a) System Equation

$$f(Y,q_{in}) = \dot{Y} = \frac{q_{in} S}{S^2 - A^2} - \sqrt{\frac{A^2}{(S^2-A^2)^2} q_{in}^2 + \frac{2A^2 g Y}{S^2 - A^2}}$$

Taylor Series Linear Approximation

$$\dot{y} \quad \dot{Y} = f(Y^*, q_{in}^*) + \frac{\partial f}{\partial Y}\Big|_{\substack{Y^* \\ q_{in}^*}} (Y - Y^*) + \frac{\partial f}{\partial q_{in}}\Big|_{\substack{Y^* \\ q_{in}^*}} (q_{in} - q_{in}^*)$$

$$f(Y^*, q_{in}^*) = \dot{Y}^* \quad ; \quad y = Y - Y^* \quad ; \quad u = q_{in} - q_{in}^*$$

$$\Rightarrow \dot{y} = \dot{Y} - \dot{Y}^*$$

$$\Rightarrow \dot{y} + \dot{Y}^* = \dot{Y}$$

$$\dot{y} + \cancel{\dot{Y}^*} = \cancel{\dot{Y}^*} + \frac{\partial f}{\partial Y}\Big|_{\substack{Y^* \\ q_{in}^*}} y + \frac{\partial f}{\partial q_{in}}\Big|_{\substack{Y^* \\ q_{in}^*}} u$$

$$\Rightarrow \dot{y} = ay + bu \quad \text{where} \quad a = \frac{\partial f}{\partial Y}\Big|_{\substack{Y^* \\ q_{in}^*}}$$

$$\mathcal{L}\downarrow \qquad\qquad b = \frac{\partial f}{\partial q_{in}}\Big|_{\substack{Y^* \\ q_{in}^*}}$$

$$s\,y(s) = a\,y(s) + b\,u(s)$$

$$\boxed{\frac{y(s)}{u(s)} = \frac{b}{s-a}}$$

$$\boxed{\begin{aligned} a &= \frac{\partial f}{\partial Y}\Big|_{\substack{Y^* \\ q_{in}^*}} = \frac{A^2 g}{(S^2-A^2)\sqrt{\frac{A^2}{(S^2-A^2)^2} q_{in}^{*2} + \frac{2A^2 g Y^*}{S^2-A^2}}} \\ b &= \frac{\partial f}{\partial q_{in}}\Big|_{\substack{Y^* \\ q_{in}^*}} = \frac{S}{S^2-A^2} - \frac{A^2 q_{in}^*}{(S^2-A^2)\sqrt{\frac{A^2}{(S^2-A^2)^2} q_{in}^{*2} + \frac{2A^2 g Y^*}{S^2-A^2}}} \end{aligned}}$$

b) $$\dot{Y} = 0 = \frac{q_{in} S}{S^2 - A^2} - \sqrt{\frac{A^2}{(S^2-A^2)^2} q_{in}^2 + \frac{2A^2 g}{S^2-A^2} Y}$$

$$\Rightarrow \left(\frac{q_{in} S}{S^2-A^2}\right)^2 = \frac{A^2}{(S^2-A^2)^2} q_{in}^2 + \frac{2A^2 g}{S^2-A^2} Y$$

$$\Rightarrow \left[\frac{S^2}{(S^2-A^2)^2} - \frac{A^2}{(S^2-A^3)^2}\right] q_{in}^2 = \frac{2A^2 g}{S^2-A^2} Y$$

$$\Rightarrow \frac{\cancel{S^2-A^2}}{(S^2-A^2)^{\cancel{2}}} q_{in}^2 = \frac{2A^2 g}{S^2-A^2} Y$$

$$\Rightarrow \frac{1}{\cancel{S^2-A^2}} q_{in}^2 = \frac{2A^2 g}{\cancel{S^2-A^2}} Y$$

$$\Rightarrow q_{in}^2 = 2A^2 g Y$$

$$\Rightarrow \cancel{g} \boxed{q_{in} = \sqrt{2A^2 g Y}}$$

The control variable q_{in} has <u>no</u> range to maintain a constant level Y for the whole working range of the level. Given $\dot{Y} = 0$, the relation between q_{in} and Y is parabolic in nature. for $q_{in} > 0$ and $Y > 0$, the relation $q_{in} = \sqrt{2A^2 g Y}$ has no range of q_{in} for which Y remains constant. The steady-state relation of Y and q_{in} is 1 to 1.