MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.
   (a) To raise money in a way that doesn't increase the total amount of money supply in the economy to prevent inflation.
   (b) Stable inflation expectations and moderate growth implying reduced expectations for interest rates to change (risk) so investors demand less premium for longer term bonds vs shorter term bonds.
   (c) Quantitative easing is a strategy to reduce interest rates to stimulate the economy by using the central bank to purchase financial assets (including bonds) thereby increasing their demand and reducing the yield. Since the beginning of the COVID-19 pandemic, uncertainty in the economy led to panic selling and the surge of yields and interest rates. Using quantitative easing, the Fed reduced long-term yields thereby encouraging borrowing and spending.

2. These are the chosen bonds based on having the same coupon dates such that linear interpolation is not required. They have maturities on either March or September which makes it convenient to compute the spot rate curve in the next section.

   i. CAN 1.25 Mar 25
   ii. CAN 0.50 Sept 25
   iii. CAN 0.25 Mar 26
   iv. CAN 1.00 Sept 26
   v. CAN 1.25 Mar 27
   vi. CAN 2.75 Sept 27
   vii. CAN 3.50 Mar 28
   viii. CAN 3.25 Sept 28
   ix. CAN 4.00 Mar 29
   x. CAN 3.50 Sept 29
   xi. CAN 2.75 Mar 30

3. Eigenvalues represent the amount of variance captured by the corresponding eigenvector. Thus, the largest eigenvalue indicates that the corresponding eigenvector corresponds to a direction where the data varies the most in terms of euclidean distance. As the eigenvalues get smaller, their respective eigenvectors capture less and less variance. Plainly, eigenvectors with large eigenvalues represent characteristic patterns in the curve with subsequent eigenvectors representing less characteristic patterns.

Empirical Questions - 75 points

4. **In all of the following, I computed and used the dirty price as the bond tracker website provided clean prices.**
   (a) Yield-to-Maturity
   Since YTM is a non-linear equation, I used `scipy.fsolve` root finding method to find the root of the calculated PV after discounting coupon cash flows and the actual PV from the bond tracker website. The underlying root finding algorithm of `scipy.fsolve` is Powell's Hybrid Method [1].

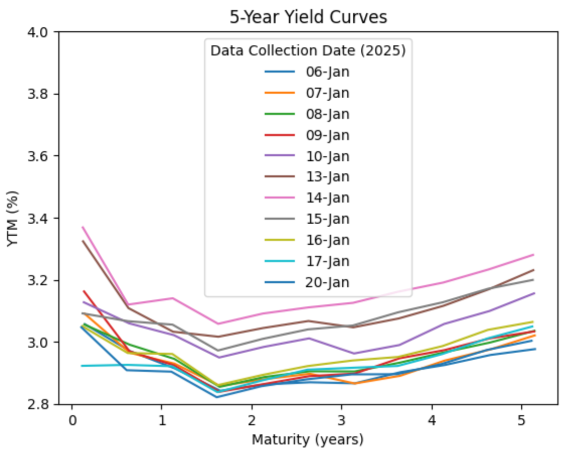

Figure 1: Yield-to-Maturity Curves

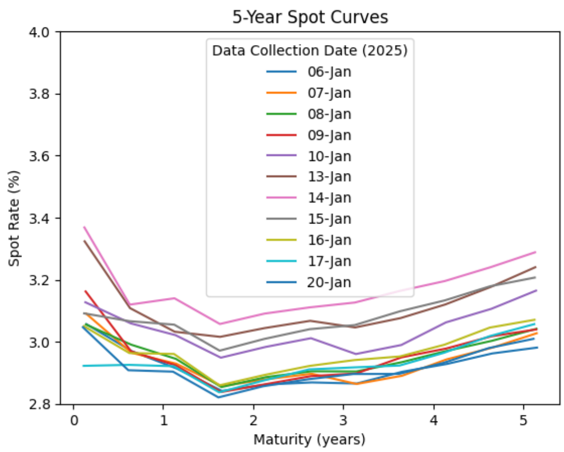

Figure 2: Spot Rate Curves

(b) Spot Rate

I carefully chose bonds so that when computing the spot rates, I did not need to linearly interpolate between known spot rates to discount coupons. I chose bonds that paid coupons in March and September.

Algorithm 1 Calculate Spot Rates

```
Initialize spot_rates_table as an empty table
Set face_value to 100
for each data_collection_date in data_collection_dates do
    for each bond in bonds_by_maturity do
        semi_annual_coupon = (bond.annual_coupon_rate * face_value) / 2
        PV = bond_table[bond, data_collection_date]
        if bond has multiple coupons then
            for each coupon in bond.coupons do
                spot_rate = spot_rates_table[coupon.date, data_collection_date]
                PV -= semi_annual_coupon * exp(-spot_rate * coupon.time_to_coupon)
        r = -log(PV / (face_value + semi_annual_coupon)) / bond.time_to_maturity
        spot_rates_table[bond.maturity, data_collection_date] = r
```

(c) Forward Rate

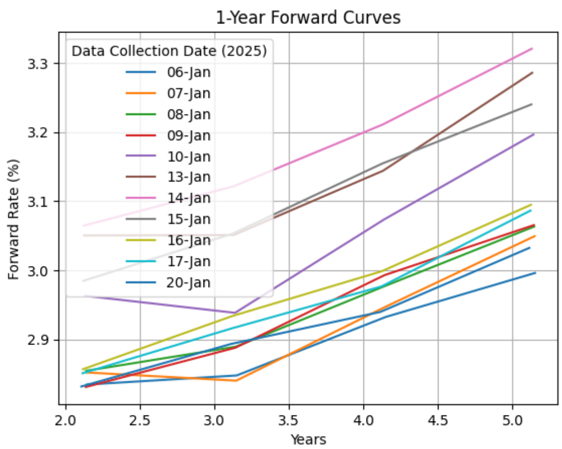

Figure 3: Forward Rate Curves

Algorithm 2 Calculate Forward Rates

```
Initialize forward_rates_table as an empty table
for each data_collection_date in data_collection_dates do
    year_1_bond = get_bond(data_collection_date, 1)
    year_1_bond_spot_rate = spot_rates_table[year_1_bond, data_collection_date]
    year_1_bond_time_to_maturity = year_1_bond.maturity - data_collection_date
    for each future_bond in 2_to_5_years_maturity_bonds do
        bond_spot_rate = spot_rates_table[future_bond, data_collection_date]
        bond_time_to_maturity = future_bond.maturity - data_collection_date
        forward_rates_table[future_bond, data_collection_date] =
              (((1 + bond_spot_rate) ** (bond_time_to_maturity + year_1_bond_time_to_maturity))
        / ((1 + year_1_bond_spot_rate) ** year_1_bond_time_to_maturity)) **
        (year_1_bond_time_to_maturity) - 1
```

5. Covariance Matrices

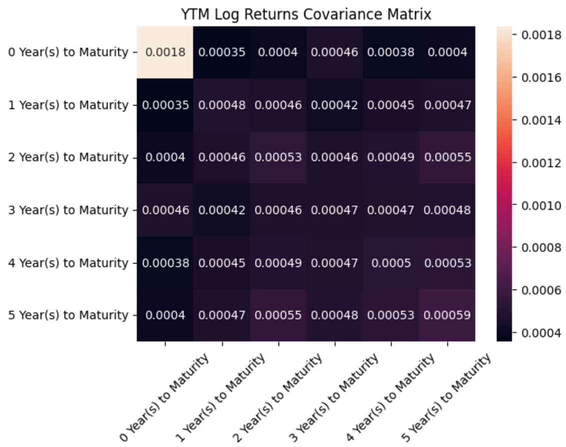

Figure 4: Yield-to-Maturity Log Returns Covariance Matrix

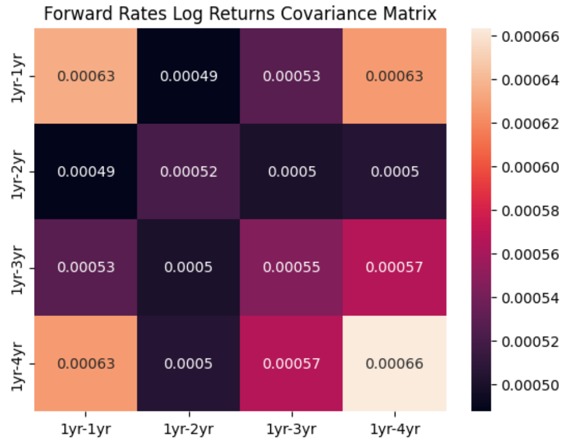

Figure 5: Forward Rates Log Returns Covariance Matrix

6. Eigenvalues & Eigenvectors of Covariance Matrices

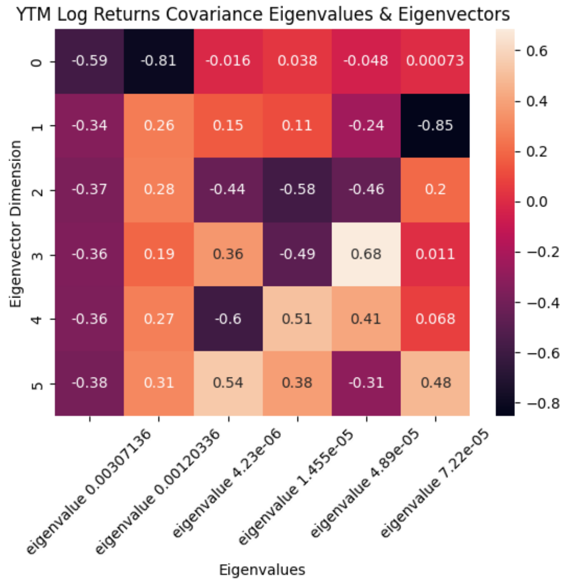

Figure 6: Yield-to-Maturity Log Returns Covariance Matrix Eigenvalues & Eigenvectors

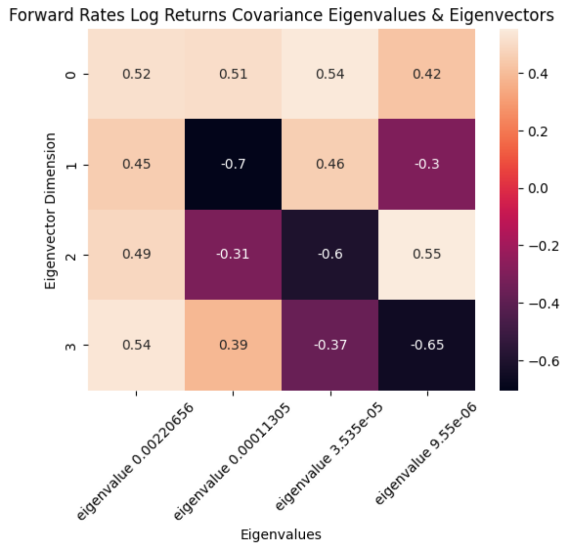

Figure 7: Forward Rates Log Returns Covariance Matrix Eigenvalues & Eigenvectors

The first eigenvector in both covariance matrices have an equal value in each dimension implying that the primary driver of day-to-day returns affects all instruments (bonds with different maturities) equally (likely indicative of a market index). Based on the magnitudes of the first eigenvalue, we can get the explained variance of the first eigenvector which is 69.6% and 94.0% for YTM and Forward Rates respectively implying that the "index" eigenvector accounts for the majority of the variance.

References and GitHub Link to Code

GitHub Link

https://github.com/Jonathan-Woo/APM466/blob/main/Assignment%201/assignment.ipynb

References

[1] Wikipedia, *Powell's dog leg method — Wikipedia, the free encyclopedia*, `http://en.wikipedia.org/w/index.php?title=Powell's%20dog%20leg%20method&oldid=1262814259`, [Online; accessed 01-February-2025], 2025.