d) avg. value of $\theta(t)$ for thrust = 1

over 1 period, min $= -46°$
max $= -90°$

$$\bar{\theta} = \frac{-46° + (-90°)}{2} = -68°$$

$$\frac{\theta(s)}{u(s)} = \frac{b}{s^2 - a}$$

thrust = 1 at time t=0 is equivalent to an input $u(t) = \text{step}(t)$ so $u(s) = \frac{1}{s}$

$$\Rightarrow \theta(s) = \frac{b}{s^2 - a} \cdot u(s)$$

$$\Rightarrow \theta(s) = \frac{b}{s^2 - a} \cdot \frac{1}{s}$$

by Final Value thm.

$$\theta(t=\infty) = \lim_{s \to 0} \cancel{s} \cdot \frac{b}{s^2 - a} \cdot \frac{1}{\cancel{s}}$$

$$\theta(\infty) = -\frac{b}{a}$$

Since the system in steady-state is oscillitory, $-\frac{b}{a}$ indicates the average value of the oscillation with reference to $\Theta^* = -90°$

$$\bar{\theta} - \Theta^* = -\frac{b}{a}$$

assuming $\Theta^* = -90$, so $a = \frac{mgl}{I} = 18.52$

$$\Rightarrow -68° - (-90°) = \frac{-b}{-18.52}$$

$$\Rightarrow 22° = \frac{b}{18.52}$$

$$\Rightarrow 0.384 \text{ rad} = \frac{b}{18.52}$$

$$b = \frac{L}{I} = 7.11$$