

Characteristic eqn. : $1 + 9.81 K_d L(s)$

$$e) L(s) = \left(\frac{K_p}{K_d} + \frac{K_i}{K_d s} + s \right) \cdot \frac{9.81}{s^2}$$

$$= \frac{s^2 + \frac{K_p}{K_d} s + \frac{K_i}{K_d}}{s^3}$$

$$s = \frac{-\frac{K_p}{K_d} \pm \sqrt{\frac{K_p^2}{K_d^2} - 4 \frac{K_i}{K_d}}}{2} = \frac{-\frac{K_p}{K_d} \pm \sqrt{\frac{K_p^2 - 4 K_i K_d}{K_d^2}}}{2}$$

$$= \frac{-K_p \pm \sqrt{K_p^2 - 4 K_i K_d}}{2 K_d}$$

Zeros at $\frac{K_p \mp \sqrt{K_p^2 - 4 K_i K_d}}{2 K_d}$

In RLTool, I set the transfer function as $\frac{1}{s^3}$, and placed two real zeros. I manipulated and tuned these zeros such that 0.7 times the largest gain within the region was also 0 in the region. I got:

$$9.81 K_d = 3.63, \quad \frac{K_p}{2 K_d} + \frac{\sqrt{K_p^2 - 4 K_i K_d}}{2 K_d} = 1.076$$

$$, \quad \frac{K_p}{2 K_d} - \frac{\sqrt{K_p^2 - 4 K_i K_d}}{2 K_d} = 0.2068$$

$$\boxed{K_d = 0.370} \Rightarrow K_p + \sqrt{K_p^2 - 1.48 K_i} = 1.076 \cdot 0.74$$

$$K_p - \sqrt{K_p^2 - 1.48 K_i} = 0.2068 \cdot 0.74 \Rightarrow K_p - 1.48 K_i = (0.796 - K_p)^2$$

$$\Rightarrow K_p - 1.48 K_i = (K_p - 0.153)^2 \Rightarrow K_p - 1.48 K_i = 0.634 - 1.59 K_p + K_p^2$$

$$\Rightarrow K_p - 1.48 K_i = K_p^2 - 3.06 K_p + 0.23 \Rightarrow -1.48 K_i = 0.634 - 2.59 K_p + K_p^2$$

$$K_i = 0.676 K_p^2 + 0.878 K_p - 0.016$$