Accelerating CUP reconstruction using NESTA optimization algorithm

Liren Zhu

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Abstract

This is a summary of theoretical and algorithmic results that are related with using the NESTA algorithm in CUP reconstruction. The motivation of using NESTA is to mitigate the major computational cost on the TV denoising proxy problem that needs to be solved in every iteration of the TwIST and FISTA optimizers. I have included the framework of NESTA, detailed factorization of using NESTA with different CUP implementations, and derivation of the major steps in this note. This document will, hopefully, help readers, including myself, to better understand the implementation of this method.

1 Introduction

Currently, Compressed Ultrafast Photography (CUP) relies on a family of Iterative Shrinkage/Thresholding Algorithms (ISTA), including the Two-step Iterative Shrinkage/Thresholding (TwIST) algorithm and the Fast Iterative Shrinkage/Thresholding Algorithm (FISTA), to reconstruct the final datacube of the dynamic scene. While these methods produce excellent results, their needs of solving the Total Variation (TV) denoising proxy problem in every iteration are shown to make up most of the computation time, as well as require the user to pick non-intuitive parameters for the proxy problem.

The NESTA optimizer, on the other hand, requires only one call to the differentiation operator and its adjoint each during on iteration, in order to realize l2 constrained TV minimization. Considering that CUP's observation operator and its adjoint (of all CUP implementations) are relatively cheap to perform, an optimizer that minimizes the computation time on TV is an ideal choice for CUP. In addition, instead of solving the TV-regularized l2 error minimization problem

$$\min_x \frac{1}{2}\|b - Ax\|_2^2 + \lambda\|x\|_{\mathrm{TV}} \tag{1}$$

NESTA solves the following, more direct problem

$$\min_x \|x\|_{\mathrm{TV}} \quad \text{s.t. } \|b - Ax\|_2 \leq \epsilon \tag{2}$$

While Eq. (1) has a rather arbitrary parameter λ, Eq. (2) requires a parameter ϵ, which is closely related with the upper bound of the measurement noises. However, it is worth noticing that theoretical analyses have shown that Eq. (1) and (2) are equivalent when λ and ϵ follow a certain relation and the true optimal points were reached in both problems.

2 The NESTA framework

Based on the technical report available at this website, the NESTA algorithm can be laid out as the following steps:

Algorithm 1 General purpose NESTA solver

Compute $\nabla f(x_k)$.
Compute y_k:
$y_k = \arg\min_{x \in Q_P} \frac{L}{2}\|x - x_k\|_2^2$