Homework 1

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(Due on: Wed, October 10 by 8:00PM via e-mail)

The aim of this homework is to introduce you to feedback control systems and for you to use your MATLAB and SIMULINK skills in the context of a simulation of a feedback control system. The control system in Fig. 1. is in detail covered in the class and presented in my lecture notes with all the parameters.

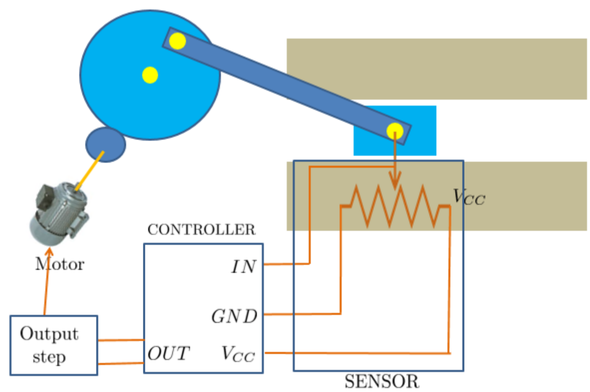

Figure 1:

a) Explain why in the feedback loop presented in the lecture notes we cannot use negative values for K_p and what the benefit of a large positive K_p value is.

Answer: The reason we cannot use negative values of K_p is that we are using negative feedback. If we used a negative K_p, the error function would be characterized by a positive real exponential instead of a negative one, meaning that the error would trend towards infinity instead of zero. This is not desirable since we want to match the output position with the desired input position, so we want the error function to trend toward zero.
Assuming K_p is negative and the error function is therefore represented by a negative exponential function, the larger K_p is, the faster the exponential will trend towards zero, meaning x_2 will reach the set desired value faster. This is because the exponential time constant τ is inversely proportional to K_p according to Equation 1.

$$\tau = 1/(1.32 * K_p) \tag{1}$$

b) Run the simulation **CE141IntroModel1.mdl** for input values 1, 2, 3,... and find the highest integer value u_{max} (for which the motor does not work). Can you explain (in words) the trend in the frequency of x_2 plots and check if the maximal and minimal values of x_2 correspond to those in the lecture notes?

Answer: The highest integer input for which the motor functions is $u = 4$. For the motor to function, u cannot exceed 5. As the input voltage increases, the frequency of oscillation of the block also increases. This is because the model for the motor is that the output angular velocity is proportional to the voltage input. This means that as the voltage increases, the motor spins faster causing the gears to rotate faster and the pin to oscillate faster in the slot. In Simulink, $x_{2min} = 3.95$ and $x_{2max} = 5.48$, compared to the lecture in which $x_{2min} = 3.95$ and $x_{2max} = 5.47$.

c) The simulink model **CE141IntroModel2.mdl** includes both the system and the proportional controller K_p. The reference for the controller changes as a square pulse from 4 to 5.4 with a period of 10s and a duty cycle of 50%. Find the largest value of the gain K_p for which the control u value does not exceed u_{max} from (b). How does the limit on u impact the performance of the feedback control loop?

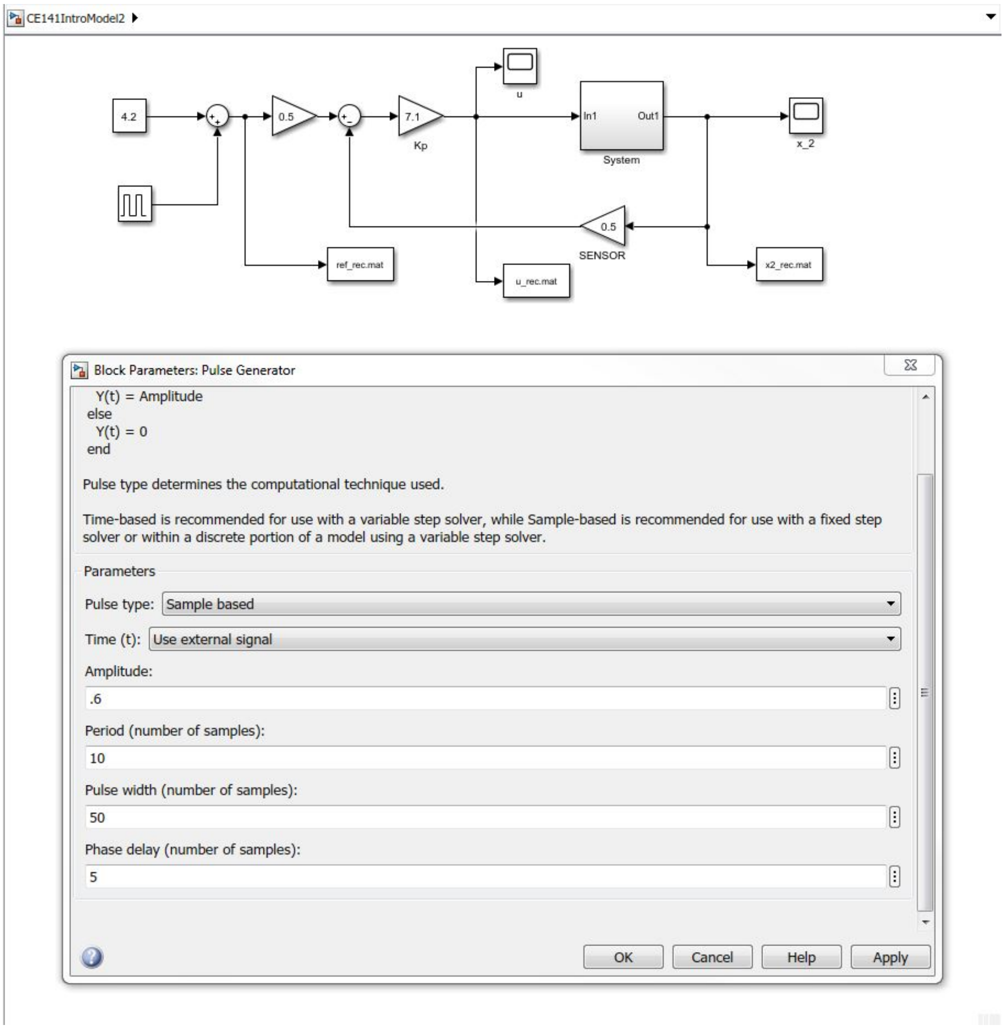

Figure 2: Model 2 modified so that the reference changes from 4.2 to 4.8 for part d)

Answer: The largest integer value of K_p for which u does not exceed 5 is $K_p = 7$. Functionally, this introduces a limit on how fast the exponential function x_2 can approach x_{2d}. The limited capability of the motor has imposed a limit on how much gain the feedback loop can supply, and thus limits the effectiveness of the correction offered by the feedback loop.
If the motor's capability weren't limited, the controller would allow x_2 to track the reference exactly, such that x_2 would approach a square wave. This is not a reasonable outcome because this would require a great amount of energy moving through the system. Real actuators; in this case: the motor, gears, arm, pin, slot; would not be able to handle this amount of energy without breaking.

d) Use K_p from (c) and adjust the model in such a way that the reference changes from 4.2 to 4.8. Include the figures in your report.

Answer: The relevant figures are Figure 2 and Figure 3.

e) The simulink model **CE141IntroModel3.mdl** models the system controlled by a digital proportional controller. Go back and forth between the simulation results of this model and of the one in **CE141IntroModel2.mdl** until you find the value of K_p that in both simulations results in a similar position (x_2) and control (u) signals. Include the figures in your report.
Answer: I started from $K_p = 7$, the results of which are shown in Figure 4, and worked backwards. For

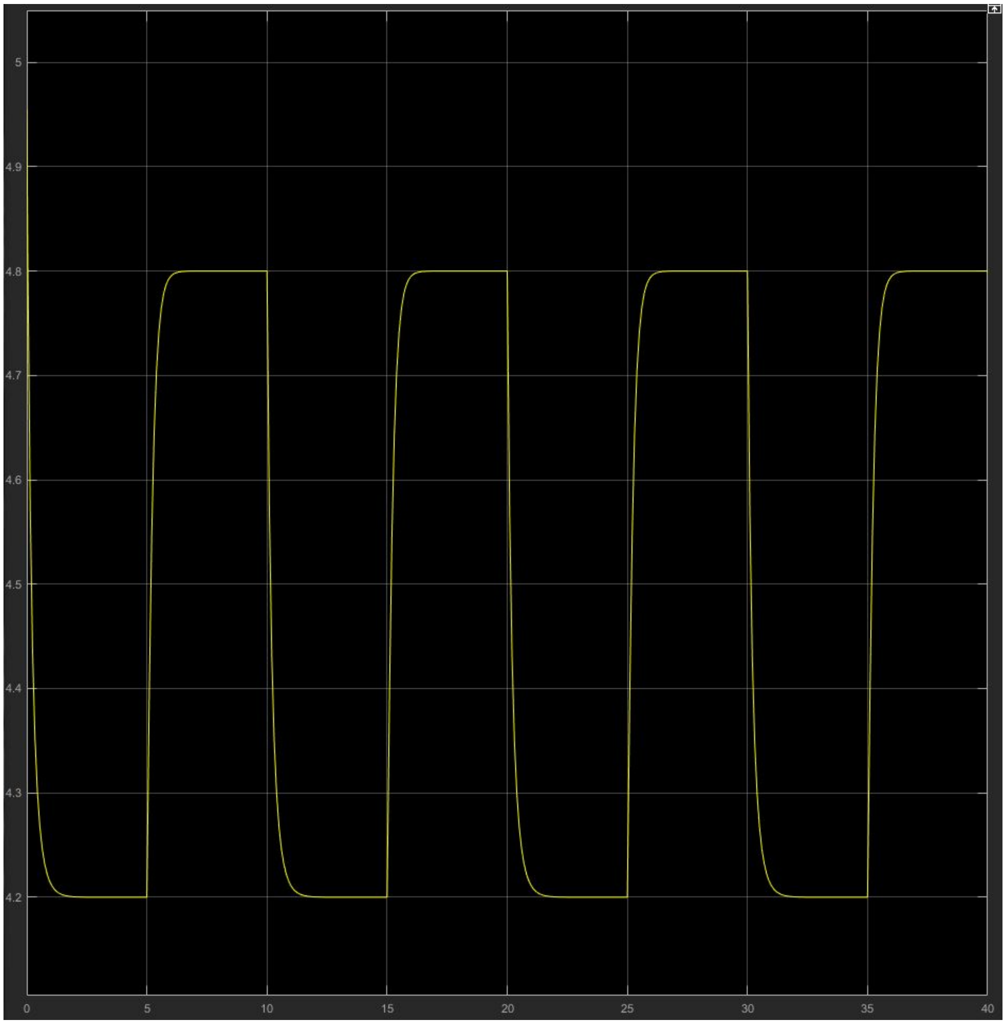

Figure 3: x_2 from analog controller (top) and digital controller (bottom) for $K_p = 7$

$K_p = 7$ shown in Figure 4, the digital controller overshot the reference signal and the analogue controller did not. For $K_p = 6$ shown in Figure 5, the digital controller performed better than the analogue controller, giving a sharper edge to the signal to better match the reference square wave. For $K_p = 5$ shown in Figure 6, x_2 from the digital controller is starting to more closely resemble x_2 from the analogue controller, enough so that I'd call the two signals 'similar'. The digital controller does not overshoot the reference, and rather than a sharp edge, the signal slowly decays to the reference value. Note that the digital controller still slightly outperforms the analogue controller in terms of decay time.

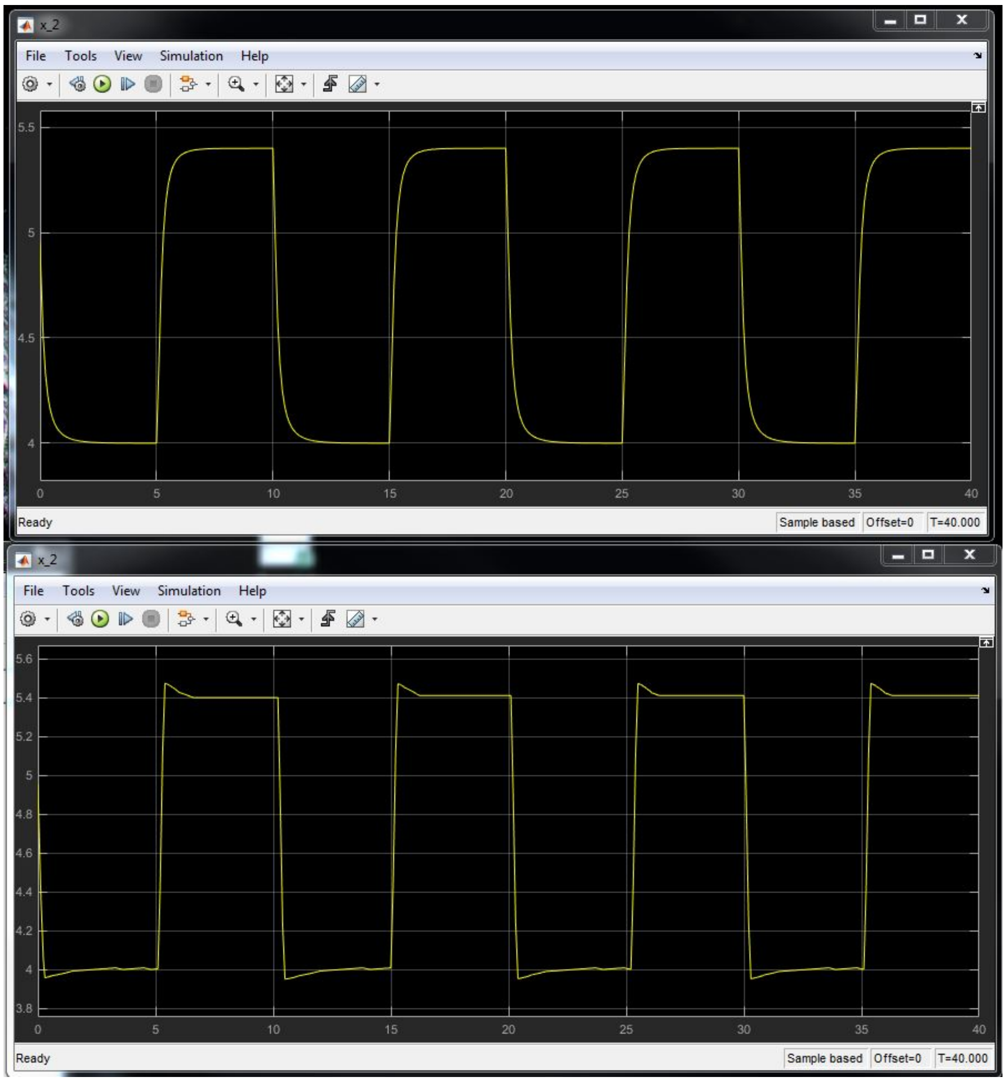

Figure 4: x_2 from analog controller (top) and digital controller (bottom) for $K_p = 7$

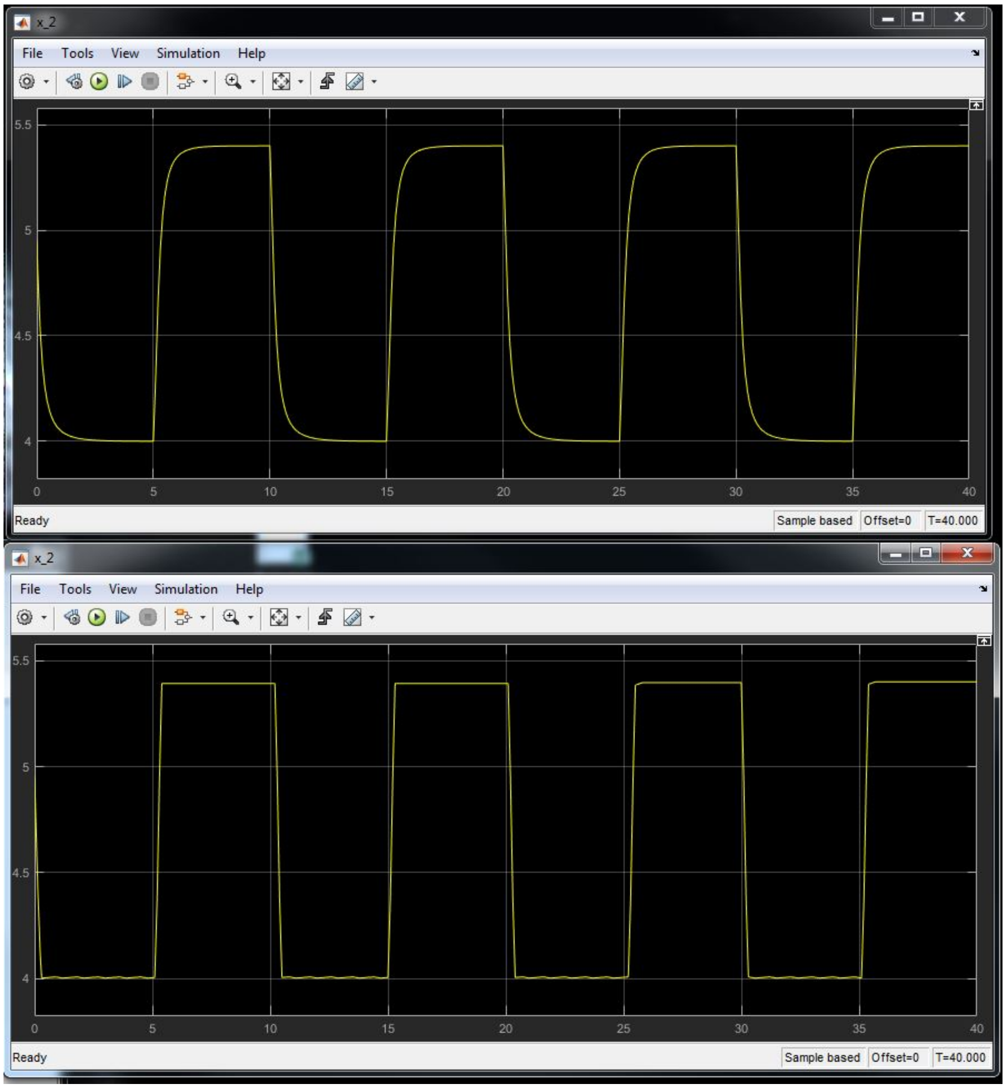

Figure 5: x_2 from analog controller (top) and digital controller (bottom) for $K_p = 6$

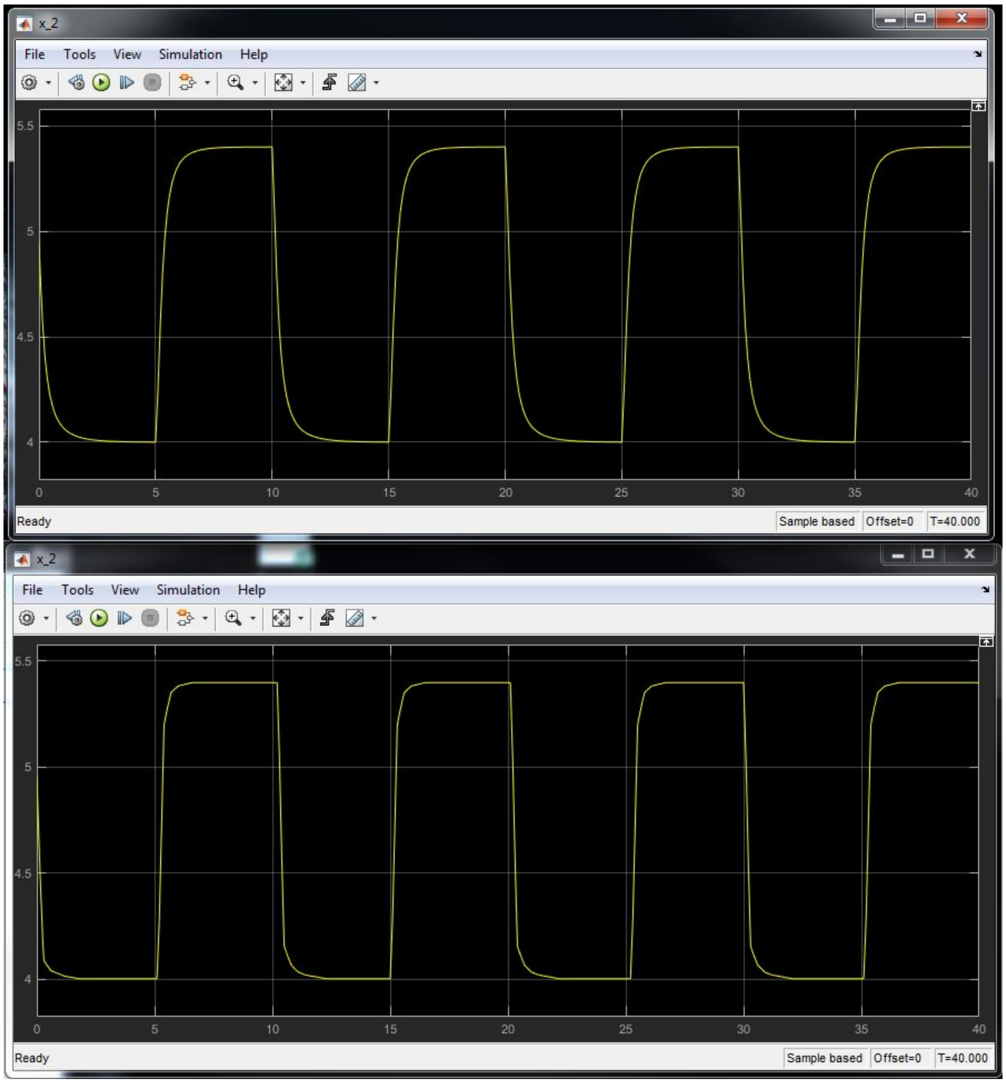

Figure 6: x_2 from analog controller (top) and digital controller (bottom) for $K_p = 5$