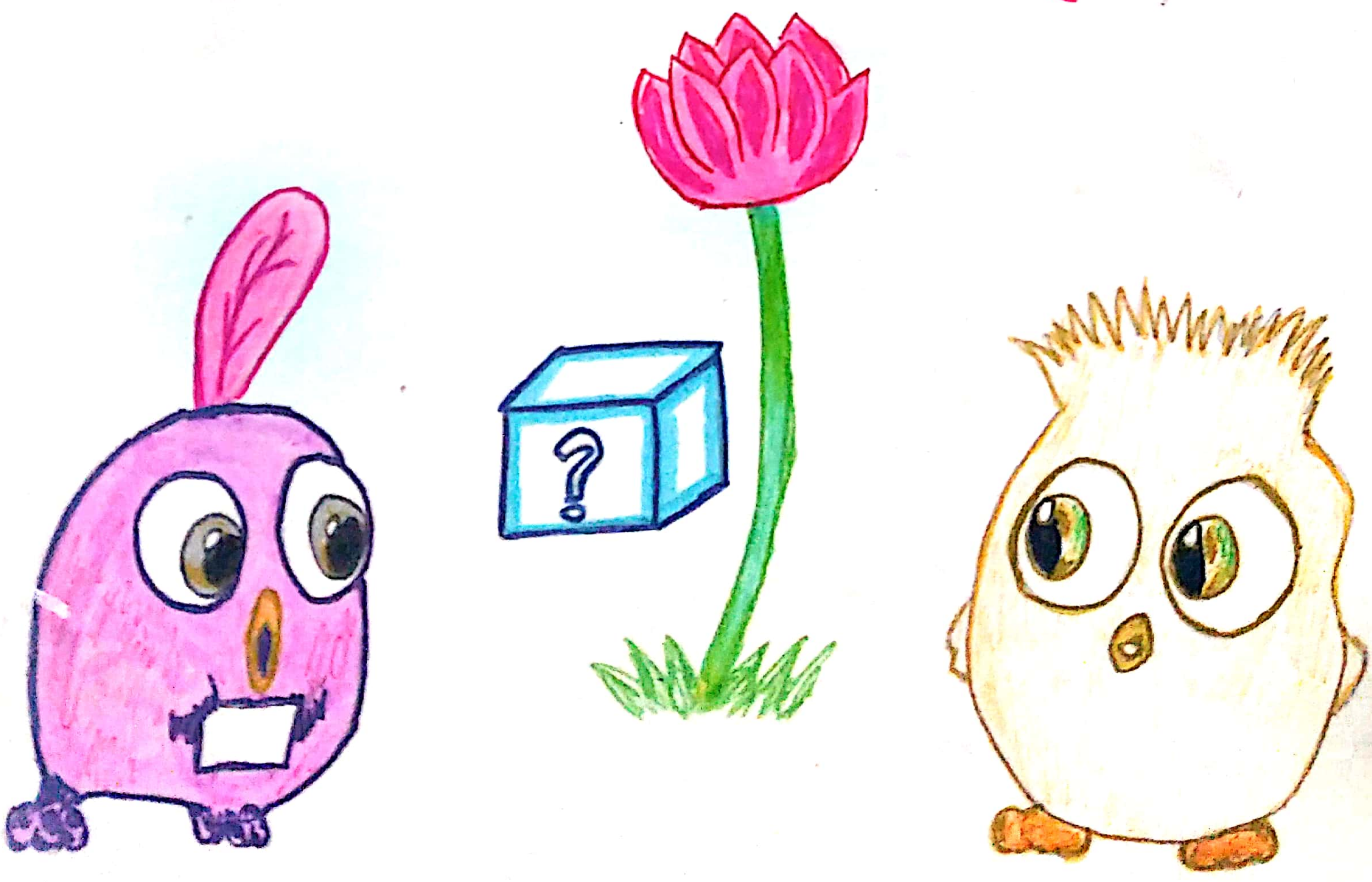
Probabilidad y Estadística



1° Cuatrimestre 2018

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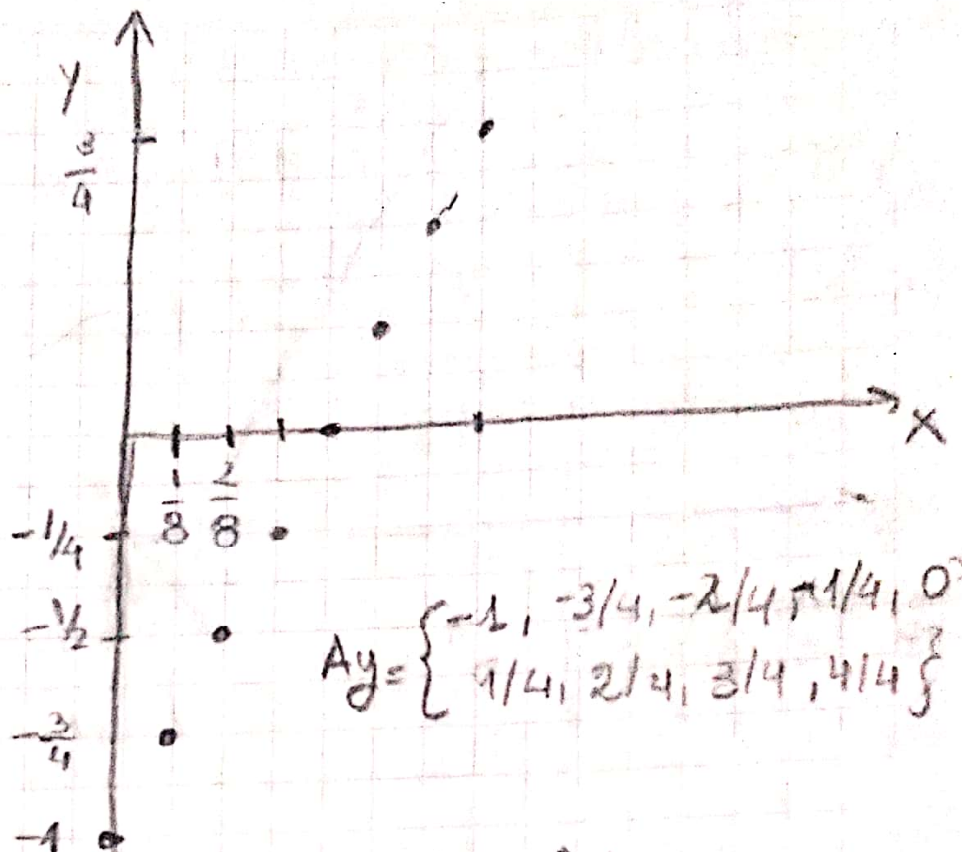
4.1

$\left\{0, \frac{1}{8}, \frac{2}{8}, \frac{3}{8} \ldots\right\}$

a)

Y	X	P_X
-1	0	0
$-3/4$	$1/8$	$\frac{2}{9}\frac{1}{8}$
$-1/4$	$\frac{3}{8}$	$\frac{2}{9}\frac{3}{8}$

$P_X = \frac{2}{9} X$



$A_y = \left\{ \begin{matrix} -1, -3/4, -2/4, -1/4, 0 \\ 1/4, 2/4, 3/4, 4/4 \end{matrix} \right\}$

$P(Y = y) = P(2X - 1 = y)$

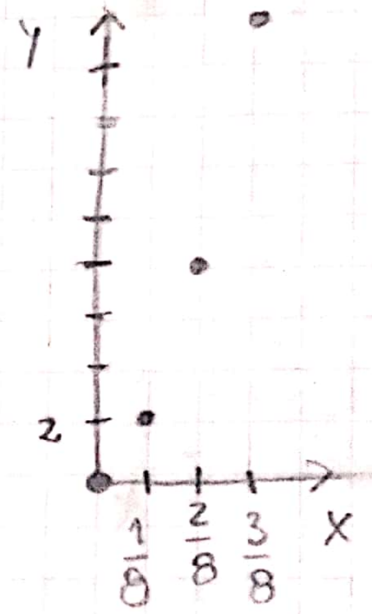
$= P\left(X = \frac{y+1}{2}\right) = \frac{2}{9}\left(\frac{y+1}{2}\right) \quad \forall y \in A_y$

b)

$P(Y = y) = P(128X^2 = y)$

$= P\left(X = \sqrt{\frac{y}{128}}\right)$

$= \frac{2}{9}\sqrt{\frac{y}{128}}$



X	Y
0	0
$1/8$	2
$2/8$	8
$3/8$	18
$4/8$	32

X	Y
$5/8$	50
$6/8$	72
$7/8$	98
$8/8$	128

$A_y = \{0, 2, 8, 18, 32, 50, 72, 98, 128\}$

c)

$P_X(a)$	X	Y
$\frac{2}{9}\cdot\frac{1}{8}$	$1/8$	9
$2/9 \cdot 2/8$	$2/8$	14
$2/9 \cdot 3/8$	$3/8$	17
$2/9 \cdot 4/8$	$4/8$	18
$2/9 \cdot 5/8$	$5/8$	17
$2/9 \cdot 6/8$	$6/8$	14
$2/9 \cdot 7/8$	$7/8$	9
$\boxed{2/9}$	$8/8$	2

X	Y
0	2

$A_y = \{2, 9, 14, 17, 18\}$

$P(Y = y) = P(-64X^2 + 64X + 2 = y)$

Busco raíces:

$-64X^2 + 64X + 2 - y = 0$

$X = \frac{-64 \pm \sqrt{36 - 4 \cdot (-64)(2-y)}}{2(-64)}$

$P_Y(2) =$

$P_Y(9) =$

$a_1 = \frac{1}{8}\left(4 + \sqrt{18 - y}\right)$

$a_2 = \frac{1}{8}\left(4 - \sqrt{18 - y}\right)$

$$P(Y=y) = P_Y(y) = \begin{cases} \frac{2}{9} & y=2 \\ 2/9 & y=9 \\ 2/9 & y=14 \\ 2/9 & y=17 \\ 1/9 & y=18 \end{cases}$$

d)

$P_X(x)$	X	Y
$\frac{2}{9}\cdot 1/8$	1/8	117
$2/9\cdot 2/8$	2/8	108
$2/9\cdot 3/8$	3/8	101
$2/9\cdot 4/8$	4/8	96
$2/9\cdot 5/8$	5/8	93
$2/9\cdot 6/8$	6/8	92
$2/9\cdot 7/8$	7/8	93
$2/9$	1	96

$$P(Y=y) = P(64X^2-96X+128=y)$$

$$X = \frac{1}{8}\left(6 \pm \sqrt{Y-92}\right)$$

$$P(Y=y) = \begin{cases} 2/9\cdot 6/8 = 1/6 & y=92 \\ 3/9 = 1/3 & y=93 \\ 3/9 = 1/3 & y=96 \\ 2/9\cdot 3/8 = 1/12 & y=101 \\ 2/9\cdot 2/8 = 1/18 & y=108 \\ 2/9\cdot 1/8 = 1/36 & y=117 \end{cases}$$

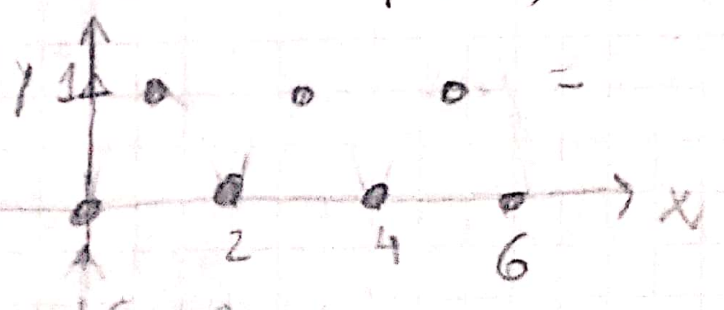
(4.2) $\sim Poi(2)$

$$Y = \left|\operatorname{sen}\left(\frac{1}{2}\pi X\right)\right| \qquad Y = f(X)$$

X	Y	P_X
0	0	0
1	1	$2/9\cdot 1$
2	0	$2/9\cdot 2$
3	1	
4	0	$2/9\cdot 4$
5	1	

$$Y = \begin{cases} 1 & \text{si } x=\{0,2,4,\dots\} \\ 0 & \text{si } x=\{1,3,5,\dots\} \end{cases}$$

$P(Y=0) = P(X=0)+P(X=2)+\dots$

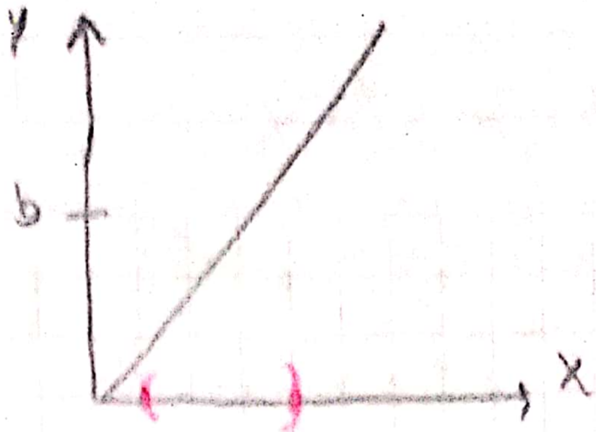


como $X>0$

$\Downarrow$

$$P(Y=y) = P_Y(y) = \begin{cases} \sum_{i=0}^{\infty} \frac{2^{(2i+1)} e^{-2}}{(2i+1)!} = \frac{sh(2)}{e^2} & y=0 \\ \sum_{i=0}^{\infty} \frac{2^{2i} e^{-2}}{(2i+1)} = \frac{ch(2)}{e^2} & y=1 \end{cases}$$

(4.3) a)



$$P(Y \leq y) = P(aX + b \leq y) = P\left(X \leq \frac{y-b}{a}\right) = F_X\left(\frac{y-b}{a}\right) = F_Y(y)$$

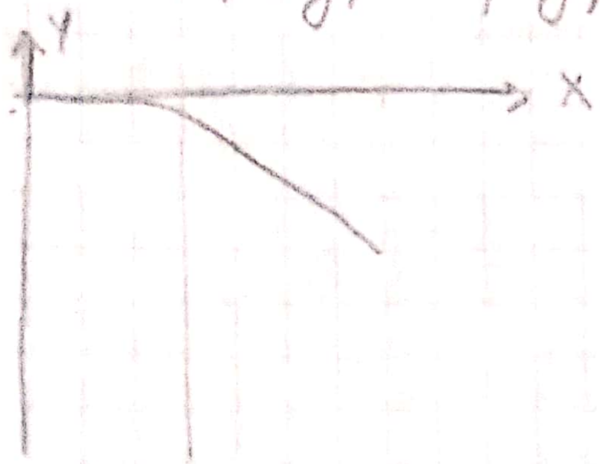
Siendo $\forall$ VAC $f_Y(y) = \frac{d\,F_Y(y)}{dy} = \frac{d\,F_X\left(\frac{y-b}{a}\right)}{dy}$

$$f_Y(y) = f_X\left(\frac{y-b}{a}\right)\frac{1}{a} = \frac{12\left(\frac{y-b}{a}\right)}{\pi^2\left(e^{\frac{y-b}{a}}+1\right)}\frac{1}{a}$$

b) $P(Y \leq y) = F_Y(y) = \begin{cases} y<0 \quad P(-X^3 \leq y) = P(X \geq \sqrt[3]{-y}) \\ 1 \text{ si } y \geq 0 \end{cases}$

$= 1 - P(X \leq \sqrt[3]{-y})$

$= 1 - F_X(\sqrt[3]{-y})$



X	Y
0	0
1	-1
2	-8
3	-27
4	-64
5	-125
6	-216
7	-343
8	-512
9	-719
10	-1000

la derivada de X es $\neq 0$ $\forall$ punto, la función es suave

$\Downarrow$

la función resultante (distrib) es continua y existe func de densidad

$$f_Y(y) = \frac{d\,F_Y(y)}{dy} = \begin{cases} 0 \text{ si } y>0 \\ \frac{d(1-F_X(\sqrt[3]{-y}))}{dy} \end{cases}$$

$$= -1 \cdot f_X(\sqrt[3]{-y}) \cdot \frac{2}{3} \cdot (-y)^{-1/3} \cdot (-1)$$

$$= \frac{2}{3} \frac{f_X(\sqrt[3]{-y})}{\sqrt[3]{-y}}$$

$$f_Y(y) = \frac{8}{\pi^2} \frac{\sqrt[3]{-y}}{\left(e^{\sqrt[3]{-y}}+1\right)\sqrt[3]{-y}}$$

$$f_Y(y) = \frac{8}{\pi^2} \frac{1}{\left(e^{\sqrt[3]{-y}}+1\right)}$$

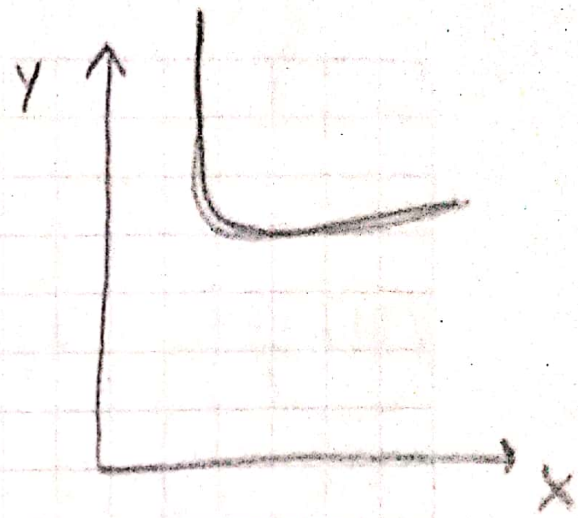
c) $Y = X + \frac{1}{X}$

$Y = \frac{x^2+x}{X} \longrightarrow YX = X^2 + X$

$x^2 - xY + 1 = 0$

$x^2 - xy = -1$

$x^2 - xy + \frac{y^2}{4} = \frac{y^2}{4} - 1$



$-\sqrt{\frac{y^2}{4} - 1} \leq \left| x - \frac{y}{2} \right| \leq \sqrt{\frac{y^2}{4} - 1} \longleftarrow \left(x - \frac{y}{2}\right)^2 = \frac{y^2}{4} - 1$

$x - \frac{y}{2} = \pm\sqrt{\frac{y^2-4}{4}}$

$$\boxed{x = \frac{y}{2} \pm \sqrt{\frac{y^2-4}{4}}}$$

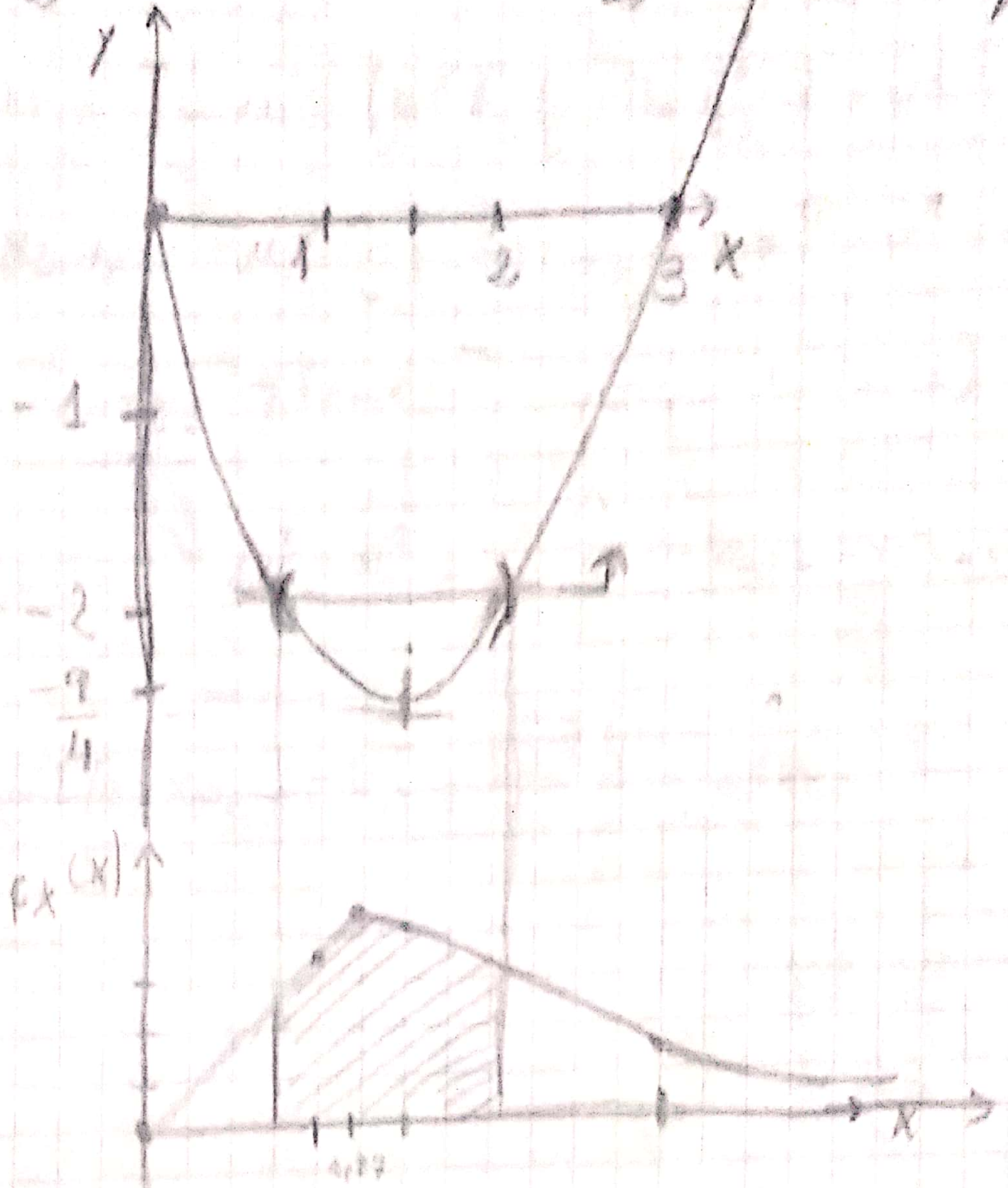
$$P(Y \leq y) = P(x + x^{-1} \leq y) = P\left(\frac{X^2+X}{X} \leq y\right) = P(X^2 + X \leq yX)$$

$$= P(X^2 + X - yX \leq 0) = P\left(\frac{y}{2} - \sqrt{\frac{y^2-4}{4}} < X \leq \frac{y}{2} + \sqrt{\frac{y^2-4}{4}}\right)$$

$$= F_X\left(\frac{y}{2} + \sqrt{\frac{y^2-4}{4}}\right) - F_X\left(\frac{y}{2} - \sqrt{\frac{y^2-4}{4}}\right) = F_Y(y)$$

d) $P(Y \leq y) = P(X^2 - 3X \leq y) = P(X^2 - 3X - y \leq 0)$

$= P\left(\frac{1}{2}\left(3 - \sqrt{4y+9}\right) \leq X \leq \frac{1}{2}\left(3 + \sqrt{4y+9}\right)\right) = F_y(y)$



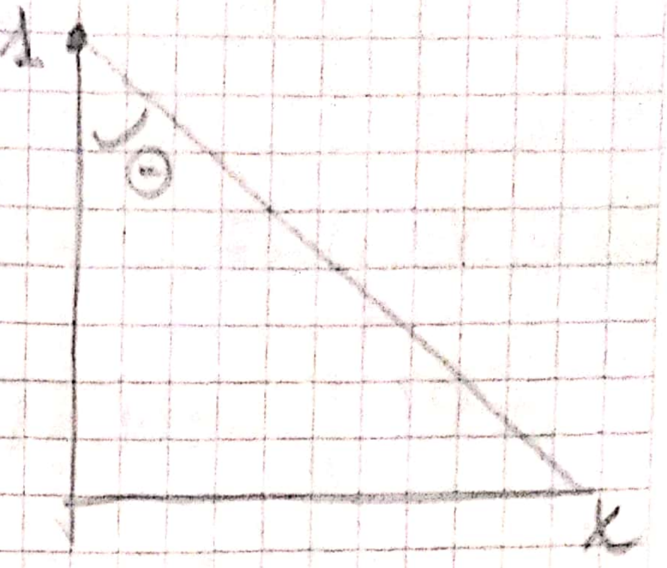
$$F_y(y) = \begin{cases} 0 & y < -9/4 \\ F_X\left(\frac{1}{2}\left(3+\sqrt{4y+9}\right)\right) - F_X\left(\frac{1}{2}\left(3-\sqrt{4y+9}\right)\right) & y \in \left[-\frac{9}{4}, 0\right) \\ F_X\left(\frac{1}{2}\left(3+\sqrt{4y+9}\right)\right) & y \geq 0 \end{cases}$$

Como Y es una VAC (la derivada de su func es func de X excepto p.c.t punto) $\Rightarrow f_y(y) = \frac{d}{dy} F_y(y)$

$$f_y(y) = \begin{cases} 0 & \text{si } y < -9/4 \\ f_X\left(\frac{1}{2}\left(3+\sqrt{4y+9}\right)\right) & \text{si } y \geq 0 \\ f_X\left(\frac{1}{2}\left(3+\sqrt{4y+9}\right)\right)\frac{1}{2}(4y+9)^{-1/2} \cdot 4 - f_X\left(\frac{1}{2}\left(3-\sqrt{4y+9}\right)\right)\left(-\frac{1}{2}\right)(4y+9)^{-1/2} \cdot 4 & -\frac{9}{4} \leq y < 0 \end{cases}$$

partir de vuelta ✓

4.4 $\Theta \sim U\left[-\pi/2, \pi/2\right]$

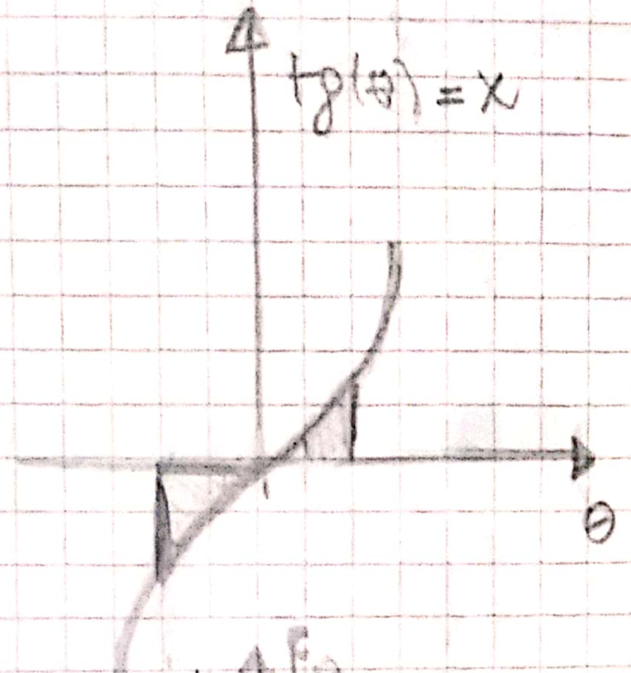


$P(X \leq x) = P(tg\,\Theta \leq x)$

$= P\left(\Theta \leq arctg_2(x)\right)$

$= F_X(x) = F_\Theta\left(arctg_2(x)\right)$

$tg(\theta) = \frac{x}{1}$

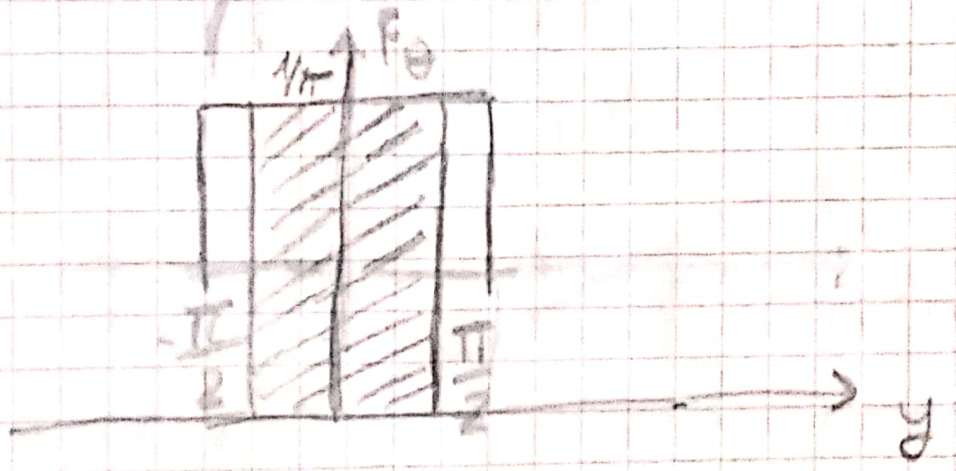


$f_\Theta = \frac{1}{\pi}$

→ Como la función tiene derivada $\neq 0$ pct punto

⇓

$f_X = \frac{d}{dx} F_X(x)$



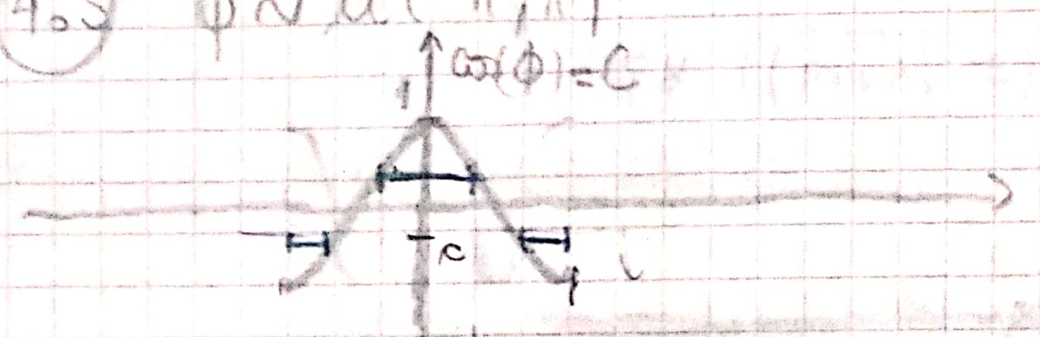
$= \frac{d}{dx} F_\Theta\left(arctg_2(x)\right)$

$= f_\Theta\left(arctg_2(x)\right) \cdot \frac{1}{x^2+1}$

buscar distr. ← $f_X = \frac{1}{\pi}\frac{1}{x^2+1} = Cau(0,1)$

4.5 $\phi \sim U(-\pi, \pi)$

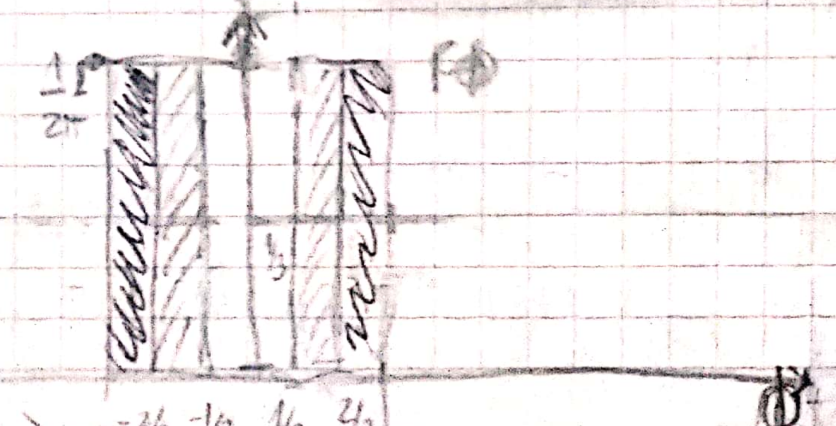
$f_\Theta = \frac{1}{2\pi}$



$P(C \leq c) = \ldots$

$* = P\left(\cancel{|\phi| \leq arccos(c)}\right)$

$= F_C(c) = F_\Theta\left(arccos(c)\right)$



$* = P(\phi < -arcos) + P(\phi > arcos)$

$f_C = \frac{1}{2\pi}\left(\frac{-1}{\sqrt{1-c^2}}\right)$

$P(|C| < 0{,}5) = P(-0{,}5 < C < 0{,}5)$

$= \frac{1}{2\pi} \cdot \frac{2}{3} = \frac{1}{3}$

→ tiempo de espera de Lucas

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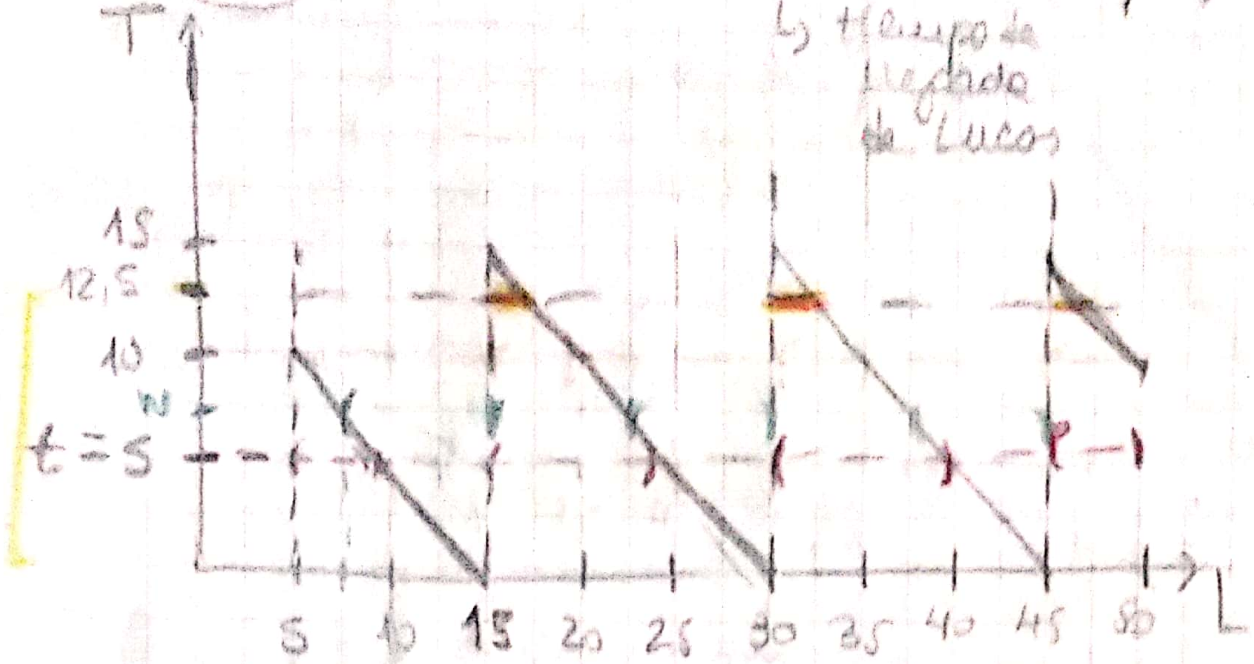
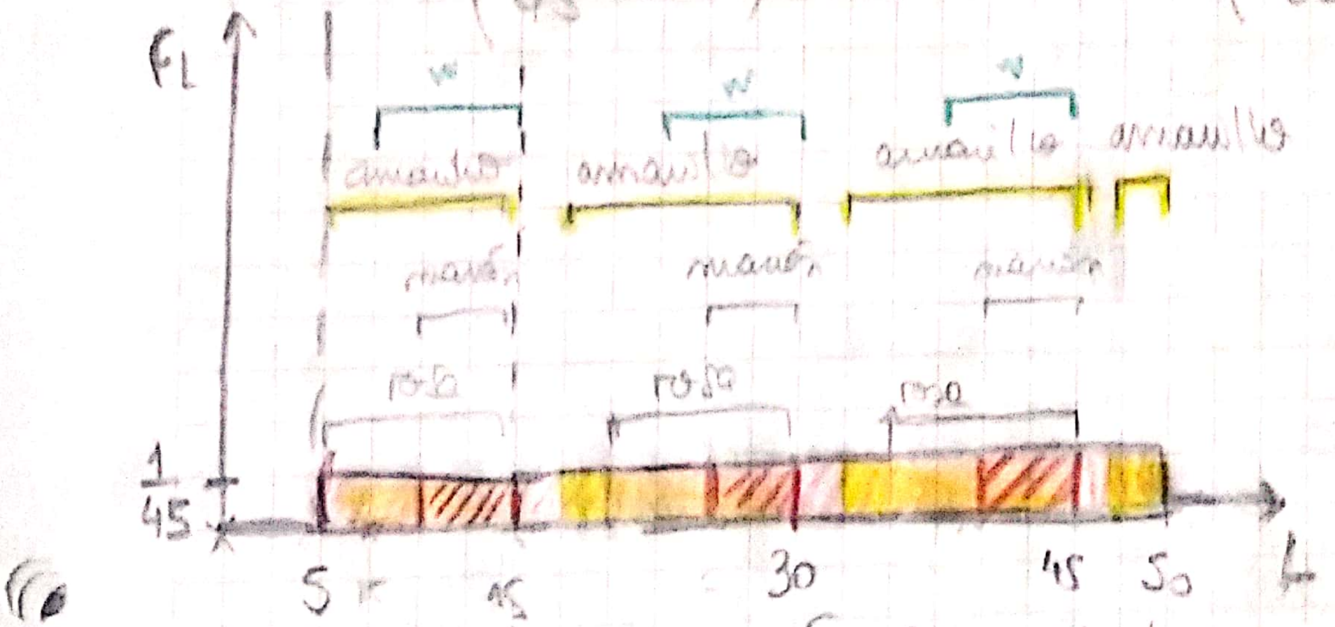
$L \sim U(5,50) \Rightarrow f_L = \frac{1}{45}$

↳ tiempo de llegada de Lucas



$$T = \left\{ \left(-\frac{10}{15}L + 5\right) \mathbb{1}\{5 \le L < 15\} + \left(-\frac{15}{30}L + 15\right)\mathbb{1}\{15 \le L < 30\} + \left(-\frac{30}{45}L + 30\right)\mathbb{1}\{30 \le L < 45\} + \left(-\frac{45}{60}L + 45\right)\mathbb{1}\{45 \le L < 50\} \right\}$$



en 12,5 amarillo acumula:

$\frac{30}{45} + \underbrace{2,5}_{(t-10)} \cdot \frac{1}{45} = 3 =$

$\frac{\Delta t}{15}$ es igual al cto de las rectas

$$P(T \le t) = F_T(t) = \begin{cases} 0 & \text{si } t < 0 \\ 1 & \text{si } t \ge 15 \\ t/15 & 0 \le t < 15 \end{cases}$$

} es extrictamente creciente cualquier función de dist

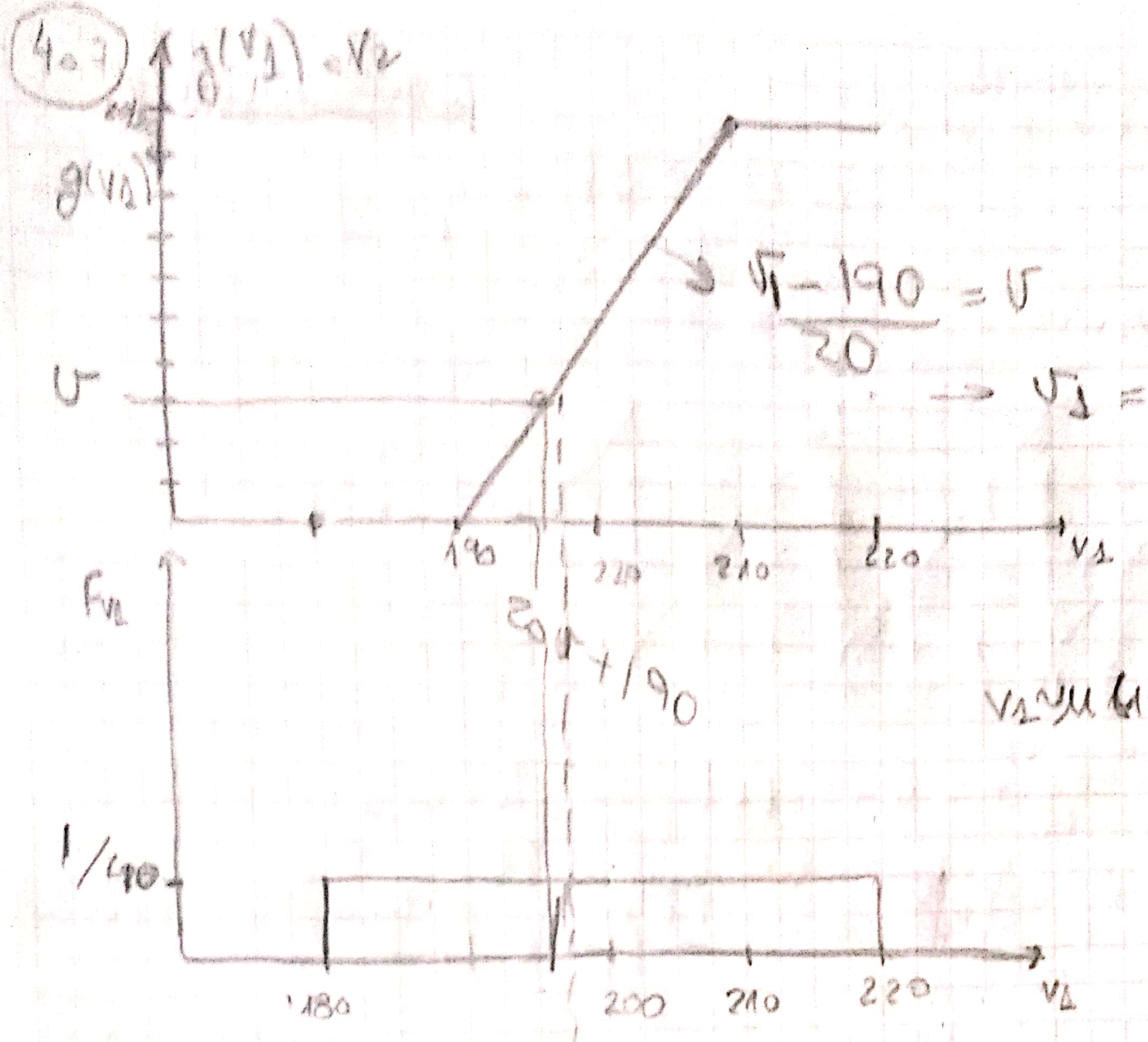
$F_T(t) = P(T \le t)$	t
15/45	5
30/45	10
1	15
3w/45	w
0	0
$5/6 = \frac{a}{15}$	amarillo = a

↓ noto que

$$\frac{dF_T(t)}{dt} = f_T(t) = \frac{3}{45} = \frac{1}{15}\, \mathbb{1}\{t \in [0,15]\}$$

Se me hace imposible hallar la función de dist si lo hago con pulsos

4.7



$\frac{v_1-190}{20} = v$

$\rightarrow v_1 = 20v+190$

$V_1 \sim U(180, 220)$

$$F_{V_2} = \begin{cases} 0 & \text{si } v_2 < 0 \\ \frac{(v_2-180)}{20} & \text{si } v_2 \geq 0,\ v_2 < 1 \\ 1 & \text{si } v_2 \geq 1 \end{cases}$$

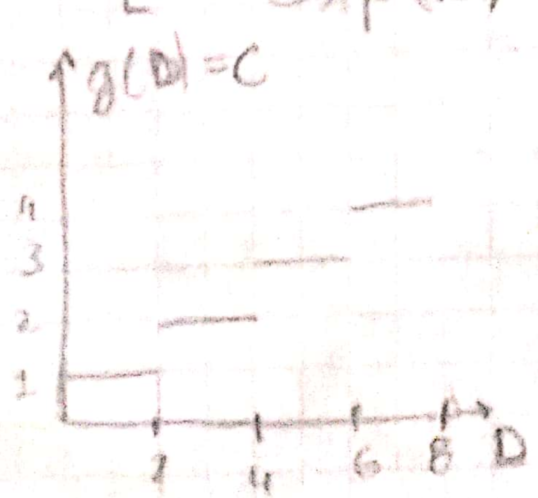
$20v-10$

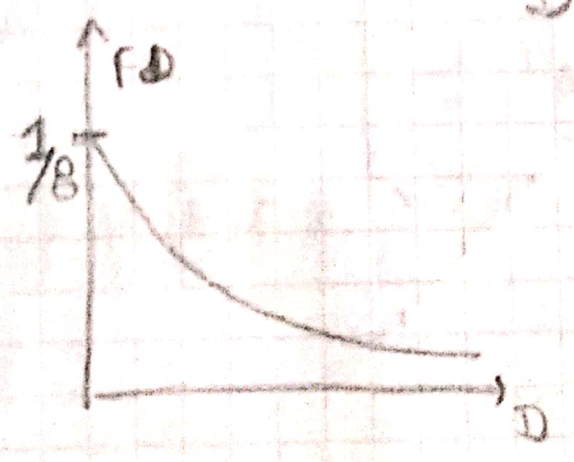
$\rightarrow \frac{20v-10}{40}$

4.8 $L \sim exp(1/8)$

D = "duración de la llamada en mins"

C = "costo de la llamada"





$l = 2$

$$C = g(D) = \begin{cases} 1 & \text{si } 0 < D \leq 2 \\ 2 & \text{si } 2 < D \leq 4 \\ 3 & \text{si } 4 < D \leq 6 \\ d & \text{si } 2d-2 < D \leq 2d \end{cases}$$

$P_{g(D)}(c) = P(C=c) = P(2d-2 < D \leq 2d)$

$= F_D(2d) - F_D(2d-2)$

$= 1 - e^{-2d} - (1 - e^{-(2d-2)})$

$= e^{-(2d-2)} - e^{-2d} = e^{-(2d-2)}(1-e^{-2})$

$g(1-e^{-2}) \cdot (e^{-2})^{d-1} (1-e^{-2})$

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X \ Y	-2	-1	1	2	P_X
-2	1/16	1/16	1/8	1/8	3/8
-1	1/16	1/16	1/8	1/8	3/8
1	0	0	1/16	1/16	1/8
2	0	0	1/16	1/16	1/8
P_Y	1/8	1/8	3/8	3/8	1

P_{UV} / $U = X$

$V = X + Y$

$P_{UV}(U,V) = P(U = u, V = v)$

$= P(X = u, X + Y = v)$

a

u \ V	-4	-3	-2	-1	0	1	3	4	2	
-2	1/16	1/16	0	1/8	1/8	0	0	0	0	3/8
-1	0	1/16	1/16	0	1/8	1/8	0	0	0	3/8
1	0	0	0	0	0	0	1/16	0	1/16	1/8
2	0	0	0	0	0	0	1/16	1/16	0	1/8
										1

b

u \ V	-2	-1	1	2	
-2	1/16	2/16	1/8	1/8	7/16
-1	0	1/16	1/8	1/8	5/16
1	0	0	1/16	2/16	3/16
2	0	0	0	1/16	1/16
	1/16	3/16	5/16	7/16	1

c

OK	U \ V	-1/2	-1	1/2	1	2	-2
1/4	8	0	1/8	0	1/8	0	0
2/4	5	+ 1/8	0	1/8	0	1/8	1/8
1/4	2	0	$\frac{1}{8}$ + 0	0	1/8	0	0

$$A^{-1}(U,V) - B = (x,y)$$

4.10 $(U,V) = A\cdot(X,Y)^T + B$ $\quad A \in \mathbb{R}^{2\times 2}$, $B \in \mathbb{R}^2$

$$F_{U,V}(u,v) = \frac{F_{X,Y}(x,y)}{|J|} = \left[\frac{F_{X,Y}(x,y)}{|\det(A)|}\right]_{(A^{-1}(U,V)-B)^T}$$

$$(U,V) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + (b_1, b_2) \quad \left[\frac{A^{-1}(u-b_1, v-b_2)}{\det(A)}\right]^T$$

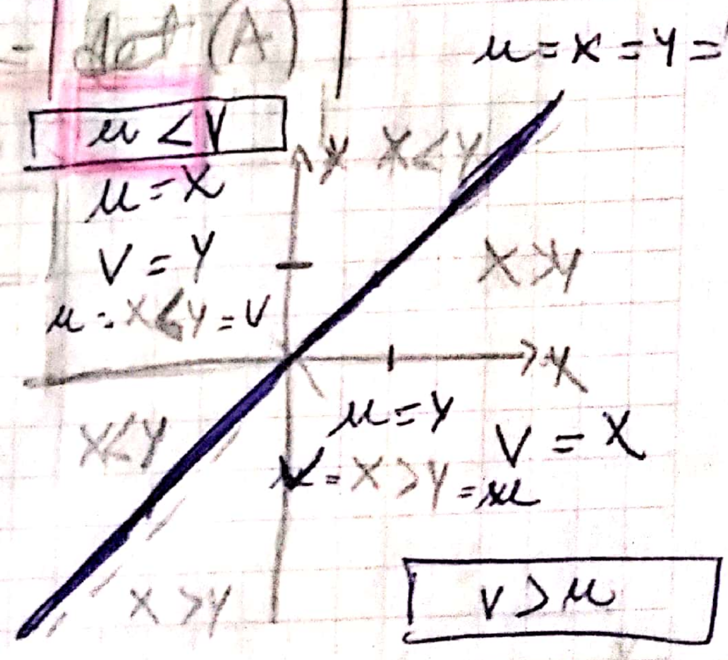
$$(U,V) = (a_{11}x + a_{12}y + b_1,\ a_{21}x + a_{22}y + b_2)$$

$$J = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \partial v/\partial x & \partial v/\partial y \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = |\det(A)|$$

b)

$$U = \begin{cases} X & X<Y \\ Y & X>Y \end{cases}$$

$$V = \begin{cases} X & X>Y \\ Y & X<Y \end{cases}$$



$$(U,V) = h(x,y) = \begin{cases} (x,y) & \text{si } X<Y = h_1(x,y) \\ (y,x) & \text{si } X>Y = h_2(x,y) \end{cases}$$

$$\|J_1\| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 \qquad \|J_2\| = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1 \qquad f_{U,V}(u,v) = \left(f_{XY}(v,u) + f_{XY}(u,v)\right)\mathbb{1}\{u \le v\}$$

c) $(U,V) = (X^2 + Y^2, Y/X)$

$$|J| = \left| \begin{vmatrix} 2X & 2Y \\ -\frac{Y}{X^2} & \frac{1}{X} \end{vmatrix} \right| = \left| 2 + \frac{2Y^2}{X^2} \right|$$

$u = x^2 + y^2 \longrightarrow u = x^2 + v^2 x^2$

$v = y/x \rightarrow y = vx$

$u = (1+v^2)\, x^2$

$x^2 = \frac{u}{1+v^2} \longrightarrow \boxed{X = \pm\sqrt{\frac{u}{1+v^2}} \quad Y = \pm v\sqrt{\frac{u}{1+v^2}}}$

$$f_{uv} = \frac{F_{xy}\left(\sqrt{\frac{u}{1+v^2}}, v\sqrt{\frac{u}{1+v^2}}\right) + F_{xy}\left(-\sqrt{\frac{u}{1+v^2}}, -v\sqrt{\frac{u}{1+v^2}}\right)}{|2 + 2v^2|}$$

$\bullet\ u > 0 \rightarrow$ falta poner el soporte

(11) 2) $(U,V) = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$

$|J| = 1$ $\quad Z_1 \sim N(0,1)$

$Z_2 \sim N(0,1)$

$(u,v) = (\cos\theta\, Z_1 - \operatorname{sen}\theta\, Z_2,\ \operatorname{sen}\theta\, Z_1 - \cos\theta\, Z_2)$

$u = \cos\theta\, Z_1 - \operatorname{sen}\theta\, Z_2$

$v = \operatorname{sen}\theta\, Z_1 + \cos\theta\, Z_2$

$u \cdot \cos\theta = Z_1 \cos^2\theta - \cos\theta \operatorname{sen}\theta\, Z_2$

$v \operatorname{sen}\theta = \operatorname{sen}^2\theta\, Z_1 + \cos\theta \operatorname{sen}\theta\, Z_2$

$\boxed{u\cos\theta + v\operatorname{sen}\theta = Z_1}$

$\boxed{Z_2 = v\cos(\theta) - u\operatorname{sen}\theta}$

$$f_{u,v}(u,v) = f_{xy}(u\cos\theta + v\,\text{sen}\,\theta,\; v\cos\theta - u\,\text{sen}\,\theta)$$

$$f_{u,v}(u,v) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(u\cos\theta + v\,\text{sen}\,\theta)^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{(v\cos\theta - u\,\text{sen}\,\theta)^2}{2}}$$

$$e^{-\frac{(u\cos\theta + v\,\text{sen}\,\theta)^2}{2}} e^{-\frac{(v\cos\theta - u\,\text{sen}\,\theta)^2}{2}}$$

$$= e^{-(u^2\cos^2\theta + 2uv\cos\theta\,\text{sen}\,\theta + v^2\,\text{sen}^2\theta)} e^{-(v^2\cos^2\theta - 2uv\cos\theta\,\text{sen}\,\theta + u^2\,\text{sen}^2\theta)}$$

$$= e^{-\frac{(u^2+v^2)}{2}} = e^{-\frac{u^2}{2}} e^{-\frac{v^2}{2}}$$

$$f_{uv}(u,v) = \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{v^2}{2}}$$

$\Rightarrow$ $U \sim N(0,1)$
$V \sim N(0,1)$

en este caso, U y V son independientes

3)

$$z_1 = \pm\sqrt{\frac{u}{1+v^2}}$$

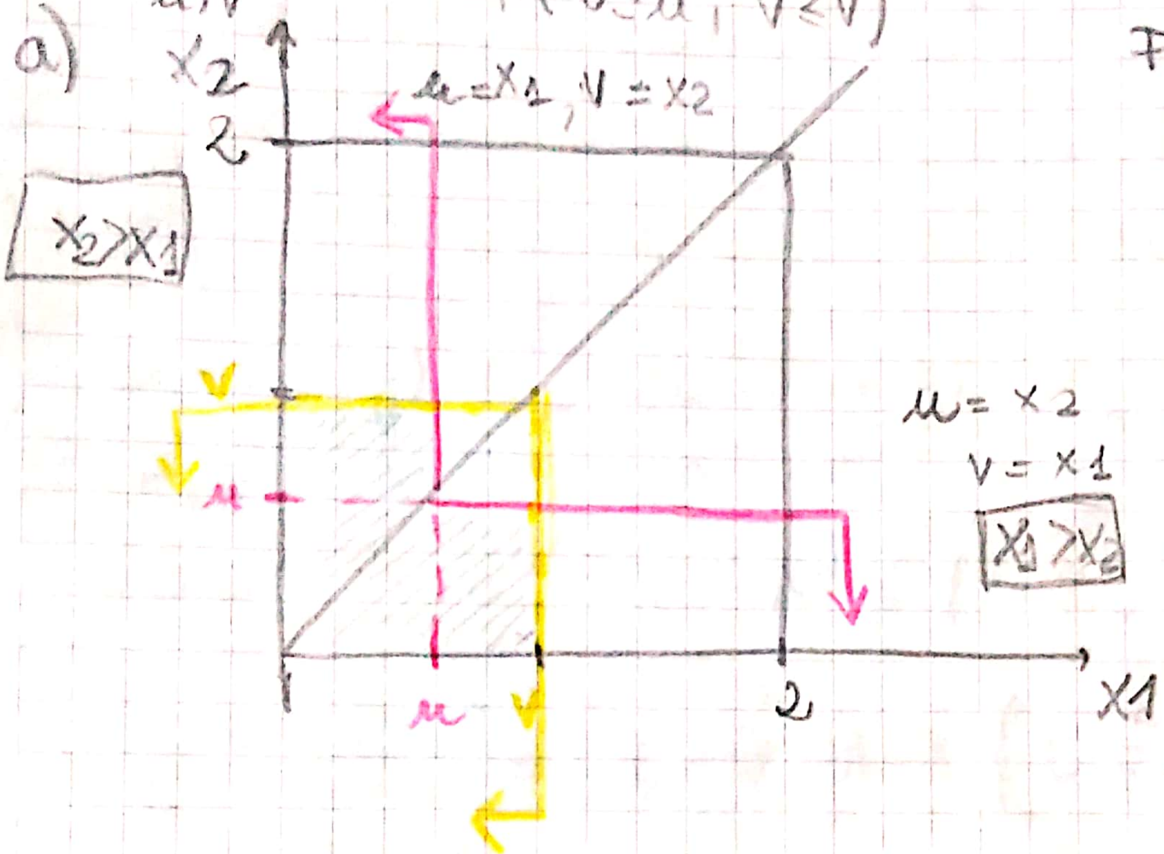
$$z_2 = \pm z_1 \sqrt{\frac{u}{1+v^2}}$$

(4.12) $X_1 \sim \mathcal{U}(0,2)$, $X_2 \sim \mathcal{U}(0,2)$, X_1, X_2 VAI

$$U = \begin{cases} X_1 & X_1 < X_2 \\ X_2 & X_1 > X_2 \end{cases}$$

$$(U,V) = \begin{cases} (X_1, X_2) & X_1 < X_2 \\ (X_2, X_1) & X_1 > X_2 \end{cases} \qquad V = \begin{cases} X_2 & X_1 < X_2 \\ X_1 & X_1 > X_2 \end{cases}$$

a) $F_{U,V}(u,v) = P(U \le u, V \le v)$



$$F_{UV}(u,v) = \begin{cases} 0 & u < 0 \\ & v < 0 \\ 1 & u \ge 2 \\ & v \ge 2 \\ (v-u)\cdot u + u \cdot v & 0 \le u \le v \le 2 \end{cases}$$

$\Downarrow$

$$f_{UV}(u,v) = \frac{\partial^2 F_{UV}}{\partial u \, \partial v}$$

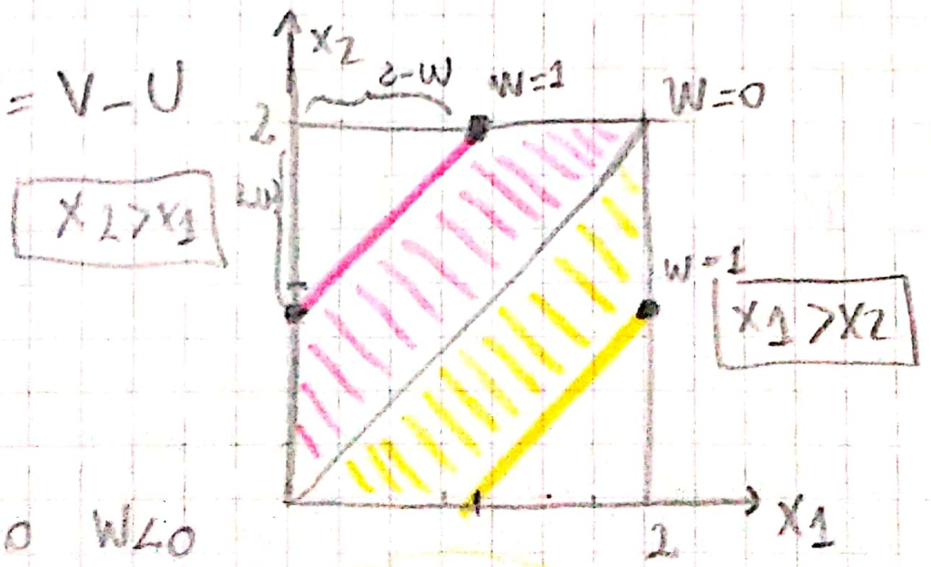
1) $\frac{\partial}{\partial u}\left((v-u)\cdot u + uv\right)$

$= \frac{\partial}{\partial v}\left(vu - u^2 + uv\right) = 2u$

2) $\frac{\partial}{\partial u}(2u) = 2$

$$f_{UV}(u,v) = \begin{cases} 0 & \text{e.o.c} \\ 2 & 0 \le u \le v \le 2 \end{cases}$$

b) $W = V - U$



$$W = \begin{cases} X_2 - X_1 & X_1 < X_2 \\ X_1 - X_2 & X_1 > X_2 \end{cases}$$

$W = 1$

$\hookrightarrow X_2 - X_1 = 1 \quad X_1 < X_2 \to X_2 = 1 + X_1$

$X_1 - X_2 = 1 \quad X_1 > X_2 \to X_1 = 1 + X_2$

$$F_W = \begin{cases} 0 & w < 0 \\ 1 & w \ge 2 \\ 1 - \frac{(2-w)^2}{4} & 0 \le w < 2 \end{cases}$$

techito: $f_{X_1 X_2}(x_1, x_2) = \frac{1}{4}$ ↓ $\boxed{\text{VA ind}}$

(4.15) $J \sim exp(5)$
$P \sim exp(10)$ > J y P independientes

Recordar que la exponencial no tiene memoria

$F_{JP} = F_J \cdot F_P$ por ser P y J VA indep.

$\Rightarrow JP \sim exp(\underbrace{10+5}_{15})$

a) La llamada va a quedar sin atender si sucede después de las 10 y que antes de 10:05 (ya que iniciaron el juego a las 10).

$$P(JP<5) = \int_0^5 15 e^{-15t} \mathbb{1}\{t>0\} \, dt = F_{JP}\left(\tfrac{1}{12}\right) = 1 - e^{-15 \cdot \frac{1}{12}} = \boxed{0,7135}$$

- $P(J<5) = F_J(1/12) = 1 - e^{-5/12} = 0,3408$

4.14 b)

b) $P(\min(J,P) = J) = P(J<P) = \frac{5}{5+10} = \frac{1}{3}$

c) $P(\min(J,P) < 5/60 \mid \cancel{\min(J,P) = J}) = P$

$U < 5/60$. $=$ 4.14 4.15 $J = J$, $J<P$

$$\min(J,P) = \begin{cases} J & J<P \\ P & J>P \end{cases}$$ v.a.c

(4.13)

$J \sim Ber(1/2)$ $\qquad J = \begin{cases} 0 & X > Y \\ 1 & X < Y \end{cases}$ $\qquad W = X+Y$

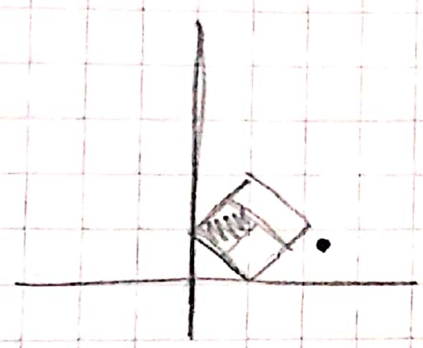
Debo chequear:

$1/2$

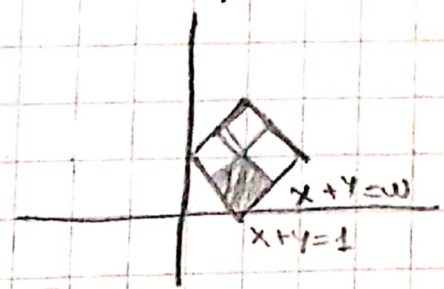
$$P(J=j \,;\, W \leq w) = P(J=j)\, P(W \leq w)$$

$$P(J=0 \,;\, W\leq w) + P(J=1 \,;\, W \leq w) = P(J=0)\, P(W\leq w) + P$$

$$j=0 \quad P(J=0, W \leq w) = P(X>Y, W \leq w) = \begin{cases} 0 & w<1 \\ \dfrac{(w-1)\sqrt{2}}{4} & e.o.c \\ 1 & w \geq 3 \end{cases}$$



$$j=1 \quad P(J=1, W \leq w) = P(X<Y, W \leq w) = \begin{cases} 0 & w<1 \\ \dfrac{(w-1)\sqrt{2}}{4} & e.o.c \\ 1 & w \geq 3 \end{cases}$$



$$P(W \leq w) = \begin{cases} 0 & si \quad w<1 \\ \dfrac{(w-1)\sqrt{2}}{2} & si \quad 1 \leq w \leq 3 \\ 1 & w>3 \end{cases}$$

4.16

$X \sim exp(\lambda) \sim \Gamma(1,\lambda)$

$Y \sim exp(\lambda) \sim \Gamma(1,\lambda)$ VA indep

$U = X+Y \qquad V = \frac{X}{X+Y}$

$\rightarrow V = \frac{X}{U}$

$\boxed{x = uv}$

$\boxed{y = u - uv}$

$$f_{UV}(u,v) = \frac{f_{XY}(x,y)}{|J|}\Bigg|_{\substack{g_x = uv \\ g_y = u-uv}}$$

$$f_{UV}(u,v) = \frac{f_{XY}(x,y)}{\left|\frac{1}{x+y}\right|}\Bigg|_{\substack{g_x = uv \\ g_y = u-uv}}$$

$$J = \begin{vmatrix} 1 & 1 \\ \frac{y}{(x+y)^2} & \frac{-x}{(x+y)^2} \end{vmatrix}$$

$$= \frac{-x-y}{(x+y)^2} = \frac{-(x+y)}{(x+y)^2}$$

$$= \frac{-1}{x+y}$$

$$\underset{\substack{\uparrow \\ \text{VA ind}}}{f_{XY}} = f_X \cdot f_Y = \lambda^2 e^{-2\lambda(x+y)} \mathbb{1}\{x>0, y>0\}$$

$$f_{UV}(u,v) = \frac{\lambda^2 e^{-2\lambda u} \mathbb{1}\{uv>0, u-uv>0\}}{\frac{1}{u}} \rightarrow \mathbb{1}\{u>0, 0<v<1\}$$

$$f_{UV}(u,v) = \lambda^2 u e^{-2\lambda u} \mathbb{1}\{u>0, 0<v<1\}$$

$$= \underbrace{\lambda^2 u e^{-2\lambda u} \mathbb{1}\{u>0\}}_{f_U} \cdot \underbrace{1\, \mathbb{1}\{0<v<1\}}_{f_V}$$

$\therefore$ U y V son VA indep ya que $f_{UV}(u,v) = f_U \cdot f_V$

4.16

$X \sim \Gamma(v_1, \lambda)$

$Y \sim \Gamma(v_2, \lambda)$

$$f_{UV}(u,v) = \left.\frac{f_{XY}(x,y)}{|J|}\right|_{\substack{g_X = uv \\ g_Y = u(1-v)}}$$

$|J| \to \left|\frac{1}{u}\right|$

$$f_{UV}(u,v) = \left.\frac{f_X(x,y)\cdot f_Y(x,y)}{|J|}\right|_{\substack{g_X = uv \\ g_Y = u(1-v)}}$$

v.A. ind

$$f_{UV}(u,v) = \left.\frac{\underbrace{\frac{\lambda^{v_1} e^{-\lambda x} x^{v_1-1}}{\Gamma(v_1)}}_{\mathbb{1}\{x>0\}} \cdot \underbrace{\frac{\lambda^{v_2} e^{-\lambda y} y^{v_2-1}}{\Gamma(v_2)}}_{\mathbb{1}\{y>0\}}}{|J|}\right|_{\substack{g_X(u,v) \\ g_Y(u,v)}}$$

$$f_{UV}(u,v) = \frac{\lambda^{(v_1+v_2)} (uv)^{v_1-1} (u(1-v))^{v_2-1} e^{-\lambda u}}{\Gamma(v_1)\,\Gamma(v_2)} \mathbb{1}\{uv>0,\ u(1-v)>0\}$$

$$f_{UV}(u,v) = \left[\frac{\lambda^{(v_1+v_2)} u^{v_1+v_2-1} e^{-\lambda u}}{\Gamma(v_1+v_2)} \mathbb{1}\{u>0\}\right] \cdot \left[\frac{\Gamma(v_1+v_2)\, v^{v_1-1} (1-v)^{v_2-1}}{\Gamma(v_1)\,\Gamma(v_2)} \mathbb{1}\, v \in (0,1)\right]$$

$U \sim \Gamma(v_1+v_2, \lambda)$

$V \sim \beta(v_1, v_2)$

tendríe que despejar R y θ pero ...

4o17

$X \sim N(0,1)$
$Y \sim N(0,1)$ > X e Y independientes

f_{xy} en coord. polares.

$X = R\cos\theta$
$Y = R\,\mathrm{sen}\,\theta$ > $R = \sqrt{x^2+y^2}$, $\theta = \mathrm{arctg}\left(\frac{y}{x}\right)$ $J = r$

$$f_{R\theta} = \left.\frac{f_{xy}(x,y)}{|J_{xy}|}\right|_{\substack{x(R,\theta)\\ y(R,\theta)}} \qquad J_{xy} = \frac{1}{J_{R\theta}}$$

$$f_{R\theta} \underset{\substack{\text{VA}\\ \text{ind}}}{=} \left.\frac{f_x(x,y)\cdot f_y(x,y)}{|J_{xy}|}\right|_{\substack{g_x(r,\theta)\\ g_y(r,\theta)}}$$

$$f_{R\theta} = \left.\frac{\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}}{|J_{xy}|}\right|_{\substack{g_x(r,\theta)\\ g_y(r,\theta)}}$$

$$f_{R\theta} = \frac{\frac{1}{\sqrt{2\pi}}e^{-\frac{r^2\cos^2\theta}{2}}\ \frac{1}{\sqrt{2\pi}}e^{\frac{r^2\mathrm{sen}^2\theta}{2}}}{1/r}$$

$$f_{R\theta} = \frac{1}{(\sqrt{2\pi})^2}e^{-\frac{r^2}{2}}\,r = \underbrace{\frac{1}{2\pi}}_{\theta\sim U[0,2\pi]}\cdot\underbrace{r\,e^{-\frac{r^2}{2}}}_{r\sim \mathrm{Ray}(1)}$$

$\boxed{\text{Box Muller}}$

$\hookrightarrow \boxed{r^2 \sim \exp\left(\frac{1}{2}\right)}$

(4b18) $U_1 \sim U(0,1)$
$U_2 \sim U(0,1)$ > VA independientes

$Z_1 = R\cos\theta$ $\quad R = \sqrt{-2\log(U_1)}$
$Z_2 = R\,\mathrm{sen}\,\theta$ $\quad \theta = 2\pi U_2$

Viene de
$\boxed{Exp(\lambda) = \frac{-\log(U(0,1))}{\lambda}}$

a)

$$f_{R\theta} = \frac{f_{U_1U_2}}{J}\Bigg|_{\substack{g_{u_1}(r,\theta)\\ g_{u_2}(r,\theta)}}$$

$$\underset{\text{VA IND}}{f_{R\theta}} = \frac{f_{U_1 \cdot U_2}}{J_{U_1U_2}}\Bigg|_{\substack{g_{u_1}(r,\theta)\\ g_{u_2}(r,\theta)}} = \frac{1}{\left|\begin{vmatrix} -e^{-\frac{r^2}{2}}r & 0 \\ 0 & \frac{1}{2\pi}\end{vmatrix}^{-1}\right|} = \underbrace{e^{-\frac{r^2}{2}}r}_{r\sim Ray(1)}\ \underbrace{\frac{1}{2\pi}}_{\theta\sim U[0,2\pi]}$$

$R^2 = -2\log(U_1)$

$-\frac{R^2}{2} = \log(U_1) \rightarrow \boxed{U_1 = e^{-\frac{R^2}{2}}}$ $\qquad J_{U_1U_2} = \frac{1}{J_{R\theta}}$

$U_2 = \frac{1}{2\pi}\theta$

b y c)

$Z_1 = R\cos\theta$
$Z_2 = R\,\mathrm{sen}\,\theta$

$$f_{Z_1Z_2} = \frac{f_{R\theta}(r,\theta)}{|J|_{R\theta}}\Bigg|_{\substack{g_r(z_1,z_2)\\ g_\theta(z_1,z_2)}} = \frac{e^{-\frac{r^2}{2}}r\,\frac{1}{2\pi}}{r}\Bigg|_{\substack{g_r(z_1,z_2)\\ g_\theta(z_1,z_2)}}$$

tiene sentido
ya que es
el camino inverso

$Z_1 \sim N(0,1)$
$Z_2 \sim N(0,1)$

$$= e^{-\frac{(z_1^2+z_2^2)}{2}}\frac{1}{2\pi} = \frac{1}{\sqrt{2\pi}}e^{-\frac{z_1^2}{2}}\frac{1}{\sqrt{2\pi}}e^{-\frac{z_2^2}{2}}$$

4b19) $X \sim U(36)$

$Y \sim U(4)$

¿P_{xy}? P_{xy} P_y

$$P_{xy} = \frac{1}{36} \cdot \frac{1}{4} \left\{ \begin{matrix} 2 \leq x + y \leq 10 \end{matrix} \right.$$

4b20) $L \sim Poi(2)$ $M \sim Poi(8)$

llamo X, llamo Y

a)
$$P(X+Y=i) = \sum_{j=0}^{i} P(X+Y=i, X=j)$$
$$= \sum_{j=0}^{i} P(Y=i-j, X=j)$$
$$= \sum_{j=0}^{i} P(Y=i-j)\, P(X=j)$$

L y M VA indep
$$= \sum_{j=0}^{i} \frac{8^{i-j} e^{-8}}{(i-j)!} \cdot \frac{2^j e^{-2}}{j!}$$

$$P(X+Y=i) = e^{-(8+2)} \sum_{j=0}^{i} \frac{8^{i-j}\, 2^j}{(i-j)!\, j!}$$

Recordar que:
$$\binom{K}{i} = \frac{K!}{(K-i)!\, i!}$$ y en esto:

$$P(X+Y=i) = \frac{e^{-(8+2)}}{i!} \sum_{j=0}^{i} \frac{i!\, 8^{i-j}\, 2^j}{(i-j)!\, j!} = \frac{e^{-(8+2)}}{i!} \sum_{j=0}^{i} \binom{i}{j} 8^{i-j} 2^j$$

Por binomio de Newton:
$$(a+b)^m = \sum_{i=0}^{m} \binom{m}{i} a^i b^{m-i}$$

$X+Y \sim Poi(10)$

$$\therefore P(X+Y=i) = \frac{e^{-(8+2)}}{i!} (8+2)^i, \; i \in \mathbb{N}_0$$

b)

$$P(M=k \mid L+M=10) = \frac{P(M=k \cap L+M=10)}{P(L+M=10)}$$

	0	1	2	3	4	5	6	7	8	9	10
0											X
1										X	
2									X		
3								X			
4							X				
5						X					
6					X						
7				X							
8			X								
9		X									
10	X										

Son casos puntos

$$P(M=k \mid L+M=10) = \frac{P(M=m,\ L=10-m)}{P(L+M=10)}$$

$m \in [0,10]$

L y M indep

$$= \frac{P(M=m)\cdot P(L=10-m)}{P(L+M=10)}$$

$$= \frac{2^m e^{-2}/m! \cdot 8^{10-m} e^{-(8)}/(10-m)!}{10^{10} \cdot e^{-10}/(10)!}$$

$$P(M=k \mid L+M=10) = \frac{2^m \cancel{e^{-10}}\, 8^{10-m}\, 10!}{m!\, \cancel{e^{-10}}\, 10^{10}\, m!\,(10-m)!}$$

$$P(M=k \mid L+M=10) = \frac{2^m\, 8^{10-m}}{10^{10}} \binom{10}{m}$$

$$= \frac{2^m\, 8^{10-m}}{10^{(10-m+m)}} \binom{10}{m} = \frac{2^m\, 8^{10-m}}{10^{10-m}\, 10^m} \binom{10}{m}$$

$$P(M=k \mid L+M=10) = \left(\frac{2}{10}\right)^m \left(\frac{8}{10}\right)^{10-m} \binom{10}{m}$$

$$P(M=k \mid L+M=10) = \left(\frac{2}{10}\right)^m \left(1-\frac{2}{10}\right)^{10-m} \binom{10}{m}$$

$$W = M \mid L+M=10 \sim \operatorname{Bin}\Big(\underbrace{10}_{k},\ \underbrace{\tfrac{2}{10}}_{\frac{2}{2+8}}\Big)$$

$2 \longrightarrow$ media de L

$2+8 \longrightarrow$ suma de medias de M y L.

$W \sim \operatorname{Bin}(10, 1/5)$

c) $P(M>2 \mid L+M=10) = \dfrac{P(M>2,\ L+M=10)}{P(L+M=10)}$

$P(W>2)$

$$= \frac{P(M>2,\ L \le 10-m)}{P(L+M=10)} \quad 2 \le m \le 10$$

$$P(M>2 \mid L+M=10) \overset{\text{VA indep}}{=} \frac{P(M>2)\, P(L \le 10-m)}{P(L+M=10)}$$

- $P(M>2) = 1 - \sum_{i=0}^{2} \frac{(8)^i}{i!} e^{-8} = 1 - \frac{\Gamma(1+2,8)}{\Gamma(1+2)}$

- $P(L \leq 10-m) = \sum_{j=0}^{10-m} \frac{2^j}{j!} e^{-2} = \frac{\Gamma(1+(10-m),2)}{\Gamma(1+(10-m))}$ media $2 \leq m \leq 10$

- $P(L+M=10) = \frac{10^{10} e^{-10}}{10!}$

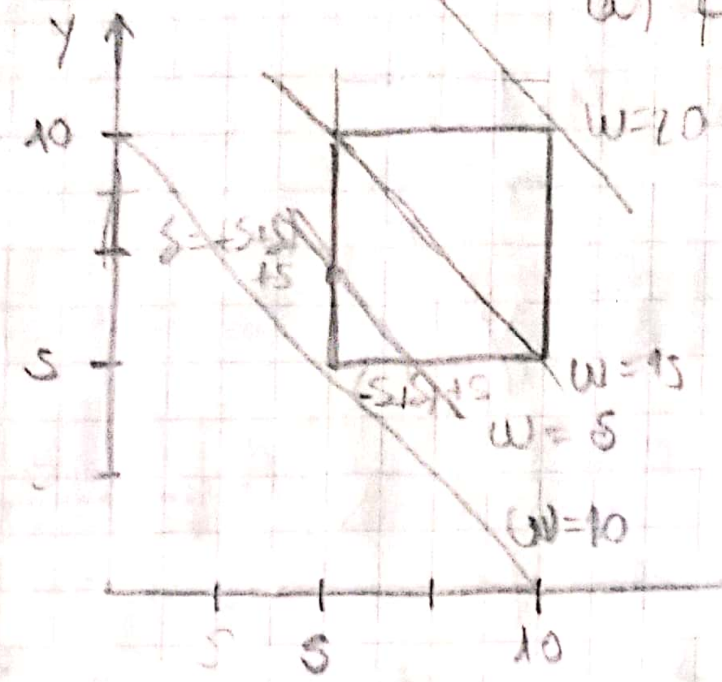
no me sirve de mucho

$$P(M>2) \cdot P(L \leq 10-m) = \left(1 - \frac{\Gamma(3,8)}{\Gamma(3)}\right) \frac{\Gamma[11-m,2]}{\Gamma(11-m)}$$

¿Hago la cuenta? No llego a nada que pueda vincular con otra cosa

4.21 $X, Y \sim U(5,10)$ X e Y indep

a) f_{XY} b) $P(X+Y<16)$

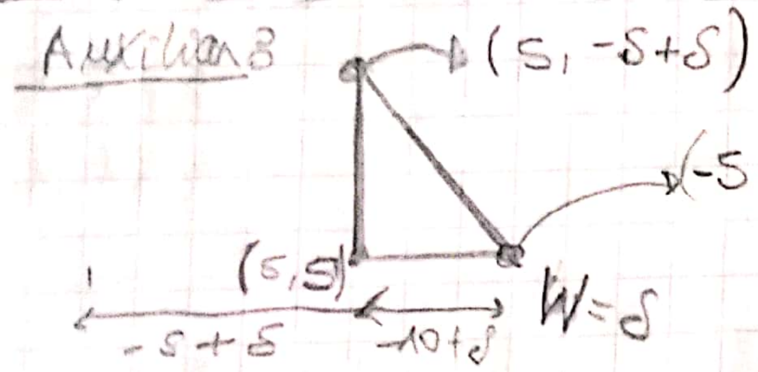


a) $f_{XY} = \begin{cases} \frac{1}{25} & 5<x<10,\ 5<y<10 \\ 0 & \text{e.o.c} \end{cases}$

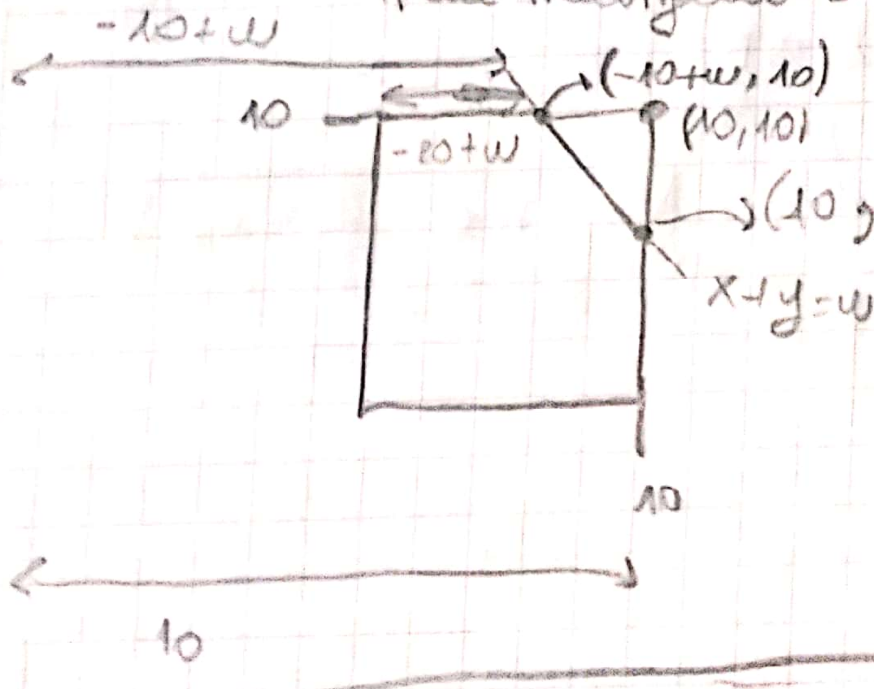
$= f_X \cdot f_Y$ ↓ indep

b) $P(W \le w) = P(X+Y \le w) = \begin{cases} 0 & \text{si } w<10 \\ \frac{(-10+w)^2}{2}\cdot\frac{1}{25} & w \ge 10,\ w<15 \\ 1-\frac{(20-w)^2}{2}\cdot\frac{1}{25} & w\ge 15,\ w<20 \\ 1 & w \ge 20 \end{cases}$

Auxiliar 8



Área triángulo $= \frac{(-5+\delta)^2}{2}$

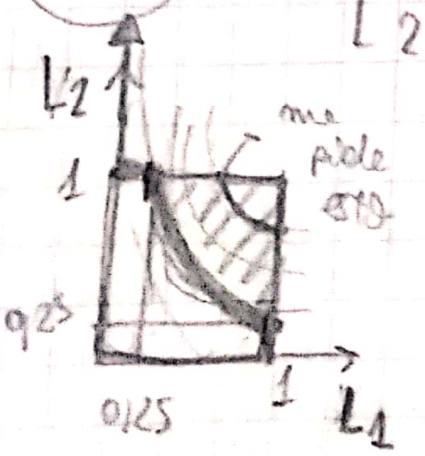


$P(W \le 16) = 1 - \frac{4^2}{2}\cdot\frac{1}{25} = 0{,}84$

$f_A = \begin{cases} 1 & 0\le x\le 1,\ 0\le y\le 1 \\ 0 & \text{e.o.c} \end{cases}$

4.22 $L_1 \sim U(0,1)$ $A = L_1 \cdot L_2$ f_A? $P(A>1/4)$?

$L_2 \sim U(0,1)$



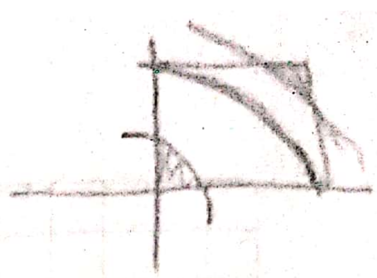
$0 \le L_1 \le 1$

$P(A \le 1/4) = P(L_1 \cdot L_2 \le 1/4) = P(L_2 \le \frac{1/4}{L_1})$

$= \left[\frac{1}{4} + \int_{1/4}^{1} \frac{1}{4}\cdot\frac{dL_1}{L_1}\right] = \frac{1}{4}\left[1 + \underbrace{\int_{1/4}^{1}\frac{1}{L_1}\,dL_1}_{1{,}3863}\right] = \boxed{0{,}5966}$

$\Rightarrow \boxed{P(A>1/4) = 1-0{,}5966 = \boxed{0{,}4034}}$

(4.23) $X \sim U(0,1)$ $Y \sim U(0,1)$ $R = \sqrt{X^2+Y^2}$



¿F_R? ¿$P(R > 1/2)$?

Idea Del (4.17)

defino X, Y como:

$X = R\cos\theta$
$Y = R\,\text{sen}\,\theta$ $\Rightarrow$ $R = \sqrt{X^2+Y^2}$ se cumple
$\theta = \text{arctg}\left(\frac{y}{x}\right)$ $\boxed{\text{invento}}$

v.a. indep

$0 \le x \le 1$
$0 \le y \le 1$

$$f_{R\theta} = \left.\frac{f_{XY}}{|J_{xy}|}\right]_{\substack{x(R,\theta)\\ y(R,\theta)}} = \left.\frac{f_X \cdot f_Y}{|J_{xy}|}\right]_{\substack{x(R,\theta)\\ y(R,\theta)}}$$

$$f_{R\theta} = \frac{1 \, \mathbb{1}\left\{\substack{0 \le r\cos\theta \le 1\\ 0 \le r\,\text{sen}\,\theta \le 1}\right\}}{1/r} = \underbrace{r}_{f_R} \cdot \underbrace{1}_{\theta \sim U(0,\frac{\pi}{4})} \mathbb{1}\{0 < r \le 1\}\, \mathbb{1}\left\{0 \le \theta \le \frac{\pi}{4}\right\}$$

$$F_R = \frac{r^2}{2}\, \mathbb{1}\{0 \le r \le 1\}$$ $\Rightarrow$ $\boxed{\text{No me parece lógico}}$

▽ siempre poner la densidad 1° así no me la olvido

(4.24)

$X \sim \mathcal{U}(5,10)$

$Y \sim \mathcal{U}(5,10)$

X e Y indep

$U = \frac{X}{2} \mathbb{1}\{X \leq Y\} + 2Y \mathbb{1}\{X > Y\}$

$f_{XY} = f_X \cdot f_Y = \frac{1}{5} \cdot \frac{1}{5} = \frac{1}{25}$

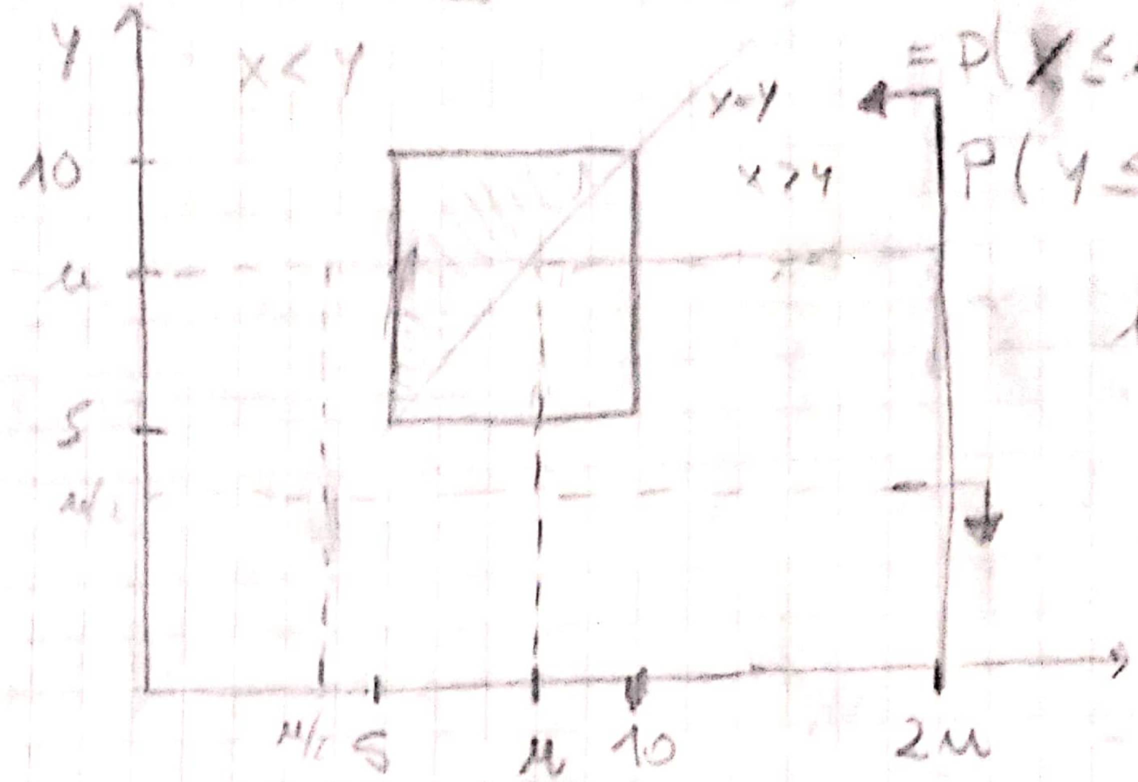
a) $F_U(u) = P(U \leq u) = P(U \leq u, X \leq Y) + P(U \leq u, X > Y)$

$F_U(u) = P\left(\frac{X}{2} \leq u, X \leq Y\right) + P(2Y \leq u, X > Y)$

$= P(X \leq 2u, X \leq Y) +$

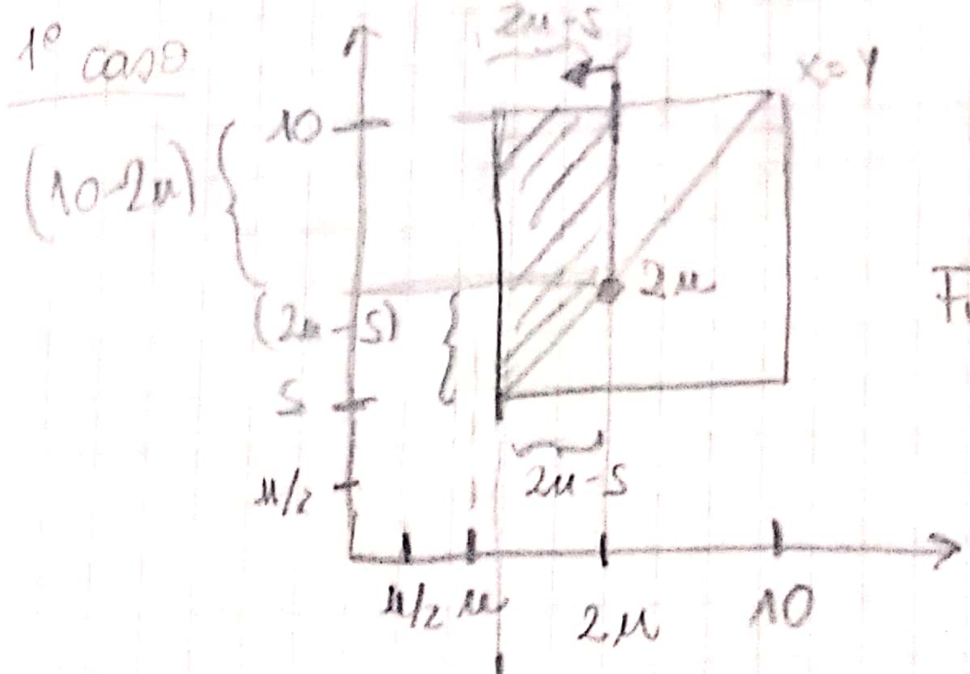
$P(Y \leq u/2, X > Y)$

Idea



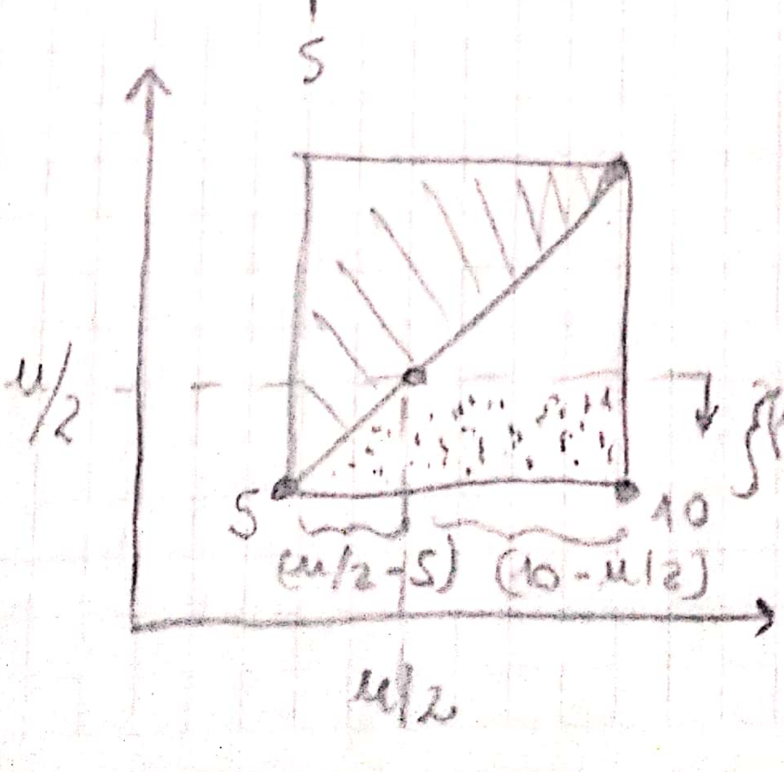
1° barre el triángulo de arriba y luego el de abajo.

1° caso



$$F_U(u) = \begin{cases} 0 & 0 \leq u < 5/2 \\ \frac{1}{25}\left[(10-2u)(2u-5) + \frac{(2u-5)^2}{2}\right] & \text{si } 5/2 < u < 5 \\ \frac{1}{2} & \text{si } 5 \leq u < 10 \\ \frac{1}{2} + \frac{1}{25} \cdot \left[\frac{(u/2-5)^2}{2} + (10-u/2)(u/2-5)\right] & 10 \leq u < 20 \\ 1 & \text{si } u \geq 20 \end{cases}$$

2° caso



$(u/2-5)$

$$f_U(u) = \frac{d}{du} F_U(u) = \begin{cases} 0 & \text{si } 0 \leq u < 5/2 \\ -\frac{4}{25}(u-5) & \text{si } 5/2 < u < 5 \\ 0 & \text{si } 5 \leq u < 10 \\ \frac{1}{5} - \frac{u}{100} & \text{si } 10 \leq u < 20 \\ 0 & \text{si } u \geq 20 \end{cases}$$

b) $V = \begin{cases} 1 & X+Y \leq 10 \\ 0 & \text{eoc} \end{cases}$

5.1

4 verdes
3 amarillas
3 rojas

} se extraen 3

X = "# bolas verdes"
Y = "# bolas rojas"

1) Hallar $P_{Y|X=x}$

$$P_{Y|X=x} = \frac{P_{XY}(x,y)}{P_X(x)}$$

$$P_{XY} = \frac{\binom{4}{x}\binom{3}{y}\binom{3}{3-x-y}}{\binom{10}{3}}, \quad x=\{0,1,2,3\} \quad y=\{0,1,2,3\} \quad x+y\leq 3$$

$$P_X = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}, \quad x=\{0,1,2,3\}$$

$$\Rightarrow P_{Y|X=x} = \frac{P(Y=y, X=x)}{P(X=x)} = \frac{\binom{3}{y}\binom{3}{3-x-y}}{\binom{6}{3-x}}, \quad x=\{0,1,2,3\} \quad y=\{0,1,2,3\} \quad x+y\leq 3$$

$$P_{Y|X=0} = \frac{\binom{3}{y}\binom{3}{3-y}}{\binom{6}{3}}, \quad A_{Y|X=0}=\{0,1,2,3\} \quad P_{Y|X=0} \sim Hip(6,3,3)$$

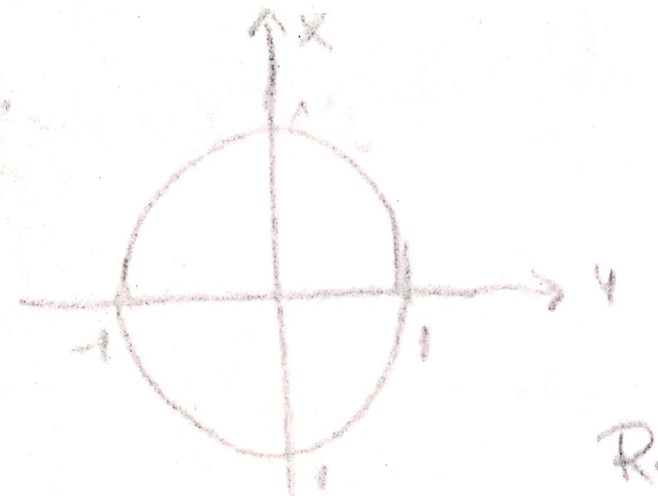
$$P_{Y|X=1} = \frac{\binom{3}{y}\binom{3}{2-y}}{\binom{6}{2}}, \quad A_{Y|X=1}=\{0,1,2\} \quad P_{Y|X=1} \sim Hip(6,3,2)$$

$$P_{Y|X=2} = \frac{\binom{3}{y}\binom{3}{1-y}}{\binom{6}{1}}, \quad A_{Y|X=2}=\{0,1\} \quad P_{Y|X=2} \sim Hip(6,3,1)$$

$$P_{Y|X=3} = \frac{\binom{3}{y}\binom{3}{0}}{\binom{6}{0}}, \quad A_{Y|X=3}=\{0\} \quad P_{Y|X=2} \sim Hip(6,3,0)$$

S.2) (X,Y) VA continua $f_{Y|X=x}$ y f_X

a)



$$f_{Y|X=x} = \frac{f_{XY}}{f_X}$$

de otra guía

Recordar ejercicio hecho en clase:

$$f_{XY} = \begin{cases} \frac{1}{\pi} & x^2+y^2 \le 1 \\ 0 & e.o.c \end{cases}$$

$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$, $-1 \le x \le 1$

$-\sqrt{1-y^2} \le x \le \sqrt{1-y^2}$, $-1 \le y \le 1$

$$f_X = \begin{cases} \frac{2}{\pi}\sqrt{1-x^2} & -1 \le y \le 1 \\ 0 & e.o.c \end{cases}$$

$$f_{Y|X=x} = \frac{\frac{1}{\pi}\mathbb{1}\{-1 \le y \le 1\}\mathbb{1}\{-\sqrt{1-y^2} \le x \le \sqrt{1-y^2}\}}{\frac{2}{\pi}\sqrt{1-x^2}\,\mathbb{1}\{-1 \le y \le 1\}}$$

$$f_{Y|X=x} = \frac{1}{2\sqrt{1-x^2}}\mathbb{1}\{-\sqrt{1-y^2} \le x \le \sqrt{1-y^2}\}$$

b) $f_{XY} = \frac{1}{2x+1} e^{-\left(2x + \frac{y}{4x+2}\right)} \mathbb{1}\{x>0\}\mathbb{1}\{y>0\}$ Corregir el +.

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}\,dy \underset{\substack{x \text{ queda} \\ \text{fijo}}}{=} \int_0^{+\infty} \frac{1}{2x+1} e^{-\left(2x - \frac{y}{4x+2}\right)} dy \, \mathbb{1}\{x>0\}$$

$$\frac{2}{2(2x+1)} \quad e^{-2x} \quad \frac{e^{-\frac{y}{4x+2}}}{2(x+1)}$$

$$= \int_0^{+\infty} \frac{1}{2x+1}$$

No es necesario. Vimos que $f_{Y|X=x}(y) = \frac{f_{XY}(x,y)}{f_X(x)}$

Entonces, la idea es ver si lo podemos separar. Si vemos $\mathbb{1}\{x>0\}$ y nos aparece un e^{-2x} podríamos ver $X \sim exp(2)$

Mostramos que $\boxed{Y|_{X=x}} \sim exp\left(\frac{1}{4x+2}\right)$

c) $f_{Y|X=x} = \dfrac{f_{XY}}{f_X}$ $\quad f_{XY} = e^{-x}\,\mathbb{1}\{0<y<x\}$

$$f_X \underset{(\ldots)}{\uparrow}= \int_0^x e^{-x}\,dy = x\,e^{-x}\,\mathbb{1}\{x>0\}$$

$$f_{Y|X=x} = \frac{e^{-x}\,\mathbb{1}\{0<y<x\}\mathbb{1}\{x>0\}}{x e^{-x}\,\mathbb{1}\{x>0\}} = \frac{1}{x}\,\mathbb{1}\{0<y<x\}$$

d) $f_{XY} = \frac{1}{6}\,x^4 y^3 e^{-xy}\,\mathbb{1}\{1<x<2\}\,\mathbb{1}\{y>0\}$

$$f_X = \int_0^\infty \underbrace{\frac{1}{3!}\,x^4 y^3 e^{-xy}}_{\Gamma[4,\,x]}\,dy\;\mathbb{1}\{1<x<2\}$$

$f_X = \mathbb{1}\{1<x<2\}$

$$f_{Y|X=x} = \frac{1}{6}\,x^4 y^3 e^{-xy}\,\mathbb{1}\{y>0\} = \Gamma(4, x)$$

So3) Y = "# litros de leche en el tanque"
X = " # litros vendidos de leche"

$f_{XY} = 0{,}5\,\mathbb{1}\{0<x<y<2\} = 0{,}5\,\mathbb{1}\{0<x<2\}\,\mathbb{1}\{x<y<2\}$

a) $$f_{X|Y=y}(x) = \frac{f_{XY}(y)}{f_Y(y)} = \frac{0{,}5\,\mathbb{1}\{0<x<2\}\mathbb{1}\{x<y<2\}}{\int_0^2 0{,}5\,dx\;\mathbb{1}\{x<y<2\}}$$

$$f_{X|Y=y}(x) = \frac{1}{2}\,\mathbb{1}\{0<x<2\} \qquad \boxed{X|_{Y=y} = \mathcal{U}(0,2)}$$

$$f_{Y|X=x}(x) = \frac{0{,}5\,\mathbb{1}\{0<x<2\}\mathbb{1}\{x<y<2\}}{\int_x^2 0{,}5\,dy\,\mathbb{1}\{0<x<2\}} = \frac{1}{2-x}\,\mathbb{1}\{x<y<2\}$$

$$\boxed{Y|_{X=x} = \mathcal{U}(x,2)}$$

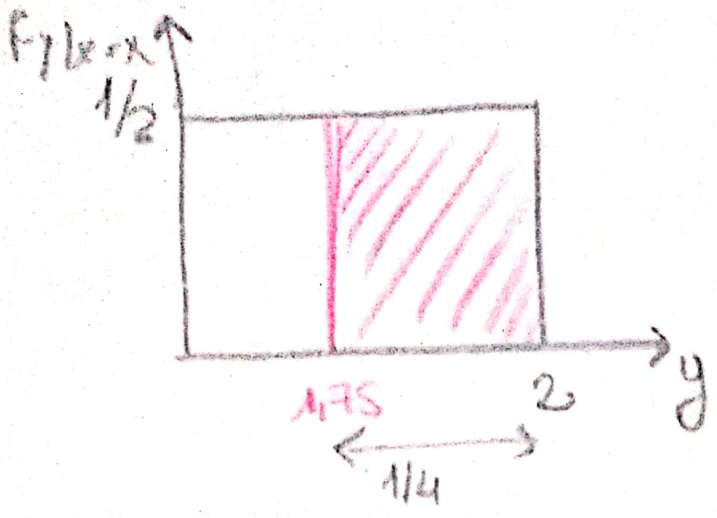
$f_{Y|X=1,5} = 2\,\mathbb{1}\{1{,}5<y<2\}$

$f_{X|Y=0,8} = \frac{1}{2}\,\mathbb{1}\{0<x<2\}$

b) $P(1{,}75 < Y < 2 \mid X = 1{,}5) = \frac{1}{8}$

$\downarrow$

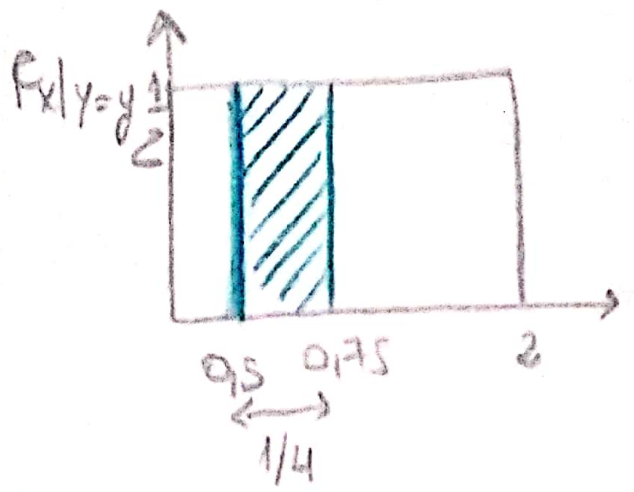
$Y \mid X = 1{,}5 \sim \mathcal{U}(1{,}5; 2)$



$P(0{,}5 < X < 0{,}75 \mid Y = 0{,}8) = \frac{1}{8}$

$\downarrow$

$X \mid Y = 0{,}8 \sim \mathcal{U}(0, 2)$



c) $f_{XY}(x,y) \neq f_X(x)\, f_Y(y) = \frac{1}{2}(2-x)\, \mathbb{1}\{0 < x < 2\}\, \mathbb{1}\{x < y < 2\}$

$\Rightarrow$ X e Y no son indep

5.4

$f_{XY}(x,y) = \frac{5}{8\pi}\, e^{-\frac{25}{32}\left(x^2 - \frac{6}{5}xy + y^2\right)}$ $\quad (X,Y) \sim N(0, 0, 1, 1, 3/5)$

a) recta de regresión de Y dada X $\quad$ 3.22 antes luego

$\mu_2 = 0 \quad \sigma_1 = 1 \quad 2\rho = \frac{6}{5} \Rightarrow \rho = \frac{6}{10} = \frac{3}{5}$

$\mu_1 = 0 \quad \sigma_2 = 1$

$\frac{-1}{2\left(1 - \left(\frac{6}{10}\right)^2\right)} = \frac{-25}{32}$ ✓ $\quad \frac{8}{5} = 2\sqrt{1-\rho^2} = 2 \cdot \frac{4}{5}$ ✓

$X \sim N(0,1)$

$Y \sim N(0,1)$

como $\rho \neq 0 \Rightarrow$ X e Y no son VA indep, $\text{cov}(X,Y) \neq 0$

$\hat{Y} = \frac{\text{cov}(X,Y)}{V(X)}(X - \underbrace{E[X]}_{0}) + \underbrace{E[Y]}_{0}$

$$\boxed{\hat{Y} = \frac{\rho\sqrt{V(X)V(Y)}}{V(X)}\, x = \boxed{\frac{3}{5}X}}$$

$\frac{5}{8\pi} = \frac{5}{4\sqrt{2\pi}\,2\pi}$

Es X mayuscula.

$$F_{XY} = F_Y \cdot F_{X|Y=y} = \frac{5}{8\pi} e^{-\frac{25}{32}\left(x^2 - \frac{6}{5}xy + y^2\right)} = \frac{5}{8\pi} e^{-\frac{1}{2(1-\rho^2)}\left(x^2 - 2\rho xy + y^2\right)}$$

$$F_{XY} = \underbrace{\frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}}_{F_Y} \cdot \underbrace{\frac{5}{4\sqrt{2\pi}} e^{-\frac{\left(x-\frac{3}{5}y\right)^2}{2\left(\frac{16}{25}\right)}}}_{F_{X|Y=y}}$$

Similarmente:

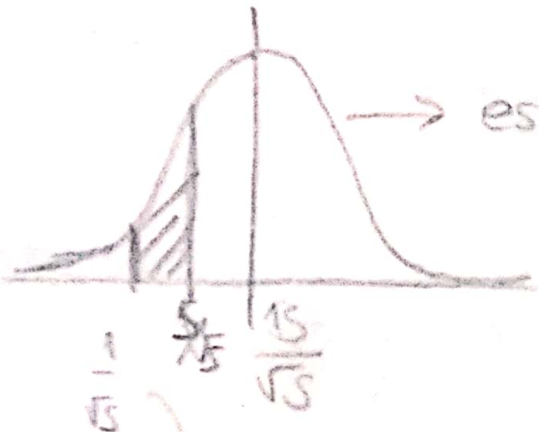
$$F_{Y|X=x} = \frac{5}{4\sqrt{2\pi}} e^{-\frac{\left(y-\frac{3}{5}x\right)^2}{2(16/25)}}$$

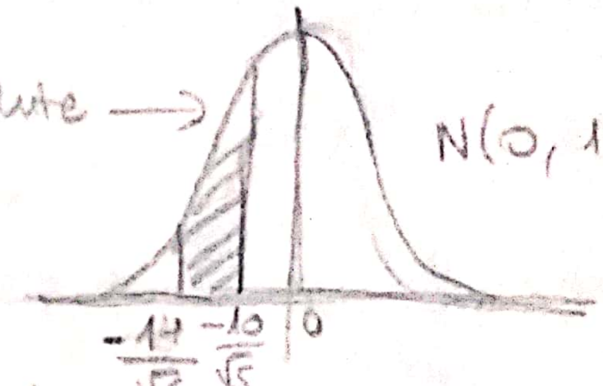
b)

$$P(1 < XY < 5 \mid X = \sqrt{5}) = \int_{1/\sqrt5}^{5/\sqrt5} F_{Y|X=\sqrt5}\, dy \longrightarrow N(3/\sqrt5, 16/25)$$

$$= \int_{1/\sqrt5}^{5/\sqrt5} \frac{5}{4\sqrt{2\pi}} e^{-\frac{\left(y - 3\cdot\frac{5}{\sqrt5}\right)^2}{2\cdot(16/25)}} dy = \int_{10/\sqrt5}^{14/\sqrt5} \frac{5}{4\sqrt{2\pi}} e^{-\frac{\cancel{t}^2\, 25}{2\cdot 16}} dy = (*)$$

(*)





achato la campana (y cálculo la análoga (*))

$P\left(\frac{1}{\sqrt5} < Y < \frac{5}{\sqrt5}\right)$

$=$

$P\left(\frac{\frac{1}{\sqrt5} - \frac{3}{\sqrt5}}{4/5} < \frac{Y - 3/\sqrt5}{4/5} < \frac{\frac{5}{\sqrt5} - \frac{3}{5}}{4/5}\right)$

$= \Phi\left(\frac{7\sqrt5}{2}\right) - \Phi(\ )$

$$\overset{(*)}{=} \int_{\frac{5\sqrt5}{2}}^{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{\theta^2}{2}} d\theta$$

Mirando (*) propondría

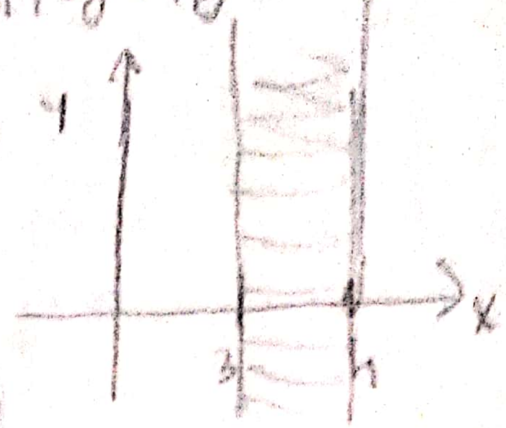
$\theta = \frac{y - \cancel{15}/\sqrt5}{4/5}$ y luego ver los límites

- si $y = 1/\sqrt5$
- si $y = 5/\sqrt5$

No sé qué pasaje hiciste como para revisar los cálculos. Parece que hay algún error ahí, porque está bien pensado, la idea es que $\theta \sim N(0,1)$

5.6 $X \sim \mathcal{U}(3,4)$ $\quad f_{XY} = f_{Y|X=x} \cdot f_X$

$Y|_{X=x} \sim N(x,1)$ (sin reemplazable de y) $\quad f_{XY} = f_{X|Y=y} \cdot f_Y$

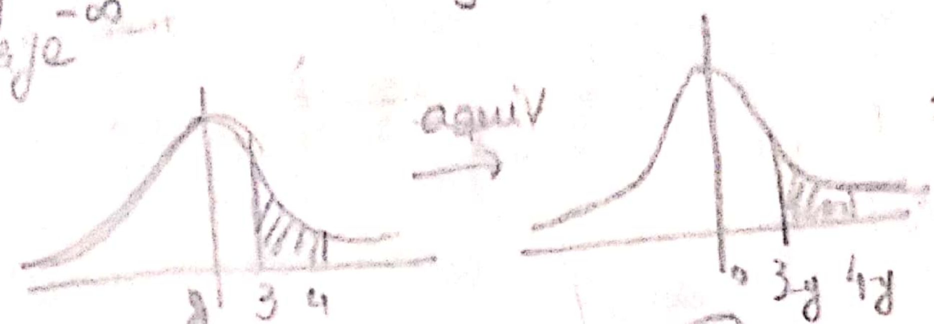


$f_Y(5)$ y $P(X>3{,}5 \mid Y=5)$

$$f_{XY} = \frac{1}{4-3} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-x)^2}{2}} \mathbb{1}\{x \in (3,4)\}$$

$$f_Y = \int_{-\infty}^{\infty} f_{XY}\, dx = \int_3^4 \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-y)^2}{2}} \, dx = \int_{3-y}^{4-y} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx$$

y fija



$= \Phi(4-y) - \Phi(3-y)$

No anda muy bien así

$p(x>3{,}5 \mid y=5)$

Tampoco así

$$f_{XY} = 1\, \mathbb{1}\{3 \le x \le 4\} \frac{1}{\sqrt{2\pi}} e^{-\frac{(y-x)^2}{2}}$$

$$f_{XY} = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-y)^2}{2}} \mathbb{1}\{3 \le x \le 4\} = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x^2-2xy+y^2)}{2}}$$

$$f_{X_1|X_2=0}(x_1) = \frac{f_{X_1X_2}(x_1,0)}{f_{X_2}(0)}$$

$$f_{X_2}(0) = \int_0^{+\infty} f_{X_1X_2}(x_1,0)\, dx_1$$

$$= \int_0^{+\infty} \frac{x_1}{2x_1} e^{-x_1} dx_1 = \frac{1}{2}$$

5.6 $\quad T_1 \sim \exp(1) \quad X_1 = T_1 + T_2 \quad X_3 = T_1/T_2$

$T_2 \sim \exp(1) \quad X_2 = T_1 - T_2$

a) $$f_{X_1X_2} = \left.\frac{f_{T_1T_2}(t_1,t_2)}{|J_{T_1T_2}|}\right|_{\substack{T_1(x_1,x_2)\\T_2(x_1,x_2)}} = \left.\frac{1 \cdot e^{-t_1} 1 e^{-t_2}}{2} \mathbb{1}\{t_1>0\}\mathbb{1}\{t_2>0\}\right|_{\substack{T_1(x_1,x_2)\\T_2(x_1,x_2)}}$$

$$|J_{T_1T_2}| = \left\|\begin{matrix} 1 & 1 \\ 1 & -1 \end{matrix}\right\| = 2$$

$$f_{X_1X_2} = \left.\frac{e^{-(t_1+t_2)}}{2}\right|_{\substack{T_1(x_1,x_2)\\T_2(x_1,x_2)}} = \frac{1}{2} \cdot e^{-x_1} \cdot \mathbb{1}\left\{\frac{x_1+x_2}{2} > 0\right\} \mathbb{1}\left\{\frac{x_1-x_2}{2} > 0\right\}$$

$$f_{X_1X_2} = \frac{1}{2} e^{-x_1} \mathbb{1}\{x_1>0\} \mathbb{1}\{-x_1 < x_2 < x_1\}$$

$$f_{X_1X_2} = 1 e^{-x_1} \mathbb{1}\{x_1>0\} \frac{1}{2} \mathbb{1}\{-x_1 < x_2 < x_1\}$$

$$f_{X_1X_2} = \underbrace{x_1 e^{-x_1} \mathbb{1}\{x_1>0\}}_{\substack{f_{X_1}\\ X_1 \sim \Gamma(2,1)}} \underbrace{\frac{1}{2(x_1)} \mathbb{1}\{-x_1 < x_2 < x_1\}}_{f_{X_2|X_1=x_1}}$$

X_1 y X_2 no son independientes

b) "Persistencia de la mala suerte"

$$f_{X_1X_3} = \left.\frac{f_{T_1T_2}(t_1,t_2)}{\left\|\begin{matrix} 1 & 1 \\ \frac{1}{T_2} & \frac{-T_1}{T_2^2} \end{matrix}\right\|}\right|_{\substack{T_1(x_1,x_3)\\T_2(x_1,x_3)}} = \left.\frac{e^{-(t_1+t_2)} \mathbb{1}\{t_1>0\}\mathbb{1}\{t_2>0\}}{\frac{t_1+t_2}{t_2^2}}\right|_{\substack{T_1(x_1,x_3)\\T_2(x_1,x_3)}}$$

$X_1 = T_1 + T_2 \to T_1 = T_2 - X_1$

$X_3 = T_1/T_2$

$\downarrow$

$X_3 = \frac{T_2 - X_1}{T_2} \to X_3 = 1 - \frac{X_1}{T_2} \to X_3 - 1 = \frac{-X_1}{T_2} \to \boxed{T_2 = \frac{-X_1}{X_3 - 1}}$

$$\boxed{T_1 = \frac{-X_1}{X_3-1} - X_1 = \left(\frac{1}{X_3-1} + 1\right)(-X_1)}$$

$$\{t_1>0, t_2>0\} = \left\{\frac{X_1X_3}{X_3+1} > 0, \frac{X_1}{X_3+1} > 0\right\} = \{X_1>0, X_3>\ldots$$

$$f_{X_1X_3} = \frac{e^{-x_1}\mathbb{1}\{x_1>0\}\mathbb{1}\{x_3>0\}}{\left(\frac{t_1+t_2}{t_2^2}\right)\Big|_{\substack{t_1(x_1,x_3)\\ t_2(x_1,x_3)}}} = \frac{x_1 e^{-x_1}\mathbb{1}\{x_1>0\}\mathbb{1}\{x_3>0\}}{(x_3+1)^2}$$

$x_1 = t_1+t_2$

$$t_2^2 = \left(\frac{x_1}{x_3+1}\right)^2 \qquad \frac{x_1\,(x_3+1)^2}{x_1^2} = \frac{(x_3+1)^2}{x_1}$$

ok x_1 me quedó con igual dist al caso anterior, eso es buena pinta

$$f_{X_1X_3} = x_1 e^{-x_1}\mathbb{1}\{x_1>0\}\,\frac{1}{(x_3+1)^2}\mathbb{1}\{x_3>0\}$$

¿Es una Cauchy?

$$\frac{\varphi}{\pi\varphi\left((x-x_0)^2+\varphi^2\right)} = \frac{1}{x^2+2x+1}$$

$$\varphi^2 x^2 + 2\varphi^2 x + \varphi^2 = \pi\varphi\left(x^2 - 2x x_0 + x_0^2 + \varphi^2\right)$$

$$\varphi x^2 + 2\varphi x + \varphi = \pi x^2 - 2\pi x x_0 + \pi x_0^2 + \pi\varphi^2$$

$\boxed{\varphi = \pi}$

$2\pi = -2\pi x_0 \rightarrow \boxed{x_0 = -1}$

$\pi + \pi^3 \neq \pi \Rightarrow$ no, no es Cauchy.

Sí, X_1 y X_3 son VA independientes.

$$f_{X_1X_2} = f_{X_1|X_2=x_2}\cdot f_{X_2} \Rightarrow f_{X_1|X_2=x_2} = \frac{f_{X_1X_2}}{f_{X_2}} \overset{\text{va indep.}}{=} \frac{f_{X_1}f_{X_2}}{f_{X_2}} = f_{X_1}$$

$$f_{X_1|X_2=x_2}(x_1) = x_1 e^{-x_1}\mathbb{1}\{x_1>0\} \qquad X_1|X_2=x_2 \sim \Gamma(2,1)$$

$f_{X_1|X_2=0}(x_1) =$

idem con $f_{X_1|X_3=1}(x_1)$ donde X_1 y X_3 tmb son indep.

$X_1|_{X_3=1} \sim \Gamma(2,1)$

d) $P(X_1 > 1 / X_2 = 0) = P(T_1 + T_2 > 1 \mid T_1 = T_2)$

$= P(2T_1 > 1)$

$P(X_1 > 1 \mid X_3 = 1) = P(X_1 > 1) = S_g(1) = \Sigma \quad = 2e^{-1}$

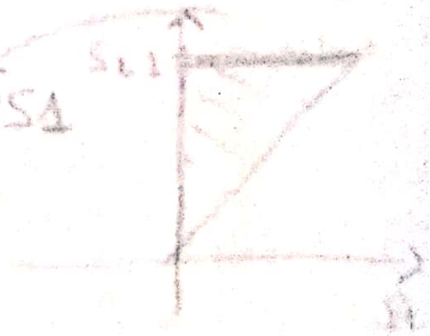
son indep

(5.7)

v.a. indep

$$f_{S_1 S_2} = \frac{f_{T_1 T_2}}{|J_{T_1 T_2}|}\Bigg|_{\substack{T_1(S_1,S_2)\\T_2(S_1,S_2)}} = \frac{f_{T_1} \cdot f_{T_2}}{\left\|\begin{matrix}1 & 0\\ 1 & 1\end{matrix}\right\|} = \lambda e^{-\lambda t_1} \cdot \lambda e^{-\lambda t_2} \mathbb{1}\{t_1 > 0\} \mathbb{1}\{t_2 > 0\}$$

$S_2 > S_1$



$$f_{S_1 S_2} = \lambda e^{-\lambda S_1} \cdot \lambda e^{-\lambda(S_2 - S_1)} \mathbb{1}\{S_1 > 0\} \mathbb{1}\{S_2 - S_1 > 0\}$$

$$f_{S_1 S_2} = \lambda e^{-\lambda S_2} \cdot \lambda \mathbb{1}\{S_2 > S_1 > 0\} / \mathbb{1}\{S_1 > 0\}$$

$$f_{S_2} = \int_{-\infty}^{\infty} f_{S_1 S_2} \, dS_1 = \int_0^{S_2} \lambda^2 e^{-\lambda S_2} \, dS_1 = S_2 \lambda^2 e^{-\lambda S_2}$$

S_2 fijo

$S_1 \mid S_2 = s_2 \sim \mathcal{U}(0, s_2)$

$S_2 \sim \Gamma(2, \lambda)$

$$f_{S_1 \mid S_2 = 2}(x) = \frac{f_{S_1 S_2}(x, 2)}{f_{S_2}(2)} = \frac{\lambda^2 e^{-\lambda x}}{\int_0}$$

$P(1/2 < S_1 < 1 \mid S_2 = 2) = 1/4$

(5.8)

$$f_{XY} = \frac{1}{5} e^{-\frac{x}{5}} \cdot \frac{1}{7} e^{-\frac{y}{7}} \mathbb{1}\{x > 0\} \mathbb{1}\{y > 0\}$$

$X \sim \exp(1/5)$, $Y \sim \exp(1/7)$ $\quad P(J=1) = \lambda$

U = tiempo del 1° llegó $\quad W = U - V = S$

$\boxed{U = \min(X, Y) = X}$

V = tiempo del que llegó + tarde

$$P(Y > X \mid V - U = S) = P(J = 1 / W = S) = \frac{f_{W \mid J=1}(s) \cdot P(J=1)}{f_{W \mid J=1}(s) \cdot P(J=1) + f_{W \mid J=2}(s) \cdot P(J=2)}$$

$\boxed{\text{Idea}}$ $\rightarrow$

(5.9) $X = S + N \quad , N \sim N(0,1)$

$P_S(s) = \frac{1}{3} \quad S = \{0,1; 0,2; 0,3\}$

$P(S=0,2 \mid X=0,87)$

Por ej.

$X \mid S=0,1 = 0,1 + N \} N(0,1,1)$

$$X = \begin{cases} 0,1+N & \text{si } S=0,1 \\ 0,2+N & \text{si } S=0,2 \\ 0,3+N & \text{si } S=0,3 \end{cases}$$

Bayes para la mezcla:

$$P(S=0,2 \mid X=0,87) = \frac{f_{X|S=0,2}(0,87) \cdot P(S=0,2)}{\left[f_{X|S=0,1}(0,87) \cdot P(S=0,1) + f_{X|S=0,2}(0,87) \cdot P(S=0,2) + f_{X|S=0,3}(0,87) \cdot P(S=0,3) \right]}$$

$$P(S=0,2 \mid X=0,87) = \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{(0,87-0,2)^2}{2}} \cdot \frac{1}{3}}{\frac{1}{3} \cdot \frac{1}{\sqrt{2\pi}} \left(e^{-\frac{(0,77)^2}{2}} + e^{-\frac{(0,67)^2}{2}} + e^{-\frac{(0,57)^2}{2}} \right)}$$

$$\boxed{P(S=0,2 \mid X=0,87) \simeq 0,334}$$

Continua 5.8 $\to$

$$P(U=X \mid \underbrace{V-V}_{W}=S) = \frac{f_{W|U=X}(S) \cdot P(U=X)}{f_{W|U=X}(S) \cdot P(U=X) + f_{W|U=Y}(S) \cdot P(U=Y)}$$

W y U indep

$$= F_W(s) \cdot \frac{\lambda_1}{\lambda_1+\lambda_2} \quad / \quad f_W(s)\left[\frac{\lambda_1}{\lambda_1+\lambda_2} + \frac{\lambda_2}{\lambda_1+\lambda_2}\right] = \boxed{\frac{\lambda_1}{\lambda_1+\lambda_2}}$$

continuus

SaS

$$f_{X|Y=5}(x) = \frac{f_{XY}(x,5)}{\underbrace{\phi(4-5)-\phi(3-5)}_{c}}$$

$$P(X>3{,}5 \mid Y=5) = \int_{3,5}^{4} \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{(x-5)^2}{2}}}{c}\, dx = \int_{-1,5}^{-1} \frac{1}{\sqrt{2\pi}} \frac{e^{-\frac{z^2}{2}}}{c}\, dz$$

$$= \frac{\phi(-1)-\phi(-1{,}5)}{\phi(-1)-\phi(-2)}$$

$P(3{,}5-5 < W-5 < 4-5)$

Solo

$(1-p) = 1/4$

X = "# de robots fallados en 6"

Y = "# detectados de robots fallados"

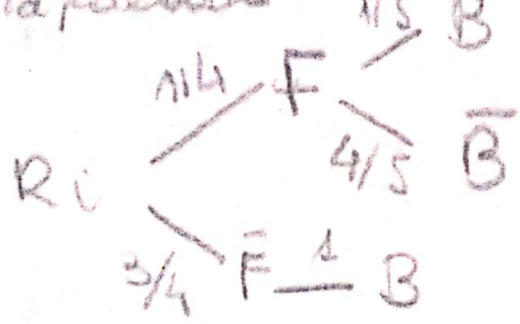
?

$X \sim \text{Bin}(6, 1/4)$

$$\mathbb{P}(\text{detección}) = \overset{4/5}{\mathbb{P}(D|F)} \cdot \overset{1/4}{\mathbb{P}(F)} + \mathbb{P}(D|\bar{F})\, \overset{3/4}{\mathbb{P}(\bar{F})}$$

F = el robot está fallado

B = es bueno



$$Y \mid X = x \sim \text{Bin}(x, 4/5)$$

$$\mathbb{E}[Y \mid X = x] = x \cdot \frac{4}{5} = \varphi(x)$$

Lo hacemos en clase

W = # no detectado de robots fallados

$X = Y + W \qquad X|_{Y=y} = y + \underbrace{W|_{Y=y}}$

$$W|_{Y=y} \sim \text{Bin}(6-y, p)$$

de robots no detectados de 6-y considerados como buenos.

$$p = P(F|B) = \frac{P(B|F)\,P(F)}{P(B)} = \frac{1/5 \cdot 1/4}{\underbrace{1/5 \cdot 1/4 + 3/4 \cdot 1}_{\frac{1}{5}\cdot\frac{1}{4} + 3/4\,\frac{5}{5}}} = \frac{1}{16}$$

$$E[X \mid Y = y] = E[y + W \mid Y = y]$$
$$= y + E[W \mid Y = y] = y + \frac{6-y}{16} = \frac{6}{16} + \frac{15}{16}y = \varphi(y)$$

(So11)

$E[Y|X]$?

a) $P(\omega) = 1/12 = F(\omega)$

1) $Y(\omega) = \omega$

$$Y(\omega) = \begin{cases} 1 & si\ \omega = 1 \\ 2 & si\ \omega = 2 \\ 3 & si\ \omega = 3 \\ 4 & si\ \omega = 4 \\ 5 & si\ \omega = 5 \\ \vdots & \end{cases} \qquad X(\omega) = \begin{cases} 1 & si\ \omega = \{1, 2\} \\ 2 & si\ \omega = \{3,4,5,6\} \\ 3 & si\ \omega = \{7, \ldots, 12\} \end{cases}$$

$$E[Y|X=x] = \sum_{y=1}^{12} y \cdot P(X=x|Y=y)$$

$P(X=x \mid Y=y)$; y's →	1	2	3	4	5	6	7	8	9	10	11	12
X = 1	1/2	1/2	0	0	0	0	0	0	0	0	0	0
X = 2	0	0	1/4	1/4	1/4	1/4	0	0	0	0	0	0
X = 3	0	0	0	0	0	0	1/6	1/6	1/6	1/6	1/6	1/6

$$E[Y] = \frac{1}{2}(2+1) + \frac{1}{4}(3+4+5+6) + \frac{1}{6}(7+8+9+10+11+12)$$

$$= \underbrace{\frac{3}{2}}_{E[Y|X=1]} + \underbrace{\frac{9}{2}}_{E[Y|X=2]} + \underbrace{\frac{57}{6}}_{E[Y|X=3]}$$

$$E[Y|X=x] = \begin{cases} 3/2 & si\ x=1 \\ 9/2 & si\ x=2 \\ 57/6 & si\ x=3 \end{cases}$$

$$E[Y|X] = \begin{cases} 3/2 & si\ X=1 \\ 9/2 & si\ X=2 \\ 57/6 & si\ X=3 \end{cases}$$

curso 29 (01.09) Bustamante, Verónica

Sol2 X = "# de resultados pares" $X \sim B(36, 1/2)$

Y = "# de " impares" no para ser Binomial debe estar definido n. en 36 tiros

$E[Y|X=x]$?

$Y + X = 36$

$Y | X=x = 36 - x$

$E[Y|X=x] = E[36-x]$
$= 36 - E[x]$
$= 36 - x$

justifique

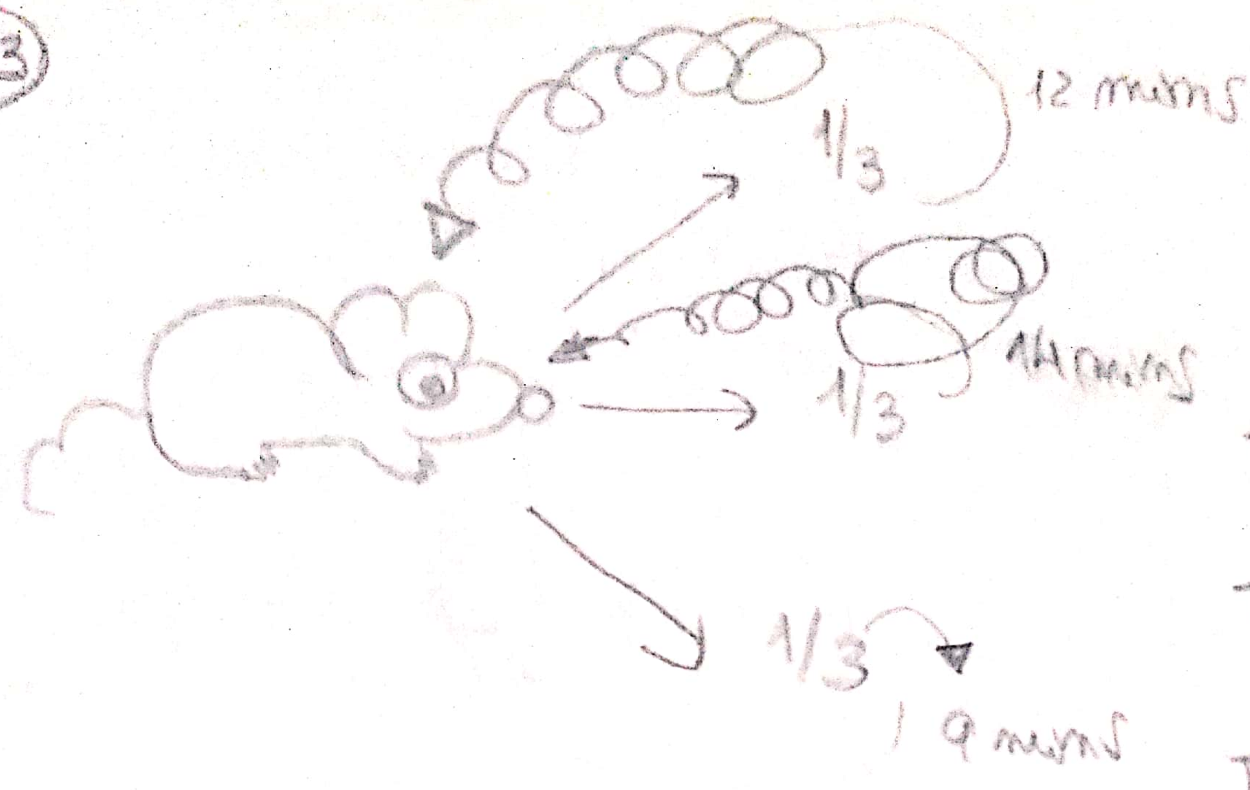
$E[Y|X] = 36 - X$

$cov(X,Y) = cov(X, E[Y|X]) = cov(X, 36-X)$
$= E[X \cdot (36-X)] - E(X) \cdot E(36-X)$
$= E(36X - X^2) - E(X) \cdot (36 - E(X))$
$= 36 E(X) - E(X^2) - 36 E(X) + E(X)^2$
$= E(X)^2 - E(X^2) = -V(X) = \boxed{-9}$

$\downarrow$

$- 36 \cdot 1/2^2$

So13



$T|_{X=1} = 12 + T$

$T|_{X=2} = 14 + T$

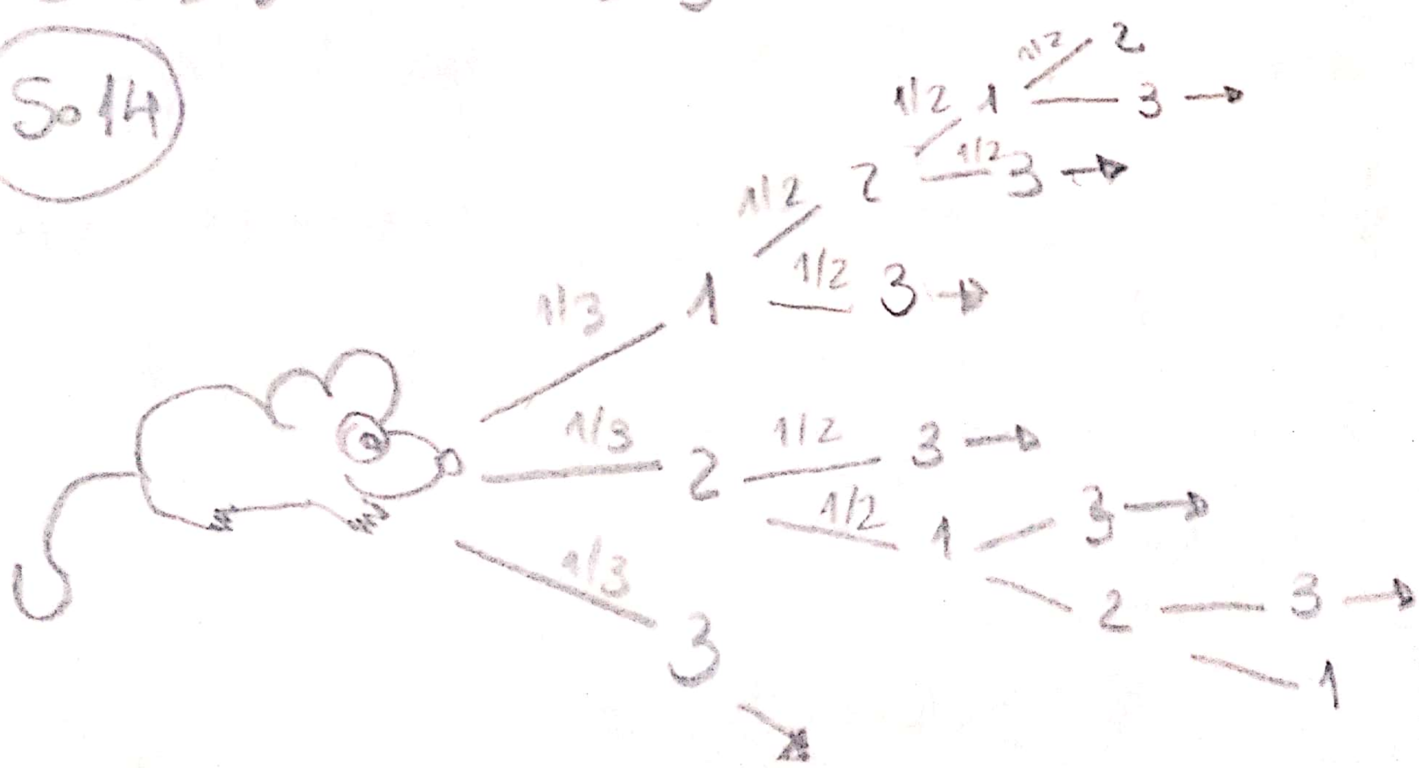
$T|_{X=3} = 9$

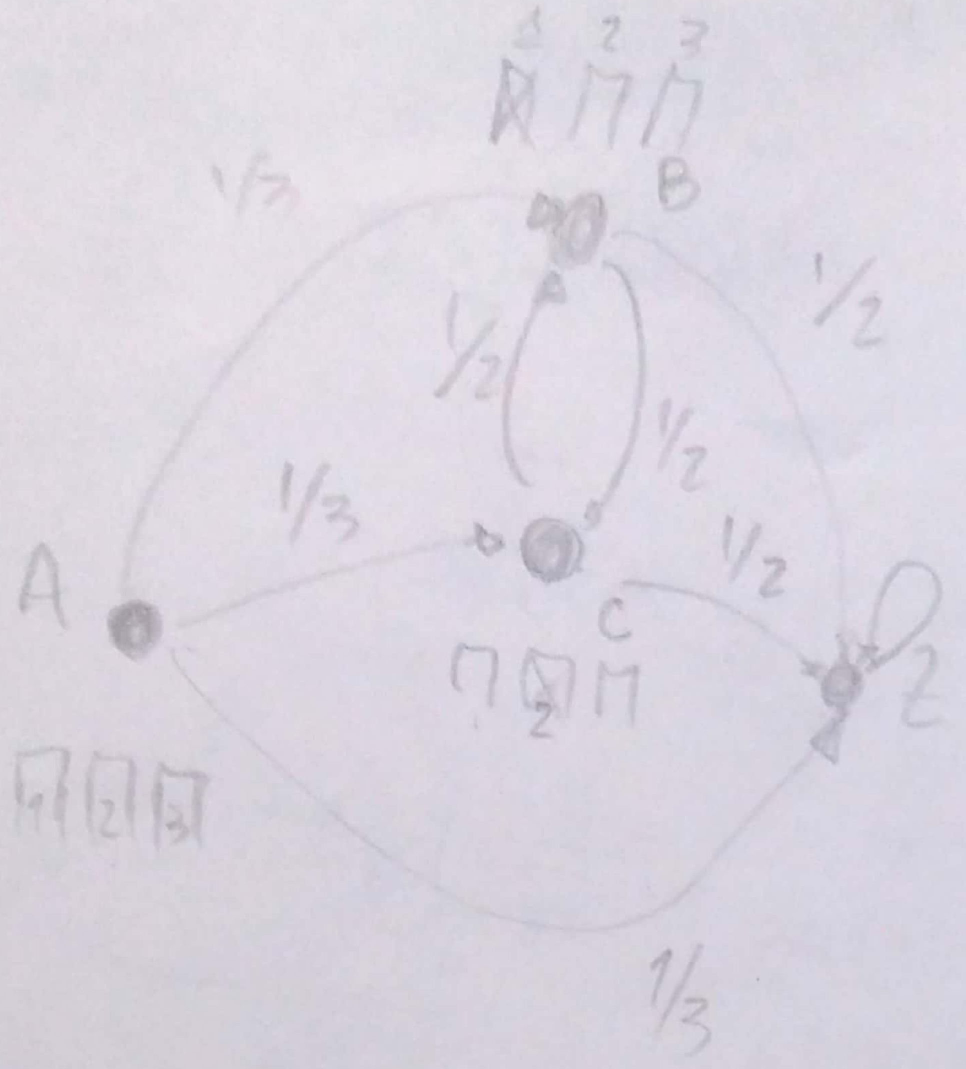
$$E[T] = E[T|X=1]\cdot P(X=1) + E[T|X=2]\cdot P(X=2) + E[T|X=3]\cdot P(X=3)$$

$$3\cdot E[T] = E[12+T] + E[14+T] + 9$$

$$3\cdot E[T] = 12 + E[T] + 14 + E[T] + 9$$

So14





T: Tiempo que tarda en salir

T_B: " " " " " (extra) si estoy en estado B

T_C: " "

Analizo T

$T|_{p_1=1} \sim 8 + T_B$

$T|_{p_1=2} \sim 13 + T_C$

$T|_{p_1=3} = 5$

Analizo T_B

$T_B|_{p_3^*=2} \sim 13 + T_C$

$T_B|_{p_3^*=3} = 5$

C

$$\begin{cases} E[T_B] = \frac{1}{2} E[13+T_C] + \frac{1}{2} \cdot 5 \\ E[T_C] = \frac{1}{2} E[8+T_B] + \frac{1}{2} \cdot 5 \end{cases}$$

$$E[T] = \frac{1}{3} E[8+T_B] + \frac{1}{3} E[13+T_C] + \frac{1}{3} \cdot 5$$

Analizo T_C

$T_C|_{p_3=1} \sim 8 + T_B$

$T_C|_{p_3=3} = 5$

5.15 $X \sim N(0,1)$ $\quad cov(X,Y) \overset{¿por qué?}{=} cov(X, E[Y|X])$

$E[Y|X] = X^2$

$= cov(X, X^2)$

$= E[X^3] - E[X]E[X^2] = \frac{4}{\sqrt{2\pi}}$

$$E[X^3] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x^3 e^{-\frac{x^2}{2}} dx = \int_0^{\infty} \frac{1}{\sqrt{2\pi}} u\, e^{-\frac{u}{2}} du$$

$u = x^2$

$du = 2x\,dx$

$$= \frac{4}{\sqrt{2\pi}} \underbrace{\int_0^{\infty} \frac{1}{4} u\, e^{-\frac{u}{2}} du}_{1}$$

$$= \frac{4}{\sqrt{2\pi}}$$

5.16 $X \sim exp(1/2)$

$E[Y|X] = X$

$Var[Y|X] = X$

Pitágoras:

$Var[Y] = E[Var(Y|X)] + Var(E[Y|X])$

Tratar de sacar conclusiones

$Var[Y] = E[X] + Var[X] = 2 + 4 = 6$

5.19 $X \sim N(1,4)$ $\quad \rho = 5/13$

$Y \sim N(0,1)$

$$Y|X=x \sim N\left(0 + \frac{5}{13} \cdot \frac{1}{2}(x-1), \frac{144}{69}\right)$$

verificar que los ~~dos~~ parámetros den iguales para que sea normal bivariada.

$$Y|X=x \sim N\left(\frac{5}{26}x - \frac{5}{26}, \frac{144}{69}\right)$$

$$E[Y|X=x] = \frac{5}{26}x - \frac{5}{26} = \varphi(x) \rightarrow \text{func de regresión}$$

$$E[Y|X] = \frac{5}{26}X - \frac{5}{26} = \varphi(X)$$

Sol 17 a) $E[Y|X=x] = \varphi(x)$ y $\phi(x) = \mathrm{var}[Y|X=x]$

b) $E[Y|X]$ y $\mathrm{var}[Y|X]$

$P_{Y|X=x} \sim Hip(6,3,x)$

a) $E[Y|X=x] = \sum_{y \in Sop} y\, P_{Y|X=x}(y)$

$P_{Y|X=0} \sim Hip(\underset{N}{6}, \underset{d}{3}, \underset{n}{3})$

$$P_{Y|X=x} = \frac{\binom{3}{y}\binom{3}{3-x-y}}{\binom{6}{3-x}} \sim Hip(6,3,3-x)$$

$x = \{0,1,2,3\}$
$y = \{0,1,2,3\}$
$x + y \leq 3$

$P(Y\|X=x)$	0	1	2	3
0	1/20	9/20	9/20	1/20
1	1/5	3/5	1/5	0
2	1/2	1/2	0	0
3	1	0	0	0

(filas: x; columnas: y)

conviene hacer minitablitas, así es compuesto

$E[Y|X=0] = 1 \cdot \frac{9}{20} + 2 \cdot 9/20 + 3 \cdot \frac{1}{20} = 3/2$

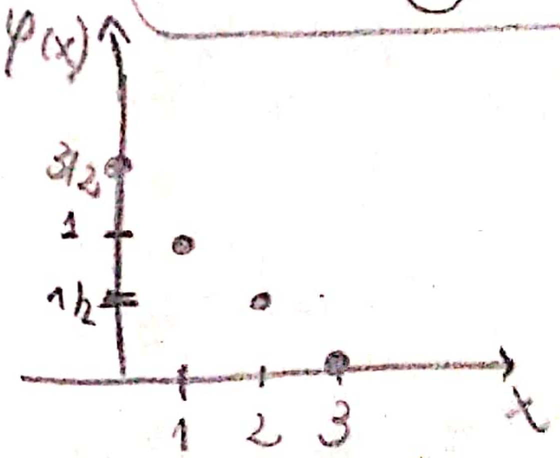
$E[Y|X=1] = 1 \cdot \frac{3}{5} + 2 \cdot 1/5 = 1$

$E[Y|X=2] = 1/2$

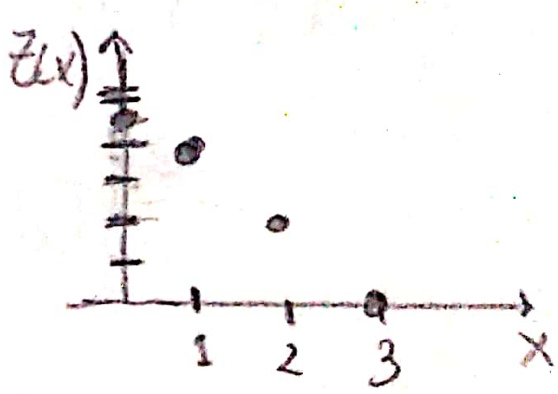
$E[Y|X=3] = 0$

da como la tabla !! :)

$$\varphi(x) = E[Y|X=x] = \begin{cases} 3/2 & \text{si } x=0 \\ 1 & \text{si } x=1 \\ 1/2 & \text{si } x=2 \\ 0 & \text{si } x=3 \end{cases}$$



$$\xi(x) = \mathrm{Var}[Y|X=x] = \begin{cases} \frac{3 \cdot 3 \cdot 3 \cdot 3}{36 \cdot 5} = \frac{9}{20} & \text{si } x=0 \\ \frac{1 \cdot 3 \cdot 4^2}{6 \cdot 5} = \frac{2}{5} & \text{si } x=1 \\ \frac{1 \cdot 3 \cdot 3 \cdot 5}{36 \cdot 5} = \frac{1}{4} & \text{si } x=2 \\ 0 & \text{si } x=3 \end{cases}$$



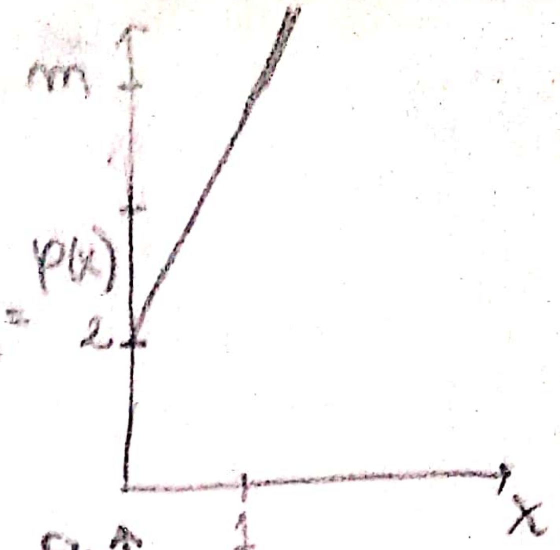
b) ~~Pero no hallás la esperanza y la varianza (caso discreto).~~

$\varphi(X) = 3/2\, \mathbb{1}\{X=0\} + \mathbb{1}\{X=1\} + 1/2\, \mathbb{1}\{X=2\} + 0\, \mathbb{1}\{X=3\}$

$\xi(X) = 9/20\, \mathbb{1}\{X=0\} + 2/5\, \mathbb{1}\{X=1\} + 1/4\, \mathbb{1}\{X=2\} + 0\, \mathbb{1}\{X=3\}$

18)

a) 1) $Y|X=x \sim \exp\left(\frac{1}{4x+2}\right)$

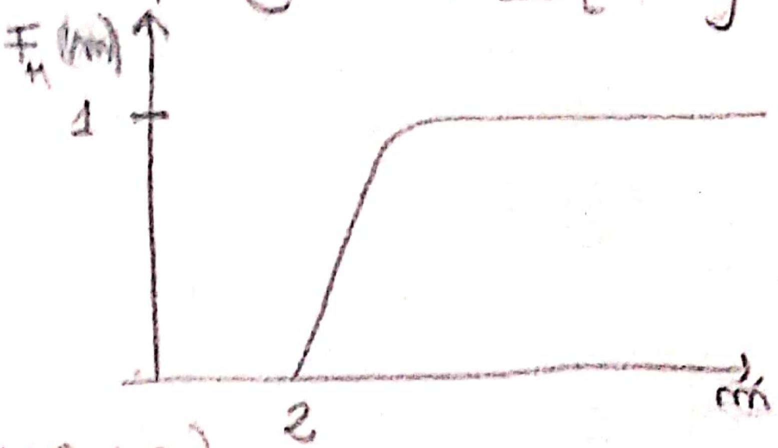


2) $E[Y|X=x] = \dfrac{1}{\left(1/(4x+2)\right)} = 4x+2 = m$

3) $E[Y|X] = 4X+2 = M$

4) $F_M(4X+2) \leq P(4X+2 \leq m) = P\left(X \leq \frac{m-2}{4}\right) = F_X\left(\frac{m-2}{4}\right)$

$= 1-e^{-\left(\frac{m-2}{4}\right)} \mathbb{1}\{m>2\}$



5) $P(1<M<2) \overset{=0}{=} P(1<4X+2<2)$

$= P(-1/4 < X < 0) = 0$

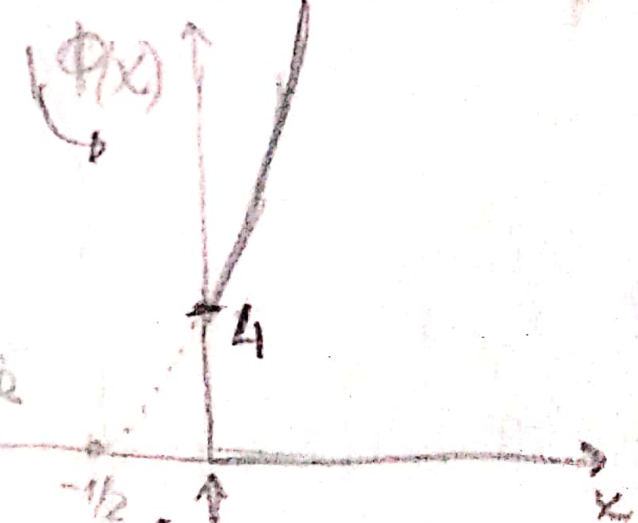
6) 7) 8) $Var[Y|X=x] = \dfrac{1}{\left(1/(4x+2)\right)^2} = (4x+2)^2 = 16x^2+16x+4 = \tilde{m}$

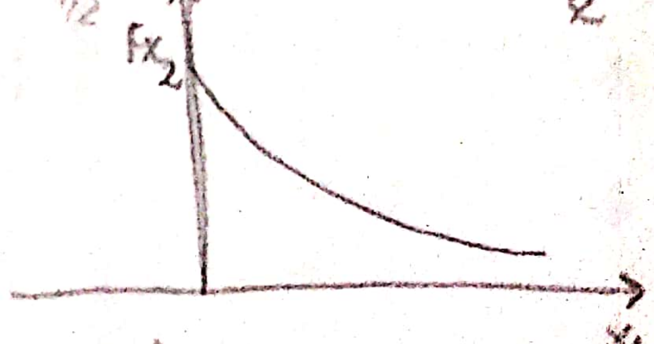
$Var[Y|X] = 16X^2+16X+4 = \tilde{M}$

$F_{\tilde{M}}(\tilde{m}) = P(16X^2+16X+4 \leq \tilde{m})$

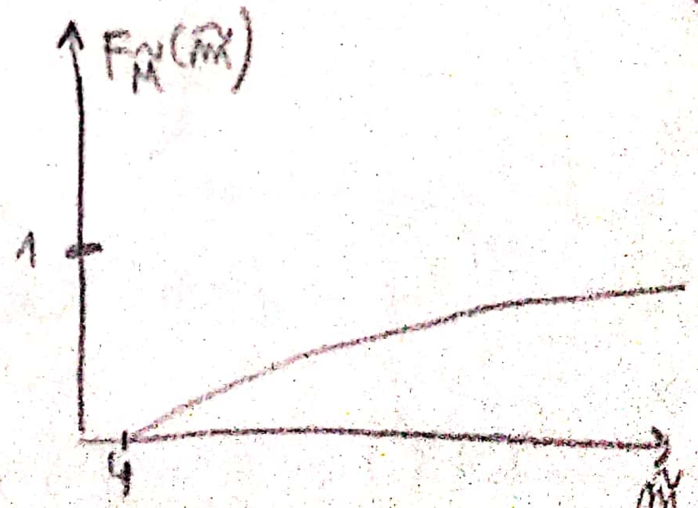
$= \begin{cases} 0 & \text{si } \tilde{m}<4 \\ F_X\left(\frac{1}{4}(\sqrt{\tilde{m}}-2)\right) & \text{si } \tilde{m}\geq 4 \end{cases}$

esto es de la guía 4 muy al inicio





$F_{\tilde{M}}(\tilde{m}) = \begin{cases} 0 & \text{si } \tilde{m}<4 \\ 1-e^{-\left(\frac{\sqrt{\tilde{m}}-2}{4}\right)} & \text{si } \tilde{m}\geq 4 \end{cases}$



9) $P(Var[Y|X]>1) = P(\tilde{M}>1) = 1$

10) $Var(Y) = E[V(Y|X)] + V[E[Y|X]]$

$= E[\tilde{M}] + V[M] =$

(So21) a) N = "# de rollos producidos hasta el 1° que mide + de 28"

L_i = "long del rollo i en m" $\sim U(20,30)$

Suponemos que la long de cada rollo es indep

$\overbrace{\sqcup \sqcup \sqcup \sqcup}^{N-1}$ E ¡éxito! $p = \frac{30-28}{30-20} = \frac{1}{5}$

$\rightarrow N \sim \mathcal{G}(1/5)$ $\rightarrow \boxed{E[N]=5}$

b) R = long total de tela para satisfacer la demanda.

$= \sum_{i=1}^{N} L_i$ $\qquad \overleftarrow{<28} \quad \overleftrightarrow{>28}$ $\sqcup \sqcup \sqcup \sqcup$ E

$$R|_{N=n} = \sum_{i=1}^{n-1} L_i | L_i < 28 + L_n | L_n > 28$$

$$E[R|N=n] = \varphi(n) = \sum_{i=1}^{n-1} E[L_i | L_i < 28] + E[L_n | L_n > 28]$$

$$E[R|N=n] = \varphi(n) = 24(n-1) + 29$$

20 24 28 $\qquad$ 28 29 30

$$E[R|N] = 29 + 24(N-1)$$

$$E[E[R|N]] = E[R] = E[29 + 24(N-1)] = 29 + 24(\overbrace{E[N]}^{5} - 1) = \boxed{125}$$

(So22) $C \sim Poi(2)$ = "# de impactos" $\boxed{E[C]=2} \rightarrow$ cantidad media de impactos por seg

P = "posición de la partícula luego de un segundo"

Buscar $E[P] = E[E[P|C]]$ donde $P = \sum_{i=1}^{C} G_i$

G_i = "posición de la part luego del golpe i"

asumimos que cada tiro es indep del otro

$$P|_{C=c} = \sum_{i=1}^{c} G_i$$

$$E[P|C=c] = \sum_{i=1}^{c} E[G_i] = c\,E[G_i]$$

$$E[G_i] = 1\cdot\left(\frac{3}{5}\right) - 1\cdot\frac{2}{5} = \frac{1}{5}$$

$$E[P|C] = \frac{C}{5} \qquad E\left[\frac{C}{5}\right] = \frac{1}{5}E[C] = \frac{2}{5}$$

5.23) P_i = "peso de bolsas de naranjas" $\sim \mathcal{U}(3,6)$

F = "peso final"

C = "# de bolsas de naranjas hasta que la balanza supere los 5 kilos"

$$F = \sum_{i=1}^{C} P_i$$

El peso de c/u es indep al resto

$$F \begin{cases} P_1 & P_1 > 5 \\ P_1 + P_2 & P_1 \leq 5 \end{cases} \text{ mezcla}$$

$$E[F] = E[F|P_1>5]\cdot P(P_1>5) + \underbrace{E(F|P_1\leq 5)}_{4}\, P(P_1 \leq 5) = K$$

$$5{,}5 \qquad \frac{1}{6-5} \qquad \frac{1}{5-3}$$

$$E[F] = 7{,}5$$

5.24) $E[X] = E[E[X|S]]$

$$X|_{S=s} = \begin{cases} 0{,}1+N & s=0{,}1 \\ 0{,}2+N & s=0{,}2 \\ 0{,}3+N & s=0{,}3 \end{cases}$$

$S = \{0{,}1;\ 0{,}2;\ 0{,}3\}$ VAD $\qquad X = S+N \qquad X|_{S=s}$ "VAC"

Supongo $E[|X|] < \infty$

$$\boxed{\text{Hoja 17 del apunte de SG}}$$

$$E[X] = E[E[X|S]] = \sum_{s\in S} E[X|S=s]\cdot P(S=s)$$

$$= \frac{1}{3}\left[E[X|_{S=0{,}1}] + E[X|_{S=0{,}2}] + E[X|_{S=0{,}3}]\right]$$

$$= \frac{1}{3}\cdot(0{,}1+0{,}2+0{,}3) = \frac{0{,}6}{3} = \boxed{0{,}2}$$

¡Esto es re importante!

Esperanza para una mezcla (como la del S.9):

$$E[X] = E[E[X|S]] = \sum_{s \in S} E[X|S=s] \cdot P(S=s)$$

Varianza condicional

$$V[X] = \underbrace{\sum_{s \in S} V(X|S=s) \cdot P(S=s)}_{E[V(X|S)]} + \underbrace{\sum_{s \in S} \left\{ \left(E[X|S=s] - E[X] \right)^2 \cdot P(S=s) \right\}}_{V[E(X|S)]}$$

⇒ para la varianza del S.24:

$$V(x) = \frac{1}{3}\left[V(X|S=0,1) + V(X|S=0,2) + V(X|S=0,3) \right]$$

$$+ \frac{1}{3}\left[\left(E[X|S=0,1] - E[X]\right)^2 + \left(E[X|S=0,2] - E[X]\right)^2 + \left(E[X|S=0,3] - E[X]\right)^2 \right]$$

$$= \frac{1}{3} \cdot (1+1+1) + \frac{1}{3}\left[(0,1-0,2)^2 + (0,2-0,2)^2 + (0,3-0,2)^2 \right]$$

$$= 1 + \frac{1}{3}\left(0,01 + 0 + 0,01\right) = 1 + 0,0066 = \boxed{1,0066}$$