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# Homework 5

1) Use Routh Criterion to find the range of K values for stable system with forward path

$$KG(s) = K\frac{s+3}{s(s+1)(s^2+4s+5)}$$

$$\frac{Y(s)}{R(s)} = \frac{K(s+3)}{s(s+1)(s^2+4s+5) + K(s+3)} = \frac{K(s+3)}{(s^2+s)(s^2+4s+5) + K(s+3)}$$

$$= \frac{K(s+3)}{s^4+s^3+4s^3+4s^2+5s^2+5s+Ks+K3}$$

$$= \frac{K(s+3)}{s^4+5s^3+9s^2+(5+K)s+3K}$$

Routh Array

| | | | |
|---|---|---|---|
| $s^4$ | $1$ | $9$ | $3K$ |
| $s^3$ | $5$ | $5+K$ | $0$ |
| $s^2$ | $8-\frac{K}{5}$ | $3K$ | $0$ |
| $s^1$ | $\frac{-K^2-40K+200}{40-K}$ | $0$ | |
| $s^0$ | $3K$ | | |

$$\frac{45-5+K}{5}$$

$$8-\frac{K}{5}$$

$$\frac{(5+K)(8-\frac{K}{5}) - 15K}{8-\frac{K}{5}}$$

$$5+K-\frac{15K}{8-\frac{K}{5}}$$

$$5+K-\frac{15K}{\frac{40-K}{5}}$$

$$5+K-\frac{75K}{40-K}$$

$$\frac{(5+K)(40-K) - 75K}{40-K}$$

$$\frac{200+35K-K^2-75K}{40-K}$$

$$\frac{-K^2-40K+200}{40-K}$$

Conditions: $8-\frac{K}{5} > 0$

$-\frac{K}{5} > -8$

$\frac{K}{5} < 8$

① $\boxed{K < 40}$

$3K > 0$

② $\boxed{K > 0}$

$$\frac{-K^2-40K+200}{40-K} > 0$$

$$\Rightarrow K+80-\frac{3000}{40-K} > 0$$

$$\Rightarrow K+80 > \frac{3000}{40-K}$$

$$\Rightarrow (K+80)(40-K) > 3000$$

$$\Rightarrow -K^2-40K+3200 > 3000$$

$$-K^2-40K+200 > 0$$

$$(K+10(2+\sqrt{6}))(K-10(\sqrt{6}-2)) > 0$$

③ $\boxed{\begin{array}{l} K > 10(\sqrt{6}-2) \\ K < 10(2+\sqrt{6}) \end{array}}$

Both already satisfied by 1 & 2

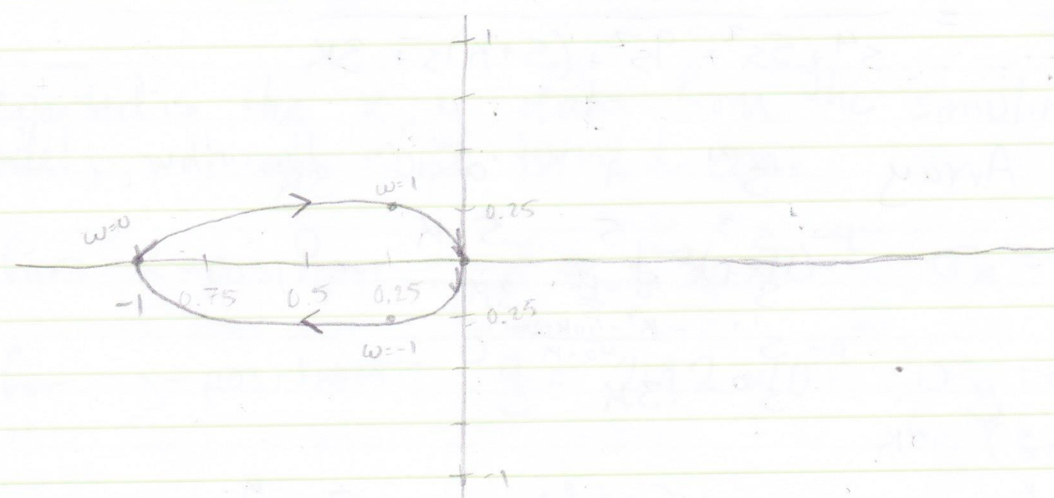
Complete conditions for stability

$$\boxed{K > 0, \quad K < 40}$$

2) Sketch the nyquist plot and find a range of stable K values for

$$G(s) = \frac{1}{(s-1)(s+1)^2}$$

$$G(j\omega) = \frac{1}{(j\omega-1)(j\omega+1)^2} = \frac{-1}{j(\omega^3+\omega)+(\omega^2+1)}$$




$\omega = 0 \qquad G(j\omega) = -1$

$\omega = 1 \qquad G(j1) = \frac{-1}{2j+2} \cdot \frac{(2-2j)}{(2-2j)} = \frac{2j-2}{8} = -\frac{1}{4} + \frac{1}{4}j$

$s = re^{j\theta} \quad r \to \infty \qquad G(re^{j\theta}) = \lim_{r\to\infty} \frac{1}{re^{j\theta} \cdot r^2 e^{2j\theta}} = 0e^{-3j\theta}$

$\theta = \frac{\pi}{2} \qquad G(\infty e^{j\theta}) = 0e^{-j\frac{3\pi}{2}}$

$\theta = -\frac{\pi}{2} \qquad G(\infty e^{j\theta}) = 0e^{j\frac{3\pi}{2}}$

$P = 1$, for $Z = 0$, $N = 1$

$-\frac{1}{K} > -1 \qquad\qquad -\frac{1}{K} < 0$

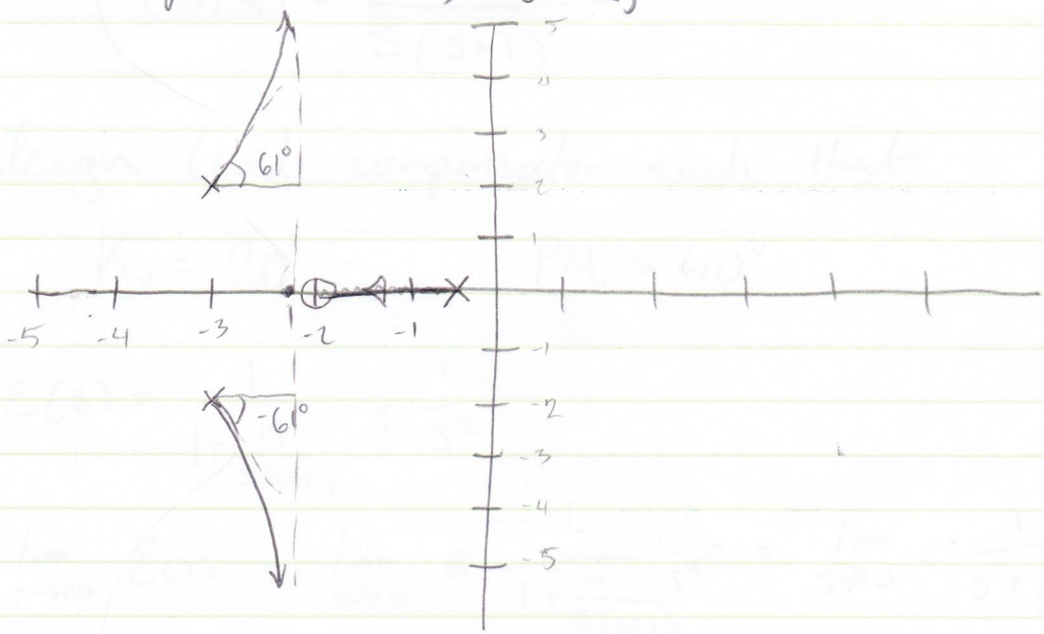
$\frac{1}{K} < 1 \qquad \boxed{K > 1} \qquad\qquad \boxed{K < \infty}$

3) Sketch the root locus of the unity feedback system where

$$K\,G(s) = K\frac{s+2}{(s+0.5)(s^2+6s+13)}$$

zeroes : $-2$

poles : $-0.5,\ -3 \pm 2j$




asymptotes: $\alpha = \dfrac{(-0.5-3-3)-(-2)}{2} = -\dfrac{9}{4}$

$$\phi_n = \frac{180° + 360°(n-1)}{2} = 90° + 180(n-1)$$

$$\phi_1 = 90°\ ,\quad \phi_2 = 270°$$

angles of departure

$$\phi_{1dep} = (117°) - (90° + 146°) - 180° = -299° = 61°$$