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Homework 2

P.1

(1) $T_m = K_T i_a$

(2) $e = K_e \dot{\theta}_1$

(3) $V_a = R_a i_a + e$

(4) $J_m \ddot{\theta}_1 + b_m \dot{\theta}_1 = T_m - T_{21}$

(5) $J_d \ddot{\theta}_2 + b_d \dot{\theta}_2 = T_{12}$

(6) $T_{21} \dot{\theta}_1 = T_{12} \dot{\theta}_2 \Rightarrow T_{21} = \frac{T_{12}}{n}$

(7) $\dot{\theta}_1 = n \dot{\theta}_2 \Rightarrow \ddot{\theta}_1 = n \ddot{\theta}_2$

Find $\frac{\Theta_2(s)}{V_a(s)}$

(2) $\xrightarrow{e}$ (3) = (3)* $V_a = R_a i_a + K_e \dot{\theta}_1$

$\Rightarrow i_a = \frac{V_a}{R_a} - \frac{K_e}{R_a} \dot{\theta}_1$

(3)* $\xrightarrow{i_a}$ (1) = (1)* $T_m = \frac{K_T V_a}{R_a} - \frac{K_T K_e}{R_a} \dot{\theta}_1$

(6) $\xrightarrow{T_{21}}$ (4) = (4)* $J_m \ddot{\theta}_1 + b_m \dot{\theta}_1 = T_m - \frac{T_{12}}{n}$

(5) $\xrightarrow{T_{12}}$ (4)* = (4)** $J_m \ddot{\theta}_1 + b_m \dot{\theta}_1 = \left(\frac{K_T}{R_a} V_a - \frac{K_T K_e}{R_a} \dot{\theta}_1\right) - \left(\frac{J_d}{n} \ddot{\theta}_2 + \frac{b_d}{n} \dot{\theta}_2\right)$

(1)* $\xrightarrow{i_m}$

(7) $\xrightarrow{\dot{\theta}_1}$ (4)**

$J_m n \ddot{\theta}_2 + b_m n \dot{\theta}_2 = \frac{K_T}{R_a} V_a - \frac{K_T K_e n}{R_a} \dot{\theta}_2 - \frac{J_d}{n} \ddot{\theta}_2 - \frac{b_d}{n} \dot{\theta}_2$

$\Rightarrow \left[J_m n + \frac{J_d}{n}\right] \ddot{\theta}_2 + \left[b_m n + \frac{K_T K_e n}{R_a} + \frac{b_d}{n}\right] \dot{\theta}_2 = \frac{K_T}{R_a} V_a$

$\mathcal{L} \downarrow$

$\left[J_m n + \frac{J_d}{n}\right] s^2 \Theta_2(s) + \left[b_m n + \frac{K_T K_e n}{R_a} + \frac{b_d}{n}\right] s \Theta_2(s) = \frac{K_T}{R_a} V_a(s)$

$$\boxed{\frac{\Theta_2(s)}{V_a(s)} = \frac{K_T}{R_a} \frac{1}{\left[J_m n + \frac{J_d}{n}\right] s^2 + \left[b_m n + \frac{K_T K_e n}{R_a} + \frac{b_d}{n}\right] s}}$$