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Homework 3

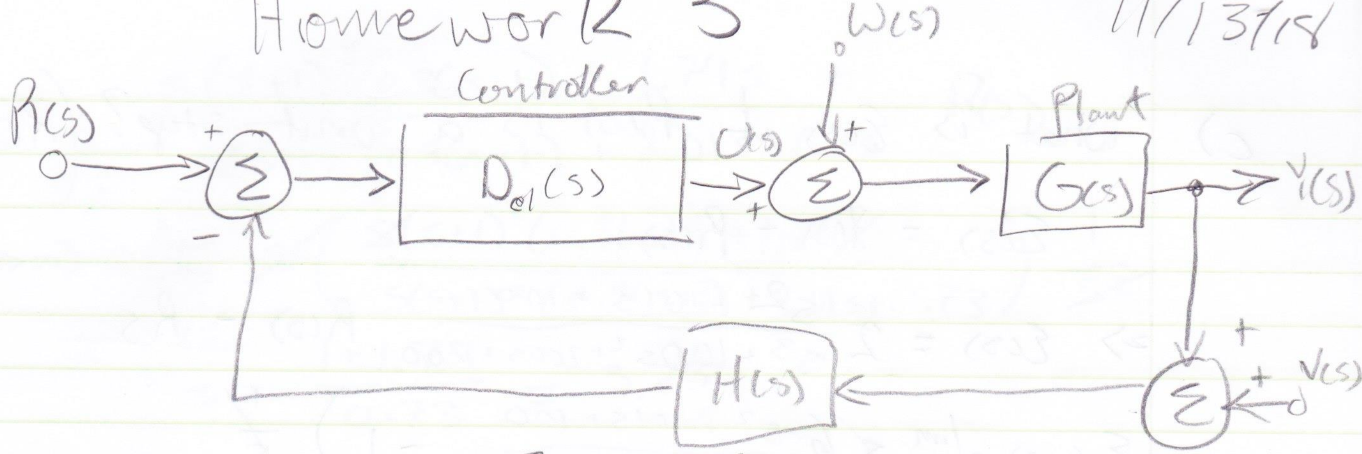


Figure 1

4.8

$$G(s) = \frac{1}{s}, \quad D_c(s) = \frac{2(s+1)}{s}, \quad H(s) = \frac{100}{s+100}$$

a) Find $\frac{Y(s)}{R(s)}$.

$$\frac{Y(s)}{R(s)} = \frac{D_c G}{1 + D_c G H} = \frac{\frac{2(s+1)}{s^2}}{1 + \frac{200(s+1)}{s^2(s+100)}}$$

$$= \frac{(2s+2)(s+100)}{s^2(s+100) + 200s + 200} = \frac{2s^2 + 202s + 200}{s^3 + 100s^2 + 200s + 200}$$

$$\boxed{\frac{Y(s)}{R(s)} = 2\frac{s^2 + 101s + 100}{s^3 + 100s^2 + 200s + 200}}$$

b) Find $\frac{Y(s)}{W(s)}$.

$$\frac{Y(s)}{W(s)} = \frac{G}{1 + D_c G H} = \frac{\frac{1}{s}}{1 + \frac{200(s+1)}{s^2(s+100)}}$$

$$\boxed{\frac{Y(s)}{W(s)} = \frac{s^2 + 100s}{s^3 + 100s^2 + 200s + 200}}$$

c) what is $\mathcal{E}(s)$ if $R(s)$ is a unit-step? $\left(R(s) = \frac{1}{s}\right)$

$$\mathcal{E}(s) = Y(s) - R(s)$$

$$\Rightarrow \mathcal{E}(s) = 2\frac{s^2+101s+100}{s^3+100s^2+200s+200} R(s) - R_s$$

$$\mathcal{E}(t{=}\infty) = \lim_{s\to 0} \cancel{s}\left(2\frac{s^2+101s+100}{s^3+100s^2+200s+200} - 1\right)\frac{1}{\cancel{s}}$$

$$\boxed{\mathcal{E}(t\to\infty) = \frac{200}{200} - 1 = 0.}$$

d) what if $R(s)$ is a unit-ramp? $\left(R(s) = \frac{1}{s^2}\right)$

$$\mathcal{E}(t{=}\infty) = \lim_{s\to 0} \cancel{s}\left(2\frac{s^2+101s+100}{s^3+100s^2+200s+200} - \frac{s^3+100s^2+200s+200}{s^3+100s^2+200s+200}\right)\frac{1}{s^{\cancel{2}}}$$

$$= \lim_{s\to 0}\left(\frac{-s^3-98s^2+2\cancel{s}}{s^4+100s^3+200s^2+200\cancel{s}}\right)$$

$$\boxed{\mathcal{E}(t{=}\infty) = \frac{1}{100}}$$

4.9 For figure 1, suppose that $G(s) = \frac{1}{s(s+1)^2}$, $D_c(s) = 0.73$, $H(s) = \frac{2.75s+1}{0.36s+1}$

(c) What is the value of the velocity error coefficient K_v?

$$\mathcal{E}(s) = R(s) - Y(s) = \left(1 - \frac{D_cG}{1+D_cGH}\right)R(s) = \left(\frac{1+D_cGH}{1+D_cGH} - \frac{D_cG}{1+D_cGH}\right)R(s)$$

$$\mathcal{E}(s) = \frac{1+D_cG(H-1)}{1+D_cGH} R(s)$$

$$\mathcal{E}(s) = \frac{1+\frac{0.73}{s(s+1)^2}\left(\frac{2.75s+1}{.36s+1} - 1\right)}{1+\frac{0.73(2.75s+1)}{s(s+1)^2(0.36s+1)}} R(s) = \frac{1+\frac{0.73}{s(s+1)^2}\cdot\frac{2.39s}{0.36s+1}}{1+\frac{0.73(2.75+1)}{s(s+1)^2(0.36s+1)}} R(s)$$

$$\mathcal{E} = \frac{s(s+1)^2(0.36s+1) + 1.74s}{s(s+1)^2(0.36s+1) + 2.01s + 0.73} R(s)$$

$$\mathcal{E}(t\to\infty) = \lim_{s\to 0} \cancel{s}\left(\frac{\cancel{s}(s+1)^2(0.36s+1) + 1.74\cancel{s}}{s(s+1)^2(0.36s+1) + 2.01s + 0.73}\right)\frac{1}{\cancel{s^2}}$$

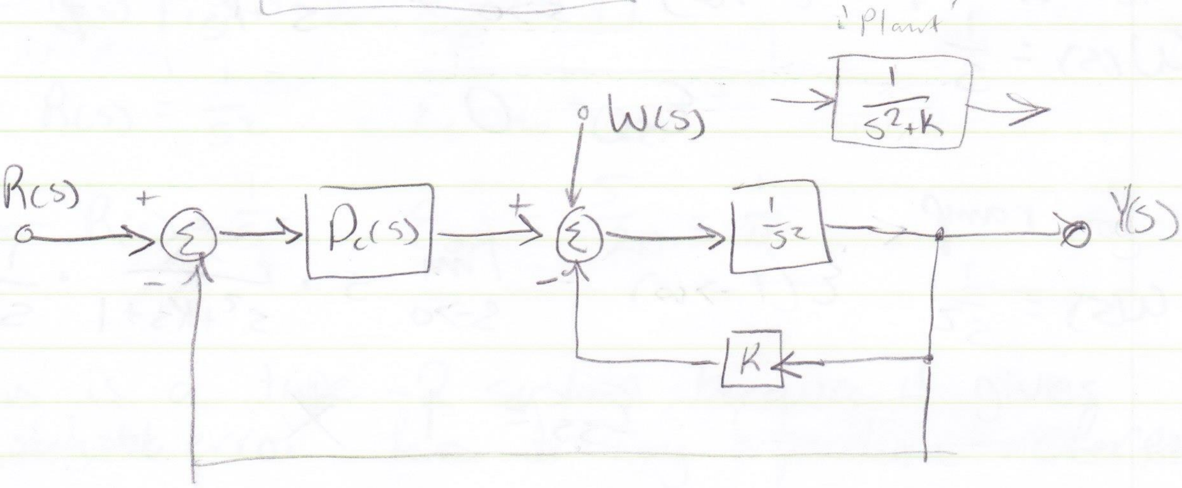
$$= \frac{1+1.74}{0.73} = 3.75$$

$$\frac{1}{K_v} = \mathcal{E}_{ss} \quad \text{for } R(s) = \frac{1}{s^2} \text{ (unit ramp)}$$

$$\frac{1}{K_v} = 3.75$$

$$\boxed{K_v = 0.267}$$

4.11



a) What condition must $D_c(s)$ satisfy so that the system can track a ramp reference with constant error

Since the Plant is a type 2 system $\left(G(s) = \frac{1}{s^2+K}\right)$, $D_c(s)$ has to include an integrator to track a ramp reference with constant error.

for disturbance $W(s)$

4.11) b) Set $R(s) = 0$, $W(s) \neq 0$

$$\mathcal{E} = \cancel{R}^{0} - Y = -Y$$

$$\Rightarrow \frac{\mathcal{E}}{W} = T_W = \frac{\frac{1}{s^2+K}}{1 + \frac{1}{s(s^2+K)}}$$

$$= \frac{1}{s^2+K+\frac{1}{s}}$$

$$\frac{\mathcal{E}(s)}{W(s)} = \frac{s}{s^3+Ks+1}$$

for unit step $W(s) = \frac{1}{s}$ $\quad \mathcal{E}(t\to\infty) = \lim_{s\to 0} \cancel{s} \cdot \frac{s}{s^3+Ks+1} \cdot \frac{1}{\cancel{s}}$

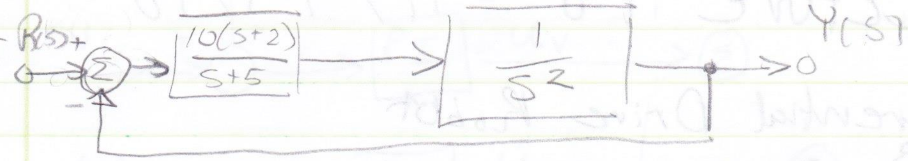
$$\mathcal{E}_{ss} = 0$$

for ramp $W(s) = \frac{1}{s^2}$ $\quad \mathcal{E}(t\to\infty) = \lim_{s\to 0} s \cdot \frac{s}{s^3+Ks+1} \cdot \frac{1}{s^2}$

$$\mathcal{E}_{ss} = 1 \quad \times$$

the system can handle step-disturbances with 0 steady-state error.

4.15



a) Find the system type for Reference tracking and the corresponding error constant for the system

$$\mathcal{E} = \frac{1}{1 + \frac{10(s+2)}{(s+5)s^2}} = \frac{s^2(s+5)}{s^2(s+5) + 10(s+2)}$$

$$= \frac{s^3 + 5s^2}{s^3 + 5s^2 + 10s + 20}$$

step for $R(s) = \frac{1}{s}$ $\quad \mathcal{E}_{ss} = 0$

ramp for $R(s) = \frac{1}{s^2}$ $\quad \mathcal{E}_{ss} = 0$

parabola for $R(s) = \frac{1}{s^3}$ $\quad \mathcal{E}_{ss} = \frac{5}{20} = \frac{1}{4}$

This is a type-2 system, because it gives constant steady state error for tracking a parabolic reference

static error constant $\quad \frac{1}{K_a} = \mathcal{E}_{ss}$

$$\boxed{K_a = \frac{1}{\mathcal{E}_{ss}} = 4}$$