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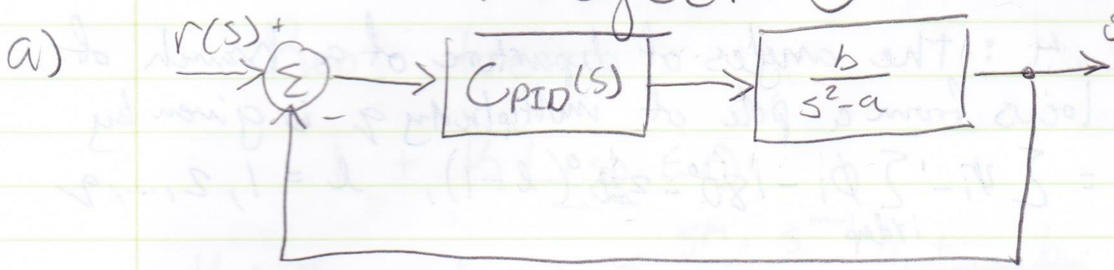
11/8/18

Project 3

a)

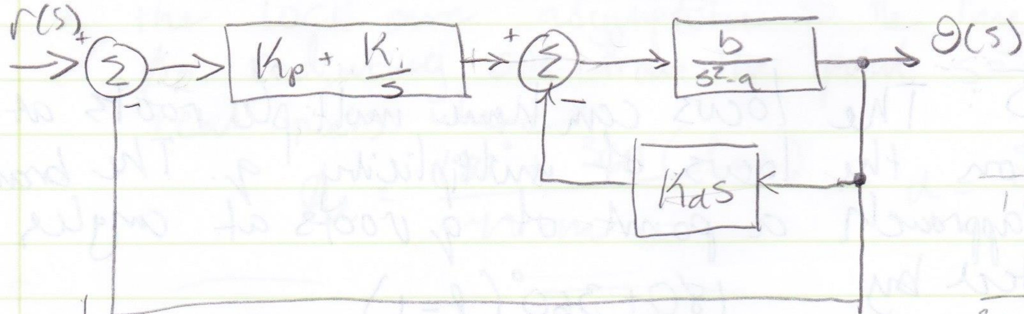


find $\frac{\theta(s)}{r(s)}$ if $C_{PID}(s) = K_p + sK_d + \frac{K_i}{s}$

$$\frac{\theta(s)}{r(s)} = \frac{\frac{b(K_p + sK_d + K_i/s)}{s^2-a}}{1 + \frac{b(K_p + sK_d + K_i/s)}{s^2-a}} = \frac{bK_p + bsK_d + bK_i/s}{s^2 - a + bK_p + bsK_d + K_i/s}$$

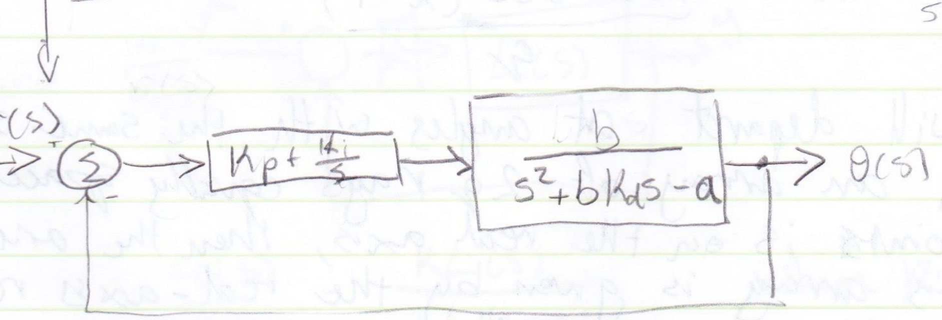
$$\boxed{\frac{\theta(s)}{r(s)} = \frac{s^2 bK_d + s\,bK_p + bK_i}{s^3 + s^2 bK_d + s(bK_p - a) + bK_i}}$$

b)



$$\frac{\frac{b}{s^2-a}}{1 + \frac{bK_d s}{s^2-a}}$$

$$\frac{b}{s^2 - a + bK_d s}$$



b) cntd.

find $\frac{\theta(s)}{r(s)} = \dfrac{\dfrac{(K_p + \frac{K_i}{s}) \cdot b}{s^2 + bK_d s - a}}{1 + \dfrac{(K_p + \frac{K_i}{s}) \cdot b}{s^2 + bK_d s - a}}$

$$= \frac{bK_p + b\frac{K_i}{s}}{s^2 + bK_d s - a + bK_p + b\frac{K_i}{s}} = \frac{bK_p s + bK_i}{s^3 + bK_d s^2 + (bK_p - a)s + bK_i}$$

$$\boxed{\frac{\theta(s)}{r(s)} = \frac{bK_p s + bK_i}{s^3 + bK_d s^2 + (bK_p - a)s + bK_i}}$$

the poles of this transfer function are the same as the poles from part a)

c) Routh Array

s^3	1	$bK_p - a$
s^2	bK_d	bK_i
s^1	$bK_p - a - \frac{K_i}{K_d}$	0
s^0	bK_i	

Stability Criteria

$bK_d > 0$

$\boxed{K_d > 0}$

$bK_i > 0$

$\boxed{K_i > 0}$

$bK_p - a - \frac{K_i}{K_d} > 0$

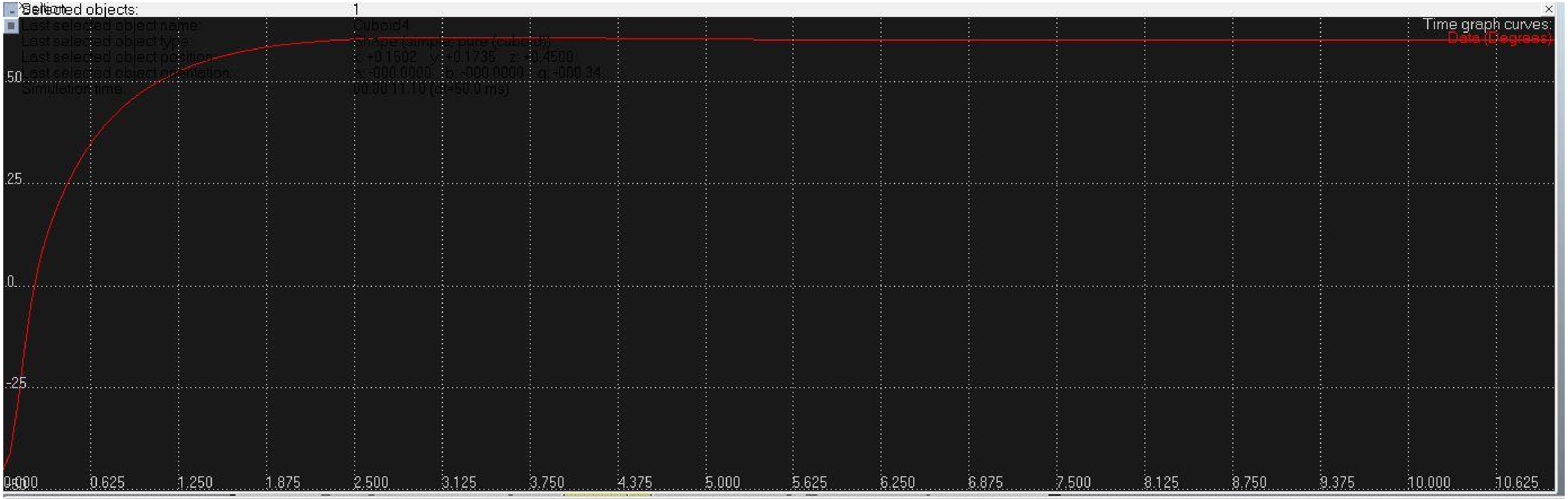
$bK_p > \frac{K_i}{K_d} + a$

For $-18.52 \leq a \leq 18.52$ and $b = 7.2$

harder criterion → $\boxed{7.2K_p > \frac{K_i}{K_d} + 18.52}$ ✓

or $7.2K_p > \frac{K_i}{K_d} - 18.52$ X

$$\boxed{\begin{aligned} K_i &= 1.5 \\ K_d &= 1.5 \\ K_p &= 5 \end{aligned}} \left(> \frac{1 + 18.52}{7.2} = 2.7 \right)$$


Selected objects: 1
Last selected object name
Cuboid4
Last selected object type
Last selected object position
Last selected object orientation
Simulation time
Time graph curves:
Data (Degrees)
50
25
0
-25
0.000
0.625
1.250
1.875
2.500
3.125
3.750
4.375
5.000
5.625
6.250
6.875
7.500
8.125
8.750
9.375
10.000
10.625


Thrust
Time graph curves:
Thrust (User unit)
7.5
5.0
2.5
0.0
0.000
0.625
1.250
1.875
2.500
3.125
3.750
4.375
5.000
5.625
6.250
6.875
7.500
8.125
8.750
9.375
10.000
10.625
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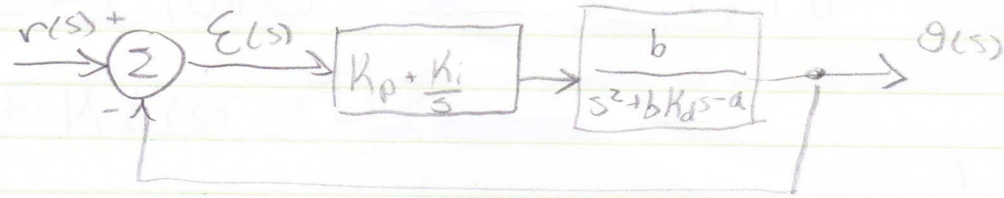
d) The PID controller forces the system to the reference level with high precision. For most angles, the error will be reduced to $\pm 0.01^\circ$ within 10 seconds, and will continue to approach zero with very high precision. This is compared to the PD controller from project 2 which, even with the non-linear compensation, would be off by $\pm 2^\circ$.

The main downside of this controller is the wind-up. For large angles, the arm will overshoot the reference by a large amount. This may cause the system to become unstable if θ becomes greater than 180° because of how the angles are mapped from -180° to 180°.

e) 

$$\frac{\varepsilon(s)}{r(s)} = \frac{1}{1 + \frac{(K_p + \frac{K_i}{s})b}{s^2 + bK_d s - a}} = \frac{s^2 + bK_d s - a}{s^2 + bK_d s - a + bK_p + \frac{bK_i}{s}}$$

$$\frac{\varepsilon(s)}{r(s)} = \frac{s^3 + bK_p s^2 - as}{s^3 + bK_d s^2 + (bK_p - a)s + bK_i}$$

By Final Value thm. for $r(s) = \frac{1}{s}$ (step function)

$$\varepsilon(t=\infty) = \lim_{s \to 0} s\varepsilon(s) = \lim_{s \to 0} \cancel{s} \cdot \frac{s^3 + bK_p s^2 - as}{s^3 + bK_d s^2 + (bK_p - a)s + bK_i} \cdot \frac{1}{\cancel{s}}$$

$$= \frac{0}{bK_i} = 0$$

The PID controller should provide error-less steady-state tracking for a step input.