PHY180 Lab Project (2020)

This document outlines the lab experiments which you will do at home during the semester. Each student will be required to build a simple pendulum (a mass swinging from a string) and then compare its performance with a specific mathematical model that theoretically should approximate the behaviour of your pendulum. The goal of this project is to quantify and analyze how accurately the mathematical model represents the actual setup.

This lab is scaffolded into sections, with separate due dates as shown below, so that you can get feedback at each step to be incorporated in subsequent steps. The final submission will be a report summarizing the entire project.

Section	Due Date	Weight
Lab 2: Setup and Q Factor	14 Oct 2020	3.0%
Lab 3: Period vs Amplitude	28 Oct 2020	3.0%
Lab 4a: Period vs Length	18 Nov 2020	1.5%
Lab 4b: Period vs Mass	18 Nov 2020	1.5%
Final Report	9 Dec 2020	13.0%

Background: Mathematical Model

Here is the mathematical model that I think should do a decent job of predicting the behaviour of your pendulum. Note that I write this before you created your pendulum so you should be skeptical. The name of the model, should you wish to research it to learn more, is 'simple harmonic motion'.

Measure the angle (in radians where 2π radians is the same as 360°) that the string makes from the vertical (with 0 being straight down) and call it θ. If you hold the pendulum at rest at some initial angle (θ_0) less than $\pi/2$ (90 degrees) and then let it go, I predict it will swing back and forth as described by the equation

$$\theta(t) = \theta_0 \, \mathrm{e}^{-t/\tau} \cos\left(2\pi \frac{t}{T} + \phi_0\right) \tag{1}$$

where t is time, ϕ_0 is called the phase constant (it would be zero if you start time at the exact instant you release the pendulum, but if there is any time delay then it will not be zero) and τ and T are quantities which are constants which depend on your set up. I have no good prediction for τ except that it is hopefully measured in seconds (as opposed to milliseconds), but it basically measures the friction of your pendulum. You want τ to be fairly large so it takes a long time for the pendulum to stop swinging. I hope your value of τ is at least 10 times larger than T. I predict that more mass will likely increase τ so I recommend you start with a heavy mass.

I can predict that T (the period of your pendulum, that is how long it takes to complete one full oscillation) will have a value of

$$T \simeq 2\sqrt{L} \tag{2}$$

where L is the length of the string in meters. Actually, it's the distance from the centre of mass of the masses at the end of the string to the pivot point, which might differ from the length of the string depending on how you build the pendulum. Note that I specifically predict that the period does not depend on how much mass you have (as long as it's much more than the mass of the string). If you do this experiment somewhere not on Earth I would have to change the value of 2 in the equation depending on the local acceleration due to gravity.

Background: Uncertainties

Very few quantities can be determined with absolute certainty. Counting small integers can usually be done with certainty. For example, I have 10 toes. However, my height does not have a singular, certain value. It depends on the time of day, and any method used to measure my height will have both random and systematic discrepancies compared with using different methods to measure my height.

There are ways to correctly determine the final uncertainty of a quantity based on all the various sources of uncertainties that were involved in finding the final quantity but you don't need to follow those this year. You can simply identify the single largest source of uncertainty and claim it as your final uncertainty. Use the following 2 rules for determining which is the largest uncertainty.

1) If adding or subtracting multiple quantities, the largest uncertainty is simply the largest uncertainty. Example: $(3.5 \pm 0.2) + (13.589 \pm 0.006) = (17.1 \pm 0.2)$ since $0.2 > 0.006$. Note that you should round your uncertainty to one place and round the value to the same accuracy as the uncertainty, so it is bad form to write 17.089 ± 0.2 as the answer.

2) For anything else (multiplication, division, logarithms, etc.), the largest uncertainty is the quantity which is the largest percentage, and that percentage uncertainty is also the percentage uncertainty of the final answer. So $(3.5 \pm 0.2) \times (13.589 \pm 0.006) = (48 \pm 3)$ because the first uncertainty is 5.7% (which is a much larger uncertainty than the second quantity), and 5.7% of 47.5615 is 3 (rounded off to one place). Again, note the rounding conventions.

You will be provided training elsewhere (not in this document) on identifying and estimating the uncertainties of measurements you make.

1 General Report Requirements

Reports for each lab activity will describe experimental methods and observations along with a discussion on analyzing the relationship between measured parameters and how this compares to the theoretical model.

These requirements apply to each of the report submissions; additional requirements specific to each lab activity are included in the lab activity descriptions.

Report Objectives

1. Describe significant elements of the experimental setup and data acquisition, with justification.

    - Your report needs to clearly document what you did and should explain why you made specific procedural design choices.

- This methods section is supposed to help someone reproduce your results by using the same experimental setup and measurement methods.
- You can assume the reader knows how to use a ruler, or stopwatch, etc. Focus on explaining what exactly you measured. For example, "The period was measured by timing how long the pendulum took to complete 2 full oscillations, starting and ending from when it was at the bottom of its swing." is better than "When the pendulum was at the bottom of its motion the start button was pushed on the stopwatch. The stop button was then pushed when the pendulum next returned to the bottom of its motion. The time was then multiplied by 2 to find the period." It is best if you also justify why you chose to measure the period when the pendulum is at the bottom of its motion rather than at the top or any other location (assuming thats what you did).

2. Present data acquired from experimental observations in a clear and concise manner (including uncertainties).

   - Data should be presented in graphs with uncertainty (error) bars and trend lines (the best fit of your data to some theoretical curve).

3. Discuss analysis and implications of experimental observations (as requested in each lab activity description) and compare with the mathematical model provided.

   - All reports need a discussion/analysis section where you highlight your most important results and provide any needed context for how your results should be interpreted. The context should, at minimum, reference your uncertainties.
   - You should clearly describe what criteria you used to reach your conclusion. For example, if you claim that your data indicates that a certain mathematical model is only a valid approximation for your pendulum for a specific range of string lengths, you should explain what criteria you used to claim validity. One possibility is that that range of string lengths produced a period which agreed with the mathematical model within one uncertainty interval (error bar).

4. Additional objectives specific to each lab activity will be provided with the activity descriptions.

Constraints

1. You must submit the assignment by the indicated due date. The late penalty is 3% per calendar day up to 5 days. After 5 days the submission will not be graded. Late penalties can only be waived due to medical and other unforeseeable issues, but not if you have a big assignment in another course.

2. Each time you submit the intermediate reports you should include the previous reports (with corrections) so that your report grows into the final report, and so the marker can quickly check any changes you made based on previous feedback.

3. You should not include a general introduction or conclusion until the final submission.

4. Reports will be automatically submitted to Turnitin for review of text-similarity. It will catch copying from each other and from websites. Please do not copy! Your work is supposed to be your own, original writing. Plagiarism is a serious academic offence.

If you are suspected of plagiarism you will likely have to explain the situation to the Dean of Engineering.

5. Do not include a hypothesis or list of materials
6. Additional constraints specific to each activity will be provided and must be followed.

Criteria

For all of the criteria, more, higher, or greater will be preferred.

- Appropriateness of experimental setup and methods.
- Quality and clarity of explanation and justification of experimental setup and methods.
- Appropriateness of data presentation and clarity of description of the data
- Quality of data analysis and assessment of uncertainties
- Quality and depth of discussion of the results and their implications
- Quality and clarity of writing and report style
- Incorporation of feedback from previous report submissions
- Additional criteria specific to each lab activity will be provided where applicable

2 Experimental Setup

You will build a simple pendulum and test how well the 'simple harmonic motion' model predicts the behaviour of your pendulum. Please note the emphasis – you are **not** being tested on how well you can make your actual set up represent the physical model, although if your pendulum is spectacularly bad (see below for examples) this will impact your results.

Pendulum Requirements

Objectives

1. Build a pendulum consisting of an adjustable string length and an adjustable mass at the bottom of the string
2. Conduct experiments using the pendulum to observe the relationships between characteristics of pendulum's motion with scientific rigour (repeatability).
3. Describe your pendulum design (including photos), and provide justification for your selected setup in the report for Lab 1.

Constraints

- Your pendulum must have an adjustable string length and you must be able to vary the mass at the bottom of the string.
- The pendulum should be attached to something which is 'fixed' in place in the sense that it does not move much while the pendulum swings.
- The string's mass should be much smaller than the variable mass at the bottom.
- Lab report 2 (on Q factor) must include a photograph and discussion of your experimental setup.

Criteria

See criteria provided in the General Report Requirements.

What qualifies as spectacularly bad?

It includes, but is not limited to, the following: you use a heavy metal chain for a string and Styrofoam balls for the "mass"; your string slips frequently, changing the length of the pendulum unpredictably; your string length cannot be adjusted; the string is attached to something which moves a lot; wind or other forces (other than gravity and the string) strongly influences the pendulum's motion. If your pendulum is spectacularly bad you can (should) fix it before the end of the semester and retake any necessary data.

This bears repeating. **You are expected to improve your experimental design and your lab report** based on feedback from the marker.

3 Lab 2: Q factor

Goal

Your first task is to find the Q (quality) factor of your pendulum. If Q is much larger than 1 then it measures how many complete oscillations it takes for your system to decrease its amplitude to about 4% (technically $e^{-\pi}$) of its original amplitude (equivalent to losing about 99.8% of its initial energy).

Overview

It is important that you know this value before the rest of the experiment because your specific value of Q should be considered when you go to measure the period. If your Q factor is large, you can measure the period more accurately by timing multiple oscillations. However, if your Q factor is small, measuring multiple oscillations is problematic as each oscillation has a significantly different amplitude, potentially confusing your results.

The Q factor can be defined by

$$Q = \pi \frac{\tau}{T}. \tag{3}$$

One way to measure the Q factor is to measure the period (T) and the time constant of the decay (τ). Another way is to count the number of oscillations until the amplitude is $\mathrm{e}^{-\pi} \sim 4\%$ of the initial amplitude, and that value is Q. Alternatively, count the number of oscillations until the amplitude is $\mathrm{e}^{-\pi/2} \sim 20\%$ and that is $Q/2$.

Experiment and Report Requirements

Objectives

1. Measure the Q factor of your pendulum using both methods (Equation (3) and counting oscillations). Note that you can use any method you wish to do this, including technologies like video cameras. Please don't spend much money on this though.

2. Present data in a graph. Remember that the x-axis is always the quantity you controlled (time or number of oscillations here) and the y-axis is always the dependent variable (the amplitude here).

3. Determine quantitatively how well your two measurements agree with each other.

4. Finally, you should consider how your Q factor might impact how you take data for the rest of the experiments. This is why the Q factor experiment came first.

Constraints

1. Your write up must include a picture of your equipment, a description of which direction you measured the Q factor (I recommend the direction be in the plane of the photograph rather than in/out of the photograph), how you measured the Q factor (both ways), and your results including uncertainties and how you determined the uncertainties.

2. You must also take data for one trial and put it into a graph which confirms or refutes the exponential decay of the amplitude as predicted in Equation (1).

Criteria

See criteria provided in the General Report Requirements.

4 Lab 3: Period versus Amplitude

Goal

I predicted that the period of oscillation of your pendulum depends **only** on the length of the string. The next 3 activities will test this prediction. First we test that the period is independent of amplitude.

Overview

Take data and plot it in a graph of period (y-axis) as a function of starting amplitude (x-axis). My prediction is that is should be a flat line (zero slope). You should fit it to a power series:

$$T = T_0 + B\theta_0 + C\theta_0^2 + \dots \tag{4}$$

where T_0 is the period for very small oscillations. Note that you need **more** data points than the number of parameters you are trying to fit (T_0, B and C here) to get a good fit. My prediction is $B = C = 0$ (if you fit higher order terms, my prediction suggests they should all be zero). If you fit your data it will never tell you that $B = 0$, it will always give you some value. You can claim that a value is 'experimentally zero' if its value is smaller than its uncertainty. If the value is up to two times larger than its uncertainty then you can still claim it is 'consistent with zero'.

It is important that you measure your angles in radians. It is also important that you take data with starting points from zero all the way to $\pi/2$ (i.e. 90 degrees).

Note that if $B \neq 0$ then something is strange with your pendulum as it is asymmetric. If you release it from the same angle on different sides you get different periods. You could design a pendulum this way but it would be highly unusual, so I expect most of you will get B is 'experimentally zero'. I make no promises about C though. **Note that you should explicitly test for an asymmetry by arbitrarily choosing one side as positive and releasing the pendulum from both positive and negative initial positions.** This data should be included in your graph, so the x-axis should have both positive and negative values. The period should always be positive.

For the rest of the activity, make sure your initial angle is small enough that C (and B) can be ignored, assuming you found that C (and B) for your setup is not consistent with zero.

Note: if you find an asymmetry in your set up, **feel free to modify your pendulum**. If you do so, you will want to retake your Q factor data; it might not change much but you should check to make sure. Also, you can retake your Q factor data if something you learned here makes you realize there was a flaw in your experimental procedure for finding the Q factor.

Experiment and Report Requirements

Objectives

1. Test whether the period of the pendulum is independent of the amplitude.
2. Identify asymmetry in pendulum and improve setup to eliminate this.
3. Describe how you took this data (specifically including the impact of the Q factor on your choices)
4. Graph and analyze the trends (you should fit your data to Equation (4) and plot both your data and the 'trend' line which is the best fit curve).
5. Discuss uncertainties as well as their impact on observations and analysis.
6. Provide a clear conclusion about whether your pendulum's period depends on amplitude. If you do find some dependence, you should clearly indicate what range (if any) of amplitudes are 'small enough' that the value of C can be ignored. Be clear as to what criteria you used to make a 'small enough' judgment.

Constraints

- Angles must be measured in radians.
- You should explicitly test for asymmetry
- You should modify your pendulum to correct for any asymmetry and address feedback from previous reports.

Criteria

See criteria provided in the General Report Requirements.

5 Lab 4a: Period Versus String Length

Goal

The goal of this activity is to examine the relationship between period and the string length of the pendulum. This activity and the next activity can be done at the same time. This is the easier of the two.

Overview

Take data and plot it for the period (y-axis) of your pendulum as a function of L (x-axis). Remember that L is not necessarily the same as the length of the string (in meters) but it should be pretty close. Fit your data to the power law function

$$T = k(L_0 + L)^n \tag{5}$$

where I predict you should get $k = 2$ and $n = 0.5$ to within your uncertainties. Ideally you will get L_0 is consistent with zero as this indicates you correctly determined the effective length of the pendulum (from pivot to centre of mass), but if it's not quite consistent with zero you do not need to retake your data. L_0 represents any bias in your experiment where you *consistently* measured the effective string length incorrectly every time.

Fit this function directly, and also plot your data with a log-log plot in which case the slope is n and the intercept tells you k.

Requirements

Objectives

1. Discuss whether equation (5) is consistent or inconsistent with your results.
2. Discuss uncertainties as well as their impact on observations and analysis.

Constraints

1. The discussion about how you did this should be informed by the results of the first 2 activities (specifically the Q factor and whether C was important).

2. Period vs string length data must be plotted and fit to the power law function in equation (5).
3. You should plot your data on log axes as well as regular axes (same data, 2 graphs).

Criteria

See criteria provided in the General Report Requirements.

6 Lab 4b: Period Versus Mass

Goal

The goal of this activity is to examine the relationship between period and the mass of the pendulum and determine whether the provided simplification is appropriate.

Overview

The theory predicts no effect, but in reality I'm sure extra mass will stretch your string which changes L. It also might change the location of the centre of mass. Finally, a more rigorous analysis suggests that the mass affects the Q factor in a way that will have some impact on the period. As you can see, this particular issue is theoretically quite complicated which is why I chose to ignore it all and assume mass has no effect. **Your job is to quantify whether my gross simplification is reasonable for your pendulum.** It is quite possible that different pendulums will disagree on this topic depending on how they were built. This is fine, don't worry if your results disagree with someone else in the class.

To get good results you will want to do what you can to keep your uncertainties as small as possible, and you will want several different mass values spread across a wide range of masses. I suggest you increase/decrease your mass exponentially (doubling or halving the masses) rather than linearly (say by having 1, then 2, then 3 identical masses).

One idea for plotting and analyzing your results is to take your data and subtract out the 'base' period which is assumed to be a constant function. This is called plotting the residuals (data - best fit from theory = residuals). Then see if there is any pattern to your residuals which you can try to analyze.

This part of the experiment will require more ingenuity and out-of-the-box thinking than the other parts. That's why it's last. You are specifically encouraged to discuss this activity with your peers.

Requirements

Objectives

1. Describe and justify experimental methods, including which masses were selected and how data was recorded.
2. Plot and analyze your data, including any investigation into residuals.

3. Discuss what your results indicate about the dependence of the period on the mass attempting to control for the incidental changes to L which you are trying to keep constant. That is to say you should determine if any period-changing measurements are consistent with just being caused by string-stretching or changing value of the centre of mass.

Constraints

1. You must test and record data for at least 5 different masses

Criteria

See criteria provided in the General Report Requirements.

7 Final Report

Goal

In terms of your overall report, remember that the goal of this project was to build a pendulum and test how well it was modelled by a specific mathematical theory which you were provided. This should be the focus of your paper. Everything you write should be aimed at **quantitatively** assessing how well the theory models your equipment.

Overview

Think of the first three reports as rough drafts and this as the final product. This report should include an introduction and a conclusion, as well as the content from the reports submitted previously.

The introduction to your final report should include a brief summary of your results. In a scientific paper, the introduction might be the only thing most people read, so it needs to catch their attention. Results (with uncertainty as appropriate) are the standard way of getting the reader's attention. The typical scientific paper starts with "We measured this value with high precision using this kind of setup. Here's why that should excite you". Granted a pendulum doesn't have much excitement value in modern science, but you should nonetheless try to emulate that introduction formula. I suggest writing your introduction last, after even your conclusion (which is probably good advice for everything you ever write at university). This will help make sure that your introduction and conclusion agree with each other and focus on the same issues. Papers where the introduction and conclusion disagree or discuss different topics are confusing to read.

The conclusion should highlight the key takeaways from the results and discussion sections of the report. The typical scientific paper will state what the key findings were, their implications on the subject of the paper, and how this may influence future work in the field. Instead of future work in the field, you should describe what the largest source of uncertainty was and how you could reduce it in future work.

Final Report Requirements

Objectives

1. Discuss how well the theoretical equation provided models your experimental setup using quantitative experimental observations.

2. Provide an introduction and conclusion to the report

3. Improve your experimental setup and methods according to feedback and/or any new ideas you may have. Include an overview of changes with justification.

Constraints

1. While you should talk with your peers, your reports must be done by yourself. Your data is your own, your analysis and graphs are your own, and the writing must be your own.

2. Your final report should include an introduction and conclusion, as well as all the data and methods that you submitted previously.

3. Any feedback you got from the first three reports must be incorporated by this stage if you want a good mark - the expectations at this step will be higher than they were before. **This specifically includes the expectation that you take new data if you received feedback about your data.** Even if you did not get such feedback, you are free to redo your data collection in light of new ideas you have.

Criteria

- Quality, clarity, and rigour of data analysis as well as the corresponding discussion comparing experiment and theory.

- Appropriateness and quality of the introduction and conclusion of the report.

- See criteria provided in the General Report Requirements.