a) Write down the gradient in closed form.

```
syms x y z
f1(x,y) = 2*x + 3*y + 1;
f2(x,y) = x^2 + y^2 - x*y - 5;
f3(x,y) = (x - 5)*cos(y-5) - (y - 5)*sin(x - 5);
grad1 = gradient(f1,[x,y])
```

```
grad1(x, y) =
```

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

```
grad2 = gradient(f2,[x,y])
```

```
grad2(x, y) =
```

$$\begin{pmatrix} 2\,x - y \\ 2\,y - x \end{pmatrix}$$

```
grad3 = gradient(f3,[x,y])
```

```
grad3(x, y) =
```

$$\begin{pmatrix} \cos(y-5) - \cos(x-5)\ (y-5) \\ -\sin(x-5) - \sin(y-5)\ (x-5) \end{pmatrix}$$

b) Produce 2-D contour plot for each function.

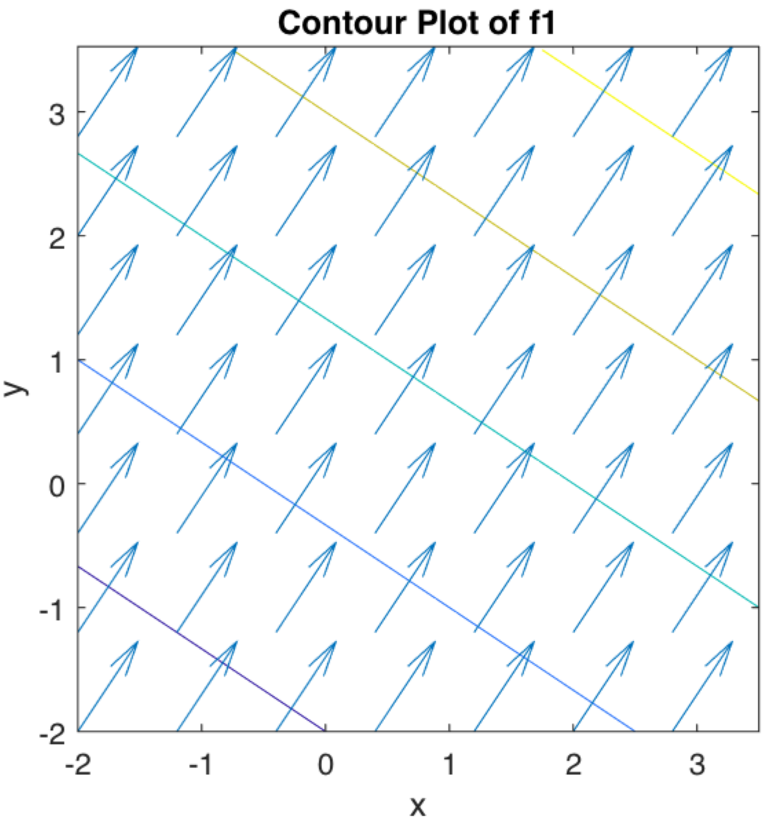
For f1:

```
x = linspace(-2, 3.5);
y = linspace(-2, 3.5);
[X,Y] = meshgrid(x,y);
Z = f1(X,Y);
contour(X,Y,Z)
hold on

spacing = 0.8;

x = -2:spacing:3.5;
y = -2:spacing:3.5;
[X,Y] = meshgrid(x,y);
Z = 2*X + 3*Y + 1;
[DX,DY] = gradient(Z, spacing);
quiver(X,Y,DX,DY)
axis equal
hold off

xlabel('x');
ylabel('y');
title('Contour Plot of f1');
```



```
%compute gradient at (1,0)
disp("Gradient at (1,0):")
```

```
Gradient at (1,0):
```

```
grad1(1,0)
```

```
ans =
```

$$\begin{pmatrix}2\\3\end{pmatrix}$$

For f2:

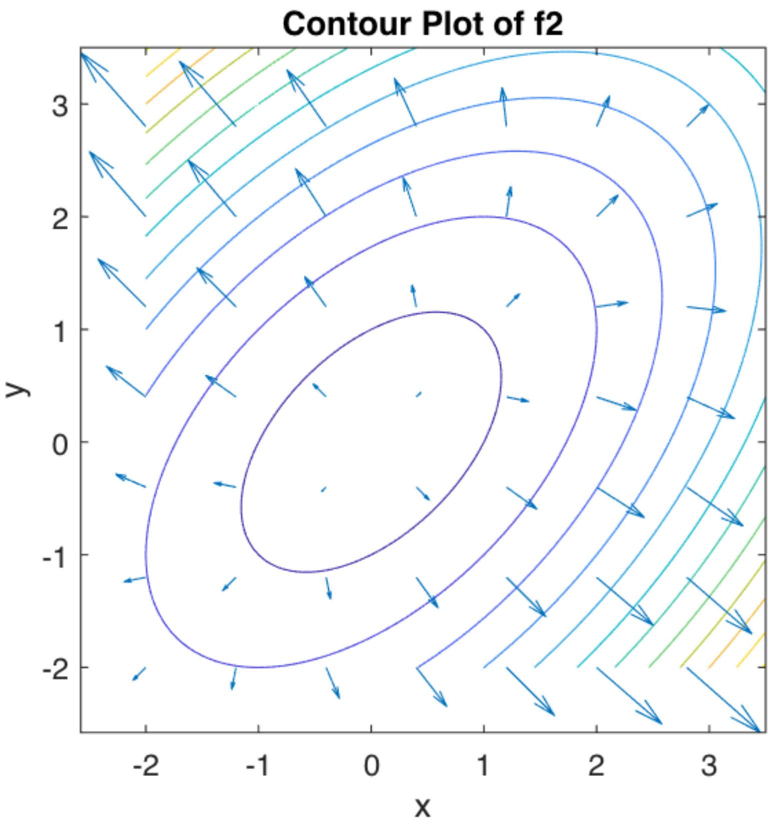
```
x = linspace(-2, 3.5);
y = linspace(-2, 3.5);
[X,Y] = meshgrid(x,y);
Z = f2(X,Y);
contour(X,Y,Z)
hold on

spacing = 0.8;

x = -2:spacing:3.5;
y = -2:spacing:3.5;
[X,Y] = meshgrid(x,y);
Z = X.^2 + Y.^2 - X.*Y - 5;
[DX,DY] = gradient(Z, spacing);
```

```
quiver(X,Y,DX,DY)
axis equal
hold off

xlabel('x');
ylabel('y');
title('Contour Plot of f2');
```



```
%compute gradient at (1,0)
disp("Gradient at (1,0):")
```

```
Gradient at (1,0):
```

```
grad2(1,0)
```

```
ans =
```

$$\begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

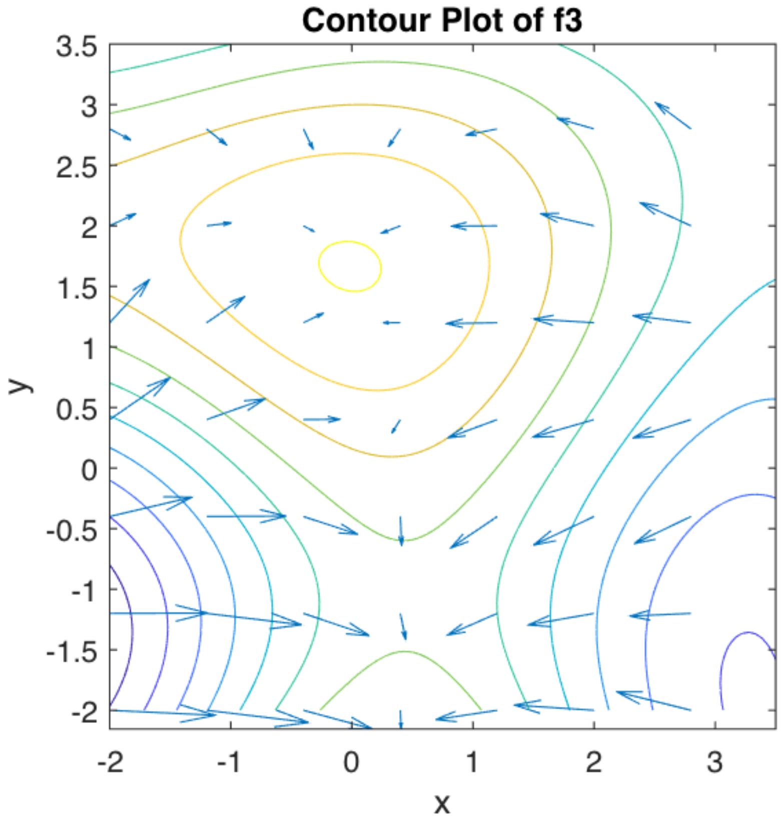
For f3:

```
x = linspace(-2, 3.5);
y = linspace(-2, 3.5);
[X,Y] = meshgrid(x,y);
Z = f3(X,Y);
contour(X,Y,Z)
hold on
```

```
spacing = 0.8;

x = -2:spacing:3.5;
y = -2:spacing:3.5;
[X,Y] = meshgrid(x,y);
Z = (X-Y).*cos(Y-5)-(Y-5).*sin(X-5);
[DX,DY] = gradient(Z, spacing);
quiver(X,Y,DX,DY)
axis equal
hold off

xlabel('x');
ylabel('y');
title('Contour Plot of f3');
```



```
%compute gradient at (1,0)
disp("Gradient at (1,0):")
```

```
Gradient at (1,0):
```

```
grad3(1,0)
```

```
ans =
```

$$\begin{pmatrix} 5\cos(4) + \cos(5) \\ \sin(4) - 4\sin(5) \end{pmatrix}$$

c) Plot 3-D linear approximation

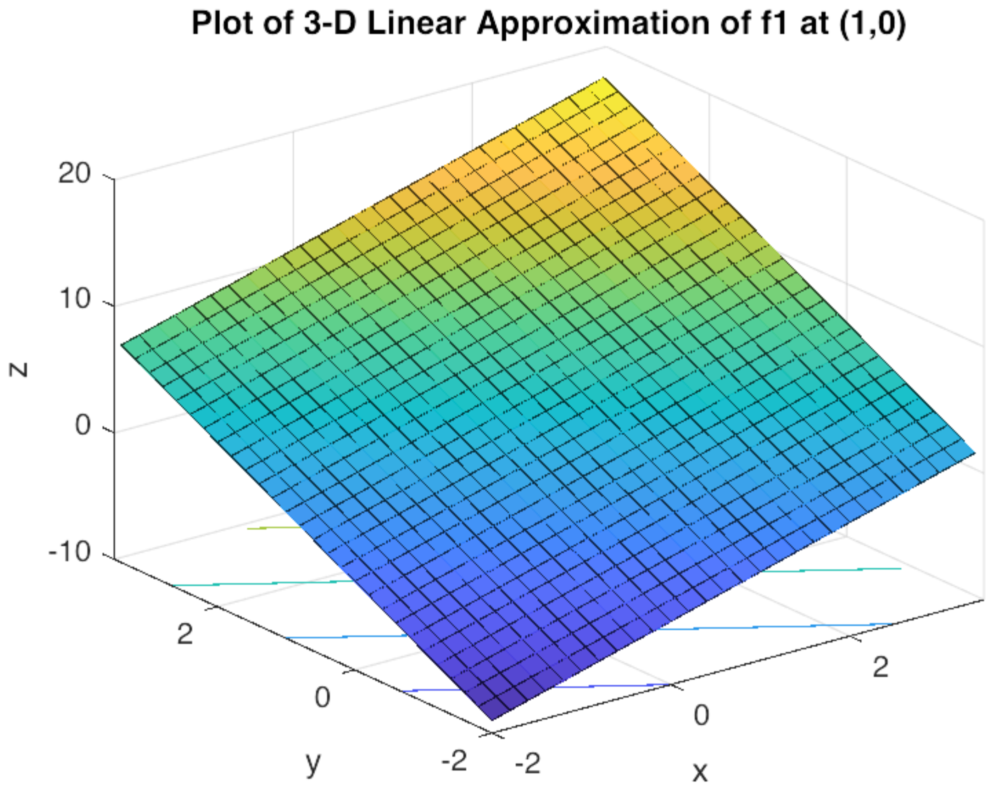
For f1:

```
f1 = @(x,y) 2.*x + 3.*y + 1;

spacing = 0.2;
[X,Y] = meshgrid(-2:spacing:3.5);
[dx,dy] = gradient(f1(X,Y),spacing);

x0 = 1;
y0 = 0;
t = (X == x0) & (Y == y0);
indt = find(t);
fx0 = dx(indt);
fy0 = dy(indt);

%compute tangent plane
f_approx = @(x,y) f1(x0,y0) + fx0*(x-x0) + fy0*(y-y0);
surf(X,Y,f_approx(X,Y),'EdgeAlpha',0.7,'FaceAlpha',0.9);
hold on
meshc(X,Y,f1(X,Y));
hold off
xlabel('x');
ylabel('y');
zlabel('z');
xlim([-2 3.5]);
ylim([-2 3.5]);
zlim([-10 20]);
title('Plot of 3-D Linear Approximation of f1 at (1,0)');
```



For f2:

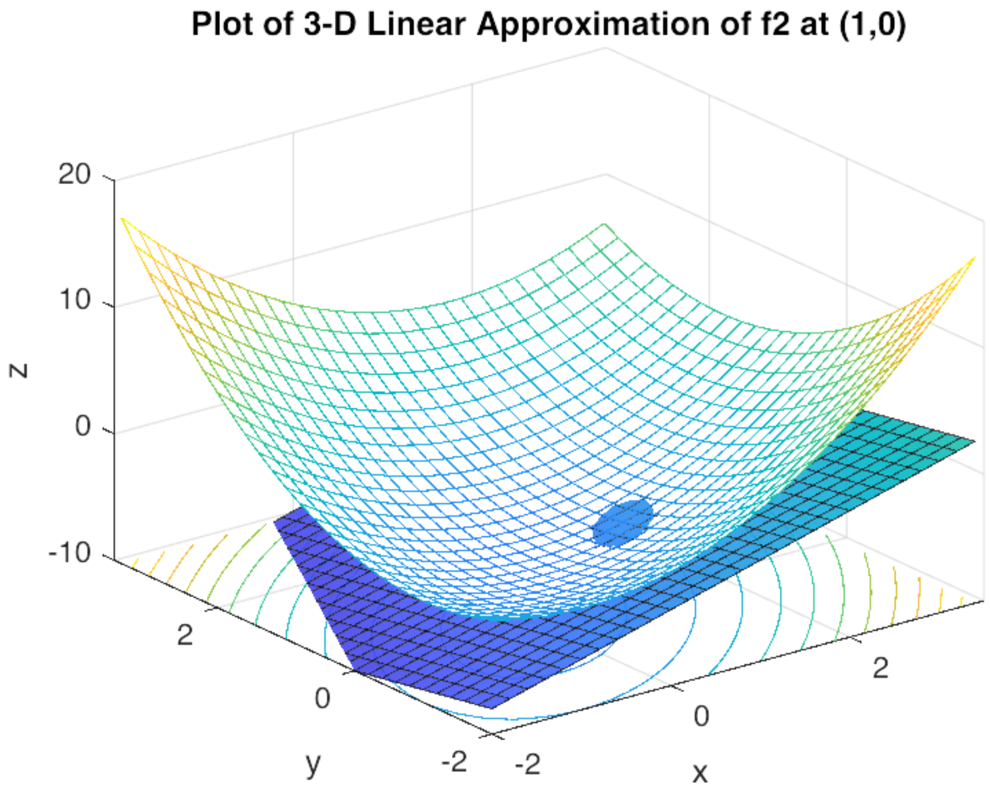
```
f2 = @(x,y) x.^2 + y.^2 - x.*y - 5;

spacing = 0.2;
[X,Y] = meshgrid(-2:spacing:3.5);
[dx,dy] = gradient(f2(X,Y),spacing);

x0 = 1;
y0 = 0;
t = (X == x0) & (Y == y0);
indt = find(t);
fx0 = dx(indt);
fy0 = dy(indt);

%compute tangent plane
f_approx = @(x,y) f2(x0,y0) + fx0*(x-x0) + fy0*(y-y0);
surf(X,Y,f_approx(X,Y),'EdgeAlpha',0.7,'FaceAlpha',0.9);
hold on
meshc(X,Y,f2(X,Y));
hold off
xlabel('x');
ylabel('y');
zlabel('z');
xlim([-2 3.5]);
ylim([-2 3.5]);
zlim([-10 20]);
```

```
title('Plot of 3-D Linear Approximation of f2 at (1,0)');
```



For f3:

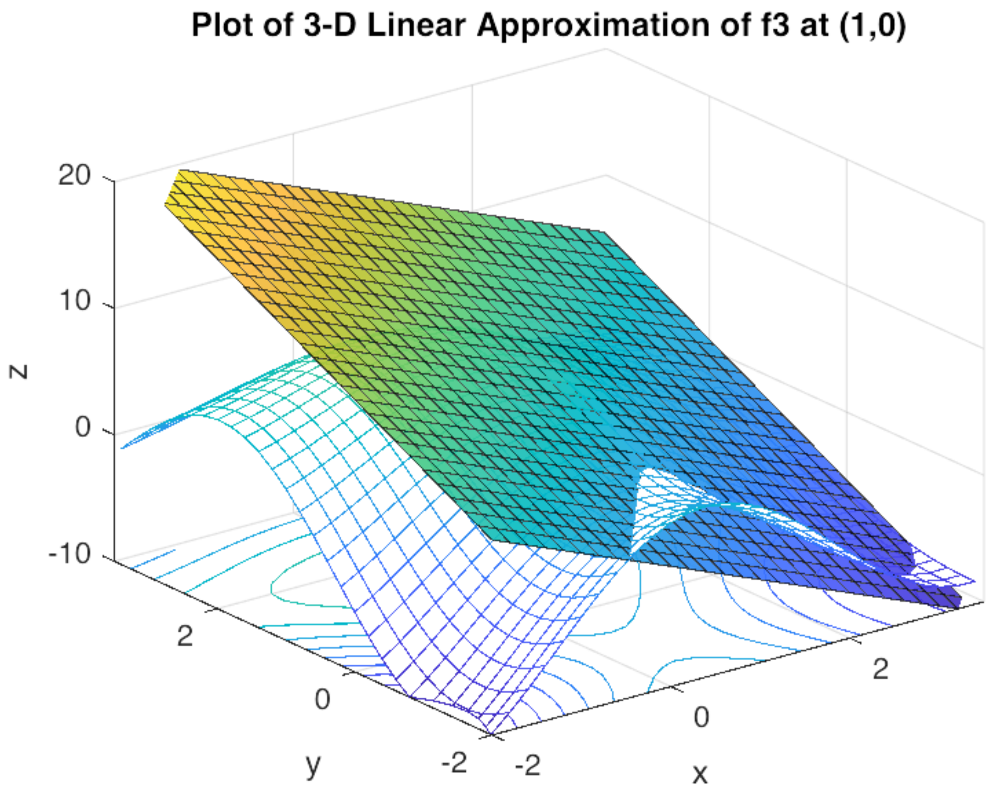
```
f3 = @(x,y) (x-5).*cos(y-5)-(y-5).*sin(x-5);

spacing = 0.2;
[X,Y] = meshgrid(-2:spacing:3.5);
[dx,dy] = gradient(f3(X,Y),spacing);

x0 = 1;
y0 = 0;
t = (X == x0) & (Y == y0);
indt = find(t);
fx0 = dx(indt);
fy0 = dy(indt);

%compute tangent plane
f_approx = @(x,y) f3(x0,y0) + fx0*(x-x0) + fy0*(y-y0);
surf(X,Y,f_approx(X,Y),'EdgeAlpha',0.7,'FaceAlpha',0.9);
hold on
meshc(X,Y,f3(X,Y));
hold off
xlabel('x');
ylabel('y');
zlabel('z');
xlim([-2 3.5]);
ylim([-2 3.5]);
```

```
zlim([-10 20]);
title('Plot of 3-D Linear Approximation of f3 at (1,0)');
```


Plot of 3-D Linear Approximation of f3 at (1,0)
z
y
x