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10/24/18

Homework 2

P.1

(1) $T_m = K_t i_a$
(2) $e = K_e \dot{\theta}_1$
(3) $V_a = R_a i_a + e$
(4) $J_m \ddot{\theta}_1 + b_m \dot{\theta}_1 = T_m - T_{21}$
(5) $J_d \ddot{\theta}_2 + b_d \dot{\theta}_2 = T_{12}$
(6) $T_{21} \dot{\theta}_1 = T_{12} \dot{\theta}_2 \Rightarrow T_{21} = \frac{T_{12}}{n}$

$\dot{\theta}_1 = n \dot{\theta}_2$ (7)
$\Rightarrow \ddot{\theta}_1 = n \ddot{\theta}_2$

Find $\frac{\Theta_2(s)}{V_a(s)}$

$(2) \xrightarrow{e} (3) = (3)^* \quad V_a = R_a i_a + K_e \dot{\theta}_1$

$\Rightarrow \quad i_a = \frac{V_a}{R_a} - \frac{K_e}{R_a} \dot{\theta}_1$

$(3)^* \xrightarrow{i_a} (1) = (1)^* \quad T_m = \frac{K_T V_a}{R_a} - \frac{K_T K_e}{R_a} \dot{\theta}_1$

$(6) \xrightarrow{T_{21}} (4) = (4)^* \quad J_m \ddot{\theta}_1 + b_m \dot{\theta}_1 = T_m - \frac{T_{12}}{n}$

$(5) \xrightarrow{T_{12}}, (1)^* \xrightarrow{T_m} (4)^* = (4)^{**} \quad J_m \ddot{\theta}_1 + b_m \dot{\theta}_1 = \left(\frac{K_T}{R_a} V_a - \frac{K_T K_e}{R_a} \dot{\theta}_1 \right) - \left(\frac{J_d}{n} \ddot{\theta}_2 + \frac{b_d}{n} \dot{\theta}_2 \right)$

$(7) \xrightarrow{\dot{\theta}_1} (4)^{**} \quad J_m n \ddot{\theta}_2 + b_m n \dot{\theta}_2 = \frac{K_T}{R_a} V_a - \frac{K_T K_e n}{R_a} \dot{\theta}_2 - \frac{J_d}{n} \ddot{\theta}_2 - \frac{b_d}{n} \dot{\theta}_2$

$\Rightarrow \left[J_m n + \frac{J_d}{n} \right] \ddot{\theta}_2 + \left[b_m n + \frac{K_T K_e n}{R_a} + \frac{b_d}{n} \right] \dot{\theta}_2 = \frac{K_T}{R_a} V_a$

$\Big\downarrow \mathcal{L}$

$$\left[J_m n + \frac{J_d}{n} \right] s^2 \Theta_2(s) + \left[b_m n + \frac{K_T K_e n}{R_a} + \frac{b_d}{n} \right] s \Theta_2(s) = \frac{K_T}{R_a} V_a(s)$$

$$\boxed{\frac{\Theta_2(s)}{V_a(s)} = \frac{K_T}{R_a} \frac{1}{\left[J_m n + \frac{J_d}{n} \right] s^2 + \left[b_m n + \frac{K_T K_e n}{R_a} + \frac{b_d}{n} \right] s}}$$

P.2

$$① \quad (m_t + m_p)\ddot{x} + m_p l \ddot{\theta}_B = u - b\dot{x}$$
$$② \quad m_p l \ddot{x} + (I + m_p l^2)\ddot{\theta}_B = -m_p l g \theta_B$$

$\mathcal{L} \downarrow$

$$① \quad (m_t + m_p)s^2 X(s) + m_p l s^2 \Theta(s) = U(s) - bsX(s)$$
$$② \quad m_p l s^2 X(s) + (I + m_p l^2)s^2 \Theta(s) = -m_p l g \Theta(s)$$

$$① \quad \left[(m_t + m_p)s^2 + bs\right] X(s) + m_p l s^2 \Theta(s) = U(s)$$

$$② \quad m_p l s^2 X(s) = -\left[(I + m_p l^2)s^2 + m_p l g\right]\Theta(s)$$

$$\Rightarrow X(s) = -\left[\frac{I\cancel{s^2}}{m_p l \cancel{s^2}} + \frac{\cancel{m_p} l^{\cancel{2}} \cancel{s^2}}{\cancel{m_p l s^2}} + \frac{\cancel{m_p l} g}{\cancel{m_p l} s^2}\right]\Theta(s)$$

$$\Rightarrow X(s) = -\left[\frac{I}{m_p l} + l + \frac{g}{s^2}\right]\Theta(s)$$

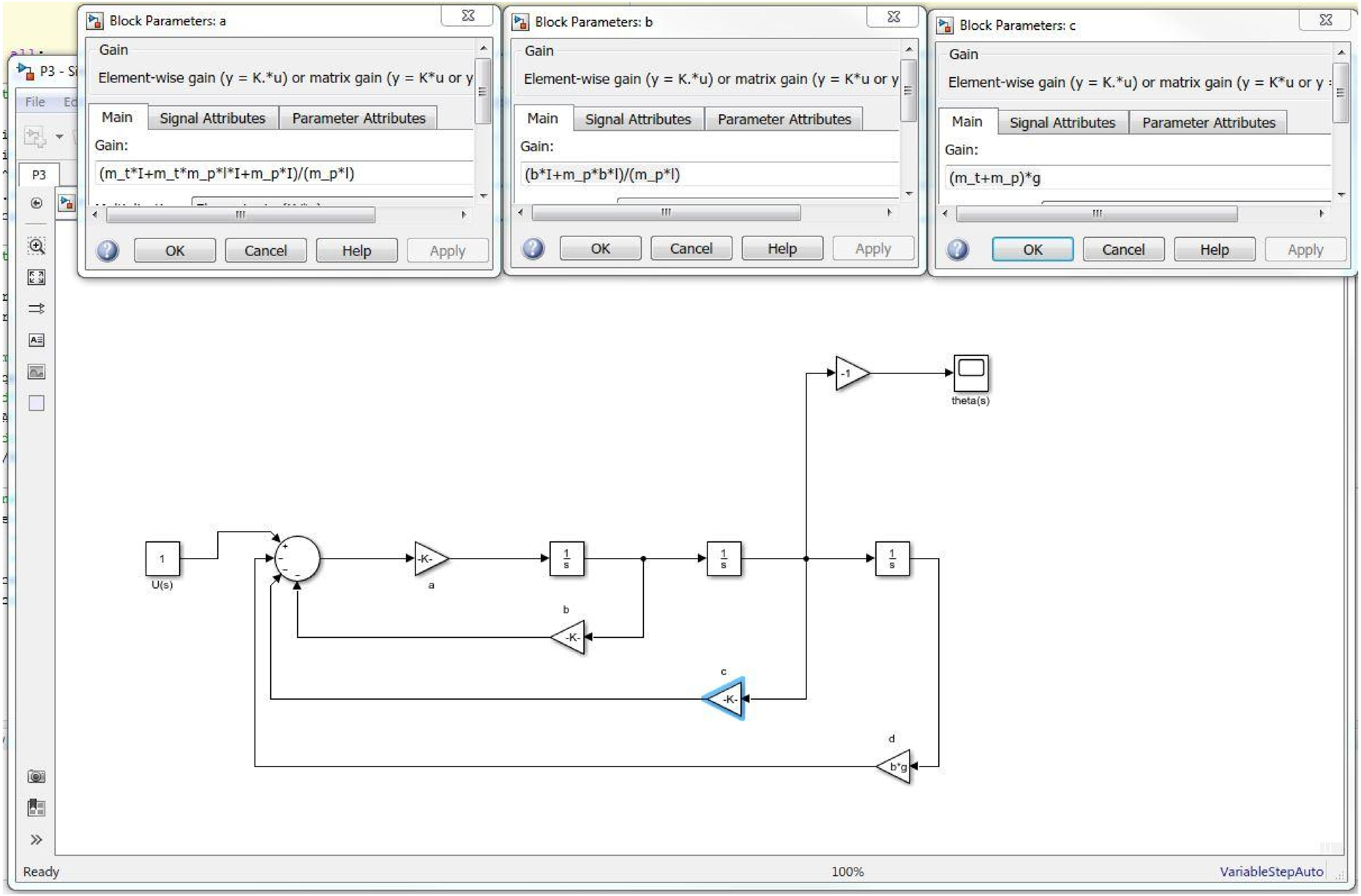
$$② \xrightarrow{X(s)} ① \quad -\left[m_t s^2 + m_p s^2 + bs\right]\left[\frac{I}{m_p l} + l + \frac{g}{s^2}\right]\Theta(s) + m_p l s^2 \Theta(s) = U(s)$$

$$\Rightarrow -\left\{\frac{m_t I}{m_p l}s^2 - \frac{m_t l}{\cancel{m_p l}}s^2 - m_t g\frac{\cancel{s^2}}{\cancel{s^2}} - \frac{\cancel{m_p} I}{\cancel{m_p} l}s^2 - m_p l s^2 - m_p g \frac{\cancel{s^2}}{\cancel{s^2}}\right.$$
$$\left. - \frac{bI}{m_p l}s - bls - bg\frac{\cancel{s}}{s^{\cancel{2}}}\right\}\Theta(s) + m_p l s^2 \Theta(s) = U(s)$$

$$③ \Rightarrow \left(\left[\cancel{m_p l} - \frac{m_t I}{m_p l} - m_t l - \frac{I}{l} - \cancel{m_p l}\right]s^2 - \left[\frac{bI}{m_p l} + bl\right]s - \left[m_t g + m_p g\right]\right.$$
$$\left. - bg\frac{1}{s}\right)\Theta(s) = U(s)$$

$$s \cdot ③ \left(-\left[\frac{m_t I + m_t m_p l I + m_p I}{m_p l}\right]s^3 - \left[\frac{bI}{m_p l} + bl\right]s^2 - \left[(m_t + m_p)g\right]s - bg\right)\theta(s) = U(s)$$

$$\boxed{\frac{\theta_B(s)}{U(s)} = \frac{-s}{\underbrace{\left(\frac{m_t I + m_t m_p l I + m_p I}{m_p l}\right)}_{a} s^3 + \underbrace{\left(\frac{bI + m_p b l}{m_p l}\right)}_{b} s^2 + \underbrace{\left((m_t + m_p)g\right)}_{c} s + \underbrace{bg}_{d}}}$$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Signal Attributes
Parameter Attributes
Gain:
(m_t*I+m_t*m_p*l*I+m_p*I)/(m_p*l)
OK
Cancel
Help
Apply
Block Parameters: b
Gain
Element-wise gain (y = K.*u) or matrix gain (y = K*u or y
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(b*I+m_p*b*l)/(m_p*l)
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Cancel
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Main
Signal Attributes
Parameter Attributes
Gain:
(m_t+m_p)*g
OK
Cancel
Help
Apply
P3
File
1
U(s)
-K-
a
1/s
-1
theta(s)
b
-K-
c
-K-
d
b*g
Ready
100%
VariableStepAuto

P.4

a) $\dot{y}_1 = y_2 + U(t)$ ①

$\dot{y}_2 = 2y_1 + y_2 - U(t)$ ②

find $\frac{Y_1(s)}{U(s)}$ and put it in form $\frac{1}{s+a}$?

$\mathcal{L} \downarrow$

$sY_1(s) = Y_2(s) + U(s)$ ①

$sY_2(s) = 2Y_1(s) + Y_2(s) - U(s)$ ②

① $Y_2(s) = sY_1 - U$

① $\xrightarrow{Y_2}$ ② $s(sY_1 - U) = 2Y_1 + (sY_1 - U) - U$

$\Rightarrow s^2Y_1 - sU = 2Y_1 + sY_1 - U - U$

$s^2Y_1 - sY_1 - 2Y_1 = sU - 2U$

$$\boxed{G(s)\ \frac{Y_1(s)}{U(s)} = \frac{s-2}{s^2-s-2}}$$

b) ~~I'm assuming~~ this question is asking ~~for the transfer function in~~ expanded partial - fraction ~~form.~~

$G(s) = \frac{\cancel{s-2}}{(s+1)\cancel{(s-2)}} = \cancel{\frac{A}{s+1} + \frac{B}{(s-2)}}$

~~$s-2 = As - 2A + Bs + B$~~

~~$1 = A + B \Rightarrow B = 1 - A$~~

~~$-2 = -2A + B \Rightarrow -2 = -2A + 1 - A$~~

~~$= -3 = -3A$~~

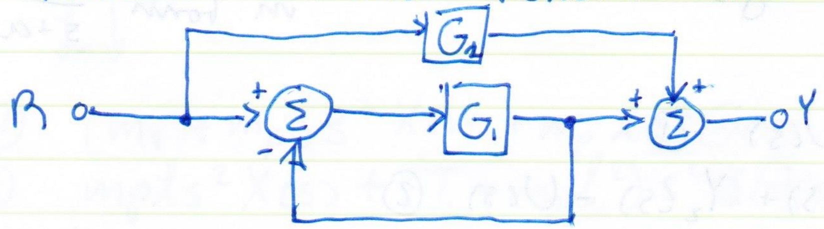
~~$A = 1$~~

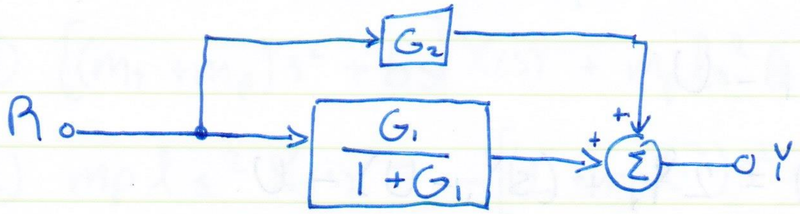
$G(s) = \frac{1}{s+1}$

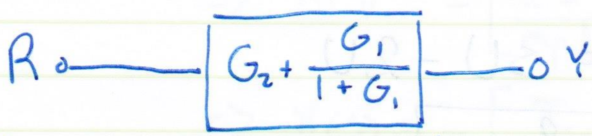
easier than I thought...

P.5 Textbook problem 3.20 a)

find the transfer function for the block diagram







$$\boxed{\frac{Y}{R} = G_2 + \frac{G_1}{1+G_1}}$$