] = \frac{E[N \mathbb{1}\{N \leq 4\}]}{P(N \leq 4)} =$$

$$= \frac{1 \cdot 10 e^{-10} + \frac{\cancel{2} \cdot 10^2 e^{-10}}{\cancel{2}} + \frac{\cancel{3} \cdot 10^3 e^{-10}}{\cancel{6}\,2} + \frac{4 \cdot 10^4 e^{-10}}{\cancel{24}\,6}}{10^0 e^{-10} + 10^1 e^{10} + \frac{10^2 e^{-10}}{2} + \frac{10^3 e^{-10}}{6} + \frac{10^4 e^{-10}}{24}}$$

$$\cong 3{,}533$$

3.6) $E[T] = \frac{1}{\lambda} = 3$

$$E[T \mid T \leq 2] = \frac{\int_0^2 t \cdot \frac{1}{3} e^{-t/3}\, dt}{1 - e^{-2/3}} = 0{,}8897$$

——— o ———

$$\underbrace{E[T]}_{3} = E[T \mid T \leq 2] \cdot \underbrace{P(T \leq 2)}_{1 - e^{-2/3}} + \underbrace{E[T \mid T > 2]}_{2+3} \cdot \underbrace{P(T > 2)}_{e^{-2/3}}$$

$$\Rightarrow E[T \mid T \leq 2] = 0{,}8897$$

.7) a) $F_X(x) = \frac{9!}{3!5!} x^3 (1-x)^5 \mathbb{1}\{0<x<1\}$

$$= 9 \cdot \binom{8}{5} x^3 (1-x)^5 \mathbb{1}\{0<x<1\}$$

$$E[X] = \int_0^1 \frac{9!}{3!\,5!} x^4 (1-x)^5 \, dx = \frac{2}{5}$$

b) $F_X(x) = \frac{1}{6} x^3 e^{-x} \mathbb{1}\{x>0\}$

$X \sim \text{GAMMA}(4,1)$

$\Rightarrow E[X] = 4$

3.8) $Z \sim N(0,1)$

a) $$E[Z \mid Z > z_0] = \frac{\int_{z_0}^{+\infty} x\varphi(x)\,dx}{P(Z>z_0)} = \frac{\int_{z_0}^{+\infty} x \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,dx}{1-\Phi(z_0)} = \frac{\frac{1}{\sqrt{2\pi}} \int_{z_0}^{+\infty} x e^{-x^2/2}\,dx}{1-\Phi(z_0)} =$$

$$= \frac{\frac{1}{\sqrt{2\pi}} e^{-z_0^2/2}}{1-\Phi(z_0)} = \frac{\varphi(z_0)}{1-\Phi(z_0)}$$

b) $$\frac{z_0 \overbrace{\cancel{\int_{z_0}^{+\infty} \varphi(x)\,dx}}^{1-\Phi(z_0)}}{\cancel{1-\Phi(z_0)}} \leqslant \frac{\int_{z_0}^{+\infty} x\varphi(x)\,dx}{1-\Phi(z_0)} = \frac{\varphi(z_0)}{1-\Phi(z_0)}$$

$$\Rightarrow 1-\Phi(z_0) \leqslant \frac{\varphi(z_0)}{z_0}$$

3.9) $P(\max(z_1,z_2)>5) \leqslant \dfrac{2e^{-\frac{25}{2}}}{5\sqrt{2\pi}}$

$$P(\max(z_1,z_2)>5) = P(z_1>5 \cup z_2>5) =$$

$$= P(z_1>5)+P(z_2>5)-P(z_1>5 \cap z_2>5)$$

$$\leqslant P(z_1>5)+P(z_2>5) = 1-\Phi(5) + 1-\Phi(5)$$

$$\leqslant \frac{\varphi(5)}{5} + \frac{\varphi(5)}{5} = \frac{2}{5}\frac{e^{-25/2}}{\sqrt{2\pi}}$$

continua 3.7)

c) $E[X|X>1/2] = \dfrac{E[X\,\mathbb{1}\{X>1/2\}]}{P(X>1/2)} =$

$$= \frac{\int_{1/2}^{+\infty} \frac{1}{6}x^4 e^{-x}\,dx}{\int_{1/2}^{+\infty} \frac{1}{6}x^3 e^{-x}\,dx} =$$

.10) $L =$ "longitud del alambre" $\sim EXP(1/60)$

a)

$$A(L) = \frac{L^2}{4\pi}$$

$$2\pi r = L \Rightarrow r = \frac{L}{2\pi} \Rightarrow A = \pi\left(\frac{L}{2\pi}\right)^2 = \frac{L^2}{4\pi}$$

$$E[A] = E[L^2/4\pi] = \frac{1}{4\pi}E[L^2] = \frac{1}{4\pi}\left(V(L) + E^2[L]\right)$$

$$= \frac{1}{4\pi}(3600 + 3600) = 572{,}96$$

b) A: "área del círculo" $\sim EXP(1/15)$

$$P(A) = 2\sqrt{\pi A}$$

$$A = \pi r^2 \Rightarrow r = \sqrt{\frac{A}{\pi}} \Rightarrow 2\pi\sqrt{\frac{A}{\pi}} = L \Rightarrow L = 2\sqrt{\pi}\sqrt{A}$$

$$E[P] = E[2\sqrt{\pi A}] = 2\sqrt{\pi}\,E[\sqrt{A}] = 2\sqrt{\pi}\underbrace{\int_0^{+\infty}\sqrt{a}\,e^{-a/15}\,dt}_{\cong 182{,}5}$$

3.12) $f_X(x) = f_Y(y) = \frac{1}{\pi} \Rightarrow f_{XY}(x,y) = \frac{1}{\pi^2}$

$$E[X\,sen(XY)] = \int_0^{\pi}\int_0^{\pi} x\,sen(xy)\,\frac{1}{\pi^2}\,dy\,dx =$$

$$= \int_0^{\pi} -\frac{1}{\pi^2}\cancel{x}\,\frac{\cos(xy)}{\cancel{x}}\Big|_0^{\pi} dx = -\frac{1}{\pi^2}\int_0^{\pi}(\cos(\pi x) - 1)\,dx =$$

$$= -\frac{1}{\pi^2}\left(\frac{sen(\pi x)}{\pi} - x\right)\Big|_0^{\pi} = -\frac{1}{\pi^2}\left(\frac{sen(\pi^2)}{\pi} - \pi\right) =$$

$$= \frac{1}{\pi} - \frac{sen(\pi^2)}{\pi^3}$$

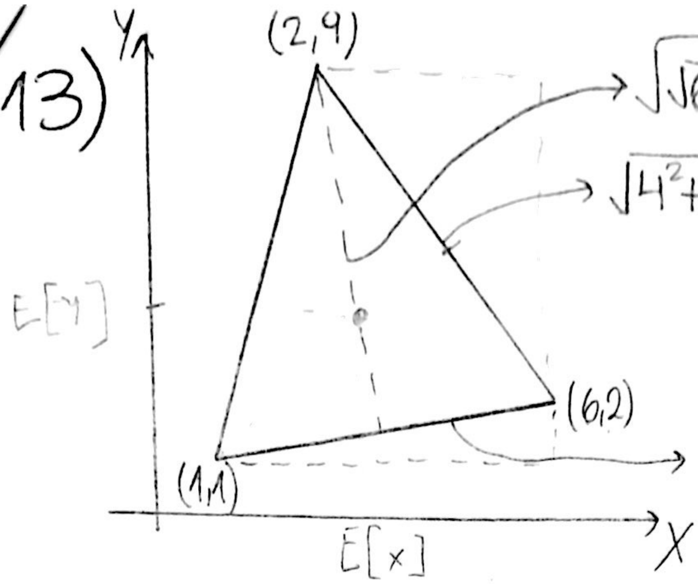
3.11) $T^* = \min(T,1)$

$$E[T^*] = \int_0^1 t^* e^{-t^*} dt^* + 1.e^{-1} = 0{,}632$$

$$V(T^*) = E[T^{*2}] - E^2[T^*]$$

$$= \int_0^1 t^{*2} e^{-t^*} dt^* + 1.e^{-1} - 0{,}632^2 = 0{,}129$$

.13)



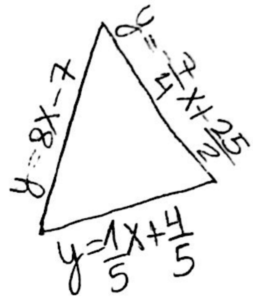
$\sqrt{\sqrt{65}^2-\left(\frac{\sqrt{26}}{2}\right)^2}=\frac{3}{2}\sqrt{26}$

$\sqrt{4^2+7^2}=\sqrt{65}$

$\sqrt{5^2+1^2}=\sqrt{26}$

$$\Rightarrow AREA = \frac{\sqrt{26}\times\frac{3\sqrt{26}}{2}}{2} = 19,5 = \frac{39}{2}$$

$$F_{xy}(x,y)=\frac{2}{39}\,\mathbb{1}\{(x,y)\in A\}$$



$$E[X]=\iint_A x\,F_{xy}\,dx\,dy =$$

$$=\int_1^2\int_{\frac{1}{5}x+\frac{4}{5}}^{8x-7} x\cdot\frac{2}{39}\,dy\,dx+\int_2^6\int_{\frac{1}{5}x+\frac{4}{5}}^{-\frac{7}{4}x+\frac{25}{2}} x\cdot\frac{2}{39}\,dy\,dx=$$

$$=\frac{2}{39}\int_1^2 \frac{39}{5}x^2-\frac{39}{5}x\,dx+\frac{2}{39}\int_2^6 -\frac{39}{20}x^2+\frac{117}{10}x\,dx=3$$

$$E[Y]=\iint_A y\,F_{xy}\,dx\,dy=\int_1^2\int_{\frac{1}{5}x+\frac{4}{5}}^{8x-7} y\frac{2}{39}\,dy\,dx+\int_2^6\int_{\frac{1}{5}x+\frac{4}{5}}^{-\frac{7}{4}x+\frac{25}{2}} y\frac{2}{39}\,dy\,dx=4$$

3.15) $X \sim U(8,10)$ $E[X] = 9$ $V(X) = \frac{1}{3}$

a) $E[Y] = E[2(X-1)] = 2E[X] - 2 = 16$

$V[Y] = V[2(X-1)] = 4V(X) = \frac{4}{3}$

b) $E[Y] = E[2X^2+1] = 2E[X^2]+1 =$

$= 2(V(X) + E^2[X]) + 1 = \frac{491}{3}$ $(V(X)+E^2[X])^2$

$V(Y) = V(2X^2+1) = 4V(X^2) = 4(E[X^4] - E^2[X^2]) =$

$= 4\left(\int_8^{10} \frac{1}{2}x^4\,dx - \frac{59536}{9}\right) = \frac{19456}{45}$

c) $E[Y] = E[2(X-1)(X-3)] = 2E[X^2 - 4X + 3] =$

$= 2(E[X^2] - 4E[X] + 3) = \frac{290}{3}$

$V(Y) = 4V(X^2 - 4X + 3) = 4(V(X^2) - 16V(X)) =$

$= 4(E[X^4] - E^2[X^2] - 16V(X)) = \frac{18496}{45}$

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d) $\min_{c\in\mathbb{R}} E[(X-c)^2] = \min_c E[X^2-2cX+c^2] =$

$= \min_c \left(E[X^2]-2cE[X]+c^2\right) =$

$= \min_c \left(\frac{244}{3} - 18c + c^2\right) \Rightarrow 2c-18=0 \Rightarrow c=9$

$\Rightarrow \min_{c\in\mathbb{R}} E[(X-c)^2] = \frac{244}{3} - 18\cdot 9 + 9^2 = -\frac{809}{13}$

e) $E[aX+b]=0$, $V(aX+b)=1$

$aE[X]+b=0$	$a^2V(X)=1$
$9a+b=0$	$\frac{1}{3}a^2=1$

$9\sqrt{3}+b=0 \leftarrow a=\sqrt{3}$

$b=-9\sqrt{3}$

3.16) $V\sim N(6,1)$ $\quad E[V]=6 \quad V(V)=1$

$E[W]=E[3V^2]=3E[V^2]=3\left(V(V)+E^2[V]\right)=$

$=3(1+6^2)=111$

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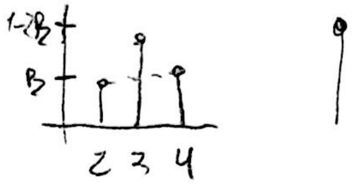
3.14) $E[X] = 3$

$$p_2 + p_3 + p_4 = 1 \rightarrow p_4 = 1 - p_2 - p_3$$

$$2p_2 + 3p_3 + 4p_4 = 3 = 2p_2 + 3p_3 + 4 - 4p_2 - 4p_3 = 4 - 2p_2 - p_3 = 3$$

$$1 = 2p_2 + p_3 \Rightarrow p_3 = 1 - 2p_2$$

$$p_4 = 1 - p_2 - 1 + 2p_2 = p_2$$



a) $\arg\max_{p_2} V[X] = \arg\max_{p_2} E[X^2] - 9 = \arg\max_{p_2} \underbrace{E[X^2]}_{\substack{4p_2 + 9p_3 + 16p_4 \\ \| \\ 4p_2 + 9 - 18p_2 + 16p_2 = 2p_2 + 9}}$

MAX $\rightarrow \boxed{p_2 = 1/2}$

min $\rightarrow \boxed{p_2 = 0}$

$$\Rightarrow p_2 = 1/2$$
$$p_3 = 0$$
$$p_4 = 1/2$$

b) $p_2 = 0$

$$p_3 = 1$$
$$p_4 = 0$$

A) $X_1 \sim BER(p_1)$ $X_2 \sim BER(p_2)$

a) $COV(X_1, X_2) = 0 = E[X_1 X_2] - E[X_1] . E[X_2]$

$\Rightarrow E[X_1 X_2] = \overbrace{E[X_1]}^{p_1} . \overbrace{E[X_2]}^{p_2}$

$\Rightarrow$ son independientes

b) $p_1 = p_2 = 0{,}7 = p$ $E[X_1] = 0{,}7$ $E[X_2] = 0{,}7$

$COV(X_1 X_2) = 0{,}1$ $V(X_1) = V(X_2) = 0{,}21$

$= p(1-p)$

hallar $\rho(Y_1, Y_2)$

datos

$Y_1 = X_1(1 - X_2)$ $Y_2 = X_2(1 - X_1)$

$$\rho(Y_1, Y_2) = \frac{COV(Y_1, Y_2)}{\sigma_{Y_1} \times \sigma_{Y_2}} = \frac{E[Y_1 Y_2] - E[Y_1] E[Y_2]}{\sqrt{E[Y_1^2] - E^2[Y_1]} \, \sqrt{E[Y_2^2] - E^2[Y_2]}}$$

$\cdot E[Y_1] = E[X_1(1 - X_2)] = E[X_1] - E[X_1 X_2]$

$\cdot E[Y_2] = E[X_2(1 - X_1)] = E[X_2] - E[X_2 X_1]$

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3.18) 3 bolas en 3 urnas C_1, C_2, C_3

X_i: "cant de bolas en c_i"

N: "cant de urnas que contienen alguna bola"

a) $E[N]$, $V(N)$, $cov(N, X_1)$

$$p_N(n) = \begin{cases} n=1 \Rightarrow X_1=3 \cup X_2=3 \cup X_3=3 \\ n=2 \Rightarrow X_1=2 \wedge X_2=1 \cup X_1=2 \wedge X_3=1 \cup \ldots \\ n=3 \Rightarrow X_1=1 \wedge X_2=1 \wedge X_3=1 \end{cases}$$

$$p_N(1) = P(X_1=3 \cup X_2=3 \cup X_3=3) =$$
$$= P(X_1=3) + P(X_2=3) + P(X_3=3) =$$
$$= \frac{1^3}{3^3} + \frac{1^3}{3^3} + \frac{1^3}{3^3} = \frac{1}{9}$$

$$p_N(2) = \frac{\binom{3}{1}\binom{3}{2}\binom{2}{1}\binom{1}{1}}{3^3} = \frac{2}{3}$$

$$p_N(3) = \frac{\binom{3}{1}\binom{2}{1}}{3^3} = \frac{2}{9}$$

$$E[N] = 1\times\frac{1}{9} + 2\times\frac{2}{3} + 3\times\frac{2}{9} = \frac{19}{9}$$

$$V(N) = E[N^2] - E^2[N]$$

$$= \left(1\times\frac{1}{9} + 4\times\frac{2}{3} + 9\times\frac{2}{9}\right) - \frac{361}{81} = \frac{26}{81}$$

$$COV(N,X_1) = E[NX_1] - E[N]\times E[X_1]$$

$$= \sum_{nx_1\in A} n x_1\, p_{NX_1}(n,x_1) - \frac{19}{9}\times\left(\sum_{x_1\in A} x_1\, p_{X_1}(x_1)\right)$$

$$p_{X_1}(x_1) = \begin{cases} x_1 = 0 \xrightarrow{3C1} 2^3/3^3 = 8/27 \\ x_1 = 1 \rightarrow \binom{3}{1} 2^2/3^3 = 4/9 \\ x_1 = 2 \rightarrow \binom{3}{2} 2^1/3^3 = 2/9 \\ x_1 = 3 \rightarrow 1^3/3^3 = 1/27 \end{cases}$$

$$E[X_1] = 1\times\frac{4}{9} + 2\times\frac{2}{9} + 3\times\cancel{\frac{1}{9}}^{1/27} = \cancel{\frac{11}{9}} \quad 1$$

$$p_{NX_1}(n,x_1) = \begin{cases} n=1, x_1=0 \rightarrow 2/27 \\ n=1, x_1=3 \rightarrow 1/27 \\ n=2, x_1=0 \rightarrow 6/27 \\ n=2, x_1=1 \rightarrow 6/27 \\ n=2, x_1=2 \rightarrow 6/27 \\ n=3, x_1=1 \rightarrow 6/27 \end{cases} \Rightarrow E[NX_1] = \frac{19}{9}$$

$$\Rightarrow COV(N,X_1) = \frac{19}{9} - \frac{19}{9}\times\frac{11}{9} = -\frac{38}{81}$$

b) para que sean independientes

$$\underbrace{p_N(1)}_{1/9} \times \underbrace{p_{X_1}(1)}_{4/9} = \underbrace{p_{NX_1}(1,1)}_{0}$$

$\Rightarrow$ no son independientes

c) $\mathrm{cov}(X_i, X_j)$, $1 \leq i \leq j \leq 3$

calculo $p_{X_2}(x_2)$, $p_{X_3}(x_3)$, $p_{X_1X_2}(x_1,x_2)$, $p_{X_1X_3}(x_1,x_3)$

y $p_{X_2X_3}(x_2,x_3)$ con sus esperanzas

$$E[X_1] = E[X_2] = E[X_3] = \frac{11}{9} \qquad E[X_1^2] = E[X_2^2] = E[X_3^2] = \frac{7}{3}$$

$$p_{X_1X_2}(x_1,x_2) = \begin{cases} x_1 = 0, x_2 = 0 \rightarrow 1/27 \\ x_1 = 0, x_2 = 1 \rightarrow 3/27 \\ x_1 = 0, x_2 = 2 \rightarrow 3/27 \\ x_1 = 0, x_2 = 3 \rightarrow 1/27 \\ x_1 = 1, x_2 = 0 \rightarrow 3/27 \\ x_1 = 1, x_2 = 1 \rightarrow 6/27 \\ x_1 = 1, x_2 = 2 \rightarrow 3/27 \\ x_1 = 2, x_2 = 0 \rightarrow 3/27 \\ x_1 = 2, x_2 = 1 \rightarrow 3/27 \\ x_1 = 3, x_2 = 0 \rightarrow 1/27 \end{cases}$$

$$E[X_1X_2] = \frac{2}{3}$$

$$E[X_1X_3] = \frac{2}{3}$$

$$E[X_2X_3] = \frac{2}{3}$$

$$COV(X_1,X_1) = V(X_1) = E[X_1^2] - E^2[X_1] = \frac{7}{3} - \frac{121}{81} = \frac{68}{81}$$

$$COV(X_1,X_2) = E[X_1X_2] - E[X_1]E[X_2] = \frac{2}{3} - \frac{11}{9} \cdot \frac{11}{9} = -\frac{67}{81}$$

$$COV(X_1,X_3) = E[X_1X_3] - E[X_1]E[X_3] = \frac{-67}{81}$$

$$COV(X_2,X_2) = V(X_2) = \frac{68}{81}$$

$$COV(X_2,X_3) = E[X_2X_3] - E[X_2]E[X_3] = \frac{-67}{81}$$

$$COV(X_3,X_3) = V(X_3) = \frac{68}{81}$$

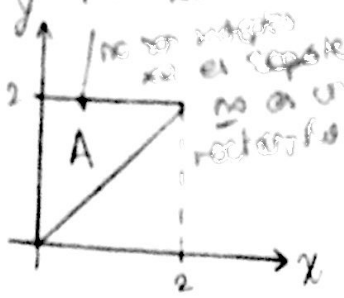
3.19) a) $COV(X,Y) < 0$

b) $COV(X,Y) = 0$

c) $COV(X,Y) > 0$

3.20) $(X,Y) \sim U(A)$

$f_{XY} = \frac{1}{Area(A)} = \frac{1}{2}$



no son independientes xq el espacio no es un rectángulo

a) $COV(X,Y) = E[XY] - E[X]E[Y] =$

$$= \int_0^2\int_0^y xy\,\frac{1}{2}\,dx\,dy - \int_0^2\int_0^y \frac{x}{2}\,dx\,dy \times \int_0^2\int_0^y \frac{y}{2}\,dx\,dy =$$

$$= 1 - \frac{2}{3} \times \frac{4}{3} = \frac{1}{9}$$

ver guia (1.2) otro camino

b) $VAR(X+Y) = E[(X+Y)^2] - E^2[X+Y] =$

$= E[X^2 + 2XY + Y^2] - E[X+Y] \times E[X+Y] =$

$= E[X^2] + E[2XY] + E[Y^2] - E[X+Y] \times E[X+Y] =$

$$= \int_0^2\int_0^y \frac{x^2}{2}\,dx\,dy + 2\int_0^2\int_0^y xy\,\frac{1}{2}\,dx\,dy + \int_0^2\int_0^y \frac{y^2}{2}\,dx\,dy - \left[\int_0^2\int_0^y \frac{(x+y)}{2}\,dx\,dy\right]^2 =$$

$$= \frac{2}{3} + 2 \times 1 + 2 - \left(\frac{2}{3} + \frac{4}{3}\right)^2 = \frac{2}{3}$$

c) $COV(3X - Y + 2, X+Y) = COV(3X,X) + COV(3X,Y) - COV(Y,X) -$

$- COV(Y,Y) + \underbrace{\cancel{COV(2,X)}}_{=0} + \underbrace{\cancel{COV(2,Y)}}_{=0} =$

$= 3\,COV(X,X) + 3\,COV(X,Y) - COV(X,Y) - COV(Y,Y) =$

$= 3V(X) + 2\,COV(X,Y) - V(Y) =$

$= 3\left(E[X^2] - E^2[X]\right) + 2 \times \frac{1}{9} - \left(E[Y^2] - E^2[Y]\right) =$

$$= 3\left[\frac{2}{3} - \left(\frac{2}{3}\right)^2\right] + \frac{2}{9} - \left[2 - \left(\frac{4}{3}\right)^2\right] = \frac{2}{3}$$

1) X: "tiempo de reacción de Juan" $\sim U(0,2)$
Y: "tiempo de reacción de María" $\sim U(0,2)$

W: "tiempo de reacción del ganador" $= \min(X,Y) = \begin{cases} x & \text{si } x<y \\ y & \text{si } y<x \end{cases}$

L: "tiempo de reacción del perdedor" $= \max(X,Y) = \begin{cases} x & \text{si } y<x \\ y & \text{si } x<y \end{cases}$

a) $$\hat{L} = \frac{COV(L,W)}{V(W)}(W - E[W]) + E[L]$$

$$E[W] = \int_0^2\int_x^2 \frac{x}{4}\,dy\,dx + \int_0^2\int_0^x \frac{y}{4}\,dy\,dx = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$E[L] = \int_0^2\int_x^2 \frac{y}{4}\,dy\,dx + \int_0^2\int_0^x \frac{x}{4}\,dy\,dx = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

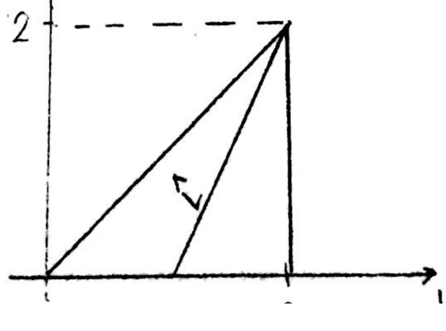
$$V(W) = E[W^2] - E^2[W] = 1/2 - (2/3)^2 = 1/18$$

$$E[W^2] = \int_0^2\int_x^2 \frac{x^2}{4}\,dy\,dx + \int_0^2\int_0^x \frac{y^2}{4}\,dy\,dx = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

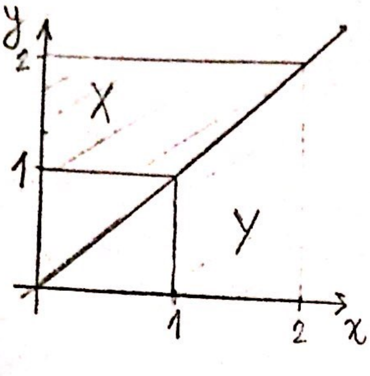
$$COV(L,W) = E[LW] - E[L]E[W] = 1 - \frac{4}{3} \times \frac{2}{3} = \frac{1}{9}$$

$$E[LW] = \int_0^2\int_0^2 \frac{xy}{4}\,dy\,dx = 1$$

$$\hat{L} = \frac{1/9}{1/18}\left(W - \frac{2}{3}\right) + \frac{4}{3} = \frac{1}{2}W + 1$$



b) $E[W|L>1] = \frac{E[W \mathbb{1}\{L>1\}]}{P(L>1)} =$



$$= \frac{\int_1^2\int_0^y \frac{x}{4}\,dx\,dy + \int_1^2\int_0^x \frac{y}{4}\,dy\,dx}{3/4} =$$

$$= \frac{7/24 + 7/24}{3/4} = \frac{7}{9}$$

3.22)

$$f_{XY}(x,y) = \frac{5}{8\pi} e^{-\frac{25}{32}(x^2 - \frac{6}{5}xy + y^2)}$$

(RESUELTO CON CONCEPTOS DE GUIA 5)

$$= \frac{1}{\pi \frac{8}{5}} e^{-\frac{25}{32}\left((y-\frac{3}{5}x)^2 + \frac{16}{25}x^2\right)} =$$

$$= \frac{1}{\sqrt{\pi}\frac{8}{5}} e^{-\frac{(y-3/5\,x)^2}{32/25}} \frac{1}{\sqrt{\pi}} e^{-\frac{1}{2}x^2} = \underbrace{\frac{1}{\sqrt{2\pi}\frac{4}{5}} e^{-\frac{1}{2}\frac{(y-3/5\,x)^2}{16/25}}}_{f_{Y|X=x} = \frac{f_{XY}}{f_X}} \times \underbrace{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}}_{f_X}$$

↪ $X \sim \underset{est}{N}(0,1)$

$$Y|X=x \sim N\left(\frac{3}{5}x, \frac{16}{25}\right)$$

tiene pinta de recta

$$E[Y|X] = \frac{3}{5}X = \varphi(X)$$

3) $E[X]=15$

demostrar $P(X\geq 60)\leq 0{,}25$

por Markov $\rightarrow P(X\geq a)\leq \dfrac{E[X]}{a}$

$$\Rightarrow P(X\geq 60)\leq \frac{15}{60}=0{,}25$$

3.24) $E[X]=10$, $V(X)=15$

demostrar $P(5<X<15)\geq 0{,}4$

por Tchebycheff $\rightarrow P(|X-E[X]|\geq\varepsilon)\leq \dfrac{V(X)}{\varepsilon^2}$

$$\Rightarrow P(|X-10|\geq\varepsilon)\leq\frac{15}{\varepsilon^2}$$

$$1-P(|X-10|<\varepsilon)\leq\frac{15}{\varepsilon^2}$$

$$1-P(|X-10|<5)\leq\frac{3}{5}$$

$$P(|X-10|<5)\geq\frac{2}{5}$$

$$\Rightarrow P(5<X<15)\geq 0{,}4$$

3.25) X "peso de paquetes de café" $\sim N(500, \sigma^2)$

$X_1, X_2, \dots, X_n \overset{iid}{\sim} X$

① hallar σ / $P(490 < \bar{X} < 510) = 0{,}99$

$$\underbrace{\bar{X} = \frac{\sum_{i=1}^{100} X_i}{100}}_{\text{peso promedio de 100 paquetes}} \qquad E[\bar{X}] = \frac{1}{100} \sum_{i=1}^{100} E[X_i] = 500$$

$$V(\bar{Y}) = V\left(\frac{\sum X_i}{100}\right) = \frac{1}{100^2} \sum_{i=1}^{100} V(X_i) = \frac{V(X_i)}{100} = \frac{\sigma^2}{100}$$

$$\Rightarrow P(490 < \bar{X} < 510) \overset{\text{estandarizo}}{=} P\left(\frac{490-500}{\sigma^2/100} < Z < \frac{510-500}{\sigma^2/100}\right) = 0{,}99$$

$$P\left(-\frac{1000}{\sigma^2} < Z < \frac{1000}{\sigma^2}\right) \approx 2\Phi\left(\frac{1000}{\sigma^2}\right) - 1 \approx 0{,}99$$

(ESTO ES GUÍA 8 PERO NO HAY QUE USAR EL =)

(ver ②)

$$2\Phi\left(\frac{100}{\sigma^2}\right) \approx 1{,}99$$

$$\Phi\left(\frac{100}{\sigma^2}\right) \approx 0{,}995$$

$$\frac{100}{\sigma^2} \approx \Phi^{-1}(0{,}995) \approx 2{,}5758$$

$$\Rightarrow \boxed{\sigma \approx 6{,}23}$$

CHEVISHEV

hallar σ / $P(490 < \bar{X} < 510) \geq 0{,}99$

$$E[\bar{X}] = 500 \qquad V[\bar{X}) = \frac{\sigma^2}{100}$$

$$P(|\bar{X}-500| < 10) \geq 0{,}99$$

$$1 - P(|\bar{X}-500| \geq 10) \geq 0{,}99$$

$$\begin{cases} P(|\bar{X}-500| \geq 10) \leq 0{,}01 \\ P(|\bar{X}-500| \geq 10) \leq \frac{\sigma^2/100}{100} \Rightarrow \frac{\sigma^2}{10000} \leq 0{,}01 \end{cases}$$

$$\Rightarrow \boxed{\sigma \leq 10}$$

3.26) $P(|X - E[X]| < 0{,}01) \geq 0{,}95$ $\quad X_i \sim Ber(p)$

$$X_i = \begin{cases} 0 & \text{si la persona } i \text{ no fuma} \\ 1 & \text{si la persona } i \text{ fuma} \end{cases}$$

promedio de funciones

$$X = \frac{\sum_{i=1}^{n} X_i}{n} \qquad E[X] = p \qquad V(X) = \frac{p(1-p)}{n}$$

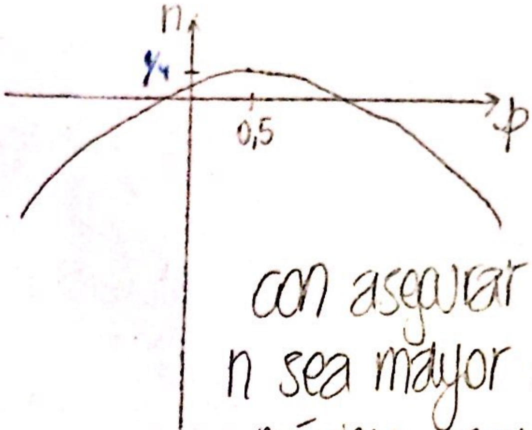
CHEVISHEV

$$= 1 - \frac{V[X]}{\varepsilon^2}, \quad \varepsilon = 0{,}01^2$$

$$\begin{cases} P(|X-p| < 0{,}01) \geq 0{,}95 \\ P(|X-p| \geq 0{,}01) \leq 0{,}05 \geq \frac{p(1-p)}{n(0{,}01)^2} \end{cases}$$

$$0{,}05 \geq \frac{p(1-p)}{n\,0{,}01^2}$$

$$n \geq \frac{p(1-p)}{0{,}05 \times 0{,}01^2}$$



$\Rightarrow$ derivo e igualo a cero*

con asegurar que n sea mayor al máximo, aseguro que sea mayor a todo p

$$\left[\frac{p(1-p)}{0{,}05 \times 0{,}01^2}\right]' = \frac{1-2p}{0{,}05 \times 0{,}01^2} = 0 \Rightarrow p = \frac{1}{2}$$

$$n \geq \frac{\frac{1}{2}\left(1-\frac{1}{2}\right)}{0{,}05 \times 0{,}01^2} \Rightarrow \boxed{n \geq 50000}$$

$V[X] = \frac{p(1-p)}{n}$ $\longrightarrow$ analizando la función se que $0 \leq p \leq 1$
y en este intervalo la función alcanza
un max en $p = \frac{1}{2}$ indepte de n