

Dependently Typed Languages in Statix

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Background: What is Spoofax?

A language designer's workbench with everything you need to design a programming language.

- Declarative, using meta-languages

Background: What is Spoofax?

Numbers.sdf3

```
17 context-free syntax
18
19 Exp.Int      = IntConst
20
21 Exp.Uminus   = [- [Exp]]
22 Exp.Times    = [[Exp] * [Exp]] {left}
23 Exp.Divide   = [[Exp] / [Exp]] {left}
24 Exp.Plus     = [[Exp] + [Exp]] {left}
25 Exp.Minus    = [[Exp] - [Exp]] {left}
26
27 Exp.Eq       = [[Exp] = [Exp]] {non-assoc}
28 Exp.Neq      = [[Exp] <> [Exp]] {non-assoc}
29 Exp.Gt       = [[Exp] > [Exp]] {non-assoc}
30 Exp.Lt       = [[Exp] < [Exp]] {non-assoc}
31 Exp.Geq      = [[Exp] ≥ [Exp]] {non-assoc}
32 Exp.Leq      = [[Exp] ≤ [Exp]] {non-assoc}
33
34 Exp.And      = [[Exp] & [Exp]] {left}
35 Exp.Or       = [[Exp] | [Exp]] {left}
```

Background: What is Spoofax?

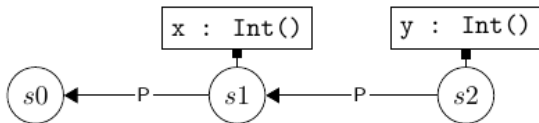
static-semantics.stx

```
356 typeOfExp(s, Int(i)) = INT() :-  
357     @i.lit := i.  
358  
359 rules // operators  
360  
361 typeOfExp(s, Uminus(e)) = INT() :-  
362     typeOfExp(s, e) = INT().  
363  
364 typeOfExp(s, Divide(e1, e2)) = INT() :-  
365     typeOfExp(s, e1) = INT(),  
366     typeOfExp(s, e2) = INT().  
367  
368 typeOfExp(s, Times(e1, e2)) = INT() :-  
369     typeOfExp(s, e1) = INT(),  
370     typeOfExp(s, e2) = INT().  
371  
372 typeOfExp(s, Minus(e1, e2)) = INT() :-  
373     typeOfExp(s, e1) = INT(),  
374     typeOfExp(s, e2) = INT().  
375
```

Background: What is Spoofax?

Example

```
int x = 5;  
int y = x + 3;  
return x + y;
```



Background: What is Spoofox?

to-ir.str ✖

```
16
17 to-ir-all = innermost(
18   to-ir +
19   to-ir-flatmap
20 )
21
22 // lhs |> rhs → lhs ; flatMap(lhs)
23 to-ir-flatmap: FlatMap(lhs, rhs) → Seq(lhs, Apply
24 // flatMap(lhs, flatMap(rhs)) → flatMap(lhs) ; fl
25 to-ir-flatmap: Apply(Var("flatMap"), [Seq(lhs, App
26   Seq(Apply(Var("flatMap"), [lhs]), Apply(Var("fla
27 // flatMap(lhs; rhs@(flatMap(_); _)) → flatMap(l
28 to-ir-flatmap: Apply(Var("flatMap"), [Seq(lhs, rhs
29   Seq(Apply(Var("flatMap"), [lhs]), rhs)
30
31 // Makes a strategy with an implicit input argumen
32 to-ir: StrategyDef(name, params, body) → Strategy
33 with inputVar := "__input" // TODO: Generate un
34
```

Background: What are Dependent Types?

- Types may depend on values!

Example

```
concat : (A: Type) -> (n m : Nat) -> Vec A n -> Vec A m  
        -> Vec A (n + m)
```

Background: What are Dependent Types?

Type checking requires evaluation

Example 1

```
let T: Type = if false then Int else Bool end;  
let b: T = true;
```


Background: What are Dependent Types?

Why are dependent types useful?

- Proof Assistants: Agda, Lean, Coq, etc

Sorted lists

```
sort : List t -> List t
sort_sorted : (v : List t) -> IsSorted (sort v)
```

Research Question

How suitable is Statix for defining a dependently-typed language?

- Will it be easier than doing it in Haskell?
- Defining system F was a challenge¹, will it even be possible?

¹Hendrik van Antwerpen et al. Scopes as types.

Why is this important?

Spoofax perspective

Developing a language with a complex type system tests the boundaries of what Spoofax can do.

Dependent Types perspective

Using a language workbench helps with rapid prototyping.

Primary Contribution: Calculus of Constructions in Statix

A lambda calculus with dependent types.

Syntax Definition

```
Expr.Type = "Type"  
Expr.Var = ID  
Expr.FnType = ID ":" Expr "->" Expr {right}  
Expr.FnConstruct = "\\\" ID ":" Expr "." Expr  
Expr.FnDestruct = Expr Expr {left}  
Expr.Let = "let" ID "=" Expr ";" Expr
```

Example

```
let f = \T: Type. \x: T. x;  
f (T: Type -> Type) (\y: Type. y)
```

Type Checking

Type checking rules

$$\boxed{\langle s \mid e \rangle : t}$$

$$\begin{array}{c}
 \frac{}{\langle s \mid \text{Type}() \rangle : \text{Type}()} \qquad \frac{\langle s \mid e \rangle : t_e \quad \langle \text{sPutSubst}(s, x, (s, e)) \mid b \rangle : t_b}{\langle s \mid \text{Let}(x, e, b) \rangle : t_b} \\
 \\
 \frac{\text{sGetName}(s, x) = \text{NType}(t)}{\langle s \mid \text{Var}(x) \rangle : t} \qquad \frac{\text{sGetName}(s, x) = \text{NSubst}(s_e, e) \quad \langle s_e \mid e \rangle : t}{\langle s \mid \text{Var}(x) \rangle : t} \\
 \\
 \frac{\langle s \mid a \rangle : t_a \quad t_a \stackrel{\beta}{=} \text{Type()} \quad \langle s \mid a \rangle \Rightarrow_{\beta} a' \quad \langle \text{sPutType}(s, x, a') \mid b \rangle : t_b \quad t_b \stackrel{\beta}{=} \text{Type}()}{\langle s \mid \text{FnType}(x, a, b) \rangle : \text{Type}()} \qquad \frac{\langle s \mid a \rangle : t_a \quad t_a \stackrel{\beta}{=} \text{Type()} \quad \langle s \mid a \rangle \Rightarrow_{\beta} a' \quad \langle \text{sPutType}(s, x, a') \mid b \rangle : t_b}{\langle s \mid \text{FnConstruct}(x, a, b) \rangle : \text{FnType}(x, a', t_b)} \\
 \\
 \frac{\langle s \mid f \rangle : t_f \quad \langle s \mid t_f \rangle \sqcap \Rightarrow_{\beta h} \langle s_f \mid \text{FnType}(x, t_{da}, t_b) \rangle \quad \langle s \mid a \rangle : t_a \quad t_a \stackrel{\beta}{=} \langle s_f \mid t_{da} \rangle \quad \langle \text{sPutSubst}(s_f, x, (s, a)) \mid t_b \rangle \Rightarrow_{\beta} t'_b}{\langle s \mid \text{FnDestruct}(f, a) \rangle : t'_b}
 \end{array}$$

Figure 4.4: Rules for type checking the Calculus of Constructions

Type Checking: From inference rules to Statix code

$$\frac{\langle s \mid e \rangle : t_e \quad \langle \text{sPutSubst}(s, x, (s, e)) \mid b \rangle : t_b}{\langle s \mid \text{Let}(x, e, b) \rangle : t_b}$$

Signature of *typeOfExpr*

```
typeOfExpr : scope * Expr -> Expr
```

Equivalent Statix code

```
typeOfExpr (s, Let(x, e, b)) = bt :-  
  typeOfExpr (s, e) == et,  
  typeOfExpr (sPutSubst (s, x, (s, e)), b) == bt
```

Type Checking: Environments & Context

Example

```
let T: Type = Bool;  
\b: T. ???
```

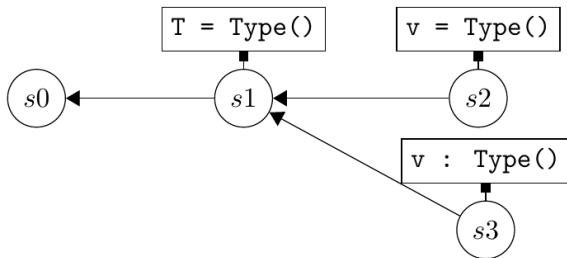
What information is needed?

- ① $T = \text{Bool}$ (Stored in Environment)
- ② $b : T$ (Stored in Context)

Type Checking: Environments & Context

How do we use scopes?

Scope graphs are used as a replacement for an environment and a context.



Type Checking: Requires Evaluation

Example from earlier

```
let T = if false then Int else Bool end;  
let b: T = true;
```

Evaluation relation

```
betaHeadReduce : scope * Expr -> scope * Expr  
betaReduce : scope * Expr -> Expr  
exactBetaEq : (scope * Expr) * (scope * Expr)
```

Type Checking: Requires Evaluation

Beta head-reduction rules

$$\langle s_1 \mid e_1 \rangle \bar{p} \Rightarrow_{\beta h} \langle s_2 \mid e_2 \rangle$$

$$\frac{}{\langle s \mid \text{Type}() \rangle \bar{\square} \Rightarrow_{\beta h} \langle s \mid \text{Type}() \rangle} \quad \frac{\langle \text{sPutSubst}(s, x, (s, e)) \mid b \rangle \bar{p} \Rightarrow_{\beta h} \langle s' \mid b' \rangle}{\langle s \mid \text{Let}(x, e, b) \rangle \bar{p} \Rightarrow_{\beta h} \langle s' \mid b' \rangle}$$

$$\frac{\text{sGetName}(s, x) = \text{NSubst}(s_e, e) \quad \langle s_e \mid e \rangle \bar{p} \Rightarrow_{\beta h} \langle s_e' \mid e' \rangle}{\langle s \mid \text{Var}(x) \rangle \bar{p} \Rightarrow_{\beta h} \langle s_e' \mid e' \rangle}$$

$$\frac{\text{sGetName}(s, x) = \text{NType}(t)}{\langle s \mid \text{Var}(x) \rangle \bar{p} \Rightarrow_{\beta h} \text{rebuild}(s, \text{Var}(x), \bar{p})} \quad \frac{}{\langle s \mid \text{FnType}(x, a, b) \rangle \bar{\square} \Rightarrow_{\beta h} \langle s \mid \text{FnType}(x, a, b) \rangle}$$

$$\frac{}{\langle s \mid \text{FnConstruct}(x, a, b) \rangle \bar{\square} \Rightarrow_{\beta h} \langle s \mid \text{FnConstruct}(x, a, b) \rangle}$$

$$\frac{\langle \text{sPutSubst}(s, x, p) \mid b \rangle \bar{p} \Rightarrow_{\beta h} \langle s' \mid e' \rangle}{\langle s \mid \text{FnConstruct}(x, _, b) \rangle (p :: \bar{p}) \Rightarrow_{\beta h} \langle s' \mid e' \rangle} \quad \frac{\langle s \mid f \rangle (a :: \bar{p}) \Rightarrow_{\beta h} \langle s' \mid e' \rangle}{\langle s \mid \text{FnDestruct}(f, a) \rangle \bar{p} \Rightarrow_{\beta h} \langle s' \mid e' \rangle}$$

Type Checking: Variable Capturing

What is the type of this expression?

$\lambda T : \text{Type}. \lambda T : T. T$

Type Checking: Variable Capturing

What is the type of this expression?

$\lambda T : \text{Type}. \lambda T : T. T$

The type

$T : \text{Type} \rightarrow T : T \rightarrow T$

Equivalent type under renaming: Not correct!

$T : \text{Type} \rightarrow x : T \rightarrow x$

Type Checking: Variable Capturing

Solutions

- 1 De Bruijn indices
- 2 Uniquifying names
- 3 Capture-avoiding substitution
- 4 **Using scopes to distinguish names**

The type

$T : \text{Type} \rightarrow T : T \rightarrow T$

The type (fixed)

$T : \text{Type} \rightarrow T : (T, s0) \rightarrow (T, s0)$

Extra contributions

Features

- ① Implemented Inference
- ② Implemented Inductive Data Types
- ③ Implemented Universes
- ④ Interpreter
- ⑤ Compiler to Clojure

Evaluation

- ① Comparison with implementation in Haskell
- ② Comparison with implementation in LambdaPi
- ③ Evaluation of Spoofax

Comparison with Haskell

Type checking

```
data EnvEntry = NType Expr | NSubst SExpr
type Env = [EnvEntry]
type SExpr = (Env , Expr)
tc :: SExpr -> Either String Expr
```

Differences

- Distribution of definitions over files
- De Bruijn Indices vs Names
- Inference built-in to Statix

On Extensibility

How to extend the language

- 1 Define parsing rules
- 2 Create a new .stx file
- 3 Define the relations: `typeOfExpr`, `betaReduceHead`, `expectBetaEq` and `betaReduce`

Term Inference

Example

```
let id = (\T : Type. \x: T. x);  
id _ true
```

How it works

- When `expectBetaEq(e1, e2)` and `e1` or `e2` is free, infer!
- But in Statix we can't query whether variables are free.
- Solution: Wrap each free constructor in `Infer`.

Term Inference: Equational Unification

- We can declare two terms to be equal if they satisfy a certain relation (such as beta equality)
- We do this using *reduction rules*, such as:

Example of a rewrite rule

$$(\lambda x : T. b) a \Rightarrow b[x := a]$$

- Comparison with LambdaPi

Conclusions

Spoofax is a great tool for developing dependently typed languages!

- We can use scopes to represent environments and contexts
- Statix can still use improvements