The Frisbee

Charles Frederic Michel and Jonathan Riso

April 2019

1 Introduction

Even though, the motion of a frisbee doesn't sound like the most interesting thing you can learn about seems to be pretty simple, it is not actually the case. Sometimes, things that look the simplest turn out to be applying different physic principal's making it far more complex. An experienced ultimate frisbee player will have to take into account the spinning rate and the angle of attack of the frisbee, the direction of the wind and much more in order for the frisbee to be received at a specific height, range or time. In fact, its trajectory is mainly the result of two concepts, aerodynamic lift and gyroscopic stability[4]. The reason we are conducting this research is to come up with a code that will answer an optimization problem related to this specific subject: At which angle should a frisbee be thrown to reach maximum range?

2 Description of the Model

2.1 The Theory behind Frisbee Flight

In other to come up with accurate results and with the right computation methods, we had to familiarize ourselves with principles in Fluids dynamics to predict the trajectory of a frisbee flight. We in fact had to take into account the ways the frisbee was going to be influenced by linear forces, but as well by rotational ones.

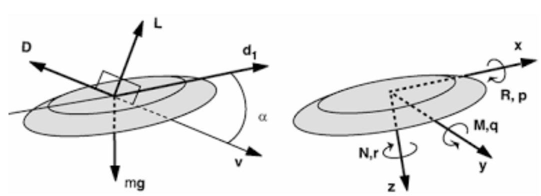

Figure 1: Free-body diagram of a Frisbee in flight [5]

2.2 Aerodynamic Forces

Lift and drag are the two aerodynamic forces as they depend on the specifications of the air and of the frisbee.

Lift, one of the two main forces that influence frisbee flight, is a force that push the frisbee higher in air and makes it stay longer in flight is explained by Kutta condition which explains that "the frisbee's shape causes the air above to travel faster than the air below it" which creates pressure difference and per the Bernoulli Principle it causes the "frisbee to hover temporarily before being overcome by gravity" [2]. Moreover, this force acts perpendicular to the velocity vector in the center of pressure (COP) which is in flight different from the COM. The equation we used to calculate the lift force and its coefficient was derived from the Bernoulli equation [4]:

$$\frac{{v_1}^2}{2} + \frac{P_1}{\rho} + gh_1 = \frac{{v_2}^2}{2} + \frac{P_2}{\rho} + gh_2$$

$$\frac{C^2{v_1}^2}{2} + \frac{P_1}{\rho} = \frac{{v_2}^2}{2} + \frac{P_2}{\rho}$$

$$F_L = \frac{1}{2}\rho v^2 A C_L$$

Where CL is the lift coefficient, which dependent on physical properties of the frisbee [4]

$$C_L = C_{L0} + C_{L\alpha}\alpha$$

The Reynolds number (Re) is an important dimensionless quantity in fluid mechanics used to help predict flow patterns in different fluid flow situations. The smaller the number, less turbulence an aerodynamic objects will face which could counter the motion of the object. This equation can be derived as willed depending on the situation, as for the flow in a pipe or the flow around airfoil, and depends on characteristics such as the density and the velocity of the fluid, the viscosity of the fluid.

The Reynolds number was found, from the specificity of the situation and the shape of the frisbee, and from that number the Prandtl relationship was chosen to calculate the drag force and coefficient. The drag force is in fact parallel and opposite to the velocity vector:

$$R = \frac{\rho v d}{\eta}$$

Where the drag coefficient, CD depends on the characteristics of the frisbee and it's angle of attack in radians.

$$F_D = -\frac{C_D \rho \pi r^2 v^2}{2} = -\frac{C_D \rho A v^2}{2}$$

$$C_D = C_{D0} + C_{D\alpha}(\alpha - \alpha_0)^2$$

Third, gravity simply results in a downward force from the centre of mass.

2.3 Gyroscopic Stability

The rotation of the frisbee is essential to a stable and long trajectory. The angular momentum, which depends on the angular speed (spinning rate) that the frisbee experiences when spinning, enables the frisbee to be easily flipped over by the torque produced by lift and drag forces acting at the center of pressure which doesn't coincide with the COM [4]. The moment's components, in the xyz axis, are respectively called the roll, the pitch and the spin down moment. Moreover, they either influence the decrease of the spin rate due to fluid viscosity or result in damping. Even though, this concept and this rotational motion of the frisbee is essential to its unique trajectory, we will not be taking it into account in our calculations [3]. Also, the location of the COP is dependent on the angle of attack.

2.4 The Wind Load

Thanks to the Generic formula, the force exerted by the wind on a static or an object in movement can be found from the pressure exerted by the wind if the speed of the wind was to be known in miles per hour [1] :

$$P = 0.00256 * v^2$$

$$F = A * P * C_D$$

Where C_D is the drag coefficient, P the pressure and A the area of the frisbee

3 Description of the Computations

3.1 Change in the vectors' direction

It's important to specify that, even though it's not in reality always the case, β in our code is set to be equal to initially be equal to the angle at which the frisbee is thrown to simplify the situation. Indeed, since β is equal δ, α is initially equal to 0 but as the velocity vector change direction, α will be then found from the difference between δ which stay the same throughout the flight and beta. As β change, the lift and the drag direction as well change with each iteration.

3.2 Methods

Optimization method and method to find the range

The initial idea was to optimize the angle with a single equation directly give us the range of the frisbee, but we quickly realized it was not possible to make such equation as the acceleration, the velocity and the angle of attack are constantly changing throughout the flight. Therefore, our only option left was to numerically find the value of the range. It's important to specify that our goal was to optimize the initial angle at which the frisbee is thrown which correspond to δ.

The main method of FrisbeeMotionOptimization is composed of the Golden Section Search (GSS) which is an effectively method of optimization that it continuously evaluates the range at each angle at which the frisbee is thrown, as it will narrow down the range in which the extremum is known to be, until the difference between the left and the right boundaries becomes less than 0.0001.

The best angle is known to be 10 degrees, so we set the initial boundary for the search to be between a throwing angle of 0 (a) to 90 (b) degrees. Moreover, since the GSS method we took from the Baywatch code already set to search for a absolute minimum, we had to make our Flight method, which returns the frisbee's range for specific throwing, instead return negative values so it would find it.

Every time, the GSS method use the Flight method, it inputs a throwing angle which will be used to initialize the initial values when the frisbee takes of the thrower hand : the direction and the values of the velocity's component, as well as the values for the acceleration in x and y.

From that takeoff point the frisbee motion is created with the Euler's method as it was proven to be an effective method to predict object's' motion in non-chaotic situations. At each time's increment of 0.0001 second, the firsbee's acceleration is found by different methods and its value from the previous increment is then used to calculate the new values for the velocity and the position.

The acceleration is in fact found through different methods :

1. First, the direction of the velocity (β) is used to calculate the new alpha which is then used to calculate the lift and the drag coefficients and to calculate the force's direction for the lift and drag
2. Then, the coefficients are used to get the values of the lift and drag forces' vectors
3. Finally, the acceleration both in x and y are return

Parameters and constants

Basic constants and parameters such as the mass of the frisbee, its diameter and the constants in the drag and lift equations which depends on the physical aspect of the frisbee, as well for the initial speed, (average speed a frisbee is thrown at) are from The Aerodynamics of Frisbee Flight by Kathleen Baumback[2] which had been taken in the first place in The Physics of Frisbees by V. R. Morrison[4]. We chose to have the same initial values to make it easier to compare with these works - except for the initial y value. Indeed, we set it at a value of 1.5 m and not 1 m because our simulation starts when the frisbee leaves the hand of the thrower which is usually higher than 1 m.

Adding the Wind

As wind usually interfere and influence the trajectory of a frisbee methods were added to calculate the force of the wind at each iteration to make our simulation more realistic and practical. Also, instructions were added on its use so the methods could be modified depending on the wind's direction. Indeed, the wind can either be set to be directed south-east or south-west.

Graphing method

Then, the value of the best angle is then inputted in FrisbeeMotionGraphing which will created five different plots : the first is for the actual trajectory of the frisbee for the best angle at which to throw the frisbee and the four others are for the changes in acceleration and velocity in x and y depending on the time.

4 Results

Our optimization model was tested by comparing our best throwing angles vs the test model's best angle. The result we obtained was .1738 rads which is equivalent to 9.96 Degrees which in relation to Kathleen's 10 degrees[2] gives us a .42% error. To test our graphing model we then graphed our best throwing angle against the test models best throwing angle. Our result has an 8% error

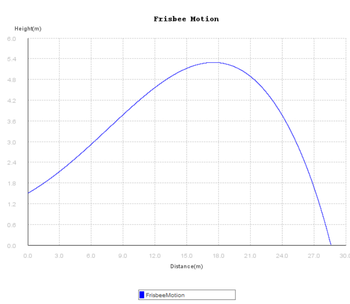

Figure 2: Graphing of Frisbee Motion in terms of Horizontal vs Vertical Distance

when compared to the test model. With our results verified, we proceeded to test and map out to further answer our question: What is best throwing angles to throw a frisbee for maximum range? As previously stated, the best angle to throw our frisbee was .1738 rads.

To further verify our results, we graphed out the final range in terms of the throwing angle. We discovered that even a slight adjustment to the throwing angle (less than .5 rads) resulted in a large increase in distance.

When taking into account wind speed, the motion of the frisbee does not change but best throwing angle does change slightly. Assuming the wind speed is

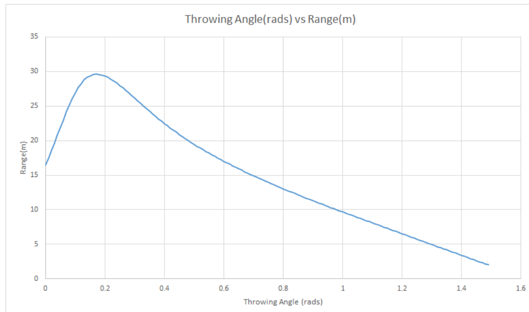

Figure 3: Graph depicting change in range in terms of the throwing angle.

32 mph and is acting on the frisbee at an angle of 45°the most optimal throwing angle becomes 9.45°.

5 Discussion

In comparison to the test model, both models agree that the best throwing angle for a frisbee is about 10 degrees. The maximum range we obtained from such a throw was larger than the test model due to our assumption that at the very instant the frisbee was thrown (t=0s), alpha was equal to 0. Additional sources of error may have been the use of a different methods and programs, the test model utilized Excel for their results and graphs while we used JAVA. Improvements to our model would be taking into account the roll, pitch and spin down moments, and wobble induced by thrower because these variables could affect the best angle to throw a frisbee. Furthermore, our model operates under the assumption that throwing angle and β are equal, which in most cases is not true.

References

[1] Kurt Andress. Wind loads. http://k7nv.com/notebook/topics/windload.html. Accessed: 2019-05-2.

[2] Kathleen Baumback. The aerodynamics of frisbee flight. *Undergraduate Journal of Mathematical Modeling: One+Two*, 3, 2010.

[3] Sarah Ann Hummel. *Frisbee flight simulation and throw biomechanics*. University of California, Davis, 2003.

[4] VR Morrison. The physics of frisbees. *Mount Allison University Physics Department*, 1, 2005.

[5] Birger Schultze. Biomechanical aspects of throwing a frisbee: A review, 2016.