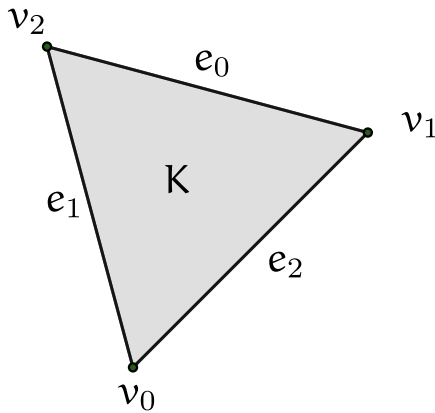# Topology — Worksheet 1
Qualifying Exam Prep Seminar 2020

1. Consider the 2-simplex K given below.

   (a) What does the chain complex

   $$C_*(K) = \cdots \xrightarrow{\partial_{n+2}} C_{n+1}(K) \xrightarrow{\partial_{n+1}} C_n(K) \xrightarrow{\partial_n} C_{n-1}(K) \xrightarrow{\partial_{n-1}} \cdots$$

   look like?

   (b) Consider the simplicial subcomplex $L = e_0 \cup e_1 \cup e_2$ of K. What does the chain complex $C_*(L)$ look like?

   (c) Recall that

   $$C_n(K, L) := C_n(K)/C_n(L)$$

   and use this to compute the chain complex $C_*(K, L)$.

   (d) Compute the **relative homology groups** $H_n(K, L)$.

2. Given a simplicial complex K and a simplicial subcomplex L of K, we get the following **long exact sequence of relative homology**:

   $$\cdots \longrightarrow H_n(L) \xrightarrow{(1)} H_n(K) \xrightarrow{(2)} H_n(K, L) \longrightarrow H_{n-1}(L) \longrightarrow \cdots$$

   Describe the maps (1) and (2).