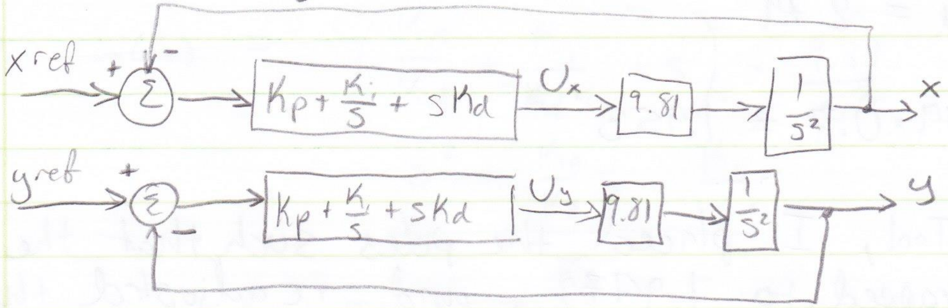
Project 5



a) Take $K_i = 0$, use RootLocus to find parameters K_p and K_d such that $\varepsilon \geq 0.7$ and all zeroes and poles are within the circle $\omega_n < \frac{1}{5T_s}$, $T_s = 0.1s$

$$\frac{X}{X_{ref}} = \frac{9.81(K_p + sK_d)(\frac{1}{s^2})}{1 + 9.81(K_p + sK_d)(\frac{1}{s^2})} = \frac{9.81(K_p + sK_d)}{s^2 + 9.81(K_p + sK_d)}$$

Characteristic eqn.

$$1 + 9.81 K_d L(s) \qquad L(s) = \frac{s + \frac{K_p}{K_d}}{s^2}$$

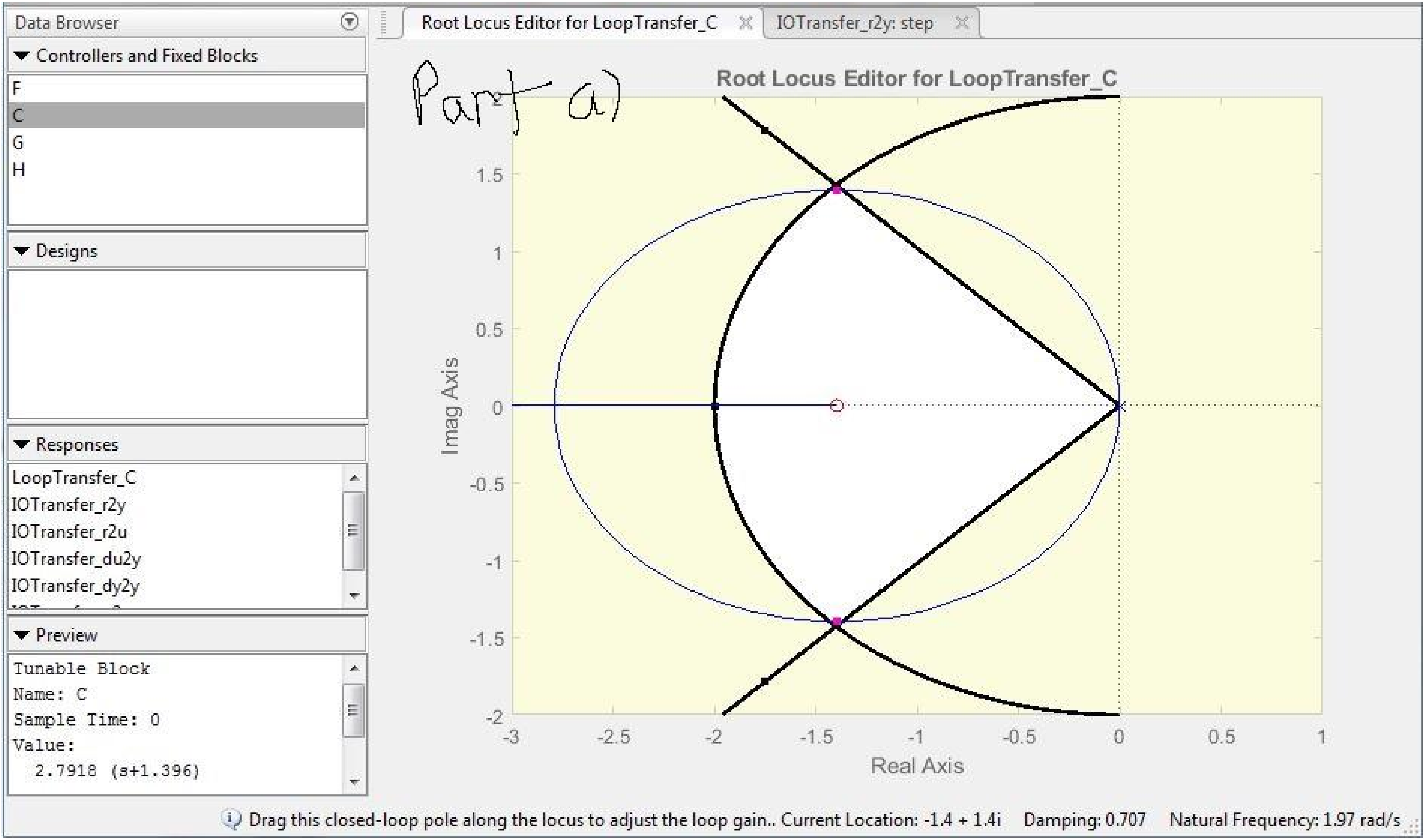
$$\omega_n < \frac{1}{5T_s} \;, \quad T_s = 0.1$$

$$\Rightarrow \omega_n < \frac{1}{0.5} \Rightarrow \omega_n < 2$$

Using RLTool, I chose the zero $\frac{K_p}{K_d}$ to be 1.40 and $9.81 \cdot K_d$ to be 2.79, based on the damping factor and frequency constraints. I had to place the zero such that part of the locus barely satasfied both constrains, and then place the poles in that range on the locus.

$$9.81 \cdot K_d = 2.79 \qquad \frac{K_p}{0.284} = 1.40$$

$$\boxed{K_d = 0.284 \qquad K_p = 0.398}$$


Part a)
Data Browser
Controllers and Fixed Blocks
F
C
G
H
Designs
Responses
LoopTransfer_C
IOTransfer_r2y
IOTransfer_r2u
IOTransfer_du2y
IOTransfer_dy2y
Preview
Tunable Block
Name: C
Sample Time: 0
Value:
2.7918 (s+1.396)
Root Locus Editor for LoopTransfer_C
IOTransfer_r2y: step
Root Locus Editor for LoopTransfer_C
Imag Axis
Real Axis
Drag this closed-loop pole along the locus to adjust the loop gain.. Current Location: -1.4 + 1.4i
Damping: 0.707
Natural Frequency: 1.97 rad/s

b) $9.81 \cdot K_d = 2.79$

$$2.79 \cdot 0.7 = 1.953$$

In RLTool, I placed the poles such that the gain dropped to 1.9472, and readjusted the zero so that even at 70% gain, the poles are still placed so they satisfy the damping factor requirement. K_d remains the same, but K_p changes

$$\frac{K_p}{0.284} = 0.9801 \qquad K_d = 0.284$$

$$K_p = 0.278$$

c) In the simulink model, for $K_i = 0$, $K_d = 0.284$ and $K_p = 0.278$, $e_x = 0.36$ and $e_y = 0$ for any constant x and y references.

d)
$$\frac{\mathcal{E}(s)}{d_x(s)} = \frac{9.81 \cdot \frac{1}{s^2}}{1 + 9.81\left(K_p + \frac{K_i}{s} + sK_d\right)\frac{1}{s^2}} = \frac{9.81\,s}{s^3 + 9.81 K_d s^2 + 9.81 K_p s + 9.81 K_i}$$

for $d_x(s) = \frac{1}{s}$ (unit step) for $K_i = 0$

$$\mathcal{E}(t=\infty) = \lim_{s \to 0} \cancel{s} \cdot \frac{9.81}{s^2 + 9.81 K_d s + 9.81 K_p} \frac{1}{\cancel{s}}$$

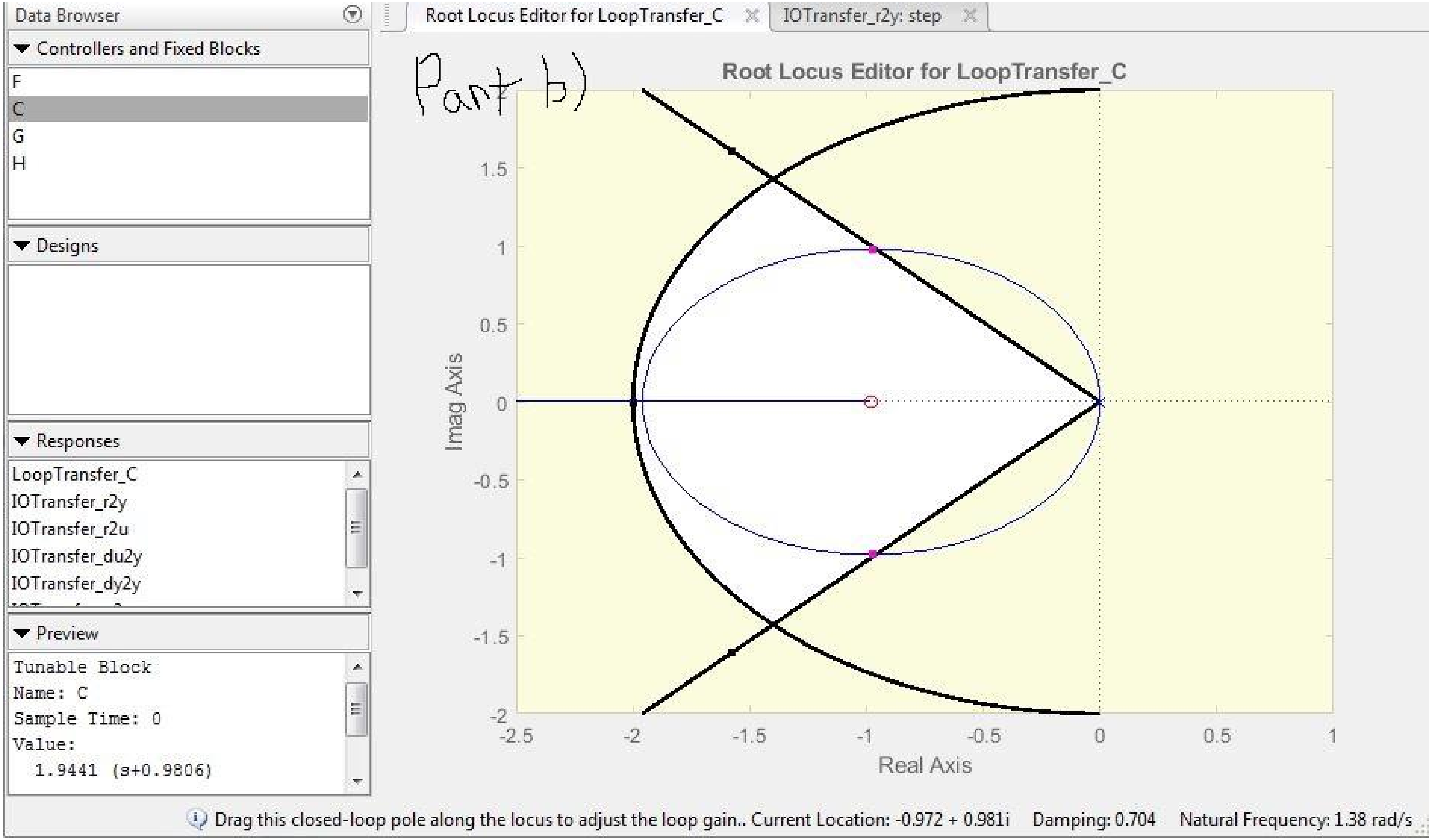
$$= \frac{1}{K_p}$$

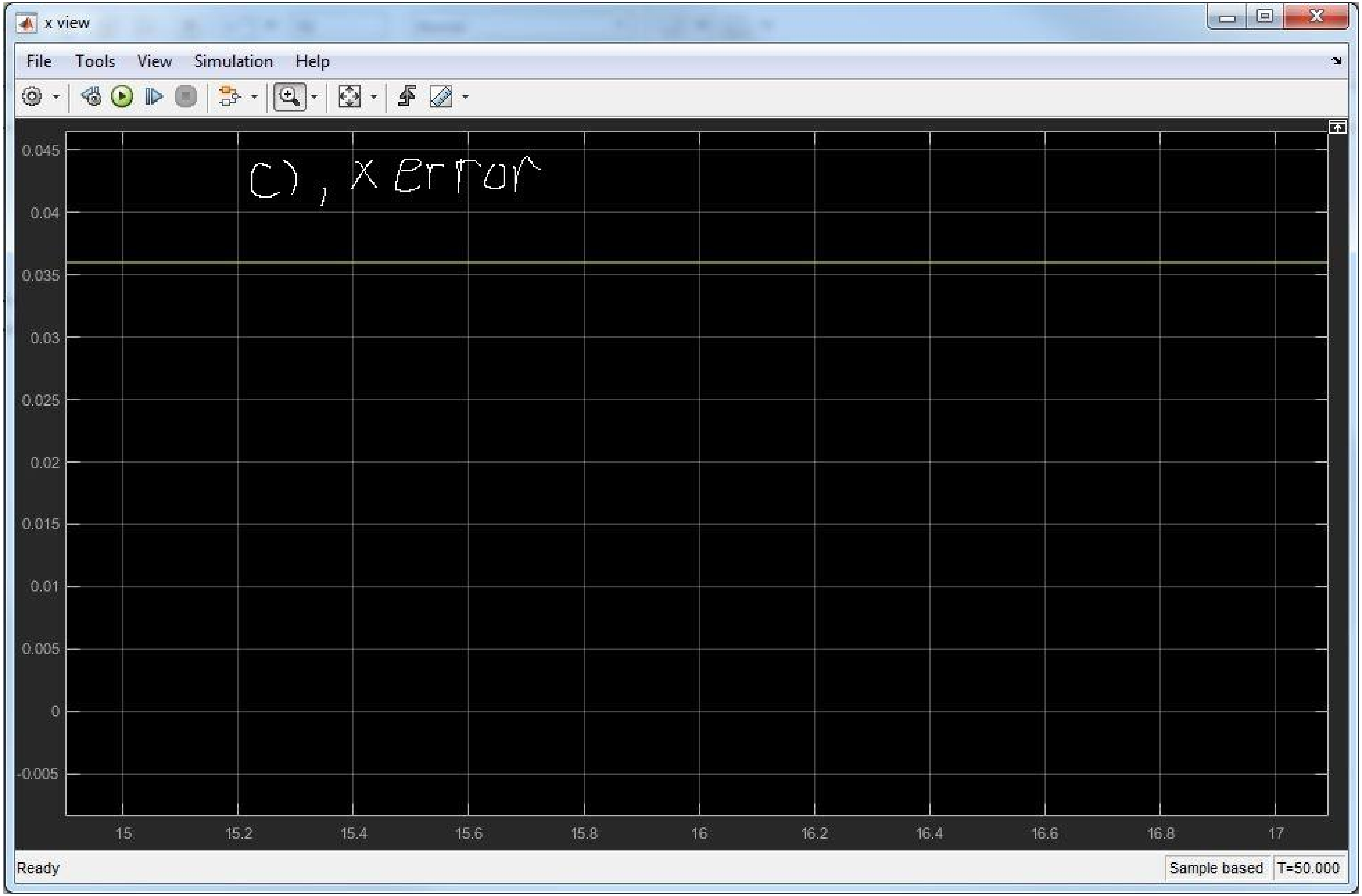
for $K_i \neq 0$

$$\mathcal{E}(t=\infty) = \lim_{s \to 0} \cancel{s} \cdot \frac{9.81 s}{s^3 + 9.81 K_d s^2 + 9.81 K_p s + K_i} \frac{1}{\cancel{s}}$$

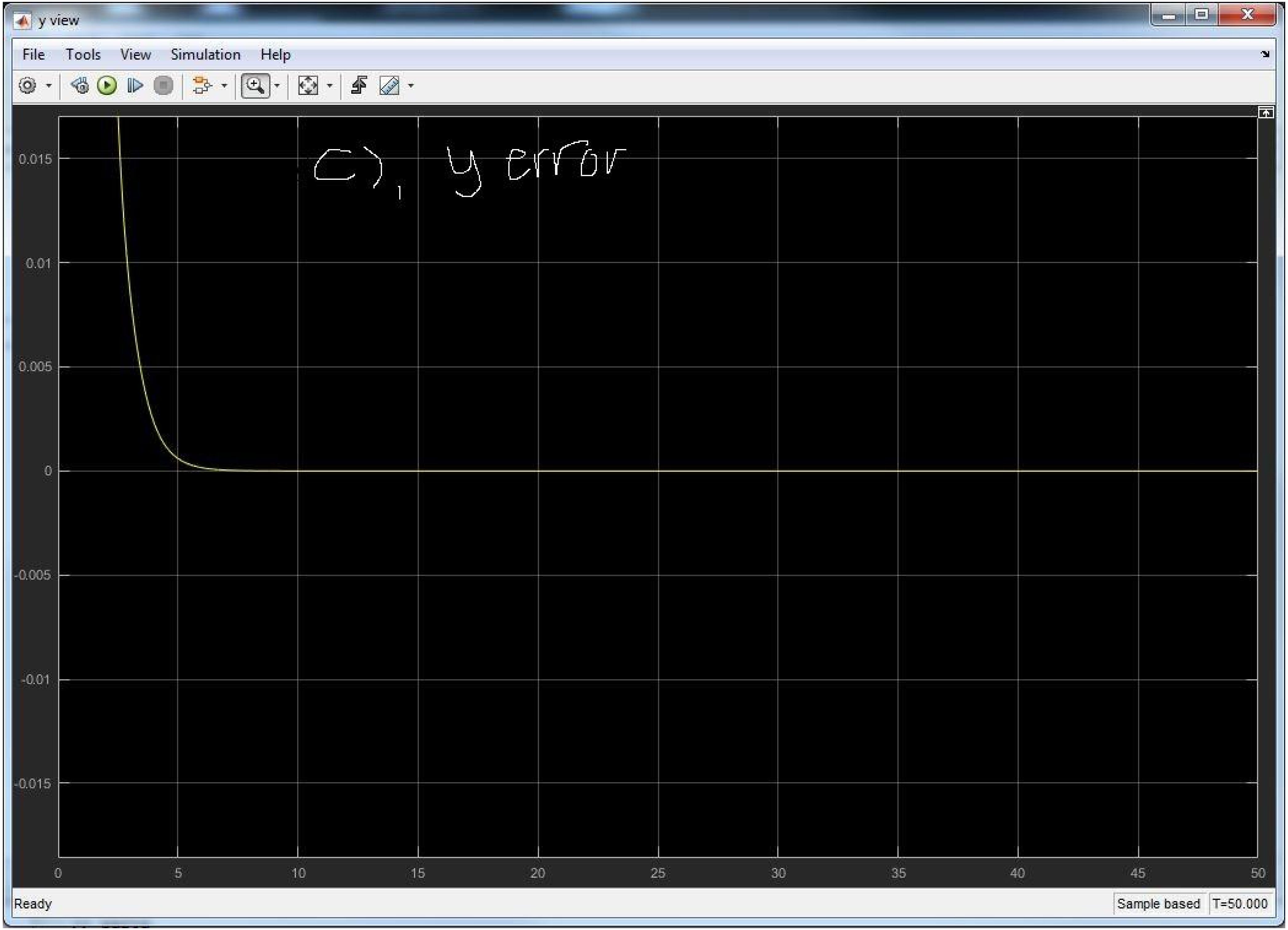
$$= 0 \quad \checkmark$$

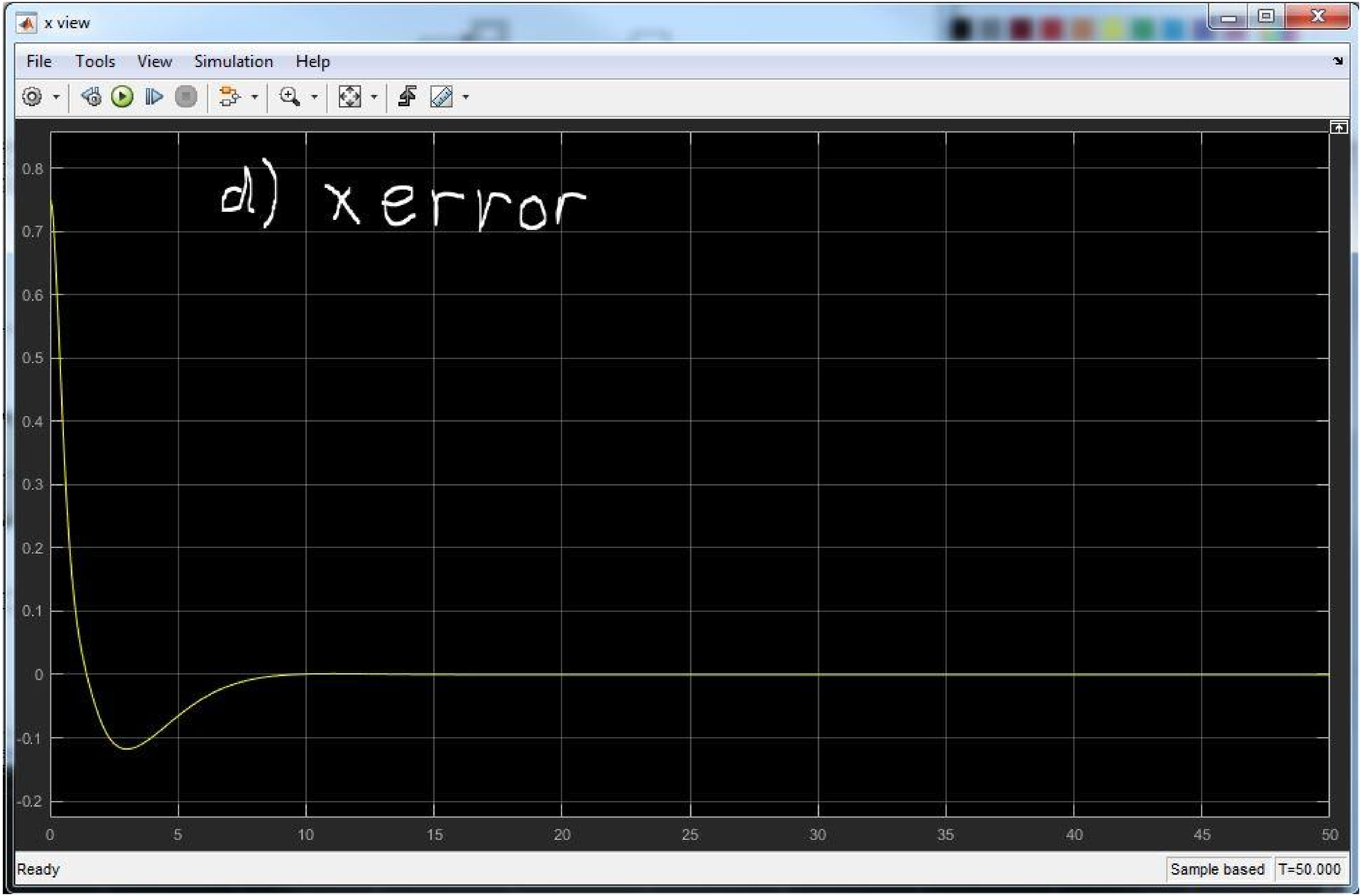
$K_i \neq 0$ does eliminate the impact of d_x / d_y, and causes the errors to go to zero. graphs are for $K_i = 0.1$, and x_{ref}, $y_{ref} = 0$

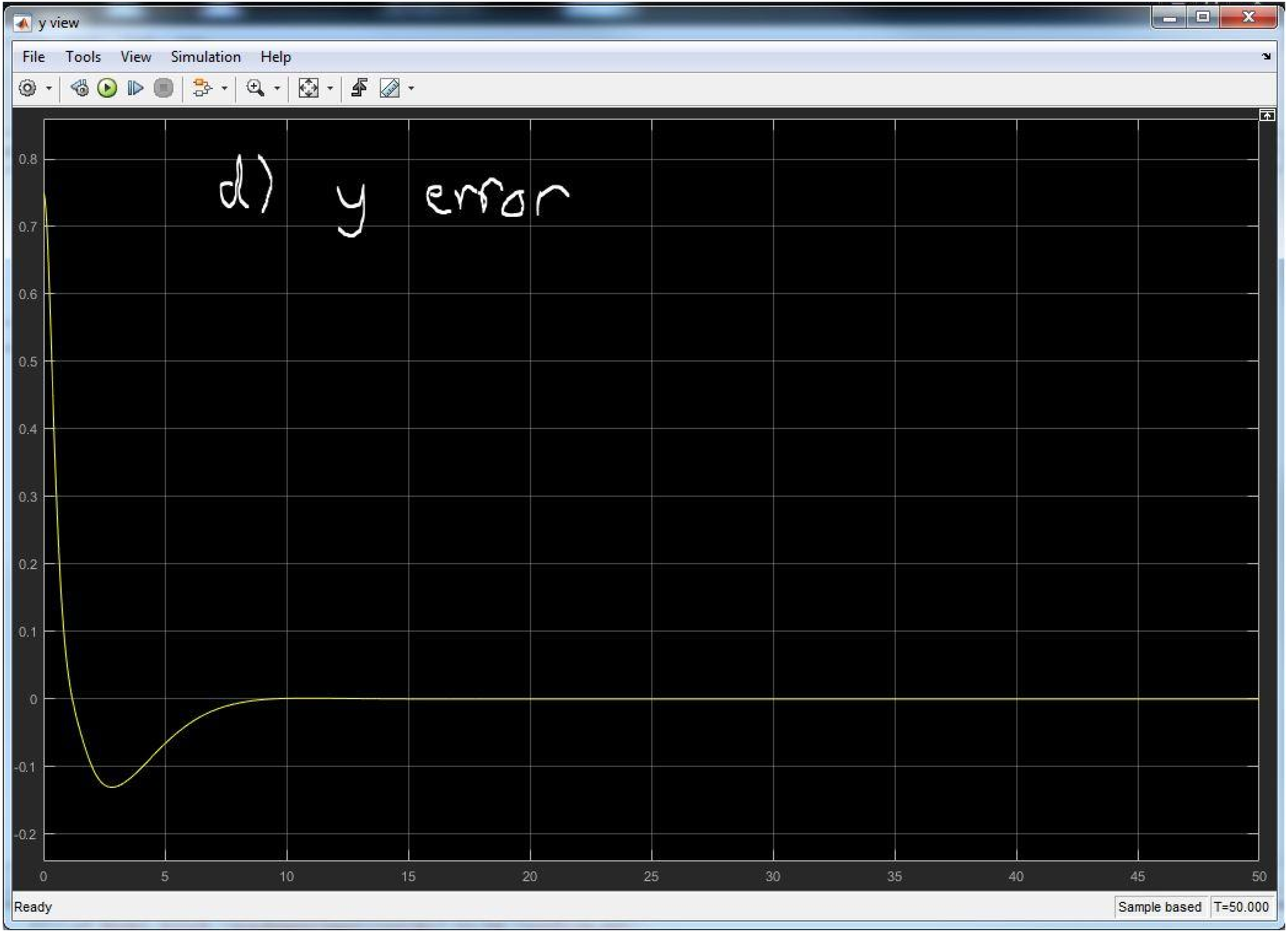

Data Browser
Controllers and Fixed Blocks
F
C
G
H
Designs
Responses
LoopTransfer_C
IOTransfer_r2y
IOTransfer_r2u
IOTransfer_du2y
IOTransfer_dy2y
Preview
Tunable Block
Name: C
Sample Time: 0
Value:
1.9441 (s+0.9806)
Root Locus Editor for LoopTransfer_C
IOTransfer_r2y: step
Part b)
Root Locus Editor for LoopTransfer_C
Imag Axis
Real Axis
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
Drag this closed-loop pole along the locus to adjust the loop gain.. Current Location: -0.972 + 0.981i
Damping: 0.704
Natural Frequency: 1.38 rad/s



x view
File Tools View Simulation Help
C), x error
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.005
15
15.2
15.4
15.6
15.8
16
16.2
16.4
16.6
16.8
17
Ready
Sample based
T=50.000


y view
File Tools View Simulation Help
c), y error
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
0 5 10 15 20 25 30 35 40 45 50
Ready
Sample based
T=50.000


x view
File Tools View Simulation Help
d) x error
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0 5 10 15 20 25 30 35 40 45 50
Ready
Sample based
T=50.000


y view
File Tools View Simulation Help
d) y error
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0 5 10 15 20 25 30 35 40 45 50
Ready
Sample based
T=50.000

Characteristic eqn. : $1 + 9.81 K_d L(s)$

e) $L(s) = \left(\frac{K_p}{K_d} + \frac{K_i}{K_d s} + s\right) \cdot \frac{9.81}{s^2}$

$$\frac{s^2 + \frac{K_p}{K_d}s + \frac{K_i}{K_d}}{s^3}$$

$$s = \frac{-\frac{K_p}{K_d} \pm \sqrt{\frac{K_p^2}{K_d^2} - 4\frac{K_i}{K_d}}}{2} = \frac{-\frac{K_p}{K_d} \pm \sqrt{\frac{K_p^2 - 4K_iK_d}{K_d^2}}}{2}$$

$$= -\frac{K_p \pm \sqrt{K_p - 4K_iK_d}}{2K_d}$$

Zeroes at $\frac{K_p}{2K_d} \mp \frac{\sqrt{K_p - 4K_iK_d}}{2K_d}$

In RLTool, I set the transfer function as $\frac{1}{s^3}$, and placed two real zeroes. I manipulated and tuned these zeroes such that 0.7 times the largest gain within the region was also in the region. I got:

$$9.81 K_d = 3.63, \quad \frac{K_p}{2K_d} + \frac{\sqrt{K_p - 4K_iK_d}}{2K_d} = 1.076$$

$$, \quad \frac{K_p}{2K_d} - \frac{\sqrt{K_p - 4K_iK_d}}{2K_d} = 0.2068$$

$\boxed{K_d = 0.370}$ $\Rightarrow$ $K_p + \sqrt{K_p - 1.48K_i} = 1.076 \cdot 0.74$

$\Rightarrow K_p - 1.48K_i = (0.796 - K_p)^2$

$\Rightarrow K_p - 1.48K_i = 0.634 - 1.59K_p + K_p^2$

$\Rightarrow -1.48K_i = 0.634 - 2.59K_p + K_p^2$

$-K_i = 0.428 - 1.75K_p + 0.676K_p^2$

$K_p - \sqrt{K_p - 1.48K_i} = 0.2068 \cdot 0.74$

$\Rightarrow K_p - 1.48K_i = (K_p - 0.153)^2$

$\Rightarrow K_p - 1.48K_i = K_p^2 - .306K_p + .023$

$K_i = 0.676K_p^2 + 0.878K_p - 0.016$

$$-K_i = 0.676 K_p^2 - 1.75 K_p + 0.428 \quad ①$$
$$K_i = -0.676 K_p^2 + 0.878 K_p - 0.016 \quad ②$$

① + ② $\quad 0 = -0.872 K_p + .412$

$$\boxed{K_p = 0.472}$$

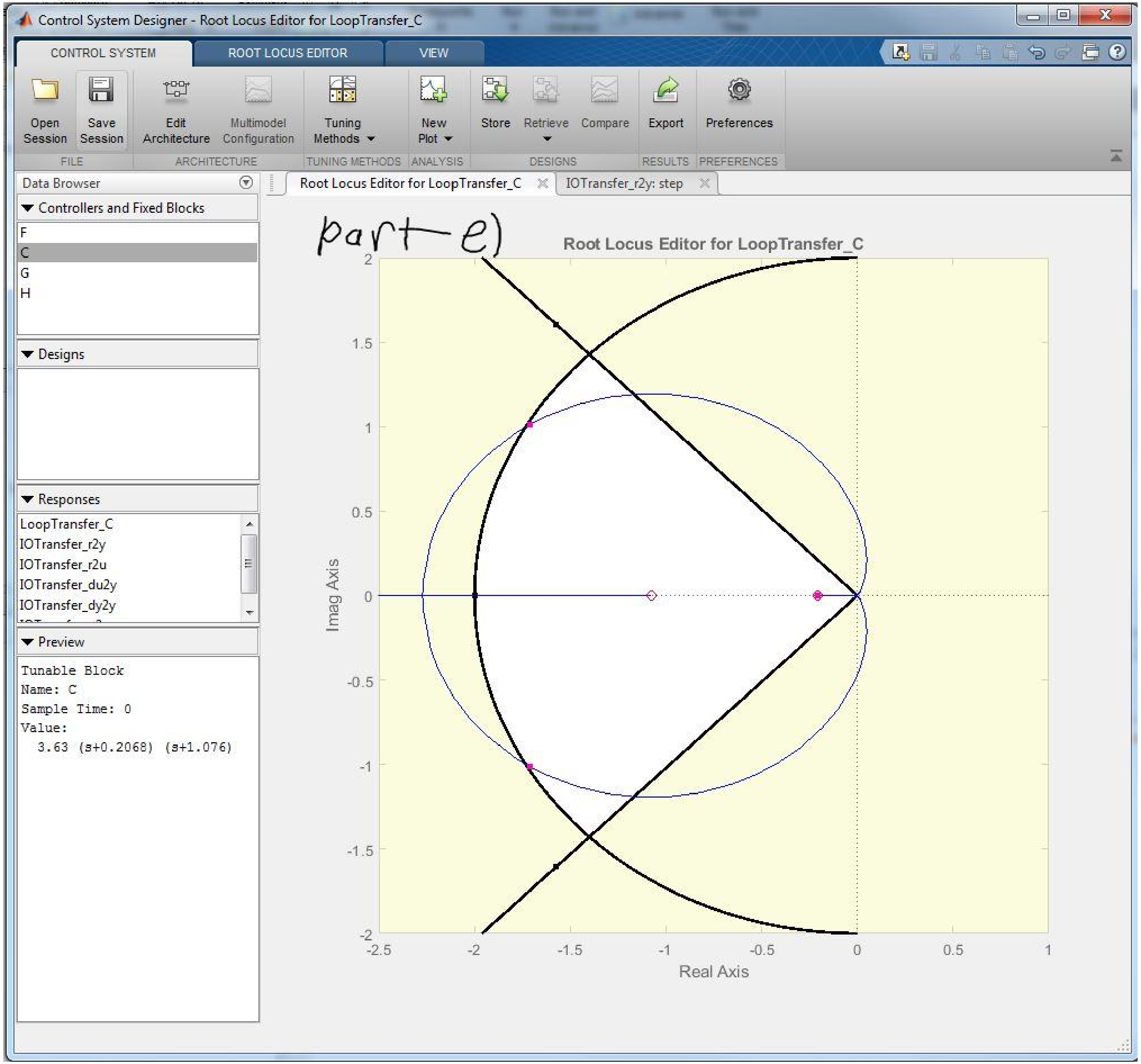
② $\quad K_i = -0.676(0.472)^2 + 0.878(0.472) - 0.016$

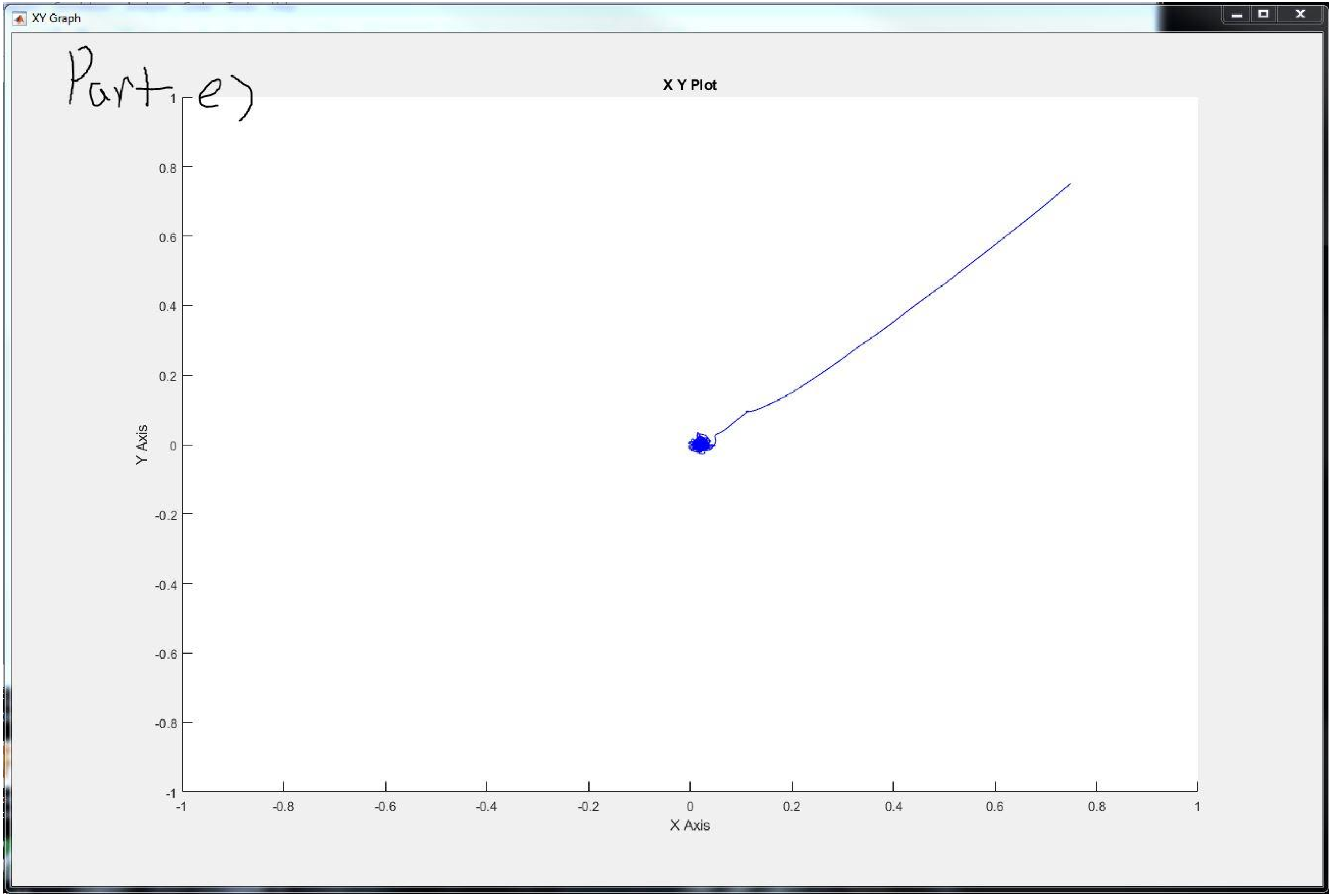
$$\boxed{K_i = 0.248}$$

Reported is the x, y data from the simulink model, with all noise turned on.

for x-position $\quad \bar{x} = 1.36 \times 10^{-4} \quad \sigma_x = 0.0754$

for y-position $\quad \bar{y} = 2.92 \times 10^{-4} \quad \sigma_y = 0.0734$


Control System Designer - Root Locus Editor for LoopTransfer_C
CONTROL SYSTEM
ROOT LOCUS EDITOR
VIEW
Open Session
Save Session
Edit Architecture
Multimodel Configuration
Tuning Methods
New Plot
Store
Retrieve
Compare
Export
Preferences
FILE
ARCHITECTURE
TUNING METHODS
ANALYSIS
DESIGNS
RESULTS
PREFERENCES
Data Browser
Controllers and Fixed Blocks
F
C
G
H
Designs
Responses
LoopTransfer_C
IOTransfer_r2y
IOTransfer_r2u
IOTransfer_du2y
IOTransfer_dy2y
Preview
Tunable Block
Name: C
Sample Time: 0
Value:
3.63 (s+0.2068) (s+1.076)
Root Locus Editor for LoopTransfer_C
IOTransfer_r2y: step
part e)
Root Locus Editor for LoopTransfer_C
Imag Axis
Real Axis
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-2
-1.5
-1
-0.5
0
0.5
1



XY Graph
Part e)
X Y Plot
Y Axis
X Axis
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1