SEBASTIAN
KOHN

SETEC
CONSULTORIA DE INTERFASES

5.1) 4V
3A
3R
→ se extraen 3
s/ REPO

$$p_{XY}(x,y) = \frac{\binom{4}{x}\binom{3}{y}\binom{3}{3-x-y}}{\binom{10}{3}}$$

X: "cant. de bolas verdes extraidas"

Y: "cant. de bolas rojas extraidas"

$$p_X(x) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$$

$$p_{Y|X=x}(y) = P(Y=y \mid X=x) = \frac{P(Y=y \wedge X=x)}{P(X=x)}$$

y \ x	0	1	2	3	p_Y
0	1/120	1/10	3/20	1/30	7/24
1	3/40	3/10	3/20	0	21/40
2	3/40	1/10	0	0	7/40
3	1/120	0	0	0	1/120
p_X	1/6	1/2	3/10	1/30	1

$$= \frac{\cancel{\binom{4}{x}}\binom{3}{y}\binom{3}{3-x-y}\cancel{\binom{10}{3}}}{\cancel{\binom{4}{x}}\binom{6}{3-x}\cancel{\binom{10}{3}}} =$$

$$= \frac{\binom{3}{y}\binom{3}{3-x-y}}{\binom{6}{3-x}}, \quad y \in [0,3], \ x \in [0,3], \ (x-y) \in [0,3]$$

HIPERGEOMÉTRICA

5.2) a) $(X,Y) \sim U(x^2+y^2 \le 1)$

$\Rightarrow (X,Y) \sim U((x,y) \in O)$

$$F_{XY}(x,y) = \frac{1}{\pi} \mathbb{1}\{(x,y) \in O\}$$

$$F_X(x) = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{1}{\pi} dy = \frac{2\sqrt{1-x^2}}{\pi} \mathbb{1}\{x \in (-\sqrt{1-y^2}, \sqrt{1-y^2})\}$$

$$F_{Y|X=x}(y) = \frac{1}{2\sqrt{1-x^2}} \mathbb{1}\{y \in (-\sqrt{1-x^2}, \sqrt{1-x^2})\}$$

b) $$F_{XY}(x,y) = \frac{1}{2x+1} e^{-\left(2x + \frac{y}{4x+2}\right)} \mathbb{1}\{x>0, y>0\}$$

$$= \underbrace{2e^{-2x}}_{F_X(x)} \cdot \underbrace{\frac{1}{4x+2} e^{-\frac{y}{4x+2}}}_{F_{Y|X=x}(y)} \mathbb{1}\{x>0, y>0\}$$

$X \sim EXP(2)$

$Y|X=x \sim EXP(1/4x+2)$

c) $$F_{XY}(x,y) = e^{-x} \mathbb{1}\{0<y<x\}$$

$$F_X(x) = \int_0^x e^{-x} dy = xe^{-x} \ \mathbb{1}\{x>0\} \Rightarrow X \sim GAMMA(2,1)$$

$$F_{Y|X=x}(y) = \frac{1}{x} \mathbb{1}\{y>0\} \Rightarrow Y|X=x \sim U(0,x)$$

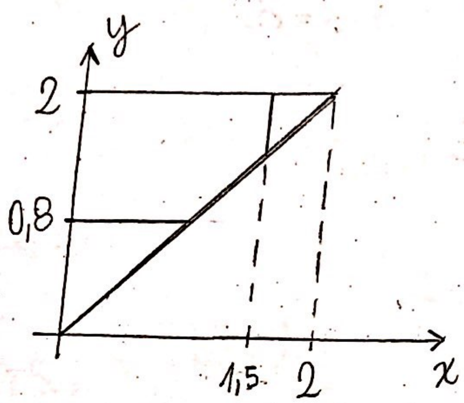
d) $F_{XY}(x,y) = \frac{1}{6} x^4 y^3 e^{-xy} \, \mathbb{1}\{1<x<2, y>0\}$

$Y|X=x \sim GAMMA(4,x) \cdot \mathbb{1}\{y>0\}$

$X \sim U(2,1) \, \mathbb{1}\{1<x<2\}$

5.3) $F_{XY}(x,y) = \frac{1}{2} \mathbb{1}\{0<x<y<2\}$

a) hallar $F_{Y|X=1,5}(y)$ y $F_{X|Y=0,8}(y)$



$Y|X=1,5 \sim U(1,5;2) \, \mathbb{1}\{1,5<y<2\}$

$X|Y=0,8 \sim U(0;0,8) \, \mathbb{1}\{0<y<0,8\}$

b) $P(1,75<Y<2|X=1,5) \rightarrow \int_{1,75}^{2} F_{Y|X=1,5}(y)$

$P(0,5<X<0,75|Y=0,8) \rightarrow \int_{0,5}^{0,75} F_{X|Y=0,8}(x)$

c) $Y|X=x \sim U(x,2) \rightarrow F_X(x) = \frac{1}{2} \cdot (2-x) = \frac{2-x}{2}$

$X|Y=y \sim U(0,y) \rightarrow F_Y(y) = \frac{1}{2} \cdot y = \frac{y}{2}$

$\Rightarrow$ no son independientes

5.4) $f_{XY}(x,y) = \frac{5}{8\pi} e^{-\frac{25}{32}(x^2 - \frac{6}{5}xy + y^2)}$

a) $Y|X=x \sim N\left(\frac{3}{5}x, \frac{16}{25}\right)$

$f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2}$ (mirar 3.22)

no son independientes

b) $P(1 < XY < 5 \mid X = \sqrt{5}) =$

$= P(1 < \sqrt{5}\,Y < 5 \mid X = \sqrt{5}) =$

$= P(1/\sqrt{5} < Y < \sqrt{5} \mid X = \sqrt{5}) = \int_{1/\sqrt{5}}^{\sqrt{5}} f_{Y|X=\sqrt{5}}(y)\, dy$

5.5) $X \sim U(3,4) \to f_X(x) = \mathbb{1}\{3 < x < 4\}$

CONT MÁS ADELANTE

$Y|X=x \sim N(x,1) \to f_{Y|X=x}(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-x)^2}{2}} \mathbb{1}\{y \in \mathbb{R}\}$

$\Rightarrow f_{XY}(x,y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-x)^2}{2}} \mathbb{1}\{y \in \mathbb{R}\}$

$f_Y(y) = \int_3^4 \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-x)^2}{2}} dx \Rightarrow f_Y(5) = \int_3^4 \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-5)^2}{2}} dx =$

$= \Phi(4-5) - \Phi(3-5) = \Phi(-1) - \Phi(-2) =$

$= 0{,}1587 - 0{,}0228 = 0{,}1359$

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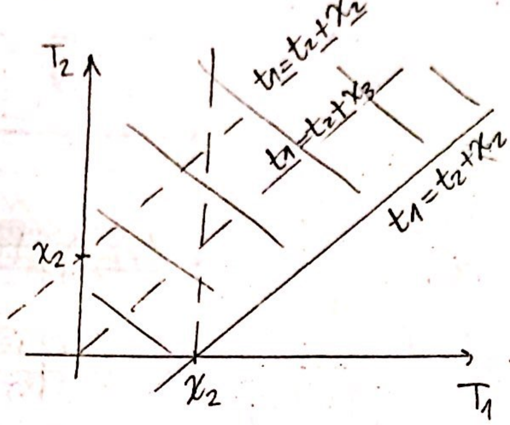
5.6) $T_1 \sim EXP(1) \quad T_2 \sim EXP(1)$ indep.

$$X_1 = T_1 + T_2 \qquad X_2 = T_1 - T_2 \qquad X_3 = T_1/T_2 \quad \Big| \quad f_{T_1T_2}(t_1,t_2) = e^{-(t_1+t_2)}$$

$$* f_{X_1}(x_1) = x_1 e^{-x_1} \, \mathbb{1}\{x_1 \geq 0\} \qquad * F_{X_2}(x_2) \Rightarrow P(X_2 \leq x_2) = P(T_1 - T_2 \leq x_2) =$$

$$= P(T_1 \leq T_2 + x_2)$$



$$F_{X_2}(x_2) = \begin{cases} x_2 < 0 \rightarrow * \\ x_2 \geq 0 \rightarrow ** \end{cases}$$

$$* \int_0^{+\infty}\int_{t_1-x_2}^{+\infty} e^{-(t_1+t_2)} \, dt_2 \, dt_1 = \int_0^{+\infty} e^{-t_1} \int_{t_1-x_2}^{+\infty} e^{-t_2} \, dt_2 \, dt_1 = \int_0^{+\infty} e^{-t_1} \left(-e^{-t_2}\right)\Big|_{t_1-x_2}^{+\infty} dt_1$$

$$= \int_0^{+\infty} e^{-t_1} e^{-t_1+x_2} \, dt_1 = \int_0^{+\infty} e^{-2t_1+x_2} \, dt_1 = -\frac{1}{2} e^{-2t_1+x_2}\Big|_0^{+\infty} = \frac{e^{x_2}}{2}$$

$$** \; 1 - \int_0^{+\infty}\int_{t_2-x_2}^{+\infty} e^{-(t_1+t_2)} \, dt_1 \, dt_2 = 1 - \int_0^{+\infty} e^{-t_1} e^{-t_1+x_2} \, dt_1 = 1 - \frac{e^{x_2}}{2}$$

$$\Rightarrow F_{X_2}(x_2) = \frac{e^{x_2}}{2} \mathbb{1}\{x_2 < 0\} - \frac{e^{x_2}}{2} \mathbb{1}\{x_2 \geq 0\}$$

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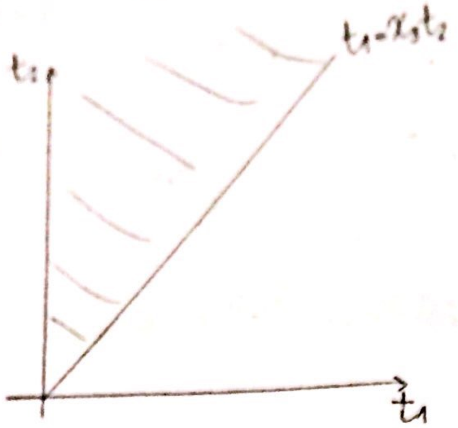
$*\ F_{X_3}(x_3) \Rightarrow P(X_3 \leq x_3) = P(T_1/T_2 \leq x_3) =$

$= P(T_1 \leq x_3 T_2) = \int_0^{+\infty}\int_0^{x_3 t_2} e^{-(t_1+t_2)}\, dt_1 dt_2$



$= \int_0^{+\infty} e^{-t_2} \int_0^{x_3 t_2} e^{-t_1}\, dt_1 dt_2 =$

$= \int_0^{+\infty} e^{-t_2} \left(-e^{-t_1}\right)\Big|_0^{x_3 t_2} dt_2 =$

$= \int_0^{+\infty} e^{-t_2}\left(1 - e^{-x_3 t_2}\right) = \int_0^{+\infty} \left(e^{-t_2} - e^{-t_2 - x_3 t_2}\right) dt_2 =$

$= \int_0^{+\infty} e^{-t_2} dt_2 + \int_0^{+\infty} -e^{-t_2(1+x_3)} dt_2 = -e^{-t_2}\Big|_0^{+\infty} + \frac{1}{(1+x_3)} e^{-t_2(1+x_3)}\Big|_0^{+\infty} =$

$= 1 - \frac{1}{1+x_3} \quad \Rightarrow f_{X_3}(x_3) = \frac{1}{(1+x_3)^2}\ \mathbb{1}\{x_3 > 0\}$

a) $\begin{matrix} X_1 = T_1 + T_2 \\ X_2 = T_1 - T_2 \end{matrix} \Rightarrow \begin{matrix} T_1 = \frac{X_1 + X_2}{2} \\ T_2 = \frac{X_1 - X_2}{2} \end{matrix} \qquad \left|\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\right| = 2$

$$f_{X_1X_2}(x_1,x_2) = \frac{f_{T_1T_2}\left(t_1(x_1,x_2), t_2(x_1,x_2)\right)}{\left|\frac{\partial(x_1,x_2)}{\partial(t_1,t_2)}\right|} = \frac{e^{-\left(\frac{x_1+x_2}{2} + \frac{x_1-x_2}{2}\right)}}{2} =$$

$$= \frac{e^{-x_1}}{2}\ \mathbb{1}\{x_1 > 0;\ x_2 \in \mathbb{R}\}$$

$\Rightarrow$ no son independientes

b) $X_1 = T_1 + T_2 \quad X_3 = \frac{T_1}{T_2} \Rightarrow T_2 = \frac{X_1}{1+X_3} \quad T_1 = \frac{X_1 X_3}{1+X_3}$

$$f_{X_1X_3}(x_1,x_3) = \frac{f_{T_1T_2}(t_1(x_1,x_3),t_2(x_1,x_3))}{\left|\frac{\partial(x_1,x_3)}{\partial(t_1,t_2)}\right|} =$$

$$= \frac{e^{-\left(\frac{x_1x_3}{1+x_3}+\frac{x_1}{1+x_3}\right)}}{\left|\begin{pmatrix}1 & 1\\ 1/t_2 & -t_1/t_2^2\end{pmatrix}\right|} = \frac{e^{-\left(\frac{x_1(1+x_3)}{1+x_3}\right)}}{\left|-\frac{t_1}{t_2^2}-\frac{1}{t_2}\right|} = \frac{e^{-\left(\frac{x_1(1+x_3)}{1+x_3}\right)}}{\frac{(1+x_3)^2}{x_1}} =$$

$$= \frac{x_1 e^{-x_1}}{(1+x_3)^2} \mathbb{1}\{x_1>0;\ x_3>0\}$$

$\Rightarrow$ son independientes

c) $$f_{X_1|X_2=0}(x_1) = \frac{f_{X_1X_2}(x_1,0)}{f_{X_2}(0)} = \frac{e^{-x_1}\mathbb{1}\{x_1>0\}}{2\cdot\frac{1}{2}} =$$

$$= e^{-x_1}\mathbb{1}\{x_1>0\}$$

$$f_{X_1|X_3=1}(x_1) = \frac{f_{X_1X_3}(x_1,1)}{f_{X_3}(1)} = \frac{x_1 e^{-x_1} \mathbb{1}\{x_1>0\}}{4 \cdot \frac{1}{4}} =$$

$$= x_1 e^{-x_1} \mathbb{1}\{x_1>0\}$$

5.7) $T_1 \sim EXP(\lambda)$ $\quad S_1 = T_1$ $\quad f_{T_1T_2}(t_1,t_2) = \lambda^2 e^{-\lambda(t_1+t_2)}$

$T_2 \sim EXP(\lambda)$ $\quad S_2 = T_1 + T_2$

a)

$$f_{S_1}(s_1) = \lambda e^{-\lambda s_1} \mathbb{1}\{s_1>0\}$$

$$f_{S_2}(s_2) = \lambda^2 s_2 e^{-\lambda s_2} \mathbb{1}\{s_2>0\}$$

$$f_{S_1S_2}(s_1,s_2) = \frac{f_{T_1T_2}(t_1(s_1,s_2), t_2(s_1,s_2))}{\left|\frac{\partial(S_1,S_2)}{\partial(T_1,T_2)}\right|} =$$

$$= \frac{\lambda^2 e^{-\lambda(s_1+(s_2-s_1))}}{\left|\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}\right|} = \frac{\lambda^2 e^{-\lambda s_2}}{1} = \lambda^2 e^{-\lambda s_2} \mathbb{1}\{s_2>0, s_1>0\}$$

$\Rightarrow$ no son independientes

b) $$P(1/2 < S_1 < 1 | S_2 = 2) = \int_{1/2}^{1} f_{S_1|S_2=2}(s_1) = \int_{1/2}^{1} \frac{1}{2}\, ds_1 =$$

$$= \frac{1}{4}$$

$$f_{S_1|S_2=s_2}(s_1) = \frac{f_{S_1S_2}(s_1,s_2)}{f_{S_2}(s_2)} = \frac{1}{s_2}$$

SETEC

CONSULTORIA DE INTERFASE

del 4.14
$$J=\begin{cases}1 & \text{si } x<y\\ 2 & \text{si } y<x\end{cases}$$

5.8) $$F_{XY}(x,y)=\frac{1}{35}e^{-\left(\frac{x}{5}+\frac{y}{7}\right)}\mathbb{1}\{x>0,y>0\}$$

$$=\underbrace{\frac{1}{5}e^{-\frac{x}{5}}}\;\underbrace{\frac{1}{7}e^{-\frac{y}{5}}}$$

DRACULA

$X\sim EXP(1/5)$ $\quad$ $Y\sim EXP(1/7)$

PENFIELD

primero que llega

$U=MIN(X,Y)$

$V=MAX(X,Y)$ último que llega

$W=V-U$ espera entre que llegó el 1ero y llega el 2do

$$P(J=1|W=5)=P(X<Y|W=5)=$$

BAYES P/ MEZCLA

$$=\frac{f_{W|J=1}(5)\cdot P(J=1)}{\underbrace{f_{W|J=1}(5)P(J=1)+f_{W|J=2}(5)P(J=2)}}=0{,}571$$

4.14

$$\Rightarrow f_W(w)=\underbrace{\lambda_2 e^{-\lambda_2 w}}_{f_{W|J=1}(w)}\underbrace{\frac{\lambda_1}{\lambda_1+\lambda_2}}_{P(J=1)}+\underbrace{\lambda_1 e^{-\lambda_1 w}}_{f_{W|J=2}(w)}\underbrace{\frac{\lambda_2}{\lambda_1+\lambda_2}}_{P(J=2)}=$$

$$=\frac{1}{7}e^{-\frac{5}{7}}\frac{\frac{1}{5}}{\frac{1}{5}+\frac{1}{7}}+\frac{1}{5}e^{-\frac{5}{5}}\frac{\frac{1}{7}}{\frac{1}{5}+\frac{1}{7}}=0{,}071$$

5.9) $X = S + N$ = Amplitud, S,N son indeptes

$$P(S=0{,}1) = P(S=0{,}2) = P(S=0{,}3) = \frac{1}{3} = \frac{\mathbb{P}(\Omega)}{3}$$

$N \sim \underset{est}{N}(0{,}1)$

PROBA CONDICIONAL P/ MEZCLA

$$P(S=0{,}2 \mid X=0{,}87) = \frac{F_{X_{0,2}}(0{,}87) \cdot P(S=0{,}2)}{F_X(0{,}87)} =$$

$\Rightarrow$ S es la variable mezcladora

$$X|_{S=0,1} = 0{,}1 + N \sim N(0{,}1;1) \longrightarrow \dots$$

$$X|_{S=0,2} = 0{,}2 + N \sim N(0{,}2;1) \rightarrow F_{X_{0,2}}(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-0{,}2)^2}$$

$$X|_{S=0,3} = 0{,}3 + N \sim N(0{,}3;1) \longrightarrow \dots$$

$$= \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(0{,}87-0{,}2)^2} \times \frac{1}{3}}{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(0{,}87-0{,}1)^2} \times \frac{1}{3} + \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(0{,}87-0{,}2)^2} \times \frac{1}{3} + \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(0{,}87-0{,}3)^2} \times \frac{1}{3}} =$$

$= 0{,}334$

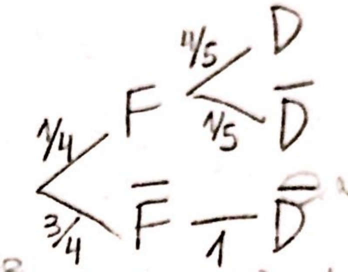
5.10) $P(\text{"un RoboCop esté fallado"}) = \frac{1}{4} = P(F_{alla})$

$P(\text{"detecte la falla"}) = \frac{4}{5} = P(D|F)$ detección

X: "cant. de Robocops fallados" $\sim BiN(6, 1/4)$

• detectar fallas en $n = 6$ robocops
• éxito = falla
• no éxito = no falla

Y: "cant detectada de RoboCops fallados"



= no detecta

(VER

a) Hallar $E[Y|X=x] = \varphi(x)$

$Y|X=x \sim BiN(x, 4/5)$ ensayo sobre x robocops fallados

$E[Y|X=x] = \frac{4}{5}x = np$

↑ proba éxito = está fallado y lo detecté

$P(\overline{D}|F) = 1/5$ $\quad P(F) = 1/4$

$$\frac{P(\text{no se detectó}|\text{falló}) \times P(\text{falló})}{P(\text{no se detectó})} = \tfrac{3}{4} \cdot \tfrac{1}{5} + \ldots$$

⇑ BAYES CONDICIONAL GUÍA 1

$P(\text{falló}|\text{no se detectó})$

b) $E[X|Y=y] = ?\ \varphi(y)$

$$\underbrace{X|Y=y}_{\text{todos los fallados}} = \underbrace{y}_{\text{nº de fallados detectados}} + \underbrace{X'|Y=y}_{\text{nº de fallados no detectados}} \rightarrow X'|Y=y \sim BiN(\underbrace{6-y}_{\text{ensayo}}, 1/16)$$

ensayo: 6 robocops – los que detecté previamente

$$E[X|Y=y] = E[y + X'|Y=y] = y + \underbrace{\frac{(6-y)}{16}}_{n \cdot p} = \frac{3}{8} + \frac{15}{16}y$$

5.11) $\cdot\ X(1) = X(2) = 1$ $\qquad P(\{\omega\}) = 1/12$, $\omega \in \Omega$

$X: \Omega \to \mathbb{R}$ $\quad \cdot\ X(3) = X(4) = X(5) = X(6) = 2$

$\cdot\ X(7) = X(8) = X(9) = X(10) = X(11) = X(12) = 3$

$\therefore X(\omega) = 1\mathbb{1}\{\omega \in (1,2)\} + 2\,\mathbb{1}\{\omega \in (3,4,5,6)\} + 3\,\mathbb{1}\{\omega \in (7,\dots,12)\}$

a) 1) $Y(\omega) = \omega$

$P(X=1) = \frac{2}{12}$

$P(X=2) = \frac{4}{12}$

$P(X=3) = \frac{6}{12}$

E[ ] para VA discreta

$$E[Y|X=x] = \sum_{\forall y} y\, p_{Y|X=x}(y)$$

TEO

$$p_{Y|X=x}(y) = \frac{p_{XY}(x,y)}{p_X(x)}$$

$$P(Y=1) = P(Y=2) = \dots = P(Y=12) = \frac{1}{12}$$

X es uniforme caso equiprobable

$$\mathbb{E}[Y|_{X=x}] = \begin{cases} 1{,}5 & \text{si } X=1 \ \left(\frac{1+2}{2}\right) \\ 4{,}5 & \text{si } X=2 \ \left(\frac{3+4+5+6}{4}\right) \\ 9{,}5 & \text{si } X=3 \ \left(\frac{7+\dots+12}{6}\right) \end{cases}$$

$\hookrightarrow E[U] = \frac{a+b+\dots}{n^\circ \text{ candidatos}}$

$Y|_{X=1} \sim U\{1,2\}$

~~$\cdot\ E[Y|X=1] = E[\omega \mid \omega \in \{1,2\}] = \frac{E[\boxed{\omega\,\mathbb{1}\{\omega \in 1,2\}}]}{\underbrace{P(\omega \in \{1,2\})}_{\frac{2}{12}}}$~~

~~$\underbrace{1 \cdot \underset{P(\omega=1)}{\ } + 2 \cdot \underset{P(\omega=2)}{\ }}$ $= \frac{\frac{3}{12}}{\frac{2}{12}}$~~

~~$\sum_{\omega} \operatorname{sen}\left(\frac{\pi}{2}\omega - \frac{\pi}{4}\right) P(\{\omega\})$~~

~~a)~~ 2) VER HOJA EXTRA

5) T: "tiempo que demora en salir"

$$T|C=1 \sim 9 \qquad P(C=1)=\frac{1}{3}$$
$$T|C=2 \sim 12+T \qquad P(C=2)=\frac{1}{3}$$
$$T|C=3 \sim 14+T \qquad P(C=3)=\frac{1}{3}$$

$$E[T]=E[T|C=1]\times P(C=1)+E[T|C=2]\times P(C=2)+$$
$$+E[T|C=3]\times P(C=3)$$

$$E[T]=E[9]\times\frac{1}{3}+E[12+T]\times\frac{1}{3}+E[14+T]\times\frac{1}{3}$$

$$E[T]=9\times\frac{1}{3}+(12+E[T])\times\frac{1}{3}+(14+E[T])\times\frac{1}{3}$$

$$E[T]=3+4+\frac{E[T]}{3}+\frac{14}{3}+\frac{E[T]}{3} \Rightarrow E[T]=35$$

T_i= "tiempo que tarda en salir del lab" → T es mezcladsra

5.14) $T|C=1 \sim 8+T_1 \rightarrow E[T|C=1]=8+E[T_1]$

$T|C=2 \sim 13+T_2 \rightarrow E[T|C=2]=13+E[T_2]$

↑ linealidad

$T|C=3 \sim 5 \rightarrow E[T|C=3]=5$

C = "camino elegido"

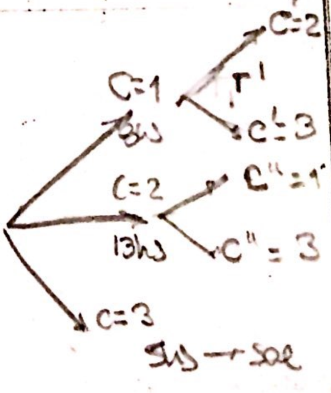
$C=1$ y $C=2$ no tienen salida, vuelven al camino 3

MEZCLA

$$E[T]=\underbrace{\frac{1}{3}}_{P(C=1)}(8+E[T_1])+\underbrace{\frac{1}{3}}_{P(C=2)}(13+E[T_2])+\underbrace{\frac{1}{3}}_{P(C=3)}\times 5$$

$$T_1|C'=2 \sim 13+T_2' \qquad T_2|C''=1 \sim 8+T_1'$$

$$T_1|C'=3 \sim 5 \qquad T_2|C''=3 \sim 5$$



* T_1 y T_1' se distribuyen igual $\rightarrow E[T_1]=E[T_1']$
* T_2 y T_2' se distribuyen igual $\rightarrow E[T_2]=E[T_2']$

$$E[T_1] \overset{= E[T_1'] =}{=} E[T_1|C'=2]\times P(C'=2)+E[T_1|C'=3]\times P(C'=3)$$

$$= E[13+T_2']\times\frac{1}{2}+E[5]\times\frac{1}{2}=$$

$$\overset{\text{linealidad}}{=}\left(13+E[T_2']\right)\times\frac{1}{2}+5\times\frac{1}{2}=$$

$$=\left(13+E[T_2]\right)\times\frac{1}{2}+\frac{5}{2}=\frac{13}{2}+\frac{E[T_2]}{2}+\frac{5}{2}$$

$$E[T_2]=E[T_2|C''=1]\times P(C''=1)+E[T_2|C'=3]\times P(C''=3)$$

$$=\left(8+E[T_1']\right)\times\frac{1}{2}+5\times\frac{1}{2}=4+\frac{E[T_1]}{2}+\frac{5}{2}$$

mi sist queda

$$\begin{cases} E[T_1]=\frac{E[T_2]}{2}+9 \rightarrow E[T_1]=\left(\frac{E[T_1]}{4}+\cancel{\frac{9}{2}}\ ^{13/4}\right)+9 \rightarrow E[T_1]=\cancel{18}\ ^{49/3} \\ E[T_2]=\frac{E[T_1]}{2}+\frac{13}{2} \rightarrow E[T_2]=\cancel{\frac{31}{2}}\ \frac{44}{3} \end{cases}$$

$$\overset{\text{ESPERANZA P/ MEZCLA}}{\Rightarrow} E[T]=\underbrace{\frac{1}{3}}_{P(C=1)}\underbrace{\left(8+\cancel{\frac{18}{2}}\ \frac{49}{3}\right)}_{E[C=1+B]}+\underbrace{\frac{1}{3}}_{P(C=2)}\underbrace{\left(13+\cancel{\frac{31}{2}}\ \frac{44}{3}\right)}_{E[C=2+B]}+\underbrace{\frac{1}{3}}_{P(C=3)}\underbrace{\times 5}_{E[C=3]}=\cancel{\frac{119}{6}\simeq 19{,}83}=19$$

5.15) $X \sim N(0,1)$

$E[Y|X] = X^2 \rightarrow E[Y] = E[E[Y|X]] = E[X^2]$

$E[Y|X=x] = x^2$

$COV(X,Y) = E[XY] - \underbrace{E[X]}_{=0} E[Y]$

$\int_{-\infty}^{\infty} x^3 \underbrace{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}}_{f_X(x)} dx$

$E[XY] = E[E[XY|X]] = E[X E[Y|X]] = E[X^3] = 0$

$(X, X^3, X^5, X^{2n+1}$ son simetricas respecto al cero$)$

la integral da $=0$

5.16) $X \sim EXP(1/2)$

$E[Y|X] = X$

$V(Y|X) = X$

$V(Y) = E[V(Y|X)] + V(E[Y|X]) = E[X] + V(X) = 2 + 4 = 6$

5.17) $p_{Y|X=x} = \dfrac{\binom{3}{y}\binom{3}{3-x-y}}{\binom{6}{3-x}}$ (mirar 5.1)

TEO $p_{Y|X=x} = \dfrac{p_{XY}}{p_X}$

a)

va discreta

$$E[Y|X=0] = \sum_{\forall y} y \cdot p_{Y|X=0}(y) = 1 \times \frac{p_{XY}(0,1)}{p_X(0)} + 2 \times \frac{p_{XY}(0,2)}{p_X(0)} +$$

$$+ 3 \times \frac{p_{XY}(0,3)}{p_X(0)} = 1 \times \frac{3/40}{1/6} + 2 \times \frac{3/40}{1/6} + 3 \times \frac{1/120}{1/6} = \frac{3}{2}$$

$$E[Y|X=1] = 1 \times \frac{p_{XY}(1,1)}{p_X(1)} + 2 \times \frac{p_{XY}(1,2)}{p_X(1)} + 3 \times \frac{p_{XY}(1,3)}{p_X(1)} =$$

$$= 1 \times \frac{3/10}{1/2} + 2 \times \frac{1/10}{1/2} + 0 = 1$$

$$E[Y|X=2] = 1 \times \frac{p_{XY}(2,1)}{p_X(2)} + 2 \times \frac{p_{XY}(2,2)}{p_X(2)} + 3 \times \frac{p_{XY}(2,3)}{p_X(2)} =$$

$$= 1 \times \frac{3/20}{3/10} + 2 \times 0 + 0 = \frac{1}{2}$$

$$E[Y|X=3] = 1 \times \frac{p_{XY}(3,1)}{p_X(3)} + 2 \times \frac{p_{XY}(3,2)}{p_X(3)} + 3 \times \frac{p_{XY}(3,3)}{p_X(3)} =$$

$$= 0$$

$$E[Y|X=x] = \begin{cases} 3/2 & \text{si } x=0 \\ 1 & \text{si } x=1 \\ 1/2 & \text{si } x=2 \\ 0 & \text{si } x\neq 0,1,2 \end{cases} = E[Y|X]$$

DISCRETA

b)

$$E[Y^2|X=x] \underset{(\text{discreta})}{=} \sum_{\forall y} y^2 \cdot p_{Y|X=x}(y)$$

solo

$$V(Y|X=x) \doteq E[Y^2|X=x] - E^2[Y|X=x]$$

$$E[Y^2|X=0] = \overbrace{1}^{y^2} \times \frac{3/40}{1/6} + \overbrace{4}^{y^2} \times \frac{3/40}{1/6} + 9 \times \frac{1/120}{1/6} = \frac{27}{10}$$

$$E[Y^2|X=1] = 1 \times \frac{3/10}{1/2} + 4 \times \frac{1/10}{1/2} + 0 = \frac{7}{5}$$

$$E[Y^2|X=2] = 1 \times \frac{3/20}{3/10} = \frac{1}{2}$$

$$E[Y^2|X=3] = 0$$

$$V(Y|X=x) = \begin{cases} \frac{27}{10} - \left(\frac{3}{2}\right)^2 = \frac{9}{20} & \text{si } x=0 \\ \frac{7}{5} - 1 = \frac{2}{5} & \text{si } x=1 \\ \frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{4} & \text{si } x=2 \\ 0 & \text{si } x\neq 0,1,2 \end{cases} = V(Y|X)$$

NOTA: en $E[Y|X]$ y $V(Y|X)$ los valores varían con los valores de $X \to X=0; X=1; X=2; X\neq 0,1,2$

5.12) $X \sim BIN(36, p_x)$

$Y \sim BIN(36, p_Y)$

$$Y = 36 - X = E[Y|X]$$

$$\Rightarrow \frac{COV(X,Y)}{V(X)} = -1$$

$$V(X) = 36 \cdot p_X (1 - p_X)$$

$$\Rightarrow COV(X,Y) = -36 \cdot p_X (1 - p_X)$$

continua 5.5)

$$P(X > 3,5 | Y = 5) = 1 - \int_3^{3,5} f_{X|Y=5}(x)\, dx$$

$$f_{X|Y=5}(x) = \frac{f_{XY}(x,5)}{f_Y(5)} = \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{(5-x)^2}{2}}}{0,1359}$$

$$\Rightarrow P(X > 3,5 / Y = 5) = 1 - \frac{1}{0,1359} \int_3^{3,5} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-5)^2}{2}} dx =$$

$$= 1 - \frac{1}{0,1359} \left(\Phi(-1,5) - \Phi(-2) \right) =$$

$$= 1 - \frac{1}{0,1359} (0,1778 - 0,1338) = 0,676$$

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3) a) $F_{XY}(x,y) = \frac{1}{2x+1} e^{-(2x+\frac{y}{4x+2})} \mathbb{1}\{x>0, y>0\}$

1) $Y|X=x \sim EXP(1/4x+2) = EXP\left(\frac{1}{2(2x+1)}\right)$ $\quad X \sim EXP(2)$

2) $E[Y|X=x] = 4x+2 = \varphi(x)$

$\uparrow$ NO es EXP

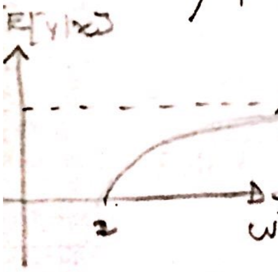
3) $E[Y|X] = 4X+2 = \varphi(x) = 2(2x+1)$

4) $F_{E[Y|X]}(w) = P(E[Y|X] \leq w) = P(4X+2 \leq w) =$

Sea $w = E[Y|x]$ $= P\left(X \leq \frac{w-2}{4}\right) = F_X\left(\frac{w-2}{4}\right) \underset{\uparrow}{=} 1-e^{-\left(\frac{w-2}{4}\right)2} \mathbb{1}\{w \geq 2\}$

$x \sim exp(2)$

5) $P(1<E[Y|X]<2) = P(1<4X+2<2) = 0$



$\cancel{= P(-1/4 \leq X < 0) = F_X(0) - F_X(-1/4) =}$

$\cancel{= e^{-2(-1/4)} - e^{-2\times 0} = 0,649}$

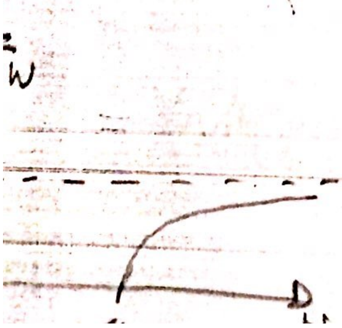
6) $V(Y|X=x) = (4x+2)^2 = 16x^2+16x+4$

7) $V(Y|X) = 16X^2+16X+4$

8) $F_{\underbrace{V(Y|X)}_{w}}(w) = P(V(Y|X) \leq w) = P((4X+2)^2 \leq w) =$

$= P(\quad 4X+2 \leq \sqrt{w}) =$

$= P(\quad \leq X \leq \frac{\sqrt{w}-2}{4}) =$



$= 1-e^{-2\left(\frac{\sqrt{w}-2}{4}\right)} \mathbb{1}\{w>4\}$

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9) $P(V(Y|X)>1) = 1$ porque $V(Y|X)$ va de 4 a $+\infty$

$F_{XY}(x,y) = 1 - e^{-(2x - \frac{y}{4x+2})}$ xq $(x,y) \sim exp$ es necesariamente <1 y $F_{XY}(x,y)$

la $P(\cdot)$ converge a 1

10) $V(Y) \overset{\checkmark \text{PITAGORAS}}{=} E[V(Y|X)] + V(E[Y|X])$

$= E[16X^2 + 16X + 4] + V(4X+2) =$

$= 16E[X^2] + 16E[X] + 4 + 16V(X) =$

$= 16(V(X) + E^2[X]) + 16E[X] + 4 + 16V(X) =$

$= 16(\frac{1}{4} + (\frac{1}{2})^2) + 16 \times \frac{1}{2} + 4 + 16 \times \frac{1}{4} =$

$= 8 + 8 + 4 + 4 = 24$

b) inciso a) $\rightarrow Y|X=x \sim U(-\sqrt{1-x^2}, \sqrt{1-x^2})$

inciso c) $\rightarrow Y|X=x \sim U(0,x)$

inciso d) $\rightarrow Y|X=x \sim GAMMA(4,x)$

y repito lo mismo que el inciso a)

a) $f_{XY}(x,y) = \frac{13}{48\pi} e^{\left(-\frac{169}{288}\left(\frac{(x-1)^2}{4} - \frac{5(x-1)y}{13} + y^2\right)\right)}$

$\mu_X = 1$ $\quad \rho = \frac{5}{13}$

$\sigma_X = 2$

$\mu_Y = 0$

$\sigma_Y = 1$

$$= \frac{1}{\pi \frac{48}{13}} e^{-\frac{169}{288}\left(\frac{(x-1)^2}{4} - \frac{5(x-1)y}{13} + y^2\right)}$$

$$= \frac{1}{2\sqrt{2\pi}} \times \frac{1}{\sqrt{2\pi}} \times \frac{1}{12/13} \times \underbrace{e^{-\frac{169}{1152}(x-1)^2 + \frac{65}{288}(x-1)y - \frac{169}{288}y^2}}$$

$$e^{-\frac{1}{2}\frac{(x-1)^2}{4}} e^{-\frac{25}{1152}(x-1)^2}$$

$$= \frac{1}{\sqrt{2\pi}\,2} e^{-\frac{1}{2}\frac{(x-1)^2}{4}} \times \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{169}{288}y^2 - \frac{65}{288}y(x-1) + \frac{25}{1152}(x-1)^2\right)} \times \frac{1}{12/13} =$$

$$= \frac{1}{2\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-1)^2}{4}} \times \frac{1}{\sqrt{2\pi}\,12/13} e^{-\frac{1}{2}\left(\frac{169}{144}y^2 - \frac{65}{144}y(x-1) + \frac{25}{576}(x-1)^2\right)} =$$

$\div 4$ $\quad \times 4$

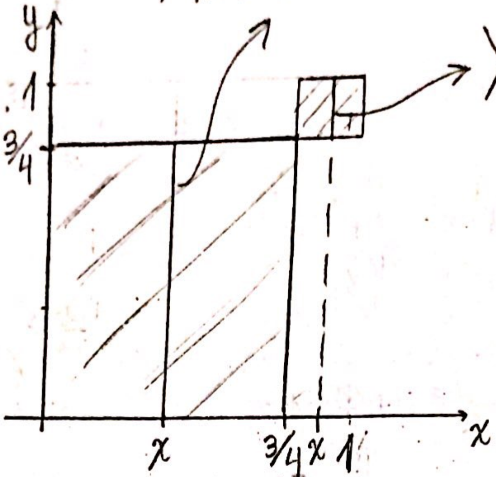
$$= \frac{1}{2\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-1)^2}{4}} \times \frac{1}{\sqrt{2\pi}\,12/13} e^{-\frac{1}{2}\underbrace{\left(\frac{169}{576}\left(4y^2 - \frac{20}{13}y(x-1) + \frac{25}{169}(x-1)^2\right)\right)}_{\frac{169}{144}\left(y^2 - \frac{10}{13}y(x-1) + \frac{25}{676}(x-1)^2\right)}} =$$

$$= \frac{1}{\sqrt{2\pi}\,2} e^{-\frac{1}{2}\frac{(x-1)^2}{4}} \times \frac{1}{\sqrt{2\pi}\,12/13} e^{-\frac{1}{2}\left(\frac{\left(y - \frac{5}{26}(x-1)\right)^2}{144/169}\right)}$$

$X \sim N(1,2)$ $\qquad Y|X=x \sim N\left(\frac{5}{26}(x-1), 12/13\right)$

$$E[Y|X] = \frac{5}{26}(X-1) = \varphi(X)$$

5.20) $Y|X=x \sim U(0, 3/4)$



$Y|X=x \sim U(3/4, 1)$ $\quad \frac{a+b}{2}$ xq es $\mathcal{N}$

$$E[Y|X=x] = \begin{cases} \frac{3}{8} & \text{si } 0<x<3/4 \\ \frac{7}{8} & \text{si } 3/4<x<1 \end{cases}$$

$$E[Y|X=x] = E[Y|X]$$

$\frac{(b-a)^2}{12}$ xq es V

3/64 no puede ser

$$V(Y|X) = \begin{cases} 3/64 & \text{si } 0<X<3/4 \\ 1/192 & \text{si } 3/4<X<1 \end{cases}$$

$$P(V(Y|X) < 1/32) = P\left(V(Y|X) = \frac{1}{192}\right)$$

area( ) $= \frac{1}{4} \cdot \frac{1}{4}$

$$= P(3/4 < X < 1) = \frac{1/16}{5/8} = \frac{1}{10}$$

area de toda la VA $=$ Area( ) $+$ Area( ) $= \frac{3}{4} \cdot \frac{3}{4} + \frac{1}{16}$

5.21) $X \sim U(20, 30)$

se requiere $X \geq 28$

N: cant. de rollos hasta fabricar uno de 28 o más

$N \sim GEOM(p)$

$$p = P(X \geq 28) = \frac{2}{10} \Rightarrow N \sim GEOM(2/10)$$

a) ~~$E[N] = \frac{10}{2} = 5$ rollos~~

a) VER CARPETA 26/10

(ej 5. 19/10)

S: "cant. de tela hasta un rollo con 28 o +"

X_i: "longitud del rollo i" $\sim U(20,30)$

n-1 fracasos

$$S = X_1 + X_2 + X_3 + \ldots + X_N = \sum_{i=1}^{N} X_i \rightarrow S/_{N=n} = \sum_{i=1}^{n-1} L_i / L_i < 28 + L_n / L_n \geq 28$$

consigo el exito en n

$$E[S] = E[E[S|N]]$$

$$E[S|N=n] \overset{\varphi(n)}{=} E\Big[\sum_{i=1}^{n-1} X_i \Big| N=n\Big] = E\Big[\sum_{i=1}^{n} X_i\Big] =$$

la saco afuera

$\sim GEOM(1/20)$

$$= E[X|X<28] \times (n-1) + E[X|X\geq 28] =$$

$$\big(X|X<28 \sim U(20,28) \text{ y } X|X\geq 28 \sim U(28,30)\big)$$

$$= 24(n-1) + 29 = 24n + 5 \quad N \sim GEOM(2/10) \rightarrow \text{TABLA} = 1/2$$

$$\Rightarrow E[S|N] = 24N + 5 \Rightarrow E[S] \overset{*}{=} 24E[N] + 5 =$$

$$= 125$$

las que van al stock = las que no vendí = todos - la que salió ≥ 28

$$c)\ E[S|N=n] = E[X|X<28] \times (n-1) = 24(n-1) =$$

$$= 24n - 24$$

$$= E[E[S|N=n]] =$$

linealidad

$$E[S] = E[24N - 24] = 24E[N] - 24 = 96$$

5.22) N: "cant de impactos por segundo" ~ Poisson(...)

P("derecha") = $\frac{3}{5}$ P("izquierda") = $\frac{2}{5}$

X: "# movimientos a derecha" podría ser a la izq también (arbitrario)

Y: Posición final

$$Y = \overset{\text{derecha}}{X} - (\overset{\text{izquierda}}{N-X}) = 2X - N$$

$$X \mid N=n \sim BIN(\overset{\text{partícula}}{n}, 3/5)$$ éxito = se desvió a la derecha

$$E[X] \overset{\cdot}{=} E[E[X|N]] = E[3/5 N] \overset{\text{linealidad}}{=} \frac{3}{5} E[N] = \frac{6}{5}$$

$X|N=n \sim BIN$ (tabla)   linealidad

$$E[Y] \overset{\cdot}{=} E[E[Y|X]] = E[2X-N] = 2E[X] - E[N] =$$

$$= 2 \times \frac{6}{5} - 2 = \frac{2}{5}$$

3) X_i: "peso de bolsa de naranja $\hat{i}$" $\sim U(3,6)$

P: "peso total"

$$P=\begin{cases} X_1 & \text{si } X_1 \geq 5 \\ X_1+X_2 & \text{si } X_1<5 \end{cases}$$

$$E[P]=E[X_1|X_1\geq 5]\times P(X_1\geq 5)+E[X_1+X_2|X_1<5]\times P(X_1<5)$$

$$=E[X_1|X_1\geq 5]\times P(X_1\geq 5)+\left(E[X_1|X_1<5]+E[X_2]\right)\times P(X_1<5)$$

$$=\frac{11}{2}\times\frac{1}{3}+\left(\frac{\cancel{7}\,4}{\cancel{2}}+\frac{9}{2}\right)\times\frac{2}{3}=\frac{43}{6}\cong\cancel{7{,}17}$$

$\hookrightarrow V[P]=E[P^2]-(E[P])^2$

E[ ] para mezcla

5.24) $E[X]=\sum_m E[X|M=m]\times P(M=m)$

$$=E[X|_{S=0,1}]\times P(S=0,1)+E[X|_{S=0,2}]\times P(S=0,2)+$$
$$+E[X|_{S=0,3}]\times P(S=0,3)=$$

$\hookrightarrow$ S es la mezcladora

$$=0{,}1\times\frac{1}{3}+0{,}2\times\frac{1}{3}+0{,}3\times\frac{1}{3}=0{,}2$$

$$V(X)\overset{(*)}{=}E[X^2]-E^2[X]=$$

$$E[X^2]=E[X^2|S=0{,}1]\times P(S=0{,}1)+E[X^2|S=0{,}2]\times P(S=0{,}2)+$$
$$+E[X^2|S=0{,}3]\times P(S=0{,}3)=$$

$$=\left(V(X_{0,1})+E^2[X_{0,1}]\right)\times P(S=0{,}1)+\ldots=$$

(*)

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$$= (1+0{,}1)^2 \times \frac{1}{3} + (1+0{,}2)^2 \times \frac{1}{3} + (1+0{,}3)^2 \times \frac{1}{3} = \frac{157}{150}$$

$$\cancel{= \frac{6}{5}}$$

$$\Rightarrow V(X) = E[X^2] - E^2[X]$$

$$= \cancel{\frac{6}{5}} - 0{,}2^2 = \cancel{\frac{29}{25}} = \frac{151}{150}$$

$$\frac{157}{150}$$