Homework 2

(Due on: Monday, October 29 by 8pm)

Problem 1. A DC motor is connected to a disk over a gearbox as presented in Fig. 1.

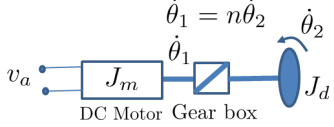

Figure 1:

The disk and motor moments of inertia are J_d and J_m, respectively. The motor model is given by

$$\begin{aligned} T_m &= K_t i_a && (1)\\ e &= K_e \dot{\theta}_1 && (2)\\ v_a &= R_a i_a + e && (3) \end{aligned}$$

where T_m is the motor torque, i_a is the motor current, e is the back emf, R_a is the motor resistance, $\dot{\theta}_1$ is the motor rotational speed and v_a is the voltage driving the motor. Because of the gear box, which reduces the speed of the motor for the factor n, we know that $\dot{\theta}_1 = n\dot{\theta}_2$ and

$$\begin{aligned} J_m\ddot{\theta}_1 + b_m\dot{\theta}_1 &= T_m - T_{21} && (4)\\ J_d\ddot{\theta}_2 + b_d\dot{\theta}_2 &= T_{12} && (5)\\ T_{21}\dot{\theta}_1 = T_{12}\dot{\theta}_2 &\Rightarrow T_{21} = \frac{T_{12}}{n} && (6) \end{aligned}$$

The first equation describes the angular acceleration of the motor and the second one describes the angular acceleration of the disk. The third equation describes the relation between the torques T_{12} and T_{21}. The torque T_{12} is the one transmitted over the gear box from the motor side to the disk side and T_{21} is the torque transmitted in the opposite direction. Use (1)-(6) to find the transfer function from v_a to θ_2, i.e., $\frac{\Theta_2(s)}{V_a(s)}$.

Problem 2. An inverted pendulum system is described by the following system of differential equations

$$\begin{aligned} (m_t + m_p)\ddot{x} + m_p l\ddot{\theta}_B &= u - b\dot{x} && (7)\\ m_p l\ddot{x} + (I + m_p l^2)\ddot{\theta}_B &= -m_p l g\theta_B && (8) \end{aligned}$$

where m_t, m_p, l, I, b and g are the system parameters. The variable x is the position of the pendulum base, θ_B is the pendulum angle and u is the force acting on the base of the pendulum, which is a control variable. Find the transfer function $\frac{\Theta_B(s)}{U(s)}$.

Problem 3. For the dynamical system given by the system of differential equations (7)-(8) in Problem 2, draw a Simulink diagram that corresponds to the realization of the transfer function $\frac{\theta_B(s)}{U(s)}$. You should use only integrators, amplifiers and summation blocks.

Problem 4. For the dynamical system

$$\dot{y}_1 = y_2 + u(t) \tag{9}$$
$$\dot{y}_2 = 2y_1 + y_2 - u(t) \tag{10}$$

(a) find the transfer function $\frac{Y_1(s)}{U(s)}$; (b) simplify the transfer function $\frac{Y_1(s)}{U(s)}$ so that its denominator is in the form $(s + a)$.

Problem 5. Textbook problem 3.20(a).