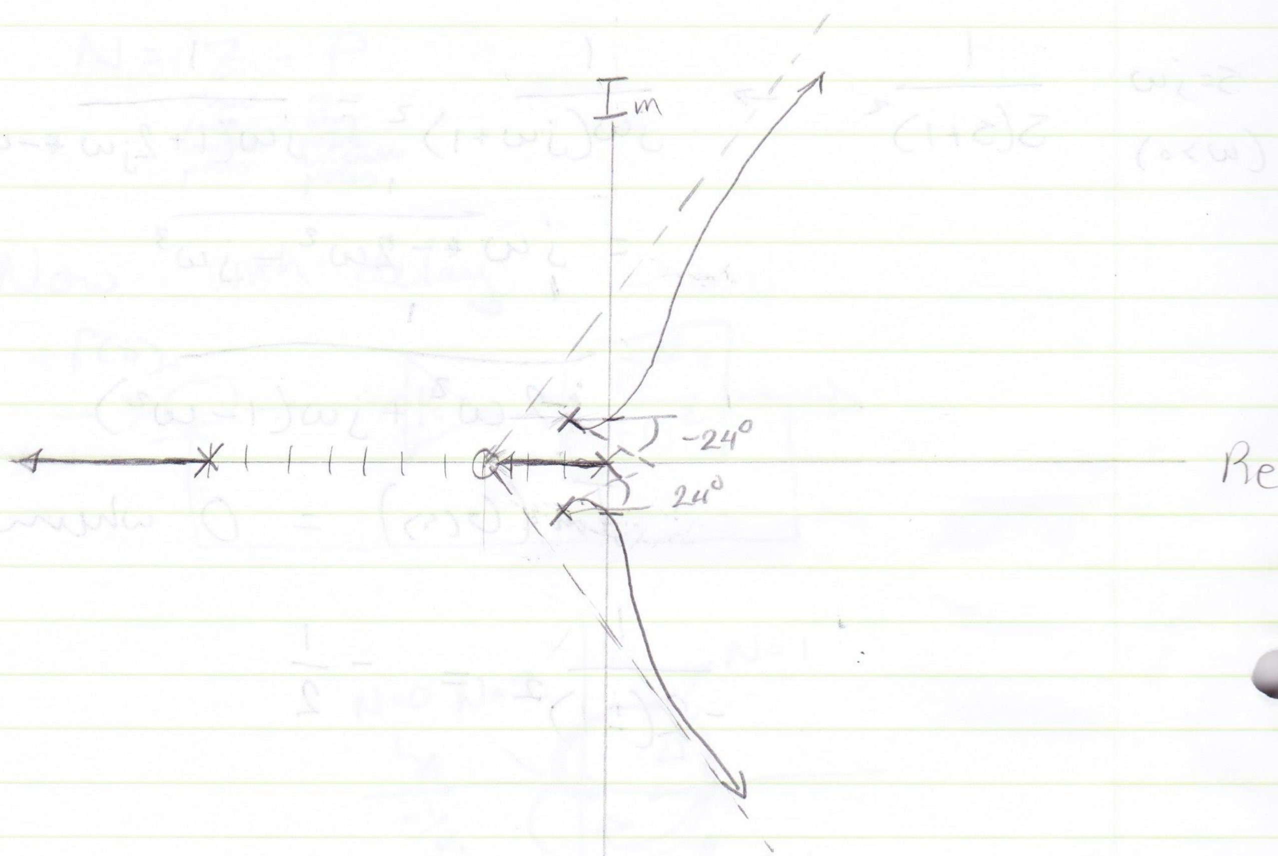
Homework 4

5.7 a) $$L(s) = \frac{s+3}{s(s+10)(s^2+2s+2)}$$



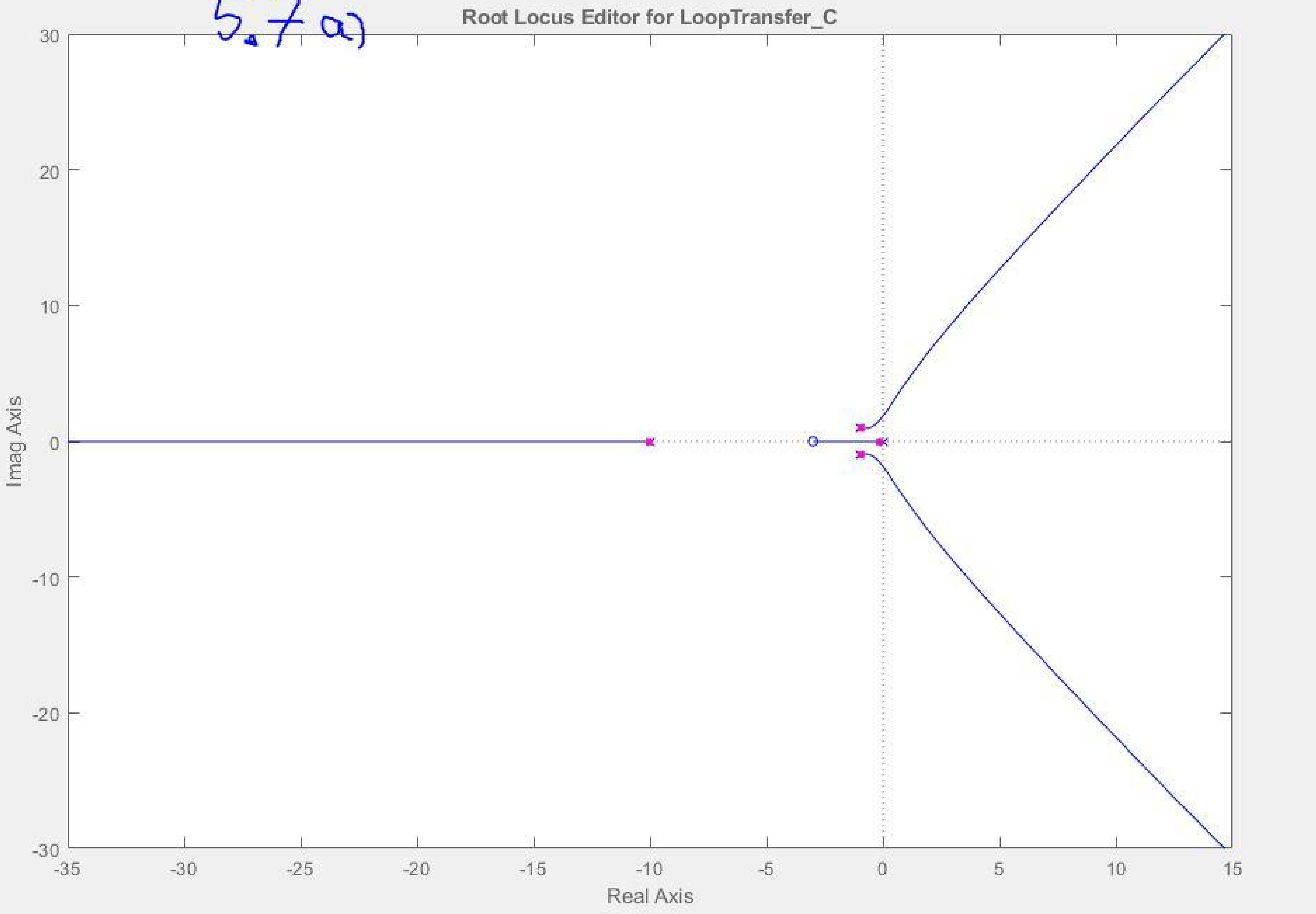
$$\alpha = \frac{(0+10+1+j+1+j)-(-3)}{4-1} = \frac{-9}{3} = -3$$

$$\phi_l = \frac{180° + 360°(l-1)}{4-1} = 60° + 120°(l-1)$$

$\phi_1 = 60°$ $\phi_2 = 180°$ $\phi_3 = 300°$

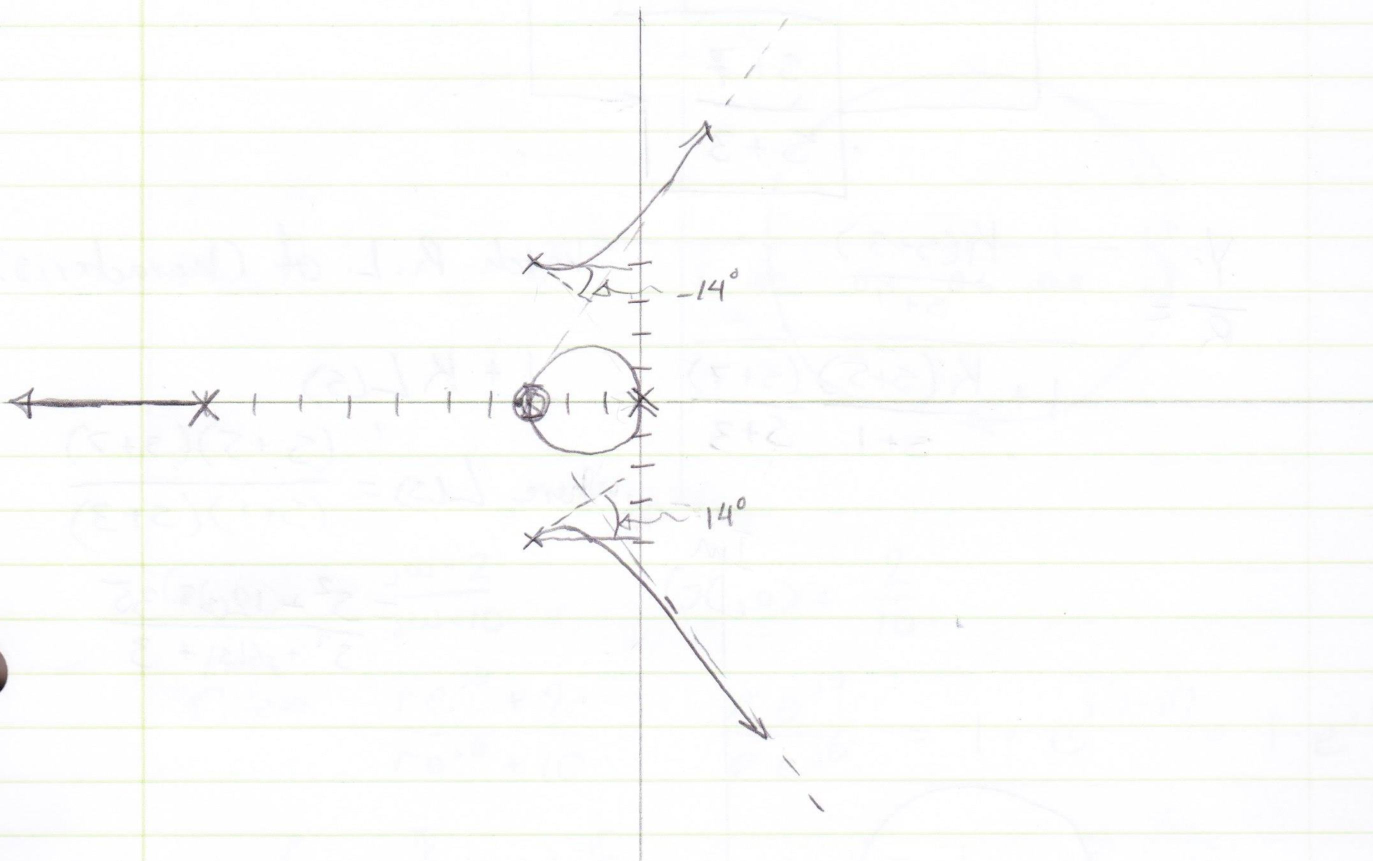
$$\phi_{l dep} = (27°) - (6° + 90° + 135°) - 180° = -24°$$

5.7 a)



Root Locus Editor for LoopTransfer_C
Imag Axis
Real Axis
30
20
10
0
-10
-20
-30
-35
-30
-25
-20
-15
-10
-5
0
5
10
15

5.7 b) $$L(s) = \frac{(s+3)^2}{s^2(s+10)(s^2+6s+25)}$$



$$\alpha = \frac{(0+0-10-3-3)-(-3-3)}{5-2} = \frac{-10}{3} = -3\tfrac{1}{3}$$

$$\phi_l = \frac{180 + 360(l-1)}{5-2} = 60^\circ + 120(l-1)$$

$$\phi_1 = 60^\circ \quad \phi_2 = 180^\circ \quad \phi_3 = 300^\circ$$

$$\phi_{dep(3+4j)} = (90+90) - (30+90+127+127) - 180 = -374^\circ = -14^\circ$$

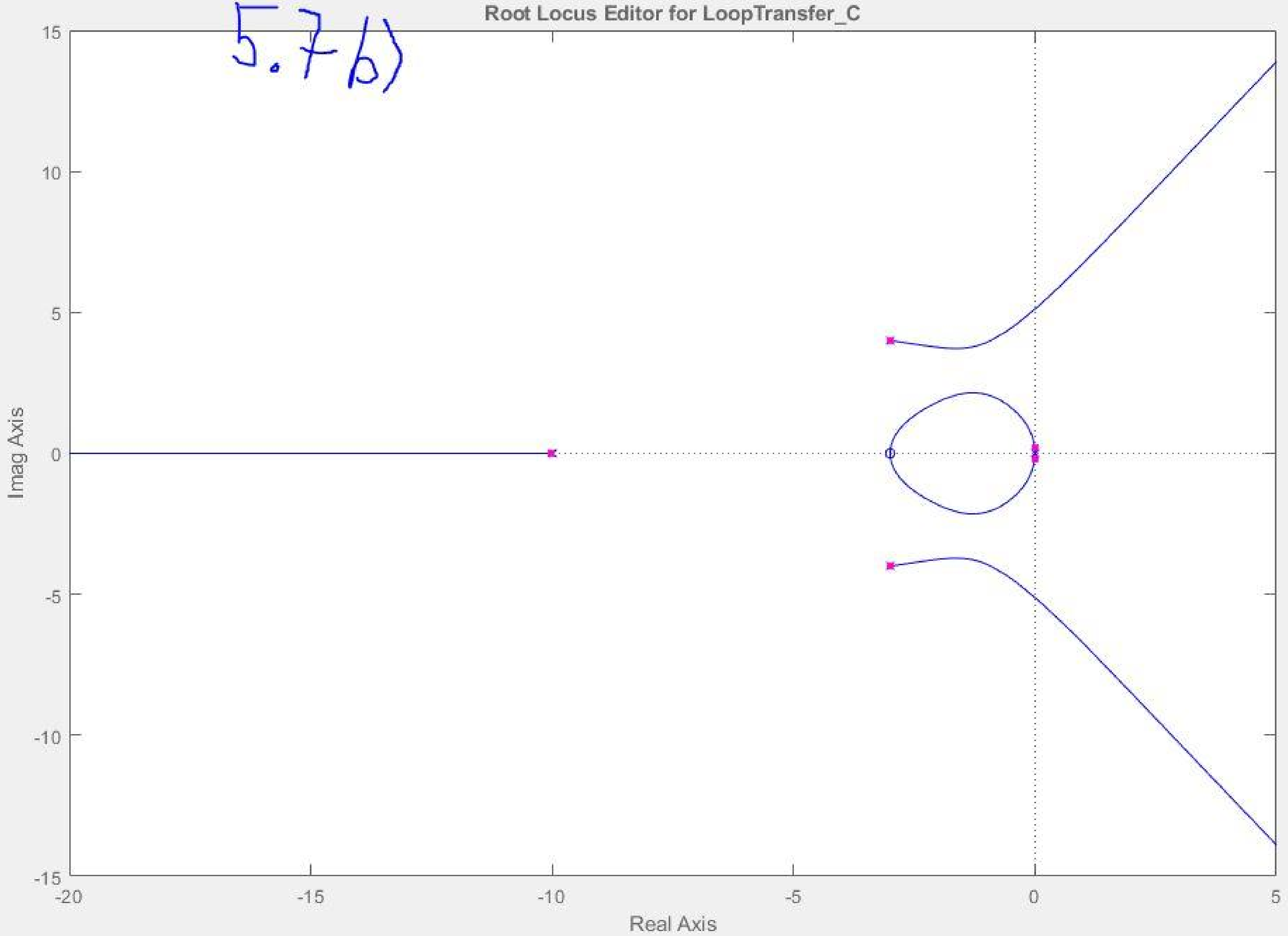
$$\phi_{dep(0)l} = \frac{(0+0) - (0-53+53) - 180 - 360(l-1)}{2}$$

$$\phi_{dep(0)1} = \frac{-180}{2} = -90^\circ \qquad \phi_{dep(0)2} = \frac{-180-360}{2} = 90^\circ$$

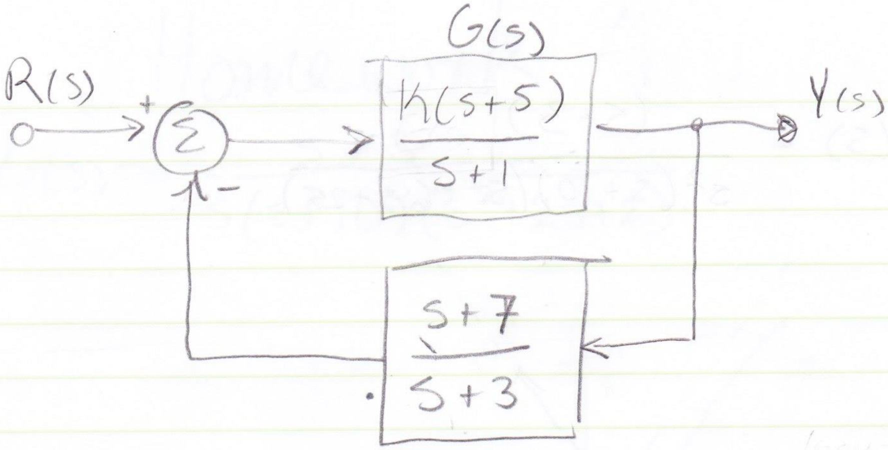
$$\psi_{arr(-3)l} = \left[(0+90-90-180-180)+(0)+180+360(l-1)\right]/2$$

$$\psi_{arr(-3)1} = \frac{-360+180}{2} = -90^\circ \qquad \psi_{arr(-3)2} = \frac{-360+180+360}{2} = 90^\circ$$

5.7 b)


Root Locus Editor for LoopTransfer_C
15
10
5
0
-5
-10
-15
Imag Axis
-20
-15
-10
-5
0
5
Real Axis

5.20 c)



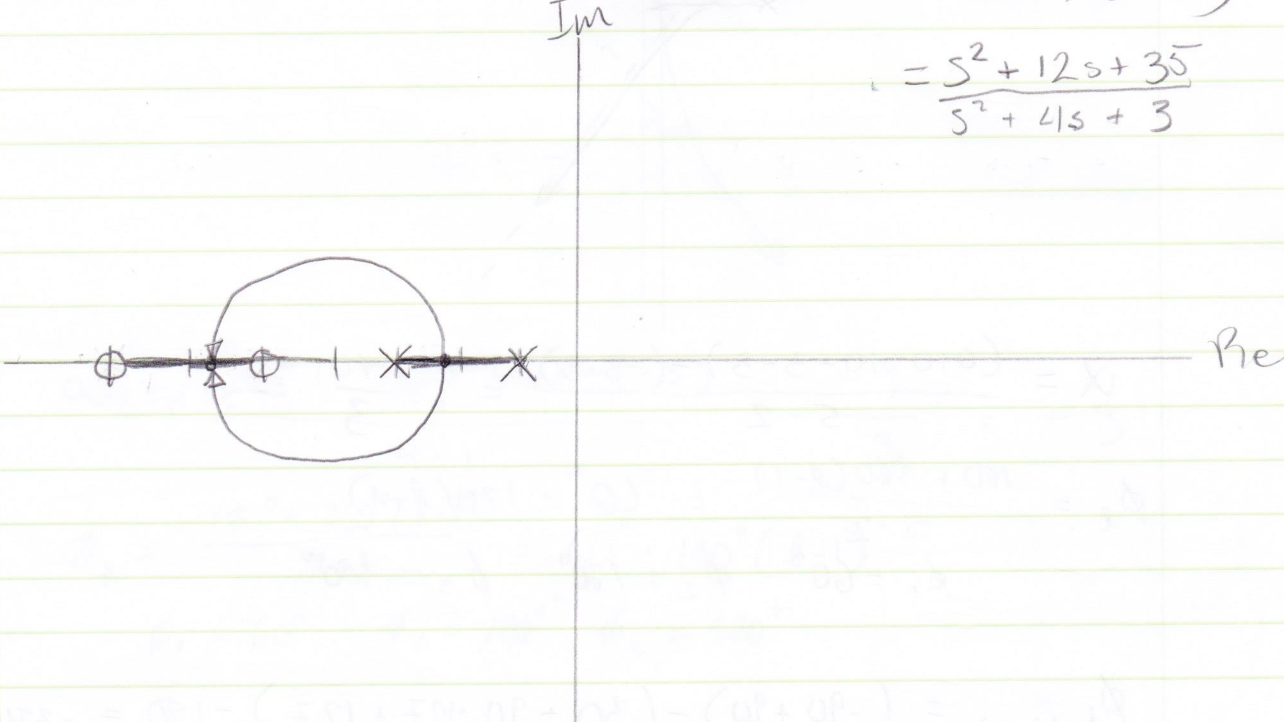
$$\frac{Y}{R} = \frac{\frac{K(s+5)}{s+1}}{1 + \frac{K(s+5)}{s+1}\frac{(s+7)}{s+3}}$$

Sketch R.L. of Characteristic egh

$1 + KL(s)$,

where $L(s) = \dfrac{(s+5)(s+7)}{(s+1)(s+3)}$

$= \dfrac{s^2+12s+35}{s^2+4s+3}$



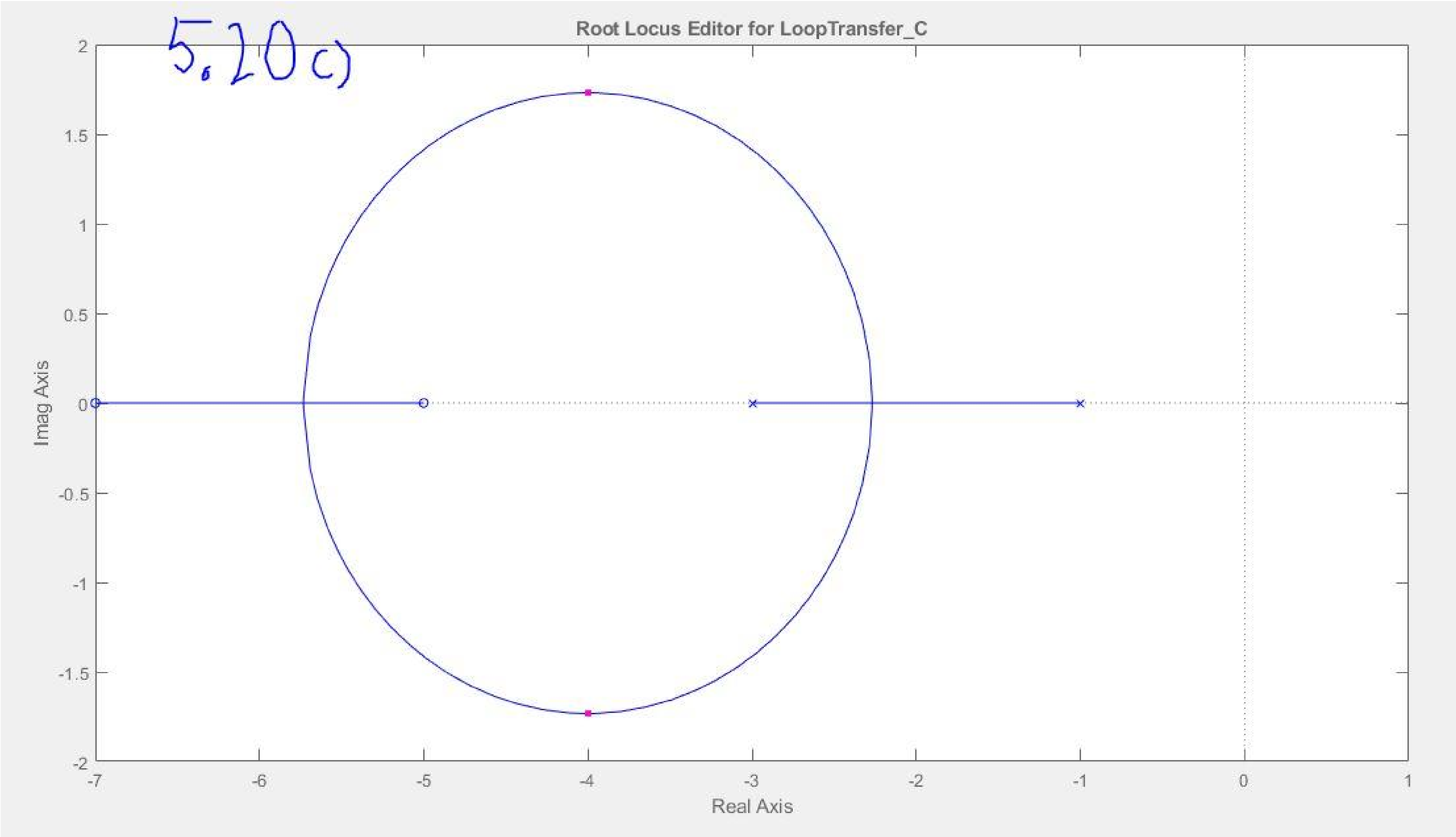
no asymptotes,

Breakaway points: $\dfrac{dL(s)}{ds} = \dfrac{(2s+12)(s^2+4s+3)-(2s+4)(s^2+12s+35)}{(s^2+4s+3)^2} = 0$

Numerator $= -8s^2-64s-104 = 0$, $s = -5.7$ & -2.3

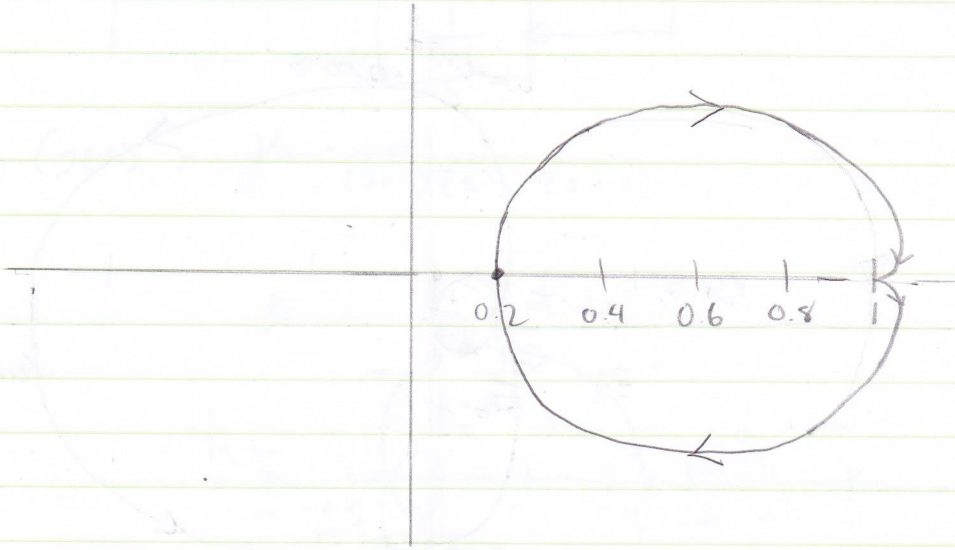
Breakaway angles $\phi_w = \dfrac{180+360(w-1)}{2}$ $\phi_1 = 90°$ $\phi_2 = -90°$

5.20 c)



Root Locus Editor for LoopTransfer_C
Imag Axis
Real Axis
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-7
-6
-5
-4
-3
-2
-1
0
1

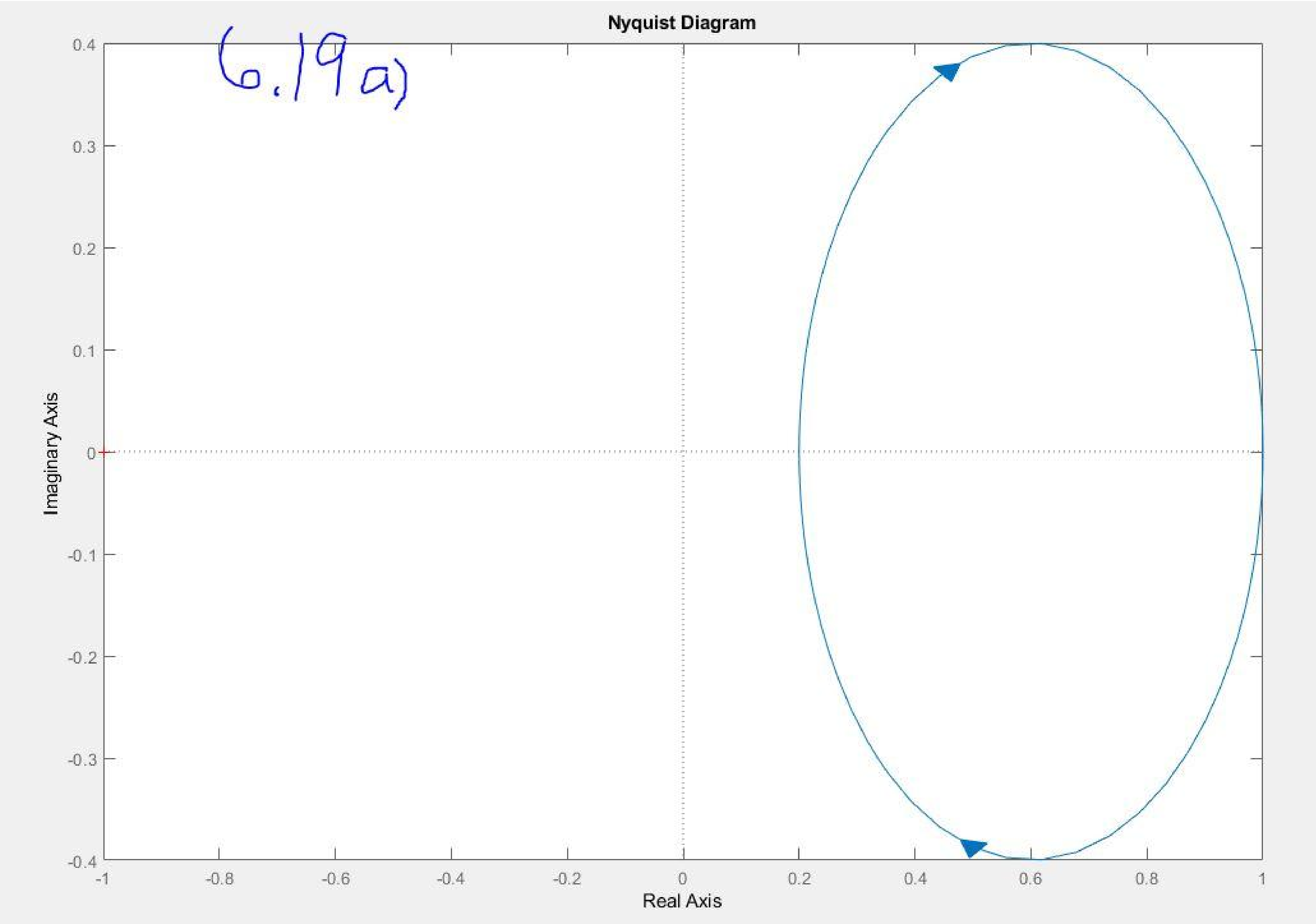
6.19 a $\quad KG(s) = K\dfrac{s+2}{s+10}$



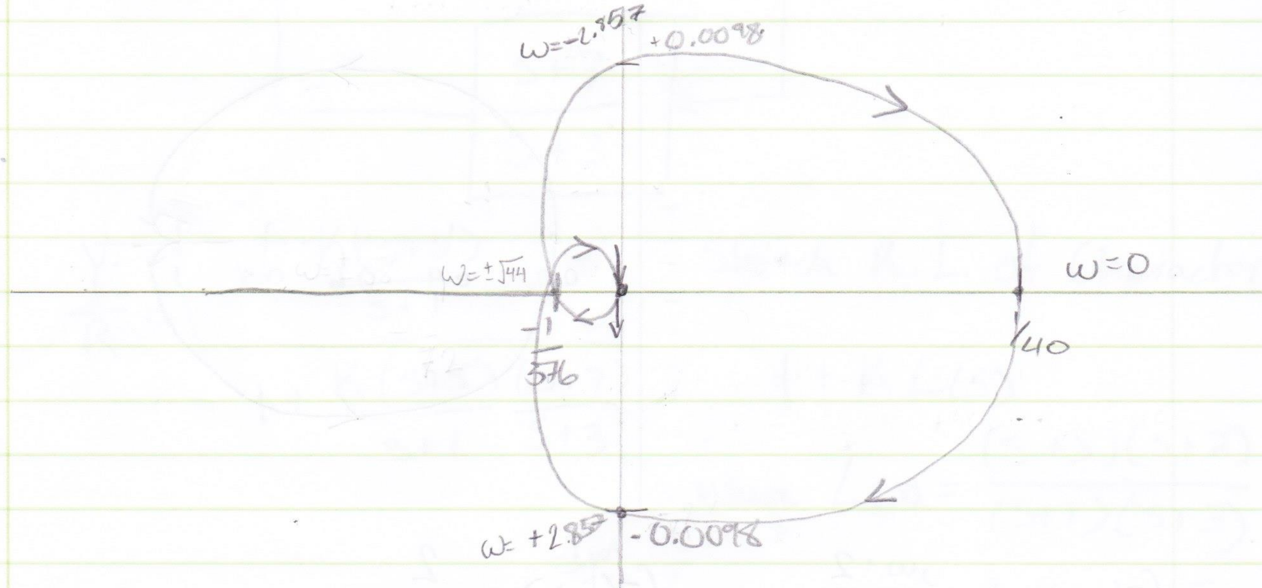
$$G(j\omega) = \frac{j\omega+2}{j\omega+10} \qquad G(j0) = \frac{2}{10}$$

$$\lim_{r\to\infty} \frac{re^{j\theta}+2}{re^{j\theta}+10} = \frac{\cancel{r}e^{j\theta}}{\cancel{r}e^{j\theta}} = 1\cdot e^{j(\theta-\theta)} = 1\cdot e^{j0}$$

for $\frac{\pi}{2} > \theta > -\frac{\pi}{2}$ $\quad G(re^{j\theta}) = 1\cdot e^{j0}$


6.19a)
Nyquist Diagram
Imaginary Axis
Real Axis
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

6.19b) $KG(s) = \dfrac{K}{(s+10)(s+2)^2}$



$$G(j\omega) = \frac{1}{(j\omega+10)(j\omega+2)^2} = \frac{1}{j(-\omega^3+44\omega)+(-14\omega^2+40)}$$

$$G(j0) = \frac{1}{10 \cdot 2^2} = \frac{1}{40} \quad ✓$$

Real intercepts : $-\omega^3+44\omega=0 \qquad \omega = 0, \pm\sqrt{44} = \pm 6.63$

$$G(j\sqrt{44}) = \frac{1}{-14 \cdot 44 + 40} = \frac{-1}{576} = -0.00174$$

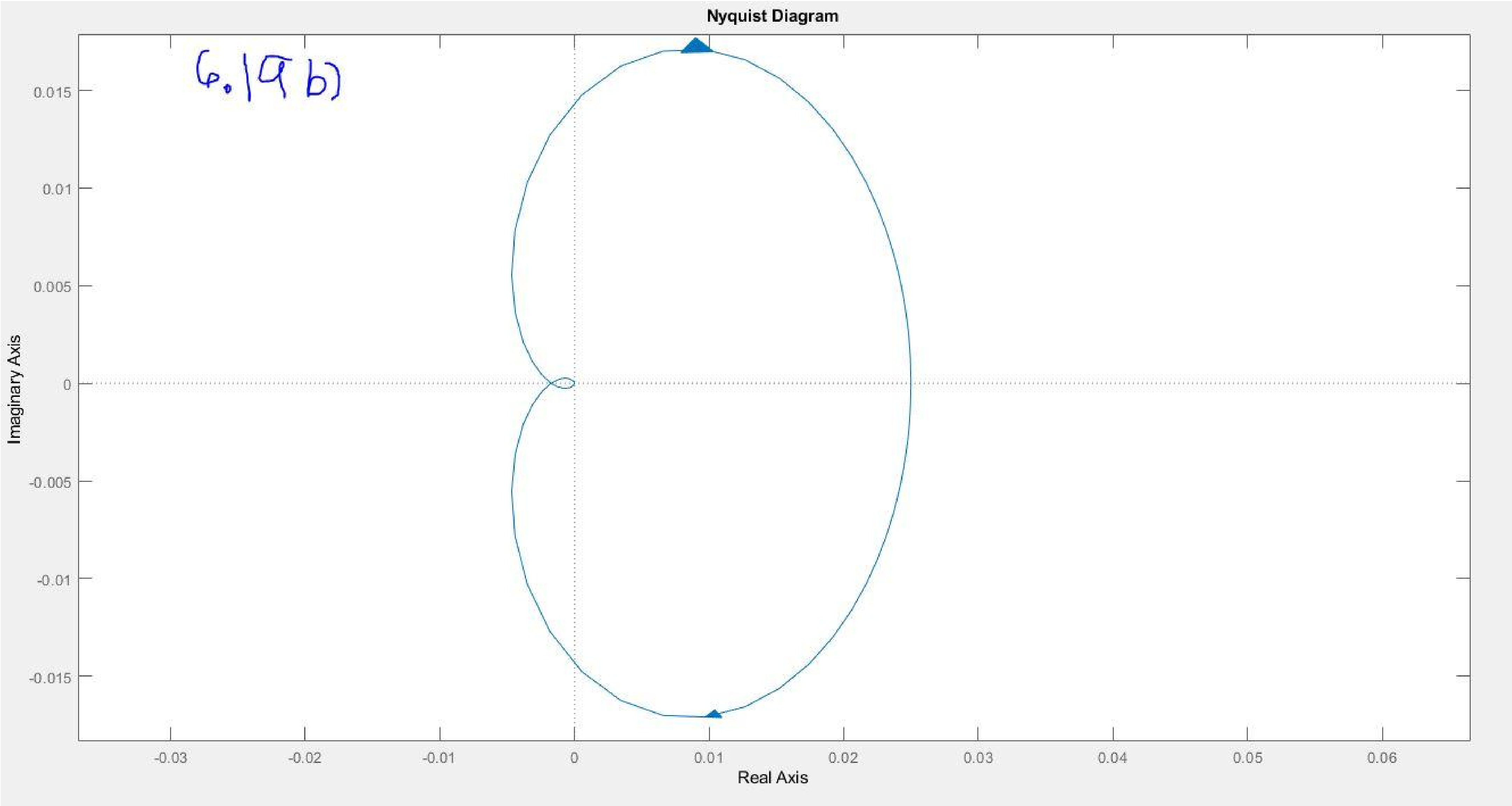
imaginary intercepts : $-14\omega^2+40=0 \qquad \omega = \pm 2.857$

$$\frac{1}{j(-(\pm 2.857)^3 + 44(\pm 2.857))} = j\,\frac{-1}{\pm 102.4} = \mp 0.0098j$$

$$G(\infty e^{j\theta}) \lim_{r\to\infty} \frac{1}{(re^{j\theta}+10)(re^{j\theta}+2)^2} = \lim_{r\to\infty} \frac{1}{r\,e^{3j\theta}} = 0 \cdot e^{-3j\theta}$$

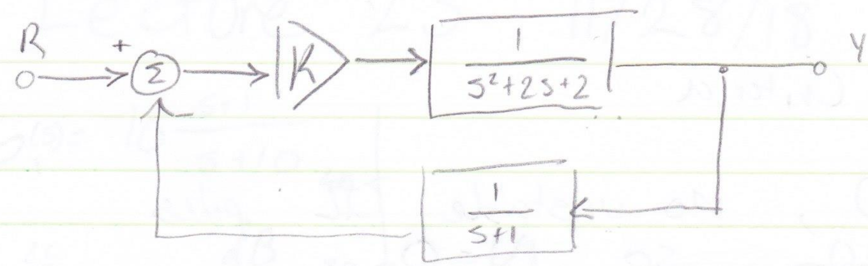
$$\theta = \frac{\pi}{2},\ G(\infty e^{j\theta}) = 0 \cdot e^{j\,-\frac{3\pi}{2}} \;\Big|\; \theta = \frac{-\pi}{2},\ G(\infty e^{j\theta}) = 0 \cdot e^{j\frac{3\pi}{2}}$$

6.19b)

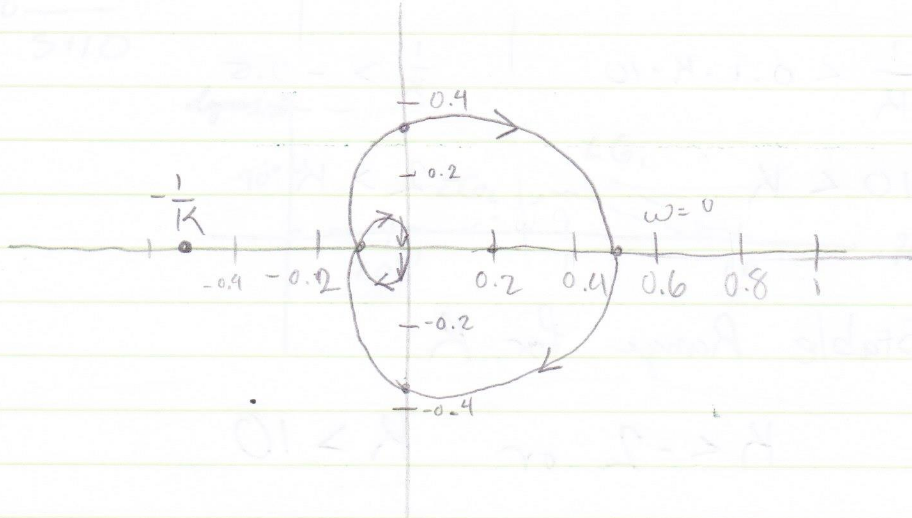


Nyquist Diagram
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
Imaginary Axis
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
0.05
0.06
Real Axis

6.21)



$$K\,G(s) = K \cdot \frac{1}{(s+1)(s^2+2s+2)}$$



$$G(j\omega) = \frac{1}{(j\omega+1)(-\omega^2+2j\omega+2)} = \frac{1}{j(-\omega^3+4\omega)+(-3\omega^2+2)}$$

$$G(j0) = \frac{1}{1\cdot 2} = \frac{1}{2}$$

Real intercepts: $-\omega^3+4\omega = 0$, $\omega = 0, \pm 2$

$$G(j\pm 2) = \frac{1}{-3(\pm 2)^2+2} = -\frac{1}{10}$$

imaginary intercepts: $-3\omega^2+2=0$, $\omega = \pm\sqrt{\frac{2}{3}}$

$$G\left(j\tfrac{\pm\sqrt{2}}{\sqrt{3}}\right) = \frac{1}{j\left(-\left(\frac{\pm\sqrt{2}}{\sqrt{3}}\right)^3+4\left(\frac{\pm\sqrt{2}}{\sqrt{3}}\right)\right)} = \frac{-j}{-(\pm 0.544)\pm 3.27} = \mp 0.367\,j$$

$$\lim_{r\to\infty} G(re^{j\theta}) = \lim_{r\to\infty}\frac{1}{(re^{j\theta}+1)(-r^2e^{2j\theta}+2re^{j\theta}+2)} = \lim_{r\to\infty}\frac{1}{re^{j\theta}(-r^2e^{2j\theta}+2re^{j\theta})}$$

$$= \lim_{r\to\infty}\frac{1}{-r^3e^{3j\theta}+\cancel{2r^2e^{2j\theta}}} = 0\cdot e^{-3j\theta} \qquad \theta=\frac{\pi}{2} \to 0\cdot e^{-j\frac{3\pi}{2}}$$

$$\theta = \frac{-\pi}{2} \to 0\cdot e^{j\frac{3\pi}{2}}$$

6.21 cntd Stability Criteria

$P = 0$, no unstable O.L poles

want $Z = 0$, so $N = 0$

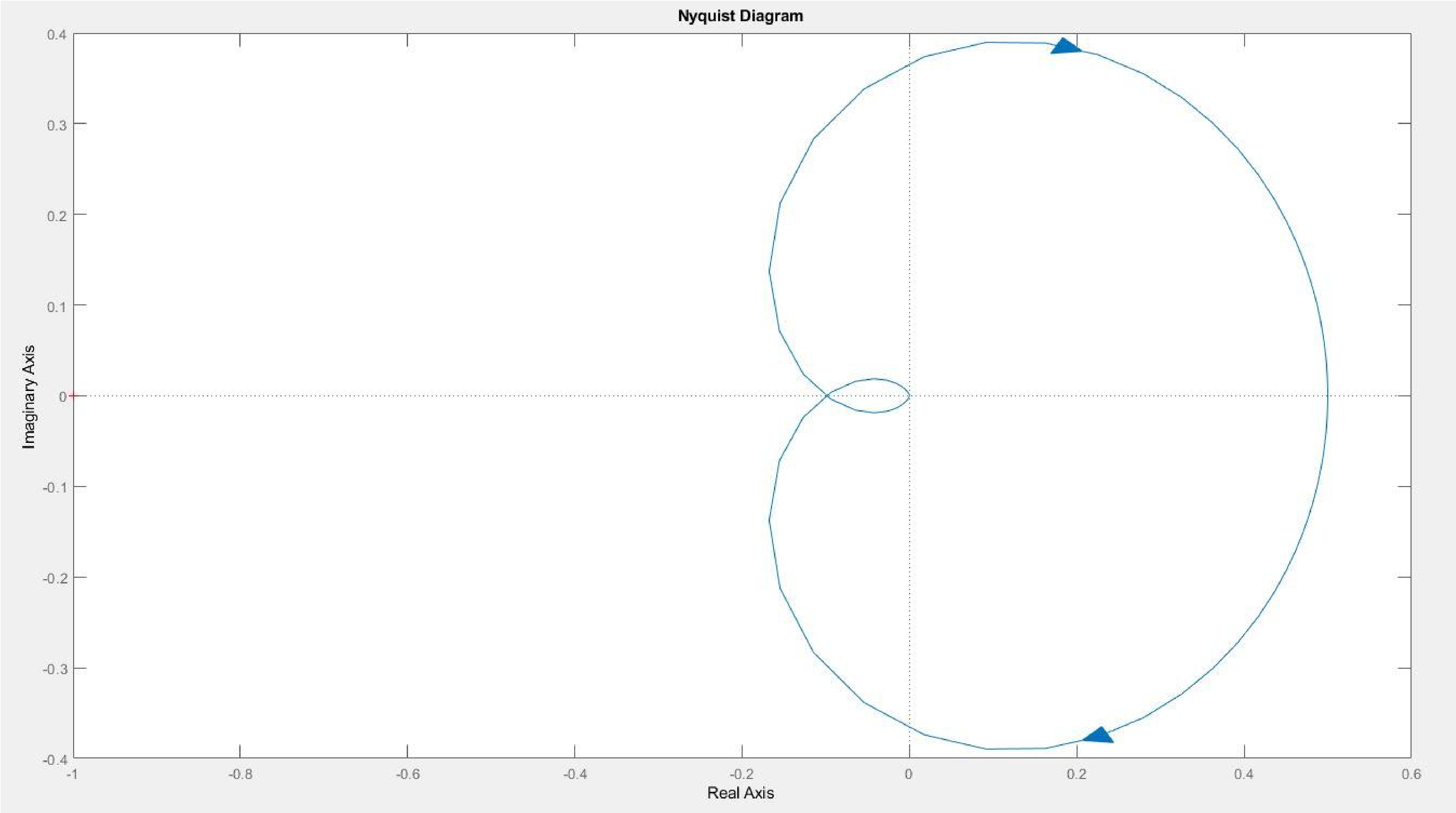
$-\frac{1}{K} < -0.1$ or $-\frac{1}{K} > 0.5$

$10 \cdot K \cdot \frac{1}{K} < 0.1 \cdot K \cdot 10$ | $\frac{1}{K} > -0.5$

$10 < K$ | $-2 > K$

Stable Range for K

$K < -2$ or $K > 10$


Nyquist Diagram
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
Imaginary Axis
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
Real Axis