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Geometry

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Based on Lectures by

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Introduction

Given a smooth surface $S \subseteq \mathbb{R}^3$, we'd like to compare and contrast with $\mathbb{R}^n$. In doing so, we may ask for analogs to lines, angles, volume, distance, length, et cetera.

In fact, we can even consider these analogs in more abstract smooth manifolds. For example:

- $\mathbb{R}P^n$
- the Torus
- the Klein Bottle

While these space can be embedded into $\mathbb{R}^n$, we don't usually consider them this way, but rather thin of them abstractly.

Riemannian Manifolds

Definition 1.1 A *Riemannian Manifold* (M, g) is a C^∞-manifold together with a smoothly varying inner product g on M.

Definition 1.2 An *Inner Product* is a map $\langle \cdot, \cdot \rangle : V \times V \to \mathbb{R}$ that is bilinear, symmetric, and positive definite.

Note Really there are inifinitely many inner products in a Riemmanian Manifold since each point has its own tangent space. We sometimes say g_p is the inner product at p. We say that g is a *Riemannian metric*, though this shouldn't be confused with standard notions of a metric.