*Lecture Notes by Jonathan Alcaraz (UCR)*

# Complex Analysis

Math 210A
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Based on Lectures by

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## The Topology of the Complex Plane

**Definition 1.1** Given $a \in \mathbb{C}$, $r > 0$, define an *open ball* by

$$B(a,r) = \{z \in \mathbb{C} : |z-a| < r\}$$

and a *closed ball* by

$$\overline{B}(a,r) = \{z \in \mathbb{C} : |z-a| \leq r\}$$

**Definition 1.2** Take sets $A \subseteq G \subseteq \mathbb{C}$. $A$ is said to be *open in* $G$ if for any $a \in A$, there is some $r > 0$ such that $B(a,r) \bigcap G \subseteq A$. $A$ is said to be *closed in* $G$ if $G \backslash A$ is open in $G$.

**Definition 1.3** A subset $G \subseteq \mathbb{C}$ is said to be *connected* if it has either of the following properties:

- If $G = A \bigcup B$ where $A$, $B$ are open and disjoint, the $A = \emptyset$ or $B = \emptyset$.
- If $A \subseteq G$ is both open in $G$ and closed in $G$, then $A = \emptyset$ or $A = G$.

**Definition 1.4** A *segment* between complex numbers $z$ and $w$, denoted $[z,w]$ is the set $\{tw + (1-t)z : t \in [0,1]\}$.

**Definition 1.5** A *polygon* from $a$ to $b$ is a set $[a,z_1] \cup [z_1,z_2] \cup \cdots \cup [z_n,b]$.

**Theorem 1.6** An open set $G$ is connected if and only if, for every $a,b \in G$ there is a polygon from $a$ to $b$.