Lecture Notes by Jonathan Alcaraz (UCR)

Complex Analysis

Math 210A
Fall 2017

Based on Lectures by

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last updated October 2, 2017

Lecture 1 29 Sep 2017

The Topology of the Complex Plane

Definition 1.1 Given $a \in \mathbb{C}$, $r > 0$, define an *open ball* by

$$B(a, r) = \{z \in \mathbb{C} : |z - a| < r\}$$

and a *closed ball* by

$$\overline{B}(a, r) = \{z \in \mathbb{C} : |z - a| \leq r\}$$

Definition 1.2 Take sets $A \subseteq G \subseteq \mathbb{C}$. A is said to be *open in* G if for any $a \in A$, there is some $r > 0$ such that $B(a, r) \bigcap G \subseteq A$. A is said to be *closed in* G if $G \backslash A$ is open in G.

Definition 1.3 A subset $G \subseteq \mathbb{C}$ is said to be *connected* if it has either of the following properties:

- If $G = A \bigcup B$ where A, B are open and disjoint, the $A = \emptyset$ or $B = \emptyset$.
- If $A \subseteq G$ is both open in G and closed in G, then $A = \emptyset$ or $A = G$.

Definition 1.4 A *segment* between complex numbers z and w, denoted $[z, w]$ is the set $\{tw + (1 - t)z : t \in [0, 1]\}$.

Definition 1.5 A *polygon* from a to b is a set $[a, z_1] \cup [z_1, z_2] \cup \cdots \cup [z_n, b]$.

Theorem 1.6 An open set G is connected if and only if, for every $a, b \in G$ there is a polygon from a to b.

Definition 1.7 Given a subset $A \subseteq \mathbb{C}$, we say $z \in \mathbb{C}$ is a *limit point* of A if there exists a sequence $\{a_n\}$ of distinct points in A such that $z = \lim_{n\to\infty} a_n$.

Corollary 1.8 A subset A is closed if and only if A contains all of its limit points.

Definition 1.9 A subset $A \subseteq \mathbb{C}$ is *complete* if every Cauchy sequencce in A converges in A.

Corollary 1.10 A is complet if and only if A is closed.

Definition 1.11 A subset A of $\mathbb{C}$ is *compact* if every open cover of A has a finite subcover. A is *sequentially compact* if every sequencec in A has a subsequence which converges in A.

Definition 1.12 A set $A \subseteq \mathbb{C}$ is *totally bounded* if for every $\varepsilon > 0$ there exists $a_1, \ldots, a_n \in A$ such that $A \subseteq \bigcup_{i=1}^{n} B(a_i, \varepsilon)$.

Theorem 1.13 The following are equivalent:

(i) A is compact;

(ii) Every infinie set in A has limit point in A;

(iii) A is sequentially compact;

(iv) A is complete and totally bounded.

Corollary 1.14 A is compact if and only if A is closed and bounded.

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