ESC103F Group Assignment

The overall objective of this assignment is to allow students to see how $A\vec{x} = \vec{b}$ can be used to formulate and analyze a real engineering problem. The specific objective is to solve for the forces in the members of the Warren truss found in your CIV102F Assignment #5, Question 2, where $\overrightarrow{AB}$ is the force in kN carried in the member which connects joints A and B, with tension denoted as positive and compression denoted as negative. Treat the leftmost joint as A and the rightmost joint as M.

1. Express the system of equations to be solved in the form $A\vec{x} = \vec{b}$. In the vector of unknowns $\vec{x}$, start with $\overrightarrow{AB}$ and end with $\overrightarrow{LM}$, so you end up with a vector of unknowns that is $23x1$ because there are 23 members:
$$\vec{x} = \begin{bmatrix} \overrightarrow{AB} \\ \overrightarrow{AC} \\ \overrightarrow{BC} \\ \overrightarrow{BD} \\ \overrightarrow{CD} \\ \vdots \\ \overrightarrow{LM} \end{bmatrix}$$
2. The size of the coefficient matrix A should be $26x23$ because there are 13 joints that produce two equations each from the x and y static force balances at each joint. The size of the constant vector $\vec{b}$ should be $26x1$. Find the true size of this matrix ($r = rank\ A$) by bringing $\left[A|\vec{b}\right]$ to its reduced normal form $\left[R|\vec{d}\right]$ using the Gaussian elimination algorithm (you may use the built-in MATLAB function rref).
3. From the reduced normal form $\left[R|\vec{d}\right]$ obtained in part 2, present the solution to the system with $\vec{x}$ in vector form and compare this to the solution you obtained in your CIV102F Assignment #5 to make sure they match.
4. In the reduced normal form of your augmented matrix $\left[R|\vec{d}\right]$ obtained in part 3, you should find three rows of $0 = 0$ at the bottom of the matrix. This means there is some redundancy in your original system of equations, but it does not tell you which three equations are redundant. Can you think of a reason for their being some redundancy? Try removing different combinations of three equations, corresponding to three rows in your original augmented matrix $\left[A|\vec{b}\right]$ until you arrive at a modified augmented system that has the same rank and solution as you found in parts 2 and 3. Which three equations did you remove and why?
5. Using the modified augmented matrix with three rows removed that you found in part 4, the coefficient matrix should now be square, i.e. $23x23$, and the constant vector should now be $23x1$, with vector $\vec{x}$ unchanged. Let us refer to this modified augmented matrix as $[A_r|b_r]$. Find the inverse of this square coefficient matrix A_r and use it to find the solution $\vec{x} = A_r^{-1}b_r$ (you may use the built-in MATLAB function inv). This solution should match your solution in part 3.
6. Let us assume the uniformly distributed load on the truss is 90 kN/m instead of 70 kN/m. In your modified augmented matrix in part 5, what changes occur in $[A_r|b_r]$? Can you think of a way to solve this modified augmented matrix without applying Gaussian elimination?
7. When you look at the different solutions to this problem, you will see that the largest force is always carried in the member that connect joints F and H and this member is always in compression. Say you wish to find the maximum value for the uniformly distributed load on the truss so that the compression force $\overrightarrow{FH}$ does not exceed 2000 kN. How would you solve this problem? Can you think of a way to reduce this problem to the solution of a single algebraic equation (hint: think about how the $\overrightarrow{FH}$ entry in $\vec{x}$ is obtained from A_r^{-1} and b_r)?

Assignment Due Date: Wednesday, November 25 at 11:59pm. One submission from each group.