Topology — Worksheet 5

Definition

A **CW complex** is the union $X = \bigcup_0^\infty X^n$ of n**-skeletons** defined inductively by:

i) X^0 is a discrete set of point;

ii) Given X^{n-1}, a collection of maps $\varphi_\alpha \colon S^{n-1} \to X^{n-1}$, and a collection of n-disks D_α^n, we define
$$X^n = X^{n-1} \sqcup_\alpha D_\alpha^n / \sim$$
where $x \sim \varphi_\alpha(x)$ for all $x \in \partial D_\alpha^n$.

If $X = X^n$ for some n, we say X is a **finite-dimensional** CW complex of **dimension** n.

A **CW structure** on a topological space Y is a CW complex X such that $X \cong Y$.

1. Construct CW structure for each of the following spaces:

   (a) S^1
   (b) T^2
   (c) S^n
   (d) $\mathbb{RP}^n$

2. Given finite CW complexes X and Y, construct a CW structure for the space $X \times Y$.

3. Compute the cellular homology groups for each of the following spaces:

   (a) S^n
   (b) $\mathbb{RP}^n$
   (c) $S^n \times S^n$