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Project 2

1 b) $\ddot{\theta} = a\theta + bu \qquad a = \frac{mgl}{I}\sin\theta^* \qquad b = \frac{L}{I}$

$\downarrow \mathcal{L}$

$s^2\theta(s) = a\theta(s) + bu(s)$

$$\boxed{\frac{\theta(s)}{u(s)} = \frac{b}{s^2 - a}}$$

c) Undamped 2nd order system equation: $\frac{1}{s^2+\omega_n^2}$

$$-a = \omega_n^2 \quad ; \quad \omega_n = \frac{2\pi}{T}$$

Measure period T

5 periods : $t_0 = 4.9$, $t_1 = 11.96$

$$T = \frac{11.96 - 4.90}{5} = 1.46s$$

assume $\theta^* = -90°$; about bottom of pendulum arc

$$a_{-90°} = \frac{mgl}{I}\sin(-90°) = -\frac{mgl}{I}$$

for $-a_{-90°} = \omega_n^2$

$$-\left(-\frac{mgl}{I}\right) = \left(\frac{2\pi}{T}\right)^2 = \left(\frac{2\pi}{1.46}\right)^2$$

$$\frac{mgl}{I} = 18.52$$

d) avg. value of $\theta(t)$ for thrust = 1

over 1 period, min $= -46^\circ$
max $= -90^\circ$

$$\bar{\theta} = \frac{-46^\circ + (-90^\circ)}{2} = -68^\circ$$

$$\frac{\theta(s)}{u(s)} = \frac{b}{s^2-a}$$

thrust = 1 at time t=0 is equivalent to an input $u(t) = \text{step}(t)$
so $u(s) = \frac{1}{s}$

$$\Rightarrow \theta(s) = \frac{b}{s^2-a} \cdot u(s)$$

$$\Rightarrow \theta(s) = \frac{b}{s^2-a} \cdot \frac{1}{s}$$

by Final Value thm.

$$\theta(t=\infty) = \lim_{s \to 0} \cancel{s} \cdot \frac{b}{s^2-a} \cdot \frac{1}{\cancel{s}}$$

$$\theta(\infty) = -\frac{b}{a}$$

Since the system in steady-state is oscillitory, $-\frac{b}{a}$ indicates the average value of the oscillation with reference to $\Theta^* = -90^\circ$

$$\bar{\theta} - \Theta^* = -\frac{b}{a}$$

assuming $\Theta^* = -90$, so $a = \frac{mgl}{I} = 18.52$

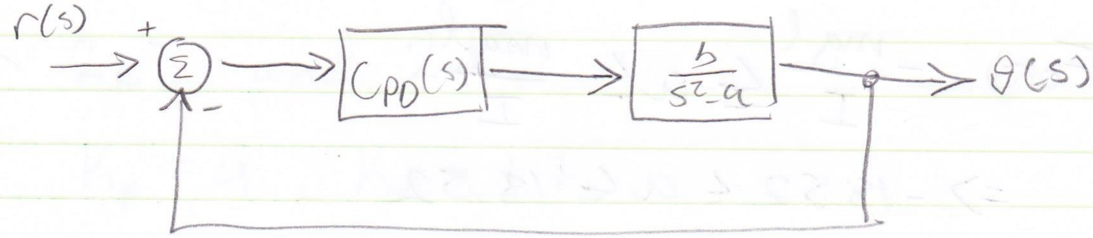
$$\Rightarrow -68^\circ - (-90^\circ) = \frac{-b}{-18.52}$$

$$\Rightarrow 22^\circ = \frac{b}{18.52}$$

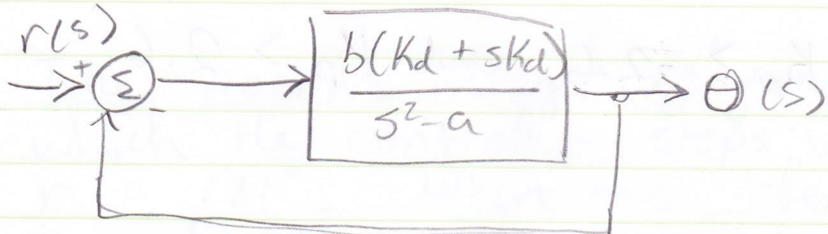
$$\Rightarrow 0.384 \text{ rad} = \frac{b}{18.52}$$

$$b = \frac{L}{I} = 7.11$$

2a)



$$C_{PD} = K_p + s K_d$$



$$\frac{\theta(s)}{r(s)} = \frac{\frac{b(K_p + sK_d)}{s^2-a}}{1 + \frac{b(K_p + sK_d)}{s^2-a}} = \frac{b(K_p + sK_d)}{s^2 - a + b(K_p + sK_d)}$$

$$\frac{\theta(s)}{r(s)} = \frac{bK_d s + bK_d}{s^2 + bK_p s + (bK_p - a)}$$

b) Routh Array

s^2	1	$bK_p - a$
s^1	bK_d	
s^0	$bK_p - a$	

Stability conditions

$bK_d > 0$ $\qquad K_p b - a > 0$

and $K_p b > a$

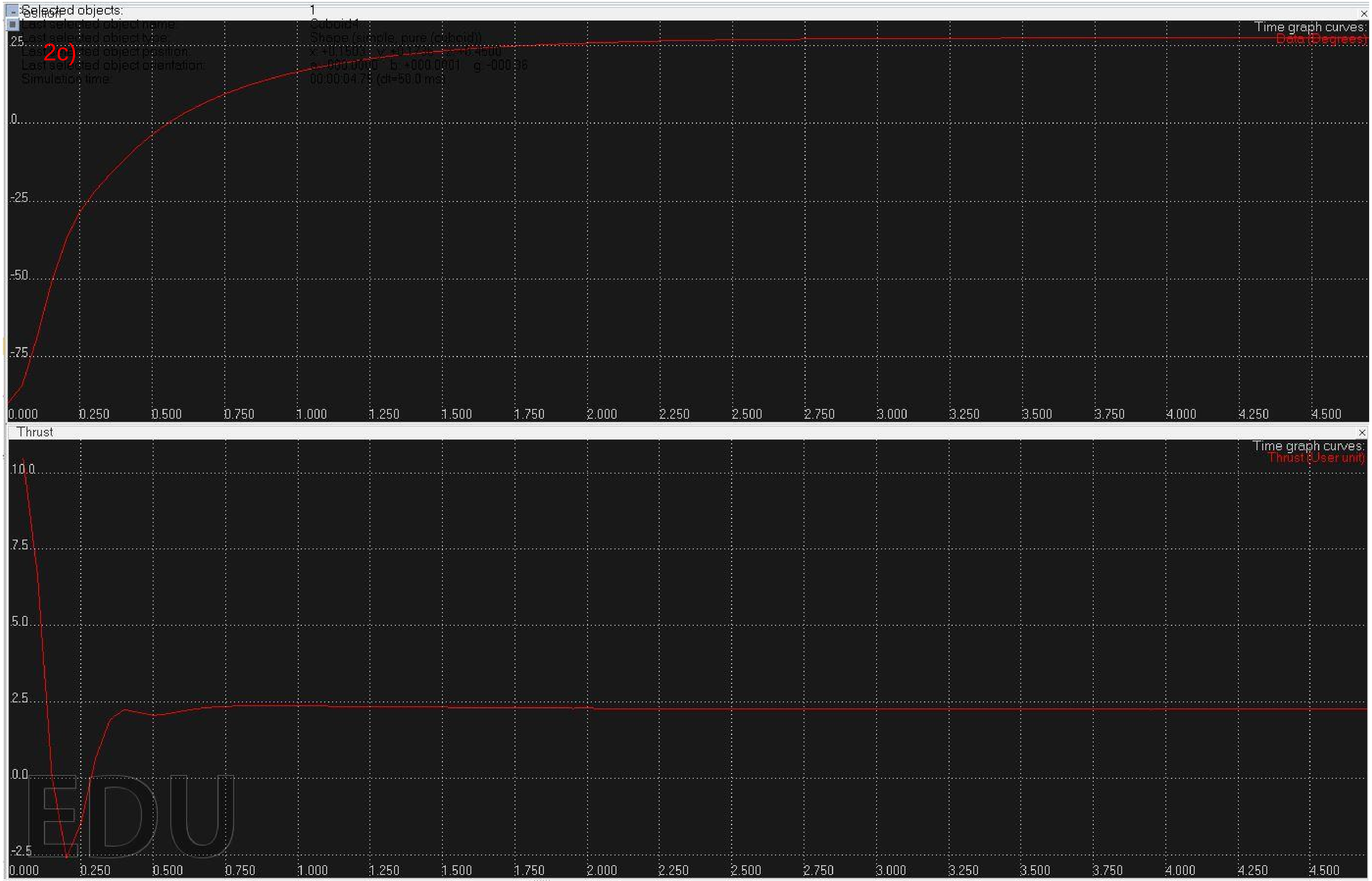
$$K_p > \frac{a}{b}$$

For $-\frac{mgl}{I} \leq a \leq \frac{mgl}{I}$

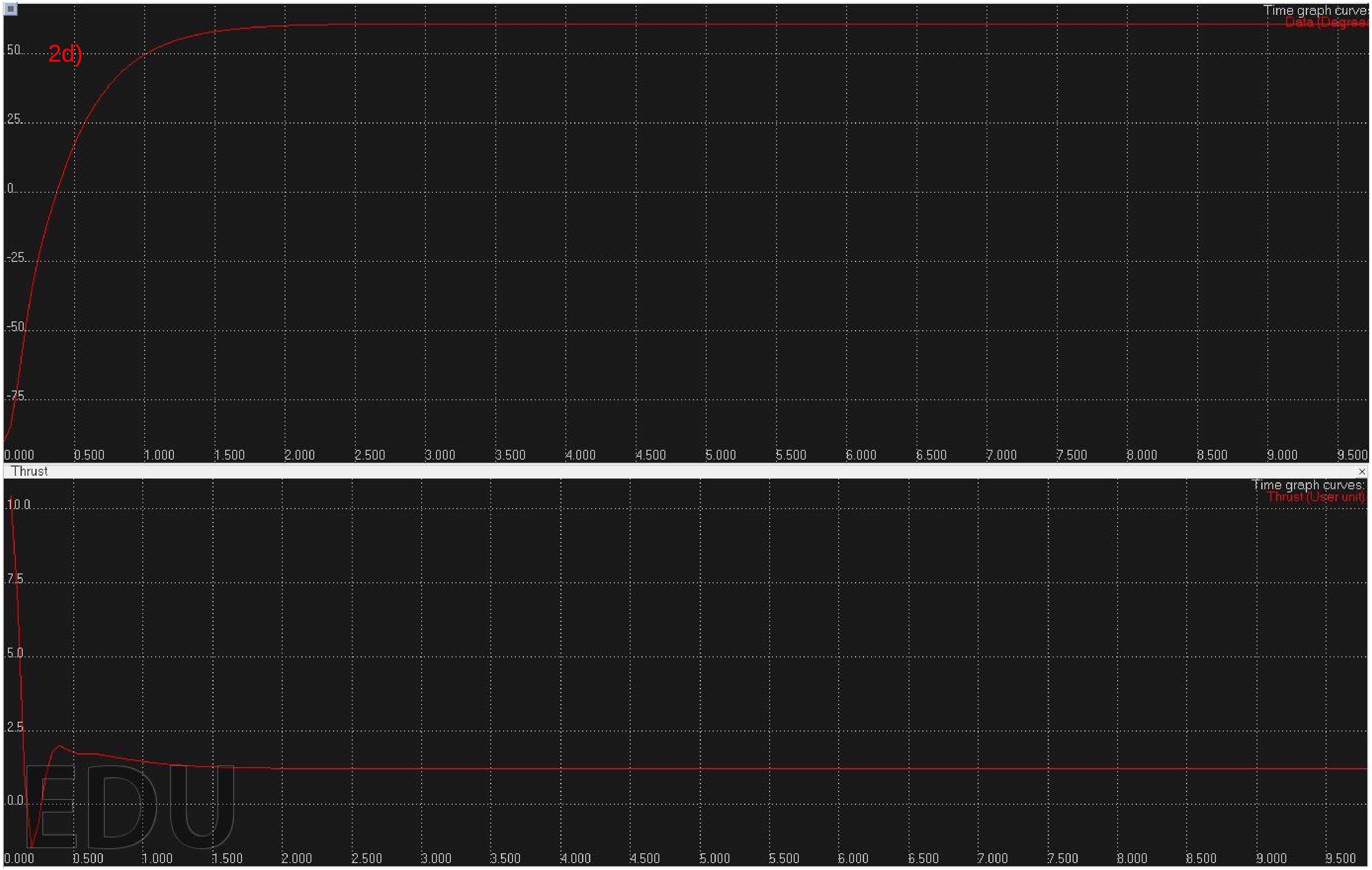
$\Rightarrow -18.52 \leq a \leq 18.52$

$K_p > \frac{-18.52}{7.11}$ and $K_p > \frac{18.52}{7.11}$

$K_d > 0$ and $K_p > -2.6$ and $K_p > 2.6$



Selected objects: 1
2c)
Time graph curves:
Data (Degrees)
Thrust
Time graph curves:
Thrust (User unit)



2d)
Time graph curves
Data (Degrees
50
25
0
-25
-50
-75
0.000
0.500
1.000
1.500
2.000
2.500
3.000
3.500
4.000
4.500
5.000
5.500
6.000
6.500
7.000
7.500
8.000
8.500
9.000
9.500
Thrust
Time graph curves:
Thrust (User unit)
10.0
7.5
5.0
2.5
0.0
EDU

For 2c) and 2d)

$K_p = 4$, $K_d = 1.7$

for 2d) $C = 2.6$

2e) The positive integer reference value for which the controller stops working is $r = 181°$. When this reference value is set, the system approaches 180° normally, then when it crosses 180° quickly loops back around to approach 180° again.
This is because of the fact that the angle measurements are defined on a scale from -180° to 180°. When the arm crossess 180° the angle is measured as -180° which gives an error of $181° - (-180°) = 361°$! The controller reacts by turning up the positive thrust proportionately to the error, causing the arm to spin around in the forward θ direction.