Project 1

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(Due on: Wed, October 17 by 8:00PM via e-mail)

a) System Equation

$$f(Y,q_{in}) = \dot{Y} = \frac{q_{in} S}{S^2-A^2} - \sqrt{\frac{A^2}{(S^2-A^2)^2} q_{in}^2 + \frac{2A^2 g Y}{S^2-A^2}}$$

Taylor Series Linear Approximation

$$\dot{y} \quad \dot{Y} = f(Y^*, q_{in}^*) + \frac{\partial f}{\partial Y}\Big|_{Y^*, q_{in}^*}(Y-Y^*) + \frac{\partial f}{\partial q_{in}}\Big|_{Y^*, q_{in}^*}(q_{in}-q_{in}^*)$$

$$f(Y^*, q_{in}^*) = \dot{Y}^* \; ; \quad y = Y - Y^* \; ; \quad u = q_{in} - q_{in}^*$$

$$\Rightarrow \dot{y} = \dot{Y} - \dot{Y}^*$$

$$\Rightarrow \dot{y} + \dot{Y}^* = \dot{Y}$$

$$\dot{y} + \cancel{\dot{Y}^*} = \cancel{\dot{Y}^*} + \frac{\partial f}{\partial Y}\Big|_{Y^*, q_{in}^*} y + \frac{\partial f}{\partial q_{in}}\Big|_{Y^*, q_{in}^*} u$$

$$\Rightarrow \dot{y} = ay + bu \quad \text{where} \quad a = \frac{\partial f}{\partial Y}\Big|_{Y^*, q_{in}^*}, \quad b = \frac{\partial f}{\partial q_{in}}\Big|_{Y^*, q_{in}^*}$$

$$\mathcal{L}\downarrow$$

$$s\,y(s) = a\,y(s) + b\,u(s)$$

$$\boxed{\frac{y(s)}{u(s)} = \frac{b}{s-a}}$$

$$\boxed{\begin{aligned} a &= \frac{\partial f}{\partial Y}\Big|_{Y^*, q_{in}^*} = \frac{A^2 g}{(S^2-A^2)\sqrt{\frac{A^2}{(S^2-A^2)^2} q_{in}^{*2} + \frac{2A^2 g Y^*}{S^2-A^2}}} \\ b &= \frac{\partial f}{\partial q_{in}}\Big|_{Y^*, q_{in}^*} = \frac{S}{S^2-A^2} - \frac{A^2 q_{in}^*}{(S^2-A^2)\sqrt{\frac{A^2}{(S^2-A^2)^2} q_{in}^{*2} + \frac{2A^2 g Y^*}{S^2-A^2}}} \end{aligned}}$$

Figure 1: Solution for part a)

b) $$\dot{Y} = 0 = \frac{q_{in} S}{S^2 - A^2} - \sqrt{\frac{A^2}{(S^2-A^2)^2} q_{in}^2 + \frac{2A^2 g}{S^2 - A^2} Y}$$

$$\Rightarrow \left(\frac{q_{in} S}{S^2 - A^2} \right)^2 = \frac{A^2}{(S^2-A^2)^2} q_{in}^2 + \frac{2A^2 g}{S^2 - A^2} Y$$

$$\Rightarrow \left[\frac{S^2}{(S^2-A^2)^2} - \frac{A^2}{(S^2-A^2)^2} \right] q_{in}^2 = \frac{2A^2 g}{S^2 - A^2} Y$$

$$\Rightarrow \frac{\cancel{S^2 - A^2}}{(S^2-A^2)^{\cancel{2}}} q_{in}^2 = \frac{2A^2 g}{S^2 - A^2} Y$$

$$\Rightarrow \frac{1}{\cancel{S^2 - A^2}} q_{in}^2 = \frac{2A^2 g}{\cancel{S^2 - A^2}} Y$$

$$\Rightarrow q_{in}^2 = 2A^2 g Y$$

$$\Rightarrow \boxed{q_{in} = \sqrt{2A^2 g Y}}$$

The control variable q_{in} has <u>no</u> range to maintain a constant level Y for the whole working range of the level. Given $\dot{Y} = 0$, the relation between q_{in} and Y is parabolic in nature. for $q_{in} > 0$ and $Y > 0$, the relation $q_{in} = \sqrt{2A^2 g Y}$ has no range of q_{in} for which Y remains constant. The steady-state relation of Y and q_{in} is 1 to 1.

Figure 2: Solution for part b)

```matlab
% EE154 Project 1
% Written by Stephen Kemp
%

close all;
clear all;
%% Set Constants

S = pi*0.1^2;
A = pi*0.005^2;
B = S^2 - A^2;
g = 9.81;
w = logspace(-6,3,1000);

%% Set Variables and Functions

Y_star = linspace(0.05,0.15,30);
q_star = sqrt(2*A*g*Y_star); % Relationship from part b)

% From part a)
C = sqrt(A^2.*q_star.^2./B^2 + 2.*A^2.*g.*Y_star./B);
% df/dY
a = -A^2.*g./B./C;
% df/dq
b = S/B - A^2.*q_star./B^2 ./ C;

%% Generate Bode plots
figure(1)
for k = 1:length(a)
    G = tf([b(k)],[1, a(k)]);
    bode(G,w);
    hold on;
end
```

Figure 3: Matlab code used to perform parts c and d

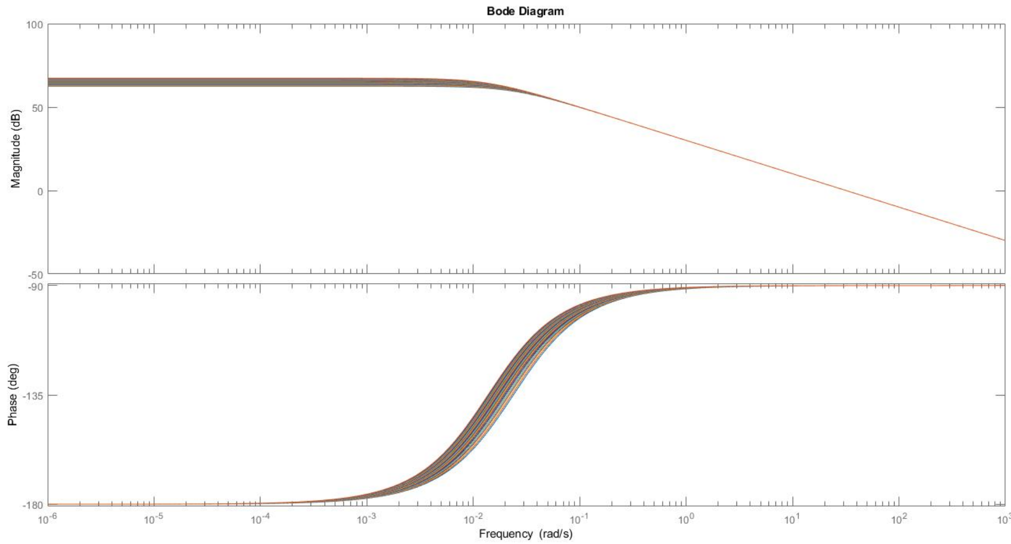

Figure 4: Bode plot showing the 30 magnitude and phase plots for $Y = [0.05, 0.15]$

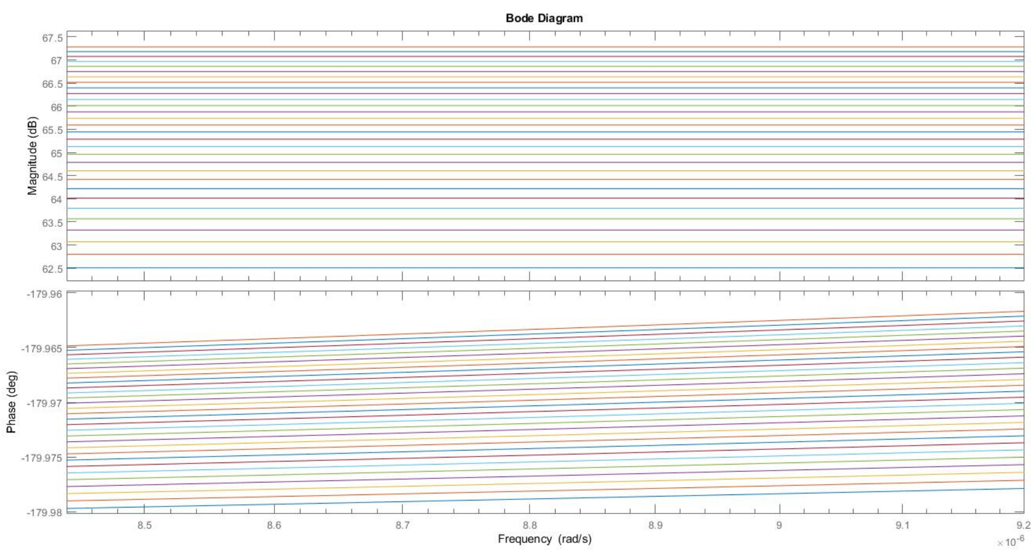

Figure 5: Zoom-in on the magnitude plot (top), showing the minimum and maximum DC gains as 62.5dB and 67.25dB respectively for the working range of $Y = [0.05, 0.15]$

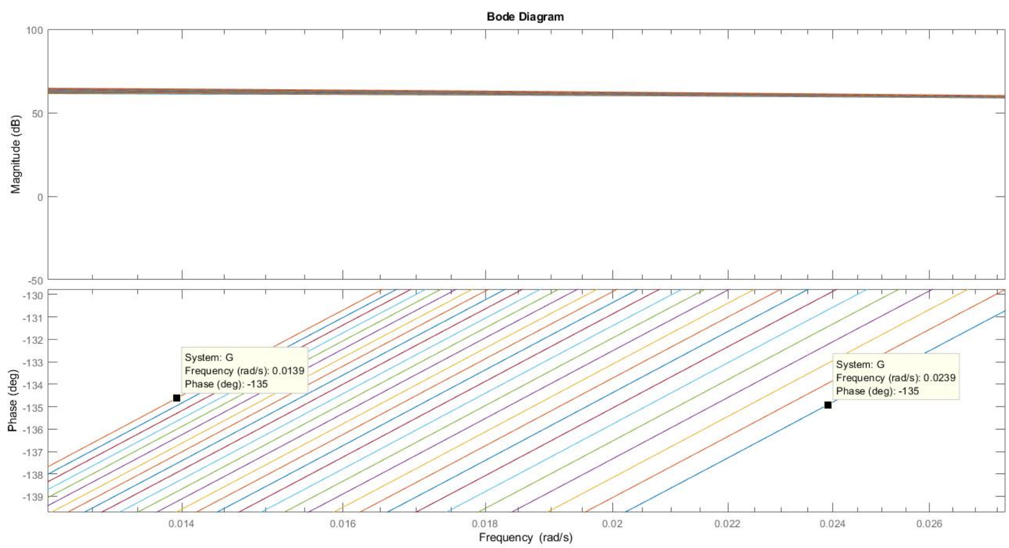

Figure 6: Zoom-in on the phase plot (bottom), showing the largest and smallest poles (points at which the phase is -135 degrees) as 0.0139rad/s and 0.0239rad/s respectively for the working range of $Y = [0.05, 0.15]$