Project 1

(Due on: Wed, October 17 by 8:00PM via e-mail)

A small water reservoir depicted in Fig. 1 has a cylindrical shape with the radius of $0.1m$ and its output pipe has the radius of $0.005m$. During exploitation, the reservoir water level Y changes in the range between $0.05m$ and $0.15m$ where the maximal volume flow rate is $q_{in} = 1.5 \cdot 10^{-4} m^3/s$. The nonlinear model for the rate of change of the level is

$$\dot{Y} = \frac{q_{in} S}{S^2 - A^2} - \sqrt{\frac{A^2}{(S^2 - A^2)^2} q_{in}^2 + \frac{2A^2 gY}{S^2 - A^2}} \tag{1}$$

(a) Linearize the system from the input q_{in} to the output Y, find the corresponding transfer function $\frac{y(s)}{u(s)}$ and write its denominator with the coefficient 1 multiplying the highest order of s. (Note: $u = q_{in} - q_{in}^*$ and $y = Y - Y^*$, where (Y^*, q_{in}^*) is the point around which we linearize the system).

(b) Find the relation between q_{in} and Y that results in $\dot{Y} = 0$. The relation defines all set points (Y, q_{in}) at which the level can be kept constant. Does the control variable q_{in} have a range to maintain a constant level Y for the whole working range of the level? Explain.

(c) For the reservoir working range $Y \in [0.05, 0.15]$, plot the magnitude of frequency characteristics $\frac{y(j\omega)}{u(j\omega)}$ for 30 equally spaced values of Y from the range $[0.05, 0.15]$. The vertical axis of the plot should be $20log_{10}|\frac{y(j\omega)}{u(j\omega)}|$ and the horizontal should be $log_{10}(\omega)$.

(d) For the reservoir working range $Y \in [0.05, 0.15]$, find the smallest and the largest DC gains, as well as the smallest and the largest poles of the transfer functions.

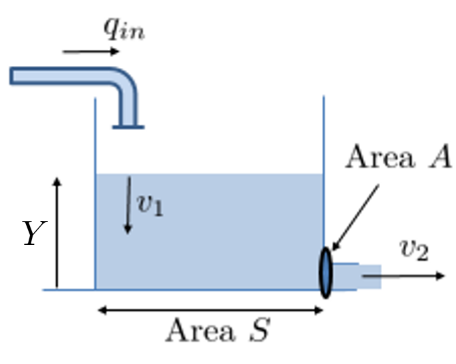

Figure 1: