$$1:\ p_a + \rho g Y(t) + \frac{\rho (v_1(t))^2}{2} = p_a + \rho g \cdot 0 + \frac{\rho (v_2(t))^2}{2}$$

$$2:\ \dot{Y}(t) = -v_1(t)$$

$$3:\ SY(t) = SY(0) + \int_0^t (q_{in}(t) - A v_2(t))dt$$

Reservoir

$$1:\ gY(t) + \frac{(v_1(t))^2}{2} = \frac{(v_2(t))^2}{2}$$

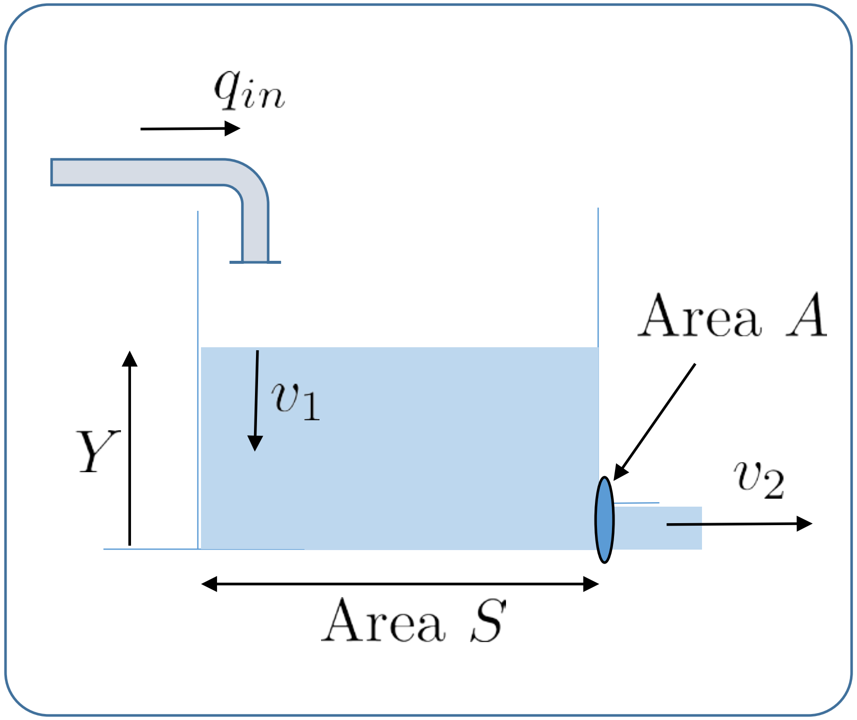
$$2:\ \dot{Y}(t) = -v_1(t)$$

$$3:\ S\dot{Y}(t) = q_{in}(t) - A v_2(t)$$

$$1:\ 2gY(t) + (v_1(t))^2 = (v_2(t))^2$$

$$2:\ \dot{Y}(t) = -v_1(t)$$

$$3:\ v_2(t) = \frac{q_{in}(t) - S\dot{Y}(t)}{A}$$



$$1:\ 2gY(t) + (\dot{Y}(t))^2 = \frac{(q_{in}(t) - S\dot{Y}(t))^2}{A^2}$$

$$2gY + \dot{Y}^2 = \frac{q_{in}^2 - 2q_{in}S\dot{Y} + S^2\dot{Y}^2}{A^2}$$

$$\left(1 - \frac{S^2}{A^2}\right)\dot{Y}^2 + \frac{2q_{in}S}{A^2}\dot{Y} + 2gY = \frac{1}{A^2}q_{in}^2$$

q_{in}: volume flow rate

Y: level

v_1: velocity

v_2: velocity

$$-\left(\frac{S^2}{A^2}-1\right)\dot{Y}^2+\frac{2q_{in}S}{A^2}\dot{Y}+2gY-\frac{1}{A^2}q_{in}^2=0$$

$$\left(\frac{S^2}{A^2}-1\right)\dot{Y}^2-\frac{2q_{in}S}{A^2}\dot{Y}-2gY+\frac{1}{A^2}q_{in}^2=0$$

$$\dot{Y}_{1,2}=\frac{\frac{2q_{in}S}{A^2}\pm\sqrt{\frac{4q_{in}^2S^2}{A^4}-4\left(\frac{S^2}{A^2}-1\right)\left(\frac{1}{A^2}q_{in}^2-2gY\right)}}{2\left(\frac{S^2}{A^2}-1\right)}$$

For $q_{in}=0$

$$\dot{Y}_1=\frac{\sqrt{8\left(\frac{S^2}{A^2}-1\right)gY}}{2\left(\frac{S^2}{A^2}-1\right)}=\sqrt{\frac{2gY}{\left(\frac{S^2}{A^2}-1\right)}}>0 \qquad \dot{Y}_2=\frac{-\sqrt{8\left(\frac{S^2}{A^2}-1\right)gY}}{2\left(\frac{S^2}{A^2}-1\right)}=-\sqrt{\frac{2gY}{\left(\frac{S^2}{A^2}-1\right)}}<0$$

therefore

$$\dot{Y}=\frac{\frac{2q_{in}S}{A^2}-\sqrt{\frac{4q_{in}^2S^2}{A^4}-4\left(\frac{S^2}{A^2}-1\right)\left(\frac{1}{A^2}q_{in}^2-2gY\right)}}{2\left(\frac{S^2}{A^2}-1\right)}$$

$$\dot{Y}=\frac{q_{in}S}{S^2-A^2}-\sqrt{\frac{q_{in}^2S^2}{(S^2-A^2)^2}-\frac{A^2\left(\frac{1}{A^2}q_{in}^2-2gY\right)}{(S^2-A^2)}}$$

$$\dot{Y}=\frac{q_{in}S}{S^2-A^2}-\sqrt{\left(\frac{S^2}{(S^2-A^2)^2}-\frac{1}{S^2-A^2}\right)q_{in}^2+\frac{2A^2gY}{S^2-A^2}}$$

$$\dot{Y}=\frac{q_{in}S}{S^2-A^2}-\sqrt{\frac{A^2}{(S^2-A^2)^2}q_{in}^2+\frac{2A^2gY}{S^2-A^2}}$$