ESC103H1F Group Assignment

Names of Group Members:

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Chan, Kwan Tung Nicole
Gupta, Stvya
Mendoza, Liam

Tutorial Session:

TUT 0112

Contributions:

As a group, we discussed the approach to the assignment and set up the framework including any required matrices and vectors in Google Sheets and preliminary code in MATLAB. From there, we divided the questions in the outline equally and combined the responses in the end into this document.

My contributions were to question 4 and 5. For those questions, I independently wrote live scripts on MATLAB to test.

Question 4 was the most challenging problem we encountered. Determining which combination of rows and equations to eliminate was difficult. I found the pattern after many iterations of trial and error. During those trials, key things I analysed were the rank of matrix $\boldsymbol{A}$ in $\boldsymbol{RNF}$ which would be *23* if appropriate rows were removed. Also, the solution to $\boldsymbol{A\vec{x}} = \boldsymbol{\vec{b}}$ after removing rows proved whether or not the row(s) removed were redundant or not. A different solution meant that too many rows were removed, or an improper combination of rows was removed. To remove rows in MATLAB, I indexed the row number and set each column value to an empty list ([]). This removed the respective row from the matrix. The same was done for vector b. Then, I used $\boldsymbol{rref}(\boldsymbol{A})$ to bring matrix A to $\boldsymbol{RNF}$ and used $\boldsymbol{rank}(\boldsymbol{A})$ to determine its rank. Finally, I used $\boldsymbol{mldivide}(\boldsymbol{A}, \boldsymbol{b})$ to solve for $\boldsymbol{\vec{x}}$ given $\boldsymbol{A\vec{x}} = \boldsymbol{\vec{b}}$.

For question 5, it was straightforward to solve the system of equations using the ***inv(*** $\boldsymbol{A_r}$***)*** * $\boldsymbol{b_r}$ function in MATLAB to solve for $\boldsymbol{\vec{x}}$. Since the solution matched that of question 3, it meant that the rows we removed were redundant.

Woo, Jonathan

I contributed by writing up our approach to questions 1 and 2. For question 1, I explained the implication of the equation $\mathrm{A}\vec{\mathrm{x}} = \vec{\mathrm{b}}$, and specifically included matrices to aid my description on it so that important features could be clearly seen – the number of columns in A must equal to the number of rows in $\vec{\mathrm{x}}$. I labelled the joints in the truss bridge provided in CIV102 Assignment 5 Q2, and listed out the vectors of unknowns, $\vec{\mathrm{x}}$, with a size of 23×1. In question 2, I filled in a table for joint forces that are split into x y components against forces of members $(\mathrm{e.g.\,AB, AC, BC, \dots\, LM})$. After inputting matrix A and $\vec{\mathrm{b}}$ into MATLAB, I could find $\vec{\mathrm{x}}$ using the $\mathrm{mldivide(A, b)}$ function and also the reduced normal form as well as the rank of A using built-in functions in MATLAB like $\mathrm{rref(A)}$ and $\mathrm{rank(A)}$. Lastly, I am responsible for formatting the whole document.

Chan, Kwan Tung Nicole

I wrote up our group's approach to question 3. Getting $\vec{x}$ from $[R|d]$ was a straightforward task. Upon comparing the results to those obtained from the methods learned in CIV102, I found that the signs for vector $\vec{x}$ were the opposite of what they should be. We realized that our assumption regarding the signs was wrong, and we went back and fixed $\vec{b}$. With this change, the results from the matrix approach to solve a truss concurred with the methods utilized in CIV102.

Gupta, Stvya

Worked together with the team during the overall process of using MATLAB to answer the given questions. Once we had developed the answers as a team, I focused on preparing the solution to questions 6 and 7. More specifically for question 6 I analysed the effects of changing the evenly distributed load. The only changes in $[\, A_{-r} \mid b_{-r} \,]$ were to the br vector since it is directly proportional to the load. Since the geometry of the truss stayed the same and only the load increased, I concluded that the forces in each member would increase proportionally to the increase in load. Then I used the mldivide function in MATLAB to solve for the new x which confirmed my conclusion as the values were the same as using the proportionality method. For question 7 I thought about how the forces in member FH are calculated. FH is the $12th$ row in the x vector so its calculation is the $12th$ row of the inverse of matrix A_{-r} dotted with matrix b_{-r}. I wrote out this dot product and removed all the 0 values to come up with an equation. Finally, I had a solution for the maximum force which could be manipulated to solve for x with different set max forces ($2000kN$ in this case).

Mendoza, Liam

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Question One

The system of equations are expressed in the form $\boldsymbol{A}\vec{\boldsymbol{x}} = \vec{\boldsymbol{b}}$, where $\boldsymbol{A}$ is the coefficient matrix, $\vec{\boldsymbol{x}}$ is the vector of unknowns and $\vec{\boldsymbol{b}}$ is the constant vector. In matrix multiplication, the number of columns in $\boldsymbol{A}$ must equal to the number of rows in $\vec{\boldsymbol{x}}$. Below shows elements like a_{11}, 11 represents the element at the 1st row and 1st column. Similarly, a_{mn} represents the element at the mth row and nth column.

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{123} & a_{2n} & \cdots & a_{mn} \end{bmatrix} \qquad \vec{\boldsymbol{x}} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{23} \end{bmatrix} \qquad \vec{\boldsymbol{b}} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

$$\text{Therefore, } \boldsymbol{A}\vec{\boldsymbol{x}} = \vec{\boldsymbol{b}} \ is \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{123} & a_{2n} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{23} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

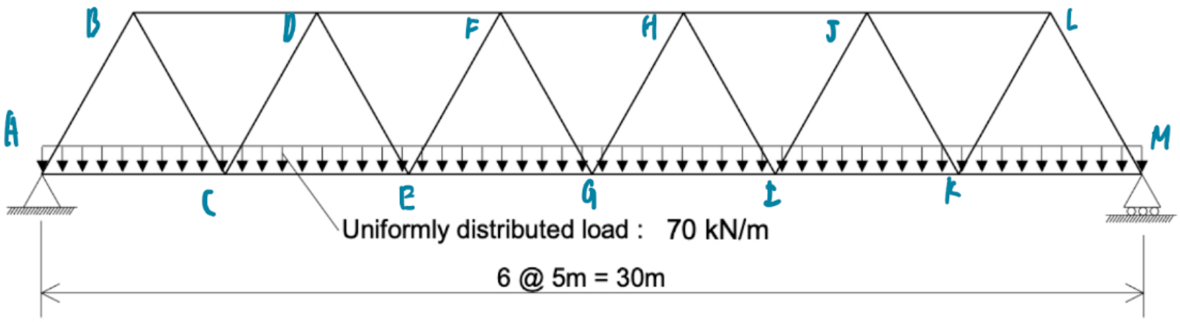
Listing the vectors of unknowns $\vec{\boldsymbol{x}}$, starting with $\overrightarrow{\boldsymbol{AB}}$ to $\overrightarrow{\boldsymbol{LM}}$:

$\vec{\boldsymbol{x}}$ **:**

$\overrightarrow{AB}$
$\overrightarrow{AC}$
$\overrightarrow{BC}$
$\overrightarrow{BD}$
$\overrightarrow{CD}$
$\overrightarrow{CE}$
$\overrightarrow{DE}$
$\overrightarrow{DF}$
$\overrightarrow{EF}$
$\overrightarrow{EG}$
$\overrightarrow{FG}$
$\overrightarrow{FH}$
$\overrightarrow{GH}$
$\overrightarrow{GI}$
$\overrightarrow{HI}$
$\overrightarrow{HJ}$
$\overrightarrow{IJ}$
$\overrightarrow{IK}$
$\overrightarrow{JK}$
$\overrightarrow{JL}$
$\overrightarrow{KL}$
$\overrightarrow{KM}$
$\overrightarrow{LM}$

Question Two

From the truss bridge below, there are 13 joints and each could produce 2 equations from the x and y direction. Therefore, the size of the coefficient matrix A is in the size 26×23, and the size of $\vec{\boldsymbol{b}}$ is 26×1.



$\vec{\boldsymbol{b}}$:

0
-875
0
0
0
350
0
0
0
350
0
0
0
350
0
0
0
350
0
0
0
350
0
0
0
-875

AB	AC	BC	BD	CD	CE	DE	DF	EF	EG	FG	FH
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0.5	1	0	0	0	0	0	0	0	0	0	0
-0.866	0	0	0	0	0	0	0	0	0	0	0
-0.5	0	0.5	1	0	0	0	0	0	0	0	0
0.866	0	0.866	0	0	0	0	0	0	0	0	0
0	-1	-0.5	0	0.5	1	0	0	0	0	0	0
0	0	-0.866	0	-0.866	0	0	0	0	0	0	0
0	0	0	-1	-0.5	0	0.5	1	0	0	0	0
0	0	0	0	0.866	0	0.866	0	0	0	0	0
0	0	0	0	0	-1	-0.5	0	0.5	1	0	0
0	0	0	0	0	0	-0.866	0	-0.866	0	0	0
0	0	0	0	0	0	0	-1	-0.5	0	0.5	1
0	0	0	0	0	0	0	0	0.866	0	0.866	0
0	0	0	0	0	0	0	0	0	-1	-0.5	0
0	0	0	0	0	0	0	0	0	0	-0.866	0
0	0	0	0	0	0	0	0	0	0	0	-1
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

GH	GI	HI	HJ	IJ	IK	JK	JL	KL	KM	LM
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0.5	1	0	0	0	0	0	0	0	0	0
0.866	0	0	0	0	0	0	0	0	0	0
-0.5	0	0.5	1	0	0	0	0	0	0	0
-0.866	0	-0.866	0	0	0	0	0	0	0	0
0	-1	-0.5	0	0.5	1	0	0	0	0	0
0	0	0.866	0	0.866	0	0	0	0	0	0
0	0	0	-1	-0.5	0	0.5	1	0	0	0
0	0	0	0	-0.866	0	-0.866	0	0	0	0
0	0	0	0	0	-1	-0.5	0	0.5	1	0
0	0	0	0	0	0	0.866	0	0.866	0	0
0	0	0	0	0	0	0	-1	-0.5	0	0.5
0	0	0	0	0	0	0	0	-0.866	0	-0.866
0	0	0	0	0	0	0	0	0	-1	-0.5
0	0	0	0	0	0	0	0	0	0	0.866

First, we find the values of matrix $\boldsymbol{A}$. We have the forces (***AB, AC, BC, … , LM***) horizontally at the top and the joint forces split into $\boldsymbol{x}$ and $\boldsymbol{y}$ components vertically.

For example, we could calculate joint force $\boldsymbol{Ax}$ with force $\boldsymbol{AB}$ by looking at the truss bridge diagram. Looking at the force $\boldsymbol{AB}$, we could split it into $\boldsymbol{x}$ and $\boldsymbol{y}$ components. Since the horizontal component of $\boldsymbol{AB}$ contributes to $\boldsymbol{A_x}$, we determined the component and associated it with the equation in matrix $\boldsymbol{A}$:

$$cos\,(60°) \;=\; 0.5$$

Another x-component force acting on joint $\boldsymbol{A}$ is member $\boldsymbol{AC}$. It is a horizontal force, so we do not need to isolate it into its components, it is purely in the x-component.

For the joint force Ay, we only consider force $\boldsymbol{AB}$ in the y direction.

$$sin\,(60°) = \frac{\sqrt{3}}{2} \;=\; 0.8660254038$$

After filling in the whole table from joint force $\boldsymbol{A_x}$ to $\boldsymbol{M_y}$, we input matrix $\boldsymbol{A}$ and $\vec{\boldsymbol{b}}$ that we have calculated into MATLAB. Negative values in the table means that it's either going to the left (x component) or down (y component).

	Forces	fAB	fAC	fBC	fBD	fCD	fCE	fDE	fDF	fEF	fEG	fFG	fFH	fGH	fGI	fHI	fHJ	fIJ	fIK	fJK	fJL	fKL	fKM	fLM
Joint Forces																								
Ax		0.5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ay		0.8660254038	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Bx		-0.5	0	0.5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
By		-0.8660254038	0	-0.8660254038	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cx		0	-1	-0.5	0	0.5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cy		0	0	0.8660254038	0	0.8660254038	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Dx		0	0	0	-1	-0.5	0	0.5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Dy		0	0	0	0	-0.8660254038	0	-0.8660254038	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ex		0	0	0	0	0	-1	-0.5	0	0.5	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Ey		0	0	0	0	0	0	0.8660254038	0	0.8660254038	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Fx		0	0	0	0	0	0	0	-1	-0.5	0	0.5	1	0	0	0	0	0	0	0	0	0	0	0
Fy		0	0	0	0	0	0	0	0	-0.8660254038	0	-0.8660254038	0	0	0	0	0	0	0	0	0	0	0	0
Gx		0	0	0	0	0	0	0	0	0	-1	-0.5	0	0.5	1	0	0	0	0	0	0	0	0	0
Gy		0	0	0	0	0	0	0	0	0	0	0.8660254038	0	0.8660254038	0	0	0	0	0	0	0	0	0	0
Hx		0	0	0	0	0	0	0	0	0	0	0	-1	-0.5	0	0.5	1	0	0	0	0	0	0	0
Hy		0	0	0	0	0	0	0	0	0	0	0	0	-0.8660254038	0	-0.8660254038	0	0	0	0	0	0	0	0
Ix		0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-0.5	0	0.5	1	0	0	0	0	0
Iy		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8660254038	0	0.8660254038	0	0	0	0	0	0
Jx		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-0.5	0	0.5	1	0	0	0
Jy		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.8660254038	0	-0.8660254038	0	0	0	0
Kx		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-0.5	0	0.5	1	0
Ky		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8660254038	0	0.8660254038	0	0
Lx		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-0.5	0	0.5
Ly		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.8660254038	0	-0.8660254038
Mx		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-0.5
My		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.8660254038

We use $\boldsymbol{x} \;=\; \boldsymbol{mldivide}\,(\boldsymbol{a}, \boldsymbol{b})$ to find $\boldsymbol{x}$ and use the built-in MATLAB function $\boldsymbol{rref}(\boldsymbol{A})$ to bring matrix $\boldsymbol{A}$ to RNF in order to find the rank of matrix A by using ***rank(A)***.

```
A = [0.5 1  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
b = [0 ; -875 ; 0 ; 0 ; 0 ; 350 ; 0 ; 0 ; 0 ; 350 ; 0 ; 0 ; 0 ; 350 ; 0 ; 0
x = mldivide (A,b)
A*x
rref(A)
rank(A)
```

Question Three

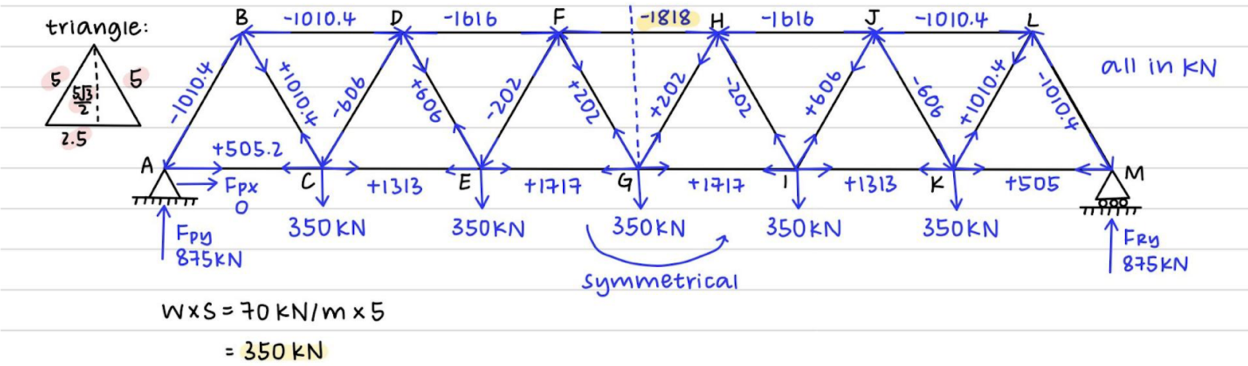
The vector which contains the unknown forces, $\vec{x}$, is the last column of the $[\boldsymbol{R}|\boldsymbol{d}]$ matrix obtained in the previous part. We can get rid of the three bottom rows as they all contain zero.

Then, $\vec{x}$ is a 23×1 vector as follows:

$\vec{x}$:

-1010.362971
505.1814855
1010.362971
-1010.362971
-606.2177826
1313.471862
606.2177826
-1616.580754
-202.0725942
1717.617051
202.0725942
-1818.653348
202.0725942
1717.617051
-202.0725942
-1616.580754
606.2177826
1313.471862
-606.2177826
-1010.362971
1010.362971
505.1814855

The values for the member forces obtained using this method agree with the values obtained using the method of joints and the method of sections.



Question Four

Member forces are determined by solving the system $\boldsymbol{A}\vec{\boldsymbol{x}} = \vec{\boldsymbol{b}}$ for $\vec{\boldsymbol{x}}$. Redundant rows in matrix $\boldsymbol{A}$ means that there are duplicate equations in matrix $\boldsymbol{A}$ which are eliminated to be a row of zeros after Gaussian elimination. In our case, we removed the $\boldsymbol{x}$ and $\boldsymbol{y}$ component at joint $\boldsymbol{A}$ and the $\boldsymbol{x}$ component force at joint $\boldsymbol{B}$.

$$\boldsymbol{x - component}:$$

$$0.5AB + AC = 0$$
$$-0.5AB + 0.5BC + BD = 0$$

$$\boldsymbol{y - component}:$$

$$\frac{\sqrt{3}}{2}AB = -875$$

Upon further testing, we found that removing any combination of 3 joint forces so as long as 2 *x*-component forces are removed and 1 *y*-component force is removed will result in a solution where A has the same rank (23) and $\boldsymbol{A}\vec{\boldsymbol{x}} = \vec{\boldsymbol{b}}$ has the same solution for *x*.

Question Five

Solving for $\vec{x}$ shows that the solution matches that of part 3 despite different A_r matrices and b_r vectors. This proves that the 3 rows we chose to remove were redundant since the solution was unchanged.

Question 5: Using the modified augmented matrix with three rows removed that you found in part 4, the coefficient matrix should now be square, i.e. 23 x 23, and the constant vector should now be 23 x 1, with vector x unchanged. Let us refer to this modified augmented matrix as [Ar|br]. Find the inverse of this square coefficient matrix Ar and use it to find the solution x = A^-1br.

```
mldivide(A_r, b_r)
```

$A_{\_r}$:

AB	AC	BC	BD	CD	CE	DE	DF	EF	EG	FG	FH
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0.5	1	0	0	0	0	0	0	0	0	0	0
-0.866	0	0	0	0	0	0	0	0	0	0	0
-0.5	0	0.5	1	0	0	0	0	0	0	0	0
0.866	0	0.866	0	0	0	0	0	0	0	0	0
0	-1	-0.5	0	0.5	1	0	0	0	0	0	0
0	0	-0.866	0	-0.866	0	0	0	0	0	0	0
0	0	0	-1	-0.5	0	0.5	1	0	0	0	0
0	0	0	0	0.866	0	0.866	0	0	0	0	0
0	0	0	0	0	-1	-0.5	0	0.5	1	0	0
0	0	0	0	0	0	-0.866	0	-0.866	0	0	0
0	0	0	0	0	0	0	-1	-0.5	0	0.5	1
0	0	0	0	0	0	0	0	0.866	0	0.866	0
0	0	0	0	0	0	0	0	0	-1	-0.5	0
0	0	0	0	0	0	0	0	0	0	-0.866	0
0	0	0	0	0	0	0	0	0	0	0	-1
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

GH	GI	HI	HJ	IJ	IK	JK	JL	KL	KM	LM
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0.5	1	0	0	0	0	0	0	0	0	0
0.866	0	0	0	0	0	0	0	0	0	0
-0.5	0	0.5	1	0	0	0	0	0	0	0
-0.866	0	-0.866	0	0	0	0	0	0	0	0
0	-1	-0.5	0	0.5	1	0	0	0	0	0
0	0	0.866	0	0.866	0	0	0	0	0	0
0	0	0	-1	-0.5	0	0.5	1	0	0	0
0	0	0	0	-0.866	0	-0.866	0	0	0	0
0	0	0	0	0	-1	-0.5	0	0.5	1	0
0	0	0	0	0	0	0.866	0	0.866	0	0
0	0	0	0	0	0	0	-1	-0.5	0	0.5
0	0	0	0	0	0	0	0	-0.866	0	-0.866
0	0	0	0	0	0	0	0	0	-1	-0.5
0	0	0	0	0	0	0	0	0	0	0.866

***b_r*:**

0
-875
0
0
0
350
0
0
0
350
0
0
0
350
0
0
0
350

0
0
0
350
0
0
0
-875

***x_r*:**

-1010.362971
505.1814855
1010.362971
-1010.362971
-606.2177826
1313.471862
606.2177826
-1616.580754
-202.0725942
1717.617051
202.0725942
-1818.653348
202.0725942
1717.617051
-202.0725942
-1616.580754
606.2177826
1313.471862
-606.2177826
-1010.362971
1010.362971
505.1814855

Question Six

When changing the uniformly distributed load of the truss the only changes in $[\,\boldsymbol{A_r} \mid \boldsymbol{b_r}]$ are due to the $\boldsymbol{b_r}$ term. The $\boldsymbol{A_r}$ term represents the ratios of $\boldsymbol{x}$ and $\boldsymbol{y}$ forces in the various members. Since the ratios are based on the trigonometry of the truss and there are no changes to the dimensions of the bridge the $\boldsymbol{A_r}$ remains unchanged. The $\boldsymbol{b_r}$ matrix however is directly dependant on the external load and is made up of the external load at the various joints. Before the change in the load $\boldsymbol{b_{r}\_70}$ is:

$\boldsymbol{b_{r}\_70}$ (23×1) :

0
0
350
0
0
0
350
0
0
0
350
0
0
0
350
0
0
0
350
0
0
0
-875

The $\mathbf{350}$ value represents the external load going down at the bottom middle joints (C, E, G, I, K) and the $\mathbf{-875}$ represents the load at the end joints (Only $\boldsymbol{M}$ since rows $\boldsymbol{Ay}$ and $\boldsymbol{Ax}$ were previously removed). The negative indicating it opposes the direction of the middle joints.

For the $\mathbf{90}\ \boldsymbol{kN/m}$ distributed load these joint forces were re-calculated using the Civ102 method used to get the $\mathbf{70}\ \boldsymbol{kN/m}$ values.

$$(5m)(90kN/m) = 450kN\ (middle\ joints)$$

$$\frac{(5\ middle\ joints)(450kN)}{2\ (half\ of\ each\ end\ joint)} = 1125kN$$

So, the new matrix $\boldsymbol{b_r}$ worked out to be:

$\boldsymbol{b_{r\_}90}$:

0
0
450
0
0
0
450
0
0
0
450
0
0
0
450
0
0
0
450
0
0
0
-1125

Now taking [$\boldsymbol{A_r}$ | $\boldsymbol{b_r\_90}$] the result follows as (column 1-12 first page, 13-24 second page):

AB	AC	BC	BD	CD	CE	DE	DF	EF	EG	FG	FH
0.8660254038	0	0.8660254038	0	0	0	0	0	0	0	0	0
0	-1	-0.5	0	0.5	1	0	0	0	0	0	0
0	0	0.8660254038	0	0.8660254038	0	0	0	0	0	0	0
0	0	0	-1	-0.5	0	0.5	1	0	0	0	0
0	0	0	0	-0.8660254038	0	-0.8660254038	0	0	0	0	0
0	0	0	0	0	-1	-0.5	0	0.5	1	0	0
0	0	0	0	0	0	0.8660254038	0	0.8660254038	0	0	0
0	0	0	0	0	0	0	-1	-0.5	0	0.5	1
0	0	0	0	0	0	0	0	-0.8660254038	0	-0.8660254038	0
0	0	0	0	0	0	0	0	0	-1	-0.5	0
0	0	0	0	0	0	0	0	0	0	0.8660254038	0
0	0	0	0	0	0	0	0	0	0	0	-1
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

GH	GI	HI	HJ	IJ	IK	JK	JL	KL	KM	LM	b
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	450
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	450
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0.5	1	0	0	0	0	0	0	0	0	0	0
0.8660254038	0	0	0	0	0	0	0	0	0	0	450
-0.5	0	0.5	1	0	0	0	0	0	0	0	0
-0.8660254038	0	-0.8660254038	0	0	0	0	0	0	0	0	0
0	-1	-0.5	0	0.5	1	0	0	0	0	0	0
0	0	0.8660254038	0	0.8660254038	0	0	0	0	0	0	450
0	0	0	-1	-0.5	0	0.5	1	0	0	0	0
0	0	0	0	-0.8660254038	0	-0.8660254038	0	0	0	0	0
0	0	0	0	0	-1	-0.5	0	0.5	1	0	0
0	0	0	0	0	0	0.8660254038	0	0.8660254038	0	0	450
0	0	0	0	0	0	0	-1	-0.5	0	0.5	0
0	0	0	0	0	0	0	0	-0.8660254038	0	-0.8660254038	0
0	0	0	0	0	0	0	0	0	-1	-0.5	0
0	0	0	0	0	0	0	0	0	0	0.8660254038	-1125

A way to solve $[\,\boldsymbol{A_r} \mid \boldsymbol{b_r}\_\boldsymbol{90}]$ would be to consider the ration of $\mathbf{90/70}$. Since every term in $\boldsymbol{b_r}\_\mathbf{90}$ is relative to br through the ratios of the uniformly distributed load the answer for $\vec{\boldsymbol{x}}\_\mathbf{90}$ will be relative to the ratio as well. In this case, $\vec{\boldsymbol{x}}\_\mathbf{90}$ was equal to $(\mathbf{90/70}) * \vec{\boldsymbol{x}}\_\mathbf{70}$. This result was confirmed by solving the above matrix through ***mldivide(*** $\boldsymbol{A_r}$***,*** $\boldsymbol{b_r}\_\mathbf{90}$***)*** and obtaining the same $\vec{\boldsymbol{x}}\_\mathbf{90}$ as through the ratio method.

Question Seven

The maximum force will come from the dot product of $\boldsymbol{A_r}^{-1}$ (23×23) and $\boldsymbol{b_r}$ (23×1) at the row corresponding to $\boldsymbol{FH}$. The row that corresponds to $\boldsymbol{FH}$ is row 12 since that is where $\boldsymbol{FH}$ appears in the $\vec{\boldsymbol{x}}$ matrix.

Row 12 of $\boldsymbol{A_r}^{-1}$ is:

0	0	0	0	0	0	0	0	0	0	0	-1	0.5774	0	1.1547	-1	1.7321	0	2.3094	-1	2.8868	0	3.4641

And the dot product with $\boldsymbol{b_r}$:

0
0
-350
0
0
0
-350
0
0
0
-350
0
0
0
-350
0
0
0
-350
0
0
0
875

Results in the equation (0 components removed):

$$1.1547(350) + 2.31(350) - 3.4641(875)$$
$$= -1818.7\ (solved\ value\ for\ FH\ previously)$$

The values in the brackets will change as the distributed load changes therefore they can be assigned to variables.

$$Let\ x = uniform\ load$$

Now using the same method as Civ102 to calculated external forces due to the load:

$$(5m)\ x = joint\ load\ (middle)$$
$$\frac{(5\ joints)(5x)}{2(2\ end\ loads)} = \frac{25x}{2} = end\ loads$$

Therefore the equation can be written as,

$$1.1547(5x) + 2.31(5x) - 3.4641\left(\frac{25x}{2}\right) = max\ compressive\ force\ (force\ in\ FH)$$

Now inputting $2000kN$ (compression so -) as the maximum load the equation works out,

$$-25.97775\,x = -2000$$
$$x = 77.0\ kN/m\ (to\ three\ decimal\ places)$$

The maximum distributed load so that the maximum compressive force does not exceed $2000kN$ is slightly less than $77kN/m$.

Conclusion

From this project, we learned that properties of a system could be solved using a system of equations given sufficient information. Previously, to analyse this truss, we used many intermediate steps. However, this assignment shows that with the use of relationships, dependencies, and equations relating values to one another, we can simplify intermediate steps. In this particular case, we found member forces through their relationship with individual component forces. Instead of solving each component force individually, we found equations relating the member forces to the component forces so that we could go from a vector of external forces b to member forces directly.

Furthermore, this shows how linear algebra can be expanded to all facets as many situations can be expressed with a system of equations. It can be utilized to solve all sorts of problems such as truss bridge problems in CIV102. We learned how the courses are all interconnected so we could use MATLAB in the future for different purposes other than ESC103.

We also learned how to cooperate with each other within the group and to ask for each other's help when we got stuck – each of us provided suggestions on how we should approach the question. For example, we did the table in Question 2 together and double checked our answers to ensure that we got the values in every cell right.

Along with the work of the assignment, the experience of working as a team was a significant lesson. A member of our group was in a different time zone and we had to coordinate around a 13 hour time difference. This was further compounded by the full schedules that each of us had. Yet, we were still able to organize by coordinating well and being flexible. Overall, this group assignment is a valuable experience for all of us and we learnt a lot from it.

Appendix

Link to google doc:
https://docs.google.com/document/d/1iRwvDYBNv2kKOfm_QaxQ0y_cmCWLyE3Dk_ezSqaK590/edit

Link to google sheets:
https://docs.google.com/spreadsheets/d/1wfyj9wWE_fjFEUS2DUe480PEUTBHyp-VaRkyJ3a7uqI/edit#gid=0