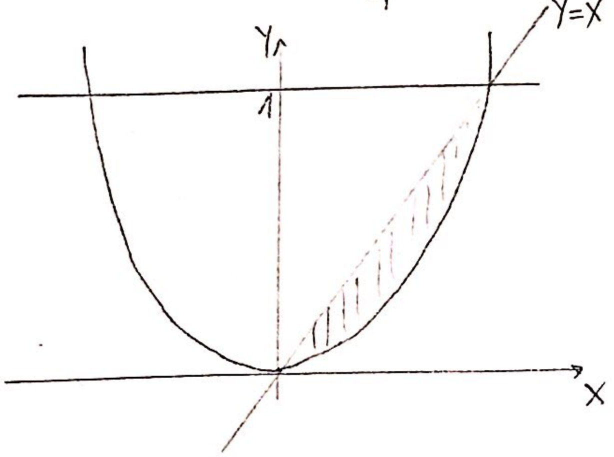
Ej $f_{XY}(x,y) = \frac{21}{4} x^2 y \, \mathbb{1}\{x^2 < y \leq 1\}$ * hallar $P(X>Y)$




$$P(X>Y) = \int_0^1 \int_{x^2}^{x} \frac{21}{4} x^2 y \, dy \, dx$$

2.1) $\Omega = \{1,2,3,4,5,6\}$ $\mathcal{A} = \{\emptyset, \{1,3,5\}, \{2,4,6\}, \Omega\}$

$X : \Omega \to \mathbb{R}$

a) $X(w) = w$ NO ES VARIABLE ALEATORIA

b) $X(w) = \mathbb{1}\{w \text{ es par}\}$ ES VARIABLE ALEATORIA

c) $X(w) = \mathbb{1}\{w \in \{1,4\}\}$ NO ES VARIABLE ALEATORIA

Una V.A. es un $f: \Omega \to \mathbb{R}$ / las ocurrencias de los valores de la función corresponden a eventos en $\mathcal{A}$ y, por lo tanto, se les puede asignar probabilidad

2.2) a)

b) $P(-2<X\leq 2)=F_X(2)-F_X(-2)=1-\frac{1}{3}=\frac{2}{3}$

$P(-2\leq X\leq 2)=F_X(2)-F_X(-2)+P(X=-2)=1-\frac{1}{3}+\frac{1}{3}=1$

$P(-2\leq X<2)=F_X(2)-F_X(-2)+P(X=-2)-P(X=2)$

$=1-\frac{1}{3}+\frac{1}{3}-\frac{1}{3}=\frac{2}{3}$

$P(-2<X<2)=F_X(2)-F_X(-2)-P(X=2)=1-\frac{1}{3}-\frac{1}{3}=\frac{1}{3}$

c) $P(X\in(-2,-1))=P(-2<X<-1)=0$

$P(|X|\leq 1)=P(-1\leq X\leq 1)=\int_{-1}^{1}\frac{1}{6}\,dx=\frac{1}{3}$

$P(X\in(1,2))=P(1<X<2)=0$

d) $P(X\leq 1{,}5\,|\,X<2)=\frac{P(X\leq 1{,}5\wedge X<2)}{P(X<2)}=\frac{P(X\leq 1{,}5)}{P(X<2)}=$

$=\frac{F_X(1{,}5)}{F_X(2)-P(X=2)}=\frac{2/3}{1-1/3}=1$

$$P(X\leq 1{,}5 \mid X\leq 2) = \frac{P(X\leq 1{,}5)}{P(X\leq 2)} = \frac{F_X(1{,}5)}{F_X(2)} = \frac{2/3}{1} = \frac{2}{3}$$

e) $$P(X=-2 \mid |X|=2) = \frac{P(X=-2)}{P(|X|=2)} = \frac{1/3}{2/3} = \frac{1}{2}$$

**2.3)** 3 verdes
5 rojas

X: "cant. de bolas verdes observables"

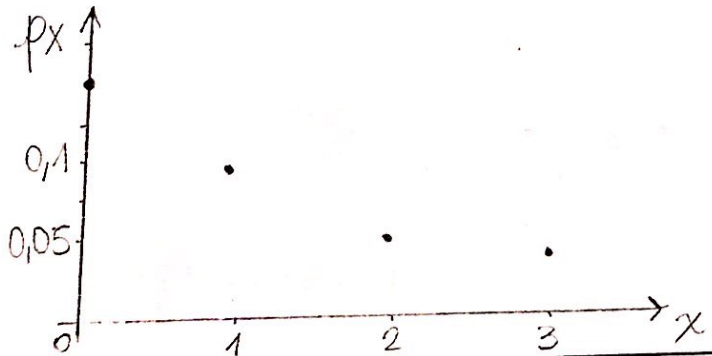
a) $p_X(0) = P(X=0) = \frac{5^4}{8^4} \cong 0{,}15$

$p_X(1) = P(X=1) = \frac{5^3 \cdot 3}{8^4} \cong 0{,}092$

$p_X(2) = P(X=2) = \frac{5^2 \cdot 3^2}{8^4} \cong 0{,}055$

$p_X(3) = P(X=3) = \frac{5 \cdot 3^3}{8^4} \cong 0{,}033$

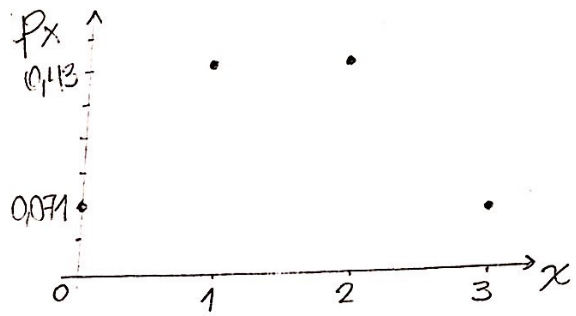
$p_X(4) = 0$



b) $p_X(0) = \dfrac{\binom{5}{4}}{\binom{8}{4}} \cong 0{,}071$ $\quad p_X(1) = \dfrac{\binom{5}{3}\binom{3}{1}}{\binom{8}{4}} \cong 0{,}43$

$p_X(2) = \dfrac{\binom{5}{2}\binom{3}{2}}{\binom{8}{4}} \cong 0{,}43$ $\quad p_X(3) = \dfrac{\binom{5}{1}\binom{3}{3}}{\binom{8}{4}} \cong 0{,}071$

$p_X(4) = 0$




2.4) $P = \frac{5}{8}$ de salir "cara"

a) $N_k$: "cant de lanzamientos hasta k-ésima cara"

$p_N(k) = \binom{n-1}{k-1}\left(\frac{5}{8}\right)^{k}\left(\frac{3}{8}\right)^{n-k}$ (V.A. DE PASCAL)

b) $P(N_1 \text{ es par}) = \left(\frac{5}{8}\right)\sum_{n=0}^{\infty}\left(\frac{3}{8}\right)^{2n+1} = \frac{15}{64}\left(\frac{1}{1-9/64}\right) = \frac{3}{11}$

c) $P(N_1=3) = \left(\frac{3}{8}\right)^{2}\cdot\frac{5}{8} \cong 0{,}088$

$P(N_2=5 \mid N_1=2) = P(N_1=3) \cong 0{,}088$

d) $P(N_1>3) = \left(\frac{3}{8}\right)^3 \cong 0{,}053$

$P(N_1>5 \mid N_1>2) = P(N_1>3) \cong 0{,}053$

e) $P(N_2>3) = \binom{3}{1}\left(\frac{3}{8}\right)^2\left(\frac{5}{8}\right) \cong 0{,}26$

$P(N_2>5 \mid N_2>2) = P(N_2>3) \cong 0{,}26$

**2.5)** N: "cant de partículas alfa emitidas (P/segundo)"

$N \sim \text{POISSON}(1/2)$

a) $P(N>3) = 1 - P(N\leq 3) = 1 - [P(N=0)+P(N=1)+P(N=2)+P(N=3)]$

$= 1 - \left[\frac{(1/2)^0 e^{-1/2}}{0!} + \frac{(1/2)^1 e^{-1/2}}{1!} + \frac{(1/2)^2 e^{-1/2}}{2!} + \frac{(1/2)^3 e^{-1/2}}{3!}\right] = 0{,}00175$

b) $P(N \text{ impar}) = \sum_{n=0}^{\infty} \frac{(1/2)^{2n+1} e^{-1/2}}{(2n+1)!} = \frac{e^{-1/2}}{2} \sum_{n=0}^{\infty} \frac{(1/4)^n}{(2n+1)!} \cong 0{,}316$

$\sum_{n=0}^{\infty} \frac{(1/2)^{2k-1} e^{-1/2}}{(2k-1)!} = \operatorname{sen}(1/2)\, e^{-1/2}$ ; $e^x = \sum_{n=0}^{\infty} x^n$

$\operatorname{sen}(x) = \sum$

$e^{i\theta} = \cos\theta + i \operatorname{sen}\theta$

F° de Euler

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2.6) debo cortar en $(0, 1/4)$ o $(3/4, 1)$

X: "longitud de corte" $\sim U(0,1)$

$$P(0<X<1/4 \cup 3/4<X<1) = \frac{1/4 + 1/4}{1} = \frac{1}{2}$$

2.7) X V.A. con $F_X(x) = 2x \, \mathbb{1}\{0 \leq x \leq 1\}$

$$P(X_1 = 2 \mid X_1 + X_2 = 3) = \frac{P(X_1 = 2 \wedge X_1 + X_2 = 3)}{P(X_1 + X_2 = 3)}$$

$$= \frac{\int_{0,21}^{0,22} 2x\,dx}{\int_{0,03}^{0,04} 2x\,dx + \int_{0,12}^{0,13} 2x\,dx + \int_{0,21}^{0,22} 2x\,dx + \int_{0,30}^{0,31} 2x\,dx} \cong 0,32$$

2.8) $T_k$: "tiempo (en seg) en emitir k partículas alfa"

$$F_{T_K}(t) = \frac{(1/2)^K}{(K-1)!} t^{K-1} e^{-t/2} \, \mathbb{1}\{t>0\} \quad T_K \sim GAMMA(K, 1/2)$$

a) $$P(T_1>3) = 1 - P(T_1 \leq 3) = 1 - \int_0^3 \frac{(1/2)}{0!} t^0 e^{-t/2} dt =$$

$$= 1 - \int_0^3 \frac{1}{2} e^{-t/2} dt \cong 0,22$$

?? / ver carpeta.

$$P(T_1>5 \mid T_1>2) = P(T_1>3) \cong 0,22$$

b) $P(T_3>3) = 1 - P(T_3 \leq 3) = 1 - \int_0^3 \frac{(1/2)^3}{2!} t^2 e^{-t/2} dt =$

$\cong 0{,}81$

$P(T_3>5 \mid T_3>2) = P(T_3>3) \cong 0{,}81$

2.9) $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$

$Z \sim N(0,1)$ ← normal estandar

$F_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

a) $\Phi(1) = 0{,}8413$ $\Phi(2) = 0{,}9772$

$\Phi(0) = 0{,}5$ $\Phi(3) = 0{,}9987$

$\Phi(4) = 1$ $\Phi(5) = \Phi(6) = 1$

b) $\Phi^{-1}(0{,}5) = 0$ $\Phi^{-1}(0{,}75) = 0{,}6745$

$\Phi^{-1}(0{,}9) = 1{,}2816$ $\Phi^{-1}(0{,}95) = 1{,}6449$

$\Phi^{-1}(0{,}99) = 2{,}3263$ $\Phi^{-1}(0{,}995) = 2{,}5758$

$\Phi^{-1}(0{,}999) = 3{,}0902$

c) $P(-0{,}43 < Z < 1{,}32) = 0{,}9066 - (1 - 0{,}6664)$
$= 0{,}573$

$P(1{,}28 < Z < 1{,}64) = 0{,}9495 - 0{,}8997 = 0{,}0498$

$P(|Z| < 1{,}64) = 0{,}9495 - (1 - 0{,}9495) = 0{,}899$

d) $P(Z < \alpha) = 0{,}05 \Rightarrow \alpha = -1{,}64419$
$P(Z > b) = 0{,}1 \Rightarrow b = -1{,}2816$
$P(|Z| < C) = 0{,}95 \Rightarrow C = 1{,}96$

2.10) $Z \sim N(0,1)$, $X = Z^2$; $F_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$

a) $\Phi(3) = \mathbb{P}(Z \leq 3)$

$$F_X(x) = \mathbb{P}(X \leq x) = \mathbb{P}(Z^2 \leq x)$$
$$= \mathbb{P}(|Z| \leq \sqrt{x})$$
$$= \mathbb{P}(-\sqrt{x} \leq Z \leq \sqrt{x})$$
$$\int_{-\infty}^{\sqrt{x}} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt + \int_{-\infty}^{-\sqrt{x}} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt \qquad = \Phi(\sqrt{x}) - \Phi(\sqrt{x}) = F_X(x)$$
$$= 2\Phi(\sqrt{x})^{-1} = F_X(x)$$

b) $f_X(x) = F_X'(x) \overset{\text{regla de la cadena}}{=} \cancel{2}\Phi'(\sqrt{x}) \underbrace{\frac{1}{\cancel{2}\sqrt{x}}}_{[\sqrt{x}]'} = \frac{1}{\sqrt{2\pi x}} e^{-\frac{1}{2}x}$

$\int_0^{+\infty} \frac{1}{\sqrt{2\pi x}} e^{-1/2 x} dx = 1$ ✓ ESTÁ BIEN definida

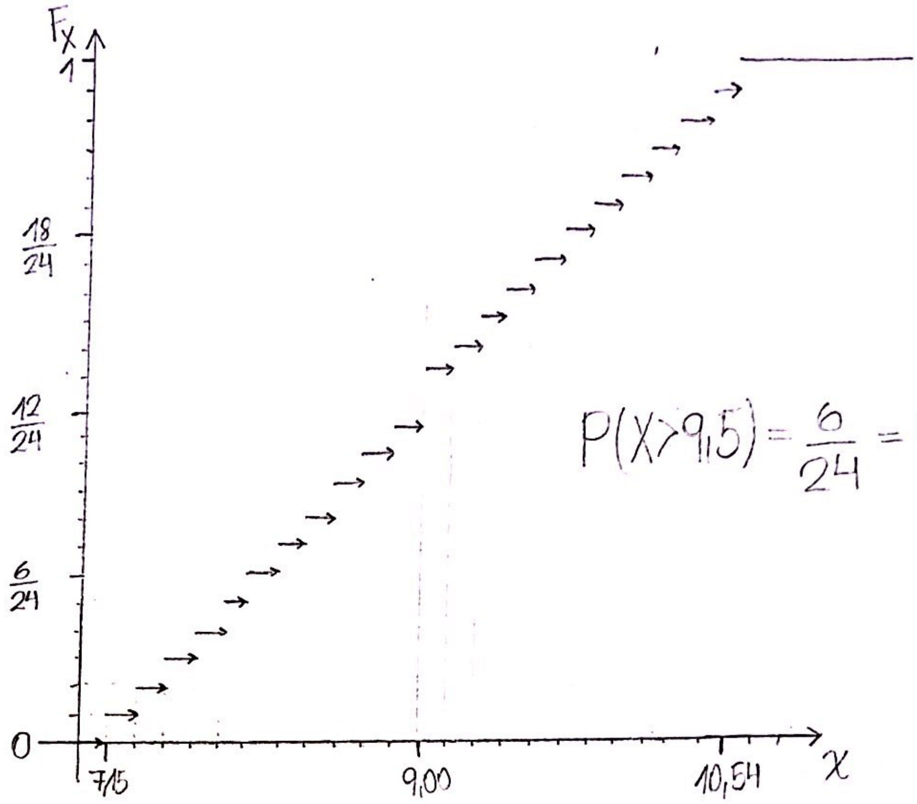
$\int_0^{+\infty} \frac{1}{\sqrt{x}} e^{-1/2 x} dx = \sqrt{2\pi}$

c) $\int_0^{+\infty} x^{-1/2} e^{-x} dx \underset{\uparrow}{=} \int_0^{+\infty} \underbrace{\frac{\sqrt{2}}{2}}_{1/\sqrt{2\pi}} y^{-1/2} e^{-y/2} dy = \frac{1}{\sqrt{2}} \sqrt{2\pi} = \sqrt{\pi}$ $\quad \int_0^{+\infty} x^{-1/2} e^{-1/2 x} dx = \sqrt{2\pi}$

$y = 2x$
$dy = 2dx$
$x^{-1/2} = \sqrt{2}\, y^{-1/2}$

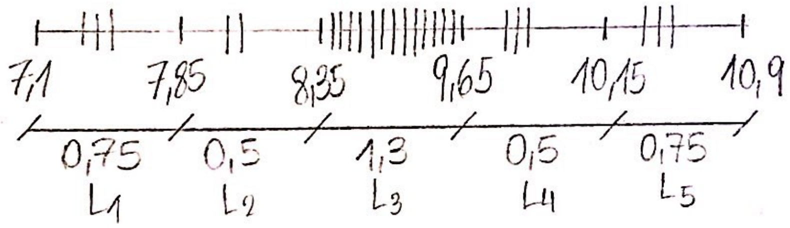
2.11)

a)




$$P(X>9{,}5)=\frac{6}{24}=0{,}25$$

b)




$p_1=\frac{3}{24}$ $\quad p_2=\frac{2}{24}$ $\quad p_3=\frac{13}{24}$ $\quad p_4=\frac{3}{24}$ $\quad p_5=\frac{3}{24}$

$$h(x)=\frac{1}{6}\mathbb{1}\{7{,}1<x\leq 7{,}85\}+\frac{1}{6}\mathbb{1}\{7{,}85<x\leq 8{,}35\}+\frac{5}{12}\mathbb{1}\{8{,}35<x\leq 9{,}65\}+$$

$$+\frac{1}{4}\mathbb{1}\{9{,}65<x\leq 10{,}15\}+\frac{1}{6}\mathbb{1}\{10{,}15<x\leq 10{,}9\}$$

---

$$P(X>9{,}5)=\frac{5}{12}(9{,}65-9{,}5)+\frac{1}{4}(10{,}15-9{,}65)+\frac{1}{4}(10{,}9-10{,}15)=0{,}375$$

2.12) $T=$ duración en años del tiempo de trabajo sin fa...

$$F_T(t) = 1-e^{-t}\,\mathbb{1}\{t>0\}$$

a) $T^* = \min(T,1)\begin{cases} T & \text{si } T<1 \\ 1 & \text{si } T\geq 1 \end{cases}$

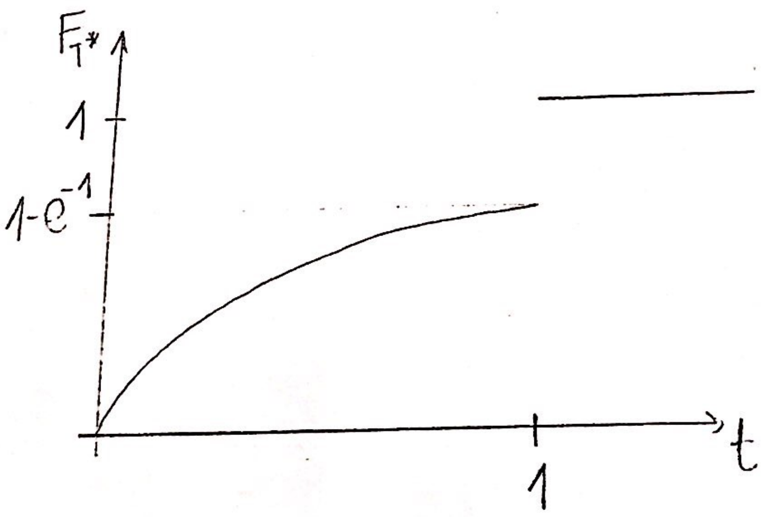
$$P(T^*\leq t) = F_{T^*}(t)$$

I) $t<0 \rightarrow P(T^*\leq t)=0$

II) $t\geq 1 \rightarrow P(T^*\leq t)=1$

III) $0\leq t<1 \rightarrow P(T^*\leq t) = P(T\leq t) = 1-e^{-t}$

$$F_{T^*}(t^*) = \begin{cases} 0 & \mathbb{1}\{t^*<0\} \\ 1-e^{-t^*} & \mathbb{1}\{0\leq t^*<1\} \\ 1 & \mathbb{1}\{t^*\geq 1\} \end{cases} \Rightarrow f_{T^*}(t^*) = e^{-t^*}\,\mathbb{1}\{0\leq t^*<1\}$$




b) $P(T^*=1) = 1-(1-e^{-1}) = e^{-1} \doteq 0{,}368$

2.13) $X$ VA / $F_X(x) = P(X \leq x)$

a) $T = G(X)$

$F_T(t) = P(T \leq t) = P(G(X) \leq t) = P(X \leq G^{-1}(t)) = F_X(G^{-1}(t))$

b) $U = F_X(X) \Rightarrow U \sim U(0,1) \Rightarrow F_U(u) = u \mathbb{1}\{0<u<1\} + \mathbb{1}\{u \geq 1\}$

c) $V = G^{-1}(F_X(X))$

$F_V(v) = P(V \leq v) = P(G^{-1}(F_X(X)) \leq v) = P(F_X(X) \leq G(v)) =$

$= P(X \leq F_X^{-1}(G(v))) = F_X(F_X^{-1}(G(v))) = G(v)$

2.14) $F_X(x) = 1 - e^{-2x}$

$F_X(X) \sim U(0,1)$

$X_1 = \frac{7}{4} \Rightarrow F_X(7/4) = 1 - e^{-2(7/4)} = 0{,}9698$

$X_2 = \frac{111}{60} \Rightarrow F_X(\frac{111}{60}) = 1 - e^{-2(111/60)} = 0{,}975$

$X_3 = \frac{137}{60} \Rightarrow F_X(\frac{137}{60}) = 1 - e^{-2(137/60)} = 0{,}9896$

2.15) $U \sim U[0,1]$ $\quad X = h(U)$

hallar $h / F_{h(U)}(u) = F_X(x)$

$$F_X(x) = \frac{1}{3}\mathbb{1}\{-2 \leq x < -1\} + \left(\frac{1}{6}x + \frac{1}{2}\right)\mathbb{1}\{-1 < x < 1\} + \frac{2}{3}\mathbb{1}\{1 \leq x < 2\} + \mathbb{1}\{x \geq 2\}$$

$F_{h(U)}(x) = F_X(x) = F_U(h^{-1}(x))$

$F_U^{-1}(F_X(x)) = F_*^{-1}(x)$ $\qquad F_U(u) = u\,\mathbb{1}\{0 \leq u \leq 1\} + \mathbb{1}\{u > 1\}$

$X = F_X^{-1}(F_U(U))$ $\qquad F_U(U) = F_X(x)$

$x = F_X^{-1}(F_U(U))$

versión alternativa $x \sim U$
en la carpeta práctica

2.16) $T^* = h(x)$ $\qquad x \sim \exp(\lambda = 1/2)$

$F_{T^*}(T^*) = F_X(x)$ $\qquad F_X(x) = 1 - e^{-\lambda x}\,\mathbb{1}\{x > 0\}, \ \lambda = \frac{1}{2}$

$T^* = F_{T^*}^{-1}(F_X(x))$

$$F_{T^*}^{-1}(u) = \begin{cases} {}^{(*)}-\ln(1-u) & 0 < u < 1 - e^{-1} \\ 1 & 1 - e^{-1} \leq u < 1 \end{cases}$$

$\lambda = \frac{1}{2}$

$(*)\ u = 1 - e^{-t}$
$e^{-t} = 1 - u$
$t = -\ln(1-u)$

$$h(x) = \begin{cases} -\ln\left(e^{-\lambda x}\right) & \text{si } 0 < 1 - e^{-\lambda x} < 1 - e^{-1} \\ 1 & \text{si } 1 - e^{-1} \leq \ \ < 1 \end{cases} \;=\; \begin{cases} x & \text{si } 0 < x < 2 \\ 1 & \text{si } 2 \leq x \end{cases}$$

$1 > e^{-\lambda x} > e^{-1}$ $\qquad 1 - e^{-1} \leq 1 - e^{-\lambda x} < 1$

$\lambda = 1/2$

$0 > -\frac{x}{2} > -1$ $\qquad e^{-1} \geq e^{-\lambda x} > 0$

$0 < \frac{x}{2} < 1$ $\qquad 1 \leq -\lambda x > -\infty$

$\lambda = 1/2$

$0 < x < 2$ $\qquad 2 \leq x$

2.17)

a) $F_T(t) = 1 - e^{-\int_0^t \frac{\beta}{\alpha}\left(\frac{u}{\alpha}\right)^{\beta-1} du} \mathbb{1}\{t>0\}$

$F_T(t) = 1 - e^{\frac{-\cancel{\frac{\beta}{\alpha}}\left(\frac{t}{\alpha}\right)^{\beta}\cdot\cancel{\alpha}}{\cancel{\beta}}} = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\beta}} \mathbb{1}\{t>0\}$

$f_T(t) = F'_T(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-(t/\alpha)^{\beta}} \mathbb{1}\{t>0\}$

b) T: "tiempo hasta pinchadura de neumatico por objeto punzante"

$\Rightarrow$

T: "tiempo hasta que se desgaste el surco y pierde agarre"

$\Rightarrow$

T: "tiempo hasta que revienta el neumatico por falla de fábrica"

$\Rightarrow$

T: "tiempo hasta que el usuario se da cuenta que salió rallada"

$\Rightarrow$

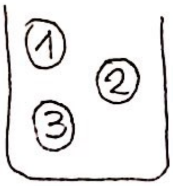
T: "tiempo hasta que falla el sistema de enfriamiento"

$\Rightarrow$

T: "tiempo hasta que el motor se quema por un cambio de tensión"

$\Rightarrow$

5)



se extraen 2 sin reposición

X: "número de primer bolilla"

Y: "número de segunda bolilla"

a) prob. conjunta y marginales

b) $P(X<Y)$

c) ¿son independientes?

a)

| $Y \backslash X$ | 1 | 2 | 3 | $p_Y$ |
|---|---|---|---|---|
| 1 | 0 | 1/6 | 1/6 | 2/6 |
| 2 | 1/6 | 0 | 1/6 | 2/6 |
| 3 | 1/6 | 1/6 | 0 | 2/6 |
| $p_X$ | 2/6 | 2/6 | 2/6 | 1 |

b) $P(X<Y) = \frac{1}{6}$

c) $p_{XY}(2,1) = 1/6$

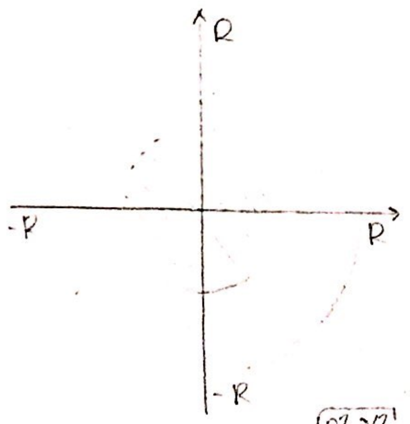
$p_X(2) = 2/6$

$p_Y(1) = 2/6$

$p_{XY}(2,1) \neq p_X(2) \cdot p_Y(1)$

$\Rightarrow$ no son independientes

Ej




DIST. UNIFORME

$$f_{XY}(x,y) = \frac{1}{|\Lambda|} \mathbb{1}\{x^2+y^2 \le R^2\}$$

$$P\left(X^2+Y^2 < \left(\tfrac{R}{2}\right)^2\right) = P((X,Y) \in A) = \frac{|A|}{|\Lambda|}$$

$$f_X(x) = \int_{-\infty}^{+\infty} f_{XY}\, dy = \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{1}{|\Lambda|}\, dy = \frac{2\sqrt{R^2-x^2}}{\pi R^2} \mathbb{1}\{R < x < R\}$$

$$f_Y(y) = \int_{-\infty}^{+\infty} f_{XY}\, dx = \int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \frac{1}{|\Lambda|}\, dx = \frac{2\sqrt{R^2-y^2}}{\pi R^2} \mathbb{1}\{-R < y < R\}$$

si el soporte es rectangular no es necesariamente independiente pero si no es rectangular no es independiente

$\Rightarrow$ X e Y no son independientes en este caso

Ej $T \sim EXP(1)$ $\quad f_T(t) = \lambda e^{-\lambda t} \mathbb{1}\{t>0\} = e^{-t} \mathbb{1}\{t>0\}$

simular 5 valores de $T^* = \min(T,1)$

$F_T(t) = F_{T^*}(t^*)$ (mirar 2.12)

$U \sim U(0,1)$

$u_1 = 0{,}433 = 1-e^{-t_1^*} \rightarrow t_1^* = \bigcirc$ $\qquad$ idem con $u_3, u_4, u_5$

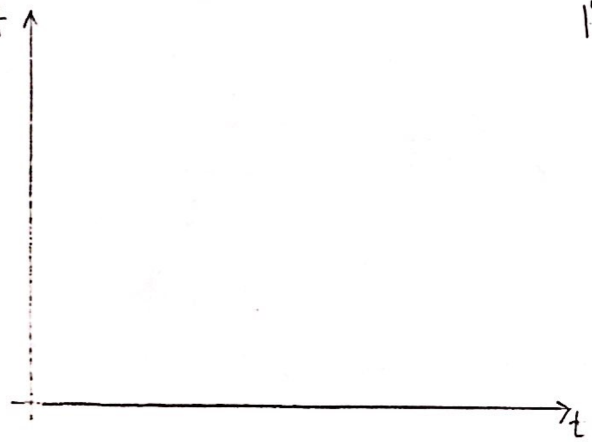
$u_2 = 0{,}933 = 1-e^{-t_2^*} \rightarrow t_2^* = 1$

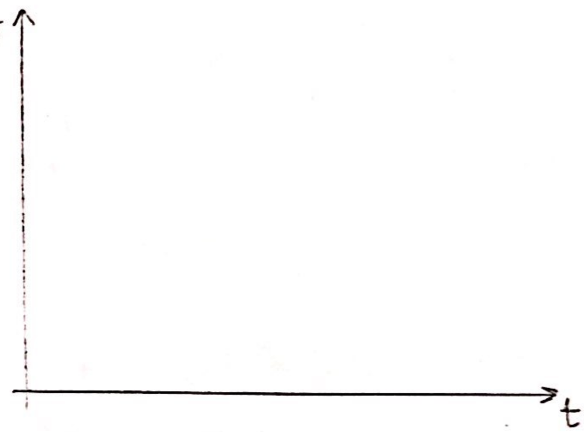
… 2.1+)

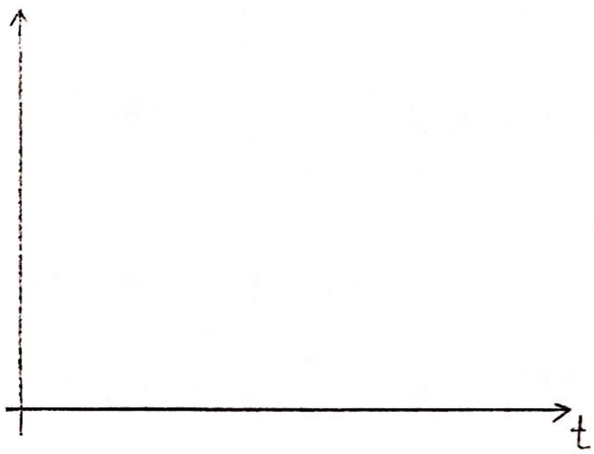
c) I) $\alpha = 1$, $\beta = 0{,}5$ $\Rightarrow F_T(t) = -\frac{1}{2} t^{-1/2} e^{t^{1/2}} \mathbb{1}\{t>0\}$

II) $\alpha = 1$, $\beta = 1$ $\Rightarrow F_T(t) = -e^{t} \mathbb{1}\{t>0\}$

III) $\alpha = 1$, $\beta = 1{,}5$ $\Rightarrow F_T(t) = -\frac{3}{2} t^{1/2} e^{t^{3/2}} \mathbb{1}\{t>0\}$

I) 


II) 


III) 


d) $P(T>1) = 1 - P(T<1) =$

I) $P(T>4 \mid T>3) = \frac{P(T>4)}{P(T>3)} =$

II) $P(T>1) = 1 - P(T \leq 1) =$

$$P(T>4 \mid T>3) = \frac{P(T>4)}{P(T>3)} =$$

III) $P(T>1) = 1 - P(T \leq 1) =$

$$P(T>4 \mid T>3) = \frac{P(T>4)}{P(T>3)} =$$

2.18) $F_T(t) = \left(1 - e^{-\sqrt{t/60}}\right) \mathbb{1}\{t>0\}$

$$P(T<60 \mid T>30) = \frac{P(T<60, T>30)}{P(T>30)} = \frac{P(30<T<60)}{P(T>30)} =$$

$$= \frac{F_T(60) - F_T(30)}{1 - F_T(30)} \simeq 0{,}254$$

2.14) X: "diámetro de arandelas"

$$f_X(x) = \frac{2x}{225} \mathbb{1}\{0 < x < 15\}$$

a) $$f_{X|X\in(3,12)}(x) = \frac{f_X(x)\mathbb{1}\{x\in(3,12)\}}{P(X\in(3,12))} =$$

$$= \frac{\frac{2x}{225}\mathbb{1}\{x\in(3,12)\}}{\int_3^{12}\frac{2x}{225}\,dx = \frac{x^2}{225}\Big|_3^{12}} =$$

$$= \frac{\frac{2x}{225}\mathbb{1}\{x\in(3,12)\}}{\frac{3}{5}} = \frac{2x}{135}\mathbb{1}\{x\in(3,12)\}$$

b) $$f_{X|X\notin(3,12)}(x) = \frac{f_X(x)\mathbb{1}\{x\notin(3,12)\}}{P(X\notin(3,12))} =$$

$$= \frac{\frac{2x}{225}\mathbb{1}\{x\notin(3,12)\}}{\int_0^{3}\frac{2x}{225}\,dx + \int_{12}^{15}\frac{2x}{225}\,dx} =$$

$$= \frac{x}{45}\mathbb{1}\{x\notin(3,12)\}$$

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2.20) X: "distancia del pto de impacto al centro de un blanco circular"

$$F_X(x) = \frac{2}{7}\mathbb{1}\{0 \le x < 2\} + \frac{10-2x}{21}\mathbb{1}\{2 \le x < 5\}$$

a) $$F_{X|X\in(0,3)}(x) = \frac{F_X(x)\,\mathbb{1}\{X\in(0,3)\}}{P(X\in(0,3))} =$$

$$= \frac{\frac{2}{7}\mathbb{1}\{0\le x<2\} + \frac{10-2x}{21}\mathbb{1}\{2\le x<3\}}{\int_0^2 \frac{2}{7}\,dx + \int_2^3 \frac{10-2x}{21}\,dx} =$$

$$= \frac{6}{17}\mathbb{1}\{0\le x<2\} + \left(\frac{10}{17} - \frac{2x}{17}\right)\mathbb{1}\{2\le x<3\}$$

b) $$F_{X|X\in(3,5)}(x) = \frac{F_X(x)\,\mathbb{1}\{X\in(3,5)\}}{P(X\in(3,5))} =$$

$$= \frac{\frac{10-2x}{21}\mathbb{1}\{3<x<5\}}{\int_3^5 \frac{10-2x}{21}\,dx} =$$

$$= \left(\frac{5}{2} - \frac{x}{2}\right)\mathbb{1}\{3<x<5\}$$

2.21) 

3 VERDES
2 AMARILLAS
3 ROJAS

$\rightarrow$ 4 bolas

a) X: "cant. de bolas verdes observables"
Y: "cant. de bolas amarillas observables"
SIN REPOSICIÓN!

| Y \ X | 0 | 1 | 2 | 3 | 4 | $p_Y$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 3/70 | 9/70 | 3/70 | 0 | 3/14 |
| 1 | 1/35 | 9/35 | 9/35 | 1/35 | 0 | 4/7 |
| 2 | 3/70 | 9/70 | 3/70 | 0 | 0 | 3/14 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| $p_X$ | 1/14 | 3/7 | 3/7 | 1/14 | 0 | 1 |

$$p_{XY}(1,0) = \frac{\binom{3}{1}\binom{2}{0}\binom{3}{3}}{\binom{8}{4}} = \frac{3}{70}$$

$$p_{XY}(2,0) = \frac{\binom{3}{2}\binom{2}{0}\binom{3}{2}}{\binom{8}{4}} = \frac{9}{70}$$

$$p_{XY}(3,0) = \frac{\binom{3}{3}\binom{2}{0}\binom{3}{1}}{\binom{8}{4}} = \frac{3}{70}$$

$$p_{XY}(0,1) = \frac{\binom{3}{0}\binom{2}{1}\binom{3}{3}}{\binom{8}{4}} = \frac{1}{35}$$

$$p_{XY}(1,1) = \frac{\binom{3}{1}\binom{2}{1}\binom{3}{2}}{\binom{8}{4}} = \frac{9}{35}$$

$$p_{XY}(2,1) = \frac{\binom{3}{2}\binom{2}{1}\binom{3}{1}}{\binom{8}{4}} = \frac{9}{35}$$

$$p_{XY}(3,1) = \frac{\binom{3}{3}\binom{2}{1}\binom{3}{0}}{\binom{8}{4}} = \frac{1}{35}$$

$$p_{XY}(0,2) = \frac{\binom{3}{0}\binom{2}{2}\binom{3}{2}}{\binom{8}{4}} = \frac{3}{70}$$

$$p_{XY}(1,2) = \frac{\binom{3}{1}\binom{2}{2}\binom{3}{1}}{\binom{8}{4}} = \frac{9}{70}$$

$$p_{XY}(2,2) = \frac{\binom{3}{2}\binom{2}{2}\binom{3}{0}}{\binom{8}{4}} = \frac{3}{70}$$

$P(X+Y\leq 2)=\frac{1}{2}$

b) CON REPOSICIÓN

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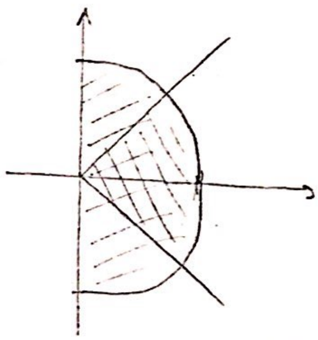
| $Y \backslash X$ | 0 | 1 | 2 | 3 | 4 | $p_Y$ |
|---|---|---|---|---|---|---|
| 0 | $\frac{81}{4096}$ | $\frac{81}{4096}$ | $\frac{81}{4096}$ | $\frac{81}{4096}$ | $\frac{81}{4096}$ | $\frac{405}{4096}$ |
| 1 | $\frac{27}{2048}$ | $\frac{27}{2048}$ | $\frac{27}{2048}$ | $\frac{27}{2048}$ | 0 | $\frac{27}{512}$ |
| 2 | $\frac{9}{1024}$ | $\frac{9}{1024}$ | $\frac{9}{1024}$ | 0 | 0 | $\frac{27}{1024}$ |
| 3 | $\frac{3}{512}$ | $\frac{3}{512}$ | 0 | 0 | 0 | $\frac{3}{256}$ |
| 4 | $\frac{1}{256}$ | 0 | 0 | 0 | 0 | $\frac{1}{256}$ |
| $p_X$ | $\frac{211}{4096}$ | $\frac{195}{4096}$ | $\frac{171}{4096}$ | $\frac{135}{4096}$ | $\frac{81}{4096}$ | $\frac{793}{4096}$ |

$P(X+Y\leq 2)=\frac{387}{4096}$

**2.22)** $(X,Y)\sim U(\Lambda)$ , $\Lambda=\{(x,y)\in\mathbb{R}^2 / x^2+y^2\leq 4, x\geq 0\}$

a) $P(|Y|<X)=\dfrac{\left(\frac{\pi 2^2}{4}\right)}{\left(\frac{\pi 2^2}{2}\right)}=\frac{1}{2}$ $\qquad f_{XY}(x,y)=\frac{1}{|\Lambda|}=\frac{1}{2\pi}$



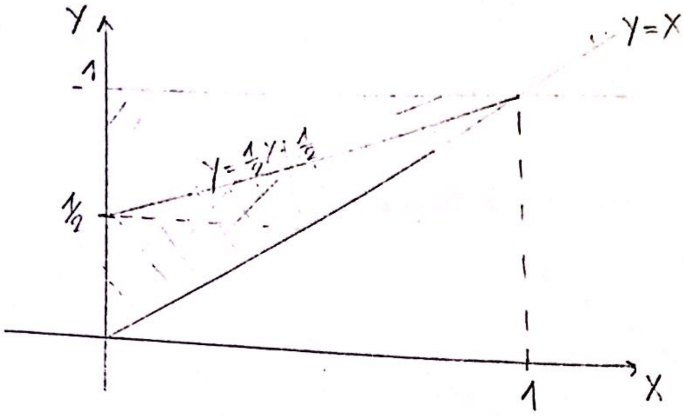
b) $f_X(x)=\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\frac{1}{2\pi}\,dy=\frac{\cancel{2}\sqrt{4-x^2}}{\cancel{2}\pi}\,\mathbb{1}\{0\leq x<2\}$

$f_Y(y)=\int_0^{\sqrt{4-y^2}}\frac{1}{2\pi}\,dx=\frac{\sqrt{4-y^2}}{2\pi}\,\mathbb{1}\{-2\leq y\leq 2\}$

c) $f_{XY}(x,y)\neq f_X(x).f_Y(y)\Rightarrow$ no son independientes

2.23) $F_{XY}(x,y) = 8xy \, \mathbb{1}\{0<x<y<1\}$

a) $P(X+1>2Y)$




$$P(X+1>2Y) = \int_0^1 \int_x^{\frac12 x+\frac12} 8xy \, dy \, dx = \int_0^1 4x \, y^2 \Big|_x^{\frac12 x+\frac12} dx =$$

$$= \int_0^1 4x\left(\left(\tfrac12 x+\tfrac12\right)^2 - x^2\right) dx = \int_0^1 4x\left(\frac{x^2}{4}+\frac{x}{2}+\frac{1}{4}-x^2\right) dx =$$

$$= \int_0^1 4x\left(-\frac{3}{4}x^2+\frac{x}{2}+\frac{1}{4}\right) dx = \int_0^1 -3x^3+2x^2+x \, dx =$$

$$= -\frac{3}{4}x^4+\frac{2}{3}x^3+\frac{x^2}{2}\Big|_0^1 = -\frac{3}{4}+\frac{2}{3}+\frac{1}{2} = \frac{5}{12} \cong 0,42$$

b) $F_X(x) = \int_x^1 F_{XY}(x,y) \, dy \cdot \mathbb{1}\{0<y<1\} = 4x-4x^3$

$F_Y(y) = \int_0^y F_{XY}(x,y) \, dx \, \mathbb{1}\{0<y<1\} = 4y^3$

c) NO SON INDEP. PORQUE EL SOPORTE NO ES RECTANGULAR

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2.24) $f_{XY}(x,y) = \frac{1}{\pi\sqrt{3}} e^{-\frac{2}{3}(x^2 - xy + y^2)}, \{(x,y) \in \mathbb{R}^2\}$

a) $x^2 - xy + y^2 = (y - \frac{1}{2}x)^2 - \frac{1}{4}x^2 + x^2 = (y - \frac{1}{2}x)^2 + \frac{3}{4}x^2$

$$\Rightarrow -\frac{2}{3}(y - \frac{1}{2}x)^2 - \frac{\cancel{2}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{4}_2} x^2 = -\frac{2}{3}(y - \frac{1}{2}x)^2 - \frac{x^2}{2}$$

$$\Rightarrow \frac{1}{\pi\sqrt{3}} e^{-\frac{2}{3}(y - \frac{1}{2}x)^2 - \frac{x^2}{2}} = \frac{1}{\pi\sqrt{3}} \cdot e^{-\frac{2}{3}(y - \frac{1}{2}x)^2} \cdot e^{-\frac{x^2}{2}}$$

quiero algo de la forma

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \Rightarrow \frac{1}{\pi\sqrt{3}} \cdot e^{-\frac{(y - \frac{1}{2}x)^2}{3/2}} \cdot e^{-\frac{x^2}{2}}$$

$$= \frac{1}{\pi\sqrt{3}} \cdot e^{-\frac{1}{2}\frac{(y - \frac{1}{2}x)^2}{3/4}} \cdot e^{-x^2/2} \rightarrow \sigma = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$= \frac{1}{2\pi\frac{\sqrt{3}}{2}} e^{-\frac{1}{2}\frac{(y - \frac{1}{2}x)^2}{3/4}} \cdot e^{-x^2/2} = \frac{\sqrt{2\pi}}{2\pi} \underbrace{\frac{1}{\sqrt{2\pi} \cdot \frac{\sqrt{3}}{2}} \cdot e^{-\frac{1}{2}\frac{(y - \frac{1}{2}x)^2}{3/4}}}_{N \sim (\frac{1}{2}x, \frac{3}{4})} \cdot e^{-x^2/2}$$

$$\Rightarrow f_X(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}, \{x \in \mathbb{R}\} \Rightarrow f_Y(y) = \frac{e^{-y^2/2}}{\sqrt{2\pi}} \; \forall \{y \in \mathbb{R}\}$$

---

b) $\Rightarrow$ NO SON INDEP PORQUE $f_{XY}(x,y) \neq f_X(x) \cdot f_Y(y)$

2.25) $X$: "cant. de fallas de tejido" $\sim Poisson(2)$

$Y$: "cant de fallas de tejido" $\sim Poisson(4)$

$X$ e $Y$ i.i.d.

a) calcular la prob. de que un rollo no tenga fallas

$P_X(x) = P(X=x) = \dfrac{2^x e^{-2}}{x!}$ $\quad$ $P_Y(y) = P(Y=y) = \dfrac{4^y e^{-4}}{y!}$

$\mathbb{P}(X=0, Y=0) \overset{\text{indep}}{=} \mathbb{P}(X=0) \cdot P(Y=0)$

$P(X=0) = \dfrac{2^0 e^{-2}}{0!} = e^{-2}$ $\quad$ $P(Y=0) = \dfrac{4^0 e^{-4}}{0!} = e^{-4}$

$P(X=0, Y=0) = e^{-2} \cdot e^{-4} = 0{,}00248$

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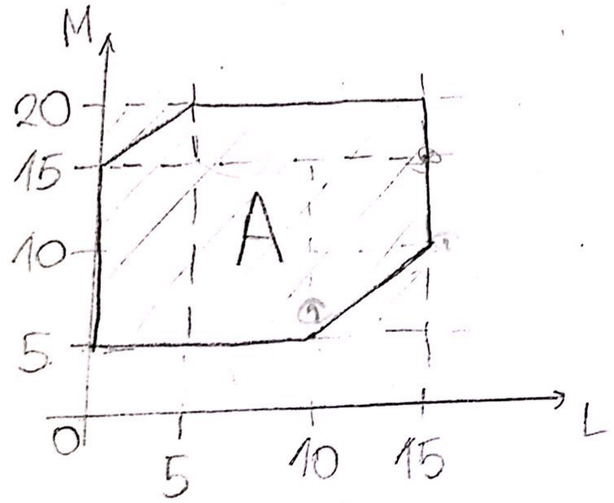
$\cancel{b)\ P(X=1 \cup Y=1) = \dfrac{2^1 e^{-2}}{1!} + \dfrac{4^1 e^{-4}}{1!} \cong 0{,}347}$

$\cancel{c)\ P(X=1 \mid X=1 \cup Y=1) = \dfrac{P(X=1, Y=1)}{P(X=1 \cup Y=1)} = \dfrac{2e^{-2} \cdot 4e^{-4}}{2e^{-2} + 4e^{-4}} \cong 0{,}058}$

2.26) L: "horario de llegada de Lucas" $\sim U(0,15)$ $+15$

M: "horario de llegada de Monk" $\sim U(5,20)$ $+5$

$$P(\text{"se encuentren"}) = P((L,M)\in A) = \frac{\frac{25}{2}+50+100+25+\frac{25}{2}}{15\times 15}$$




$$= \frac{8}{9} \cong 0{,}89$$