Topology — Worksheet
Qualifying Exam Prep Seminar 2020

1. Write the fundamental group of a space $X \times Y$ in terms of the spaces X and Y. Prove your statement.

2. **Van Kampen's Theorem** says that given certain subsets A and B of a space X, you get that
$$\pi_1(X) \cong \frac{\pi_1(A) * \pi_1(B)}{N}$$

    i. What extra hypotheses should be said about A and B?

    ii. What does the subgroup N look like?

3. Use van Kampen's theorem to compute the fundamental groups of the following spaces:

    i. $T^2 \wedge S^1$

    ii. The real projective plane, $\mathbb{RP}^2$.

    iii. The Klein Bottle.

    iv. The torus, T^2.

    v. The genus 2 surface, Σ_2.

    vi. The genus g surface, Σ_g.

4. Given connected, locally simply connected spaces X and Y, write the fundamental group of the wedge $X \wedge Y$ in terms the fundamental groups of X and Y. Prove your statement.