

Dependently Typed Languages in Statix

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March 15, 2023

Background: What are Dependent Types?

- Types may depend on values!

Example

```
concat : (A: Set) -> (n m : Nat) -> Vec A n -> Vec A m  
        -> Vec A (n + m)
```

- Proof assistants
- For example: Agda, Coq, Lean, ...

Research Question

How suitable is Statix for defining a dependently-typed language?

Why is this important?

Spoofax perspective

Developing a language with a complex type system tests the boundaries of what Spoofax can do.

Dependent Types perspective

Using a language workbench helps with rapid prototyping.

Primary Contribution

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Calculus of Constructions

A lambda calculus with dependent types.

Example 1

```
(\v: Type. v) T
```

Example 2

```
let f = \T: Type. \x: T. x;  
f (T: Type -> Type) (\y: Type. y)
```

Type Checking

Type checking relation

`typeOfExpr : scope * Expr -> Expr`

How do we use scopes?

A scope is used as a combination of an environment and a context. One relation `name → NameEntry`, `NameEntry` is either:

- `NType`: Stores a type -> Corresponds with context
- `NSubst`: Stores a substitution -> Corresponds with environment

Type Checking: Requires Evaluation

Example 1

```
let T = if false then Int else Bool end;  
let b: T = true;
```

Evaluation relation

```
betaHeadReduce : scope * Expr -> scope * Expr  
betaReduce : scope * Expr -> Expr  
exactBetaEq : (scope * Expr) * (scope * Expr)
```


Type Checking: From inference rules to Statix code

tododododododod From inference rules to Statix code

Beta head-reduction rules

$$\langle s_1 \mid e_1 \rangle \bar{p} \Rightarrow_{\beta h} \langle s_2 \mid e_2 \rangle$$

$$\frac{}{\langle s \mid \text{Type}() \rangle \bar{p} \Rightarrow_{\beta h} \langle s \mid \text{Type}() \rangle} \quad \frac{\langle \text{sPutSubst}(s, x, (s, e)) \mid b \rangle \bar{p} \Rightarrow_{\beta h} \langle s' \mid b' \rangle}{\langle s \mid \text{Let}(x, e, b) \rangle \bar{p} \Rightarrow_{\beta h} \langle s' \mid b' \rangle}$$

$$\frac{\text{sGetName}(s, x) = \text{NSubst}(s_e, e) \quad \langle s_e \mid e \rangle \bar{p} \Rightarrow_{\beta h} \langle s_{e'} \mid e' \rangle}{\langle s \mid \text{Var}(x) \rangle \bar{p} \Rightarrow_{\beta h} \langle s_{e'} \mid e' \rangle}$$

$$\frac{\text{sGetName}(s, x) = \text{NType}(t)}{\langle s \mid \text{Var}(x) \rangle \bar{p} \Rightarrow_{\beta h} \text{rebuild}(s, \text{Var}(x), \bar{p})} \quad \frac{}{\langle s \mid \text{FnType}(x, a, b) \rangle \bar{p} \Rightarrow_{\beta h} \langle s \mid \text{FnType}(x, a, b) \rangle}$$

$$\frac{}{\langle s \mid \text{FnConstruct}(x, a, b) \rangle \bar{p} \Rightarrow_{\beta h} \langle s \mid \text{FnConstruct}(x, a, b) \rangle}$$

$$\frac{\langle \text{sPutSubst}(s, x, p) \mid b \rangle \bar{p} \Rightarrow_{\beta h} \langle s' \mid e' \rangle}{\langle s \mid \text{FnConstruct}(x, _, b) \rangle (p :: \bar{p}) \Rightarrow_{\beta h} \langle s' \mid e' \rangle} \quad \frac{\langle s \mid f \rangle (a :: \bar{p}) \Rightarrow_{\beta h} \langle s' \mid e' \rangle}{\langle s \mid \text{FnDestruct}(f, a) \rangle \bar{p} \Rightarrow_{\beta h} \langle s' \mid e' \rangle}$$

Figure 4.2: Rules for beta head reducing the Calculus of Constructions

Type checking rules

$$\langle s \mid e \rangle : t$$

$$\frac{}{\langle s \mid \text{Type}() \rangle : \text{Type}()} \quad \frac{\langle s \mid e \rangle : t_e \quad \langle \text{sPutSubst}(s, x, (s, e)) \mid b \rangle : t_b}{\langle s \mid \text{Let}(x, e, b) \rangle : t_b}$$

$$\frac{\text{sGetName}(s, x) = \text{NType}(t)}{\langle s \mid \text{Var}(x) \rangle : t} \quad \frac{\text{sGetName}(s, x) = \text{NSubst}(s_e, e) \quad \langle s_e \mid e \rangle : t}{\langle s \mid \text{Var}(x) \rangle : t}$$

$$\frac{\langle s \mid a \rangle : t_a \quad t_a =_{\beta} \text{Type}() \quad \langle s \mid a \rangle \Rightarrow_{\beta} a'}{\langle \text{sPutType}(s, x, a') \mid b \rangle : t_b \quad t_b =_{\beta} \text{Type}()} \quad \frac{\langle s \mid a \rangle : t_a \quad t_a =_{\beta} \text{Type}() \quad \langle s \mid a \rangle \Rightarrow_{\beta} a' \quad \langle \text{sPutType}(s, x, a') \mid b \rangle : t_b}{\langle s \mid \text{FnConstruct}(x, a, b) \rangle : \text{FnType}(x, a', t_b)}$$

$$\frac{}{\langle s \mid \text{FnType}(x, a, b) \rangle : \text{Type}()} \quad \frac{\langle s \mid f \rangle : t_f \quad \langle s \mid t_f \rangle \bar{p} \Rightarrow_{\beta h} \langle s_f \mid \text{FnType}(x, t_{da}, t_b) \rangle}{\langle s \mid \text{FnDestruct}(f, a) \rangle (t :: \bar{p}) : t'}$$

Extra contributions

Features

- ① Implemented Inference
- ② Implemented Inductive Data Types
- ③ Implemented Universes
- ④ Interpreter
- ⑤ Compiler to Clojure

Evaluation

- ① Comparison with implementation in Haskell
- ② Comparison with implementation in LambdaPi
- ③ Evaluation of Spoofax

Conclusions

Spoofax is a great tool for developing dependently typed languages!¹
- using scopes as env+context todo beperkingen noemen - statix enforces syntactic uniqueness of metavariable solutions but dependently typed languages use a weaker notion of equality - full thesis coming soonTM

¹But there is still room for improvement