a) System Equation

$$f(Y, q_{in}) = \dot{Y} = \frac{q_{in} S}{S^2 - A^2} - \sqrt{\frac{A^2}{(S^2-A^2)^2} q_{in}^2 + \frac{2A^2 g Y}{S^2 - A^2}}$$

Taylor Series Linear Approximation

$$\dot{Y} = f(Y^*, q_{in}^*) + \left.\frac{\partial f}{\partial Y}\right|_{\substack{Y^* \\ q_{in}^*}} (Y - Y^*) + \left.\frac{\partial f}{\partial q_{in}}\right|_{\substack{Y^* \\ q_{in}^*}} (q_{in} - q_{in}^*)$$

$$f(Y^*, q_{in}^*) = \dot{Y}^* \quad ; \quad y = Y - Y^* \quad ; \quad u = q_{in} - q_{in}^*$$

$$\Rightarrow \dot{y} = \dot{Y} - \dot{Y}^*$$

$$\Rightarrow \dot{y} + \dot{Y}^* = \dot{Y}$$

$$\dot{y} + \dot{Y}^* = \dot{Y}^* + \left.\frac{\partial f}{\partial Y}\right|_{\substack{Y^* \\ q_{in}^*}} y + \left.\frac{\partial f}{\partial q_{in}}\right|_{\substack{Y^* \\ q_{in}^*}} u$$

$$\Rightarrow \dot{y} = a y + b u \quad \text{where} \quad a = \left.\frac{\partial f}{\partial Y}\right|_{\substack{Y^* \\ q_{in}^*}}$$

$$b = \left.\frac{\partial f}{\partial q_{in}}\right|_{\substack{Y^* \\ q_{in}^*}}$$

$\mathcal{L}\downarrow$

$$s\,y(s) = a\,y(s) + b\,u(s)$$

$$\boxed{\frac{y(s)}{u(s)} = \frac{b}{s - a}}$$

$$a = \left.\frac{\partial f}{\partial Y}\right|_{\substack{Y^* \\ q_{in}^*}} = \frac{A^2 g}{(S^2 - A^2)\sqrt{\frac{A^2}{(S^2-A^2)^2} q_{in}^{*2} + \frac{2A^2 g Y^*}{S^2 - A^2}}}$$

$$b = \left.\frac{\partial f}{\partial q_{in}}\right|_{\substack{Y^* \\ q_{in}^*}} = \frac{S}{S^2 - A^2} - \frac{A^2 q_{in}^*}{(S^2 - A^2)\sqrt{\frac{A^2}{(S^2-A^2)^2} q_{in}^{*2} + \frac{2A^2 g Y^*}{S^2 - A^2}}}$$