

# Name Binding and Name Resolution

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# Type Constraints

# Predicates are Defined by Rules

Predicate

`typeOfExp : scope * Exp → TYPE`

Rule

```
typeOfExp(s, Add(e1, e2)) = INT() :-  
  typeOfExp(s, e1) = INT(),  
  typeOfExp(s, e2) = INT().
```

Head

Premises

For all s, e1, e2

If the premises are true, the head is true

# Representing Name Binding with Scope Graphs

## rules

```
typeOfExp(s, e@Add(e1, e2)) = INT() :-  
  typeOfExp(s, e1) = INT() | error $[integer expected]@e1,  
  typeOfExp(s, e2) = INT() | error $[integer expected]@e2,  
  @e.type := INT().
```

error message on constraint failure

set type attribute

\$ 9 \* 10 + 3

Type: INT

x19 \$ 1 \* true

integer expected

> INT() == BOOL()

> statics/base!typeOfExp(Scope("", "s\_1-1"), True(), INT())

> statics/base!typeOfExp(Scope("", "s\_1-1"), Mul(Int("1"), True()), INT())

> statics/base!declOk(Scope("", "s\_1-1"), Exp(Mul(Int("1"), True())))

> statics/base!declsOk(Scope("", "s\_1-1"), [Exp(Mul(Int(...), True()))])

> ... trace truncated ...

Type: BOOL

# Constraints with Error Messages

```
constraint | error $[message [term]]@origin
```

# Programs with Names

# Programs with Names

```
module Names {  
  
  module Even {  
    import Odd  
    def even = fun(x) {  
      if x == 0 then true else odd(x - 1)  
    }  
  }  
  
  module Odd {  
    import Even  
    def odd = fun(x) {  
      if x == 0 then false else even(x - 1)  
    }  
  }  
  
  module Compute {  
    type Result = { input : Int, output : Bool }  
    def compute = fun(x) {  
      Result{ input = x, output = Odd@odd x }  
    }  
  }  
}
```

Name binding key in programming languages

Many name binding patterns

Deal with erroneous programs

Name resolution complicates type checkers, compilers

Ad hoc non-declarative treatment

A systematic, uniform approach to name resolution?

# Formalizing Name Binding in Type Systems

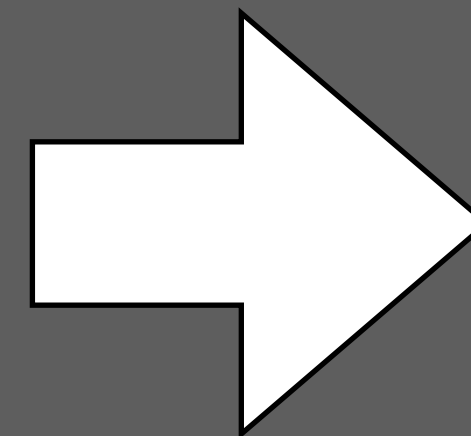
$$\frac{O(id) = T, \text{ where } T \text{ is not a function type.}}{O, M, C, R \vdash id : T} \quad [\text{VAR-READ}]$$



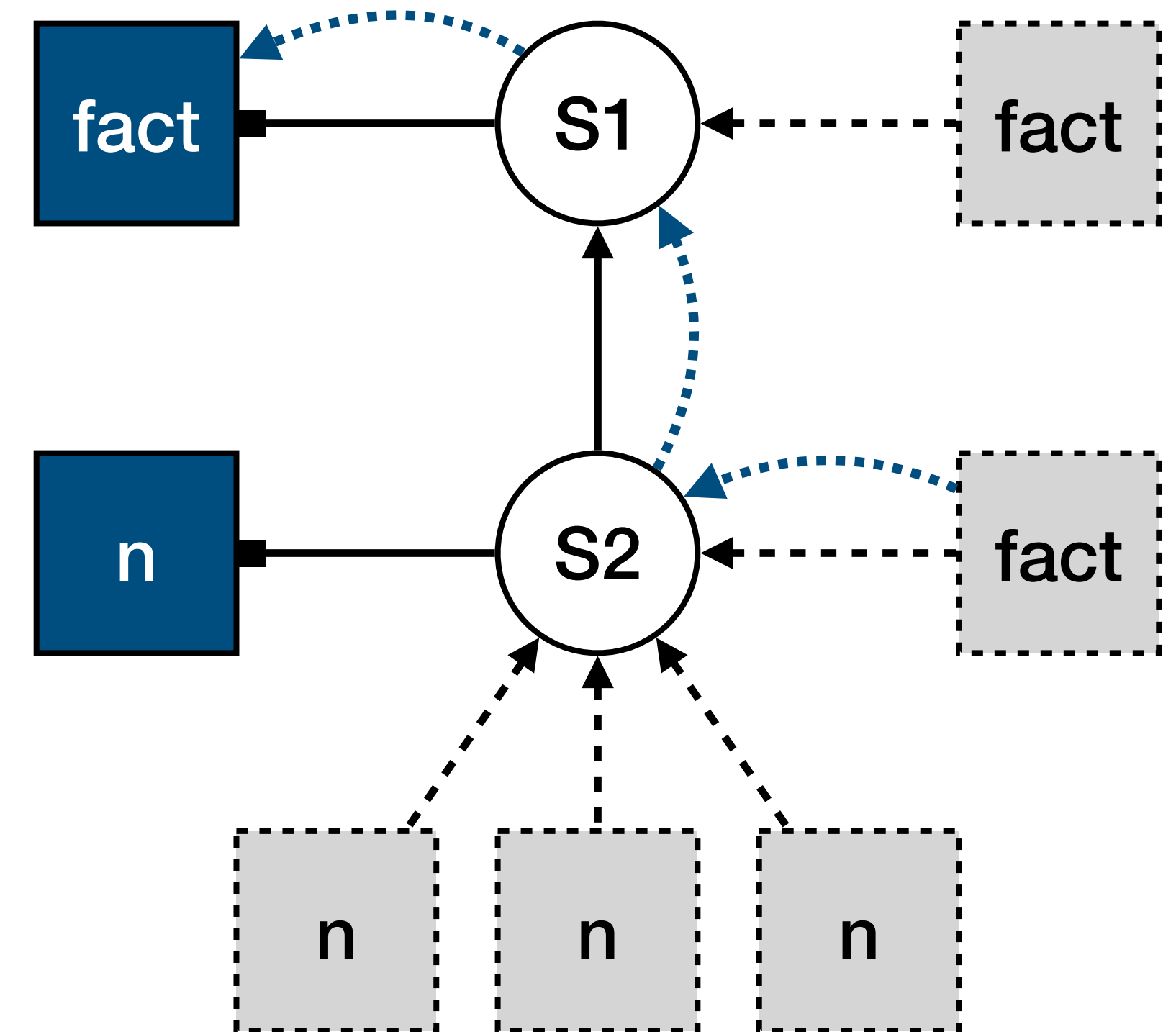
# Name Resolution with Scope Graphs

## Program

```
let function fact(n : int) : int =  
  if n < 1 then  
    1  
  else  
    n * fact(n - 1)  
  in  
    fact(10)  
end
```



## Scope Graph



## Name Resolution

# Name Resolution with Scope Graphs in Statix

Declarations and References

Lexical Scope

Modules

Records

Permission to Extend

Scheduling Resolution

# Declaring and Resolving Names

# Declarations and References

## signature

### constructors

```
Var    : ID → Exp
Bind   : ID * Exp → Bind
BindT  : ID * Type * Exp → Bind
Def    : Bind → Decl
```

## rules

```
declOk : scope * Decl
declsOk maps declOk(*, list(*))

bindOk : scope * scope * Bind
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

```
def a : Int = 0
def b : Int = a + 3
def c : Int = a + b
> a + b + c
```

typed declarations

```
def a = true
def b : Int = a
def c = 1 + b
def e = b && c
```

type mismatch

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
```

undefined variable

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
> a + b + c
def a = 0
def c = a + b
def b = a + 1
```

use before definition

# Declarations and References

## signature

### constructors

```
Var    : ID → Exp
Bind   : ID * Exp → Bind
BindT  : ID * Type * Exp → Bind
Def    : Bind → Decl
```

## rules

```
declOk : scope * Decl
declsOk maps declOk(*, list(*))

bindOk : scope * scope * Bind
```

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

```
def a : Int = 0
def b : Int = a + 3
def c : Int = a + b
> a + b + c
```

typed declarations

## rules

```
typeOfExp(s, Var(x)) = T :-
  typeOfVar(s, x) = T.
```

```
declOk(s, Def(bind)) :-
  bindOk(s, s, bind).
```

```
bindOk(s_bnd, s_ctx, BindT(x, t, e)) :- {T1 T2}
  typeOfType(s_ctx, t) = T1,
  declareVar(s_bnd, x, T1),
  typeOfExp(s_ctx, e) = T2,
  subtype(T2, T1).
```

```
bindOk(s_bnd, s_ctx, Bind(x, e)) :- {T}
  typeOfExp(s_ctx, e) = T,
  declareVar(s_bnd, x, T).
```

```
def a = true
def b : Int = a
def c = 1 + b
def e = b && c
```

type mismatch

```
def a = 0
def b = a + 1
def c = a + d
> a + e + c
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undefined variable

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
> a + b + c
def a = 0
def c = a + b
def b = a + 1
```

use before definition

# Representing Name Binding with Scope Graphs

## rules

declareVar : scope \* ID \* TYPE

typeOfVar : scope \* ID  $\rightarrow$  TYPE

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

# Scope Graphs: Declarations

**signature**

**relations**

var : ID → TYPE

**rules**

declareVar : scope \* ID \* TYPE

typeOfVar : scope \* ID → TYPE

declareVar(s, x, T) :-  
!var[x, T] in s.

declaration relation

variable x is declared in scope s  
with type T

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference

# Scope Graphs: Declarations

**signature**

**relations**

var : ID  $\rightarrow$  TYPE

**rules**

declareVar : scope \* ID \* TYPE

typeOfVar : scope \* ID  $\rightarrow$  TYPE

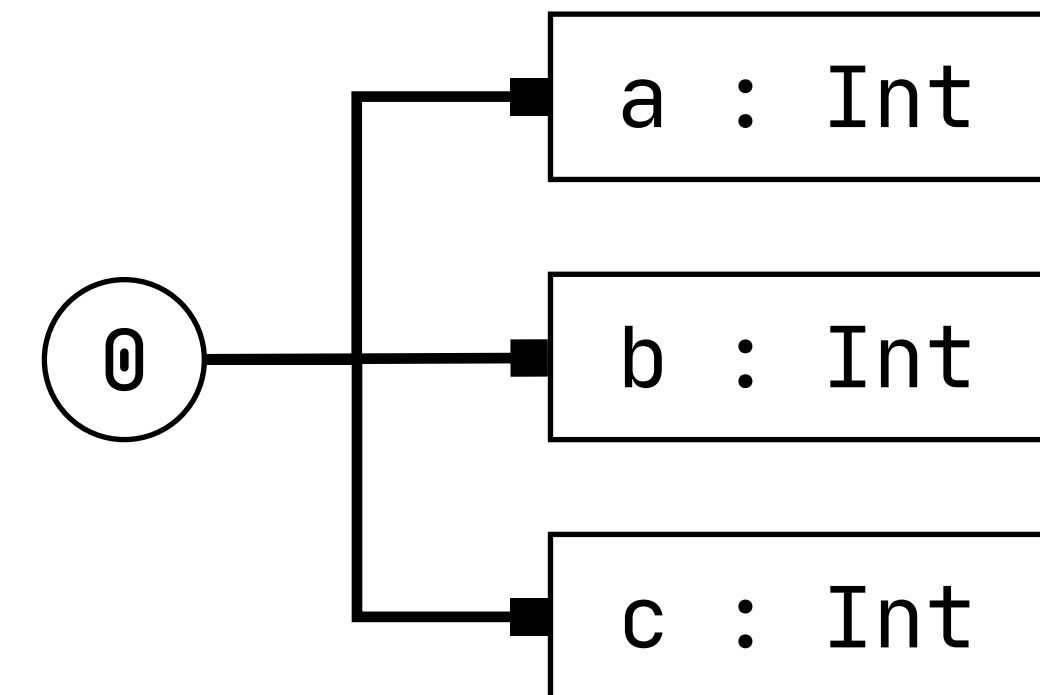
declareVar(*s*, *x*, *T*) :-  
!var[*x*, *T*] in *s*.

declaration relation

variable *x* is declared in scope *s*  
with type *T*

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference





# Scope Graphs: Name Resolution Queries

**signature**

**relations**

var : ID  $\rightarrow$  TYPE

**rules**

declareVar : scope \* ID \* TYPE

resolveVar : scope \* ID  $\rightarrow$  list((path \* (ID \* TYPE)))

typeOfVar : scope \* ID  $\rightarrow$  TYPE

declareVar(s, x, T) :-

!var[x, T] in s.

resolveVar(s, x) = ps :-

query var

filter e and { x' :- x' = x }

min and true

in s  $\mapsto$  ps.

typeOfVar(s, x) = T :- {x'}

resolveVar(s, x) = [(\_, (x', T))].

declaration relation

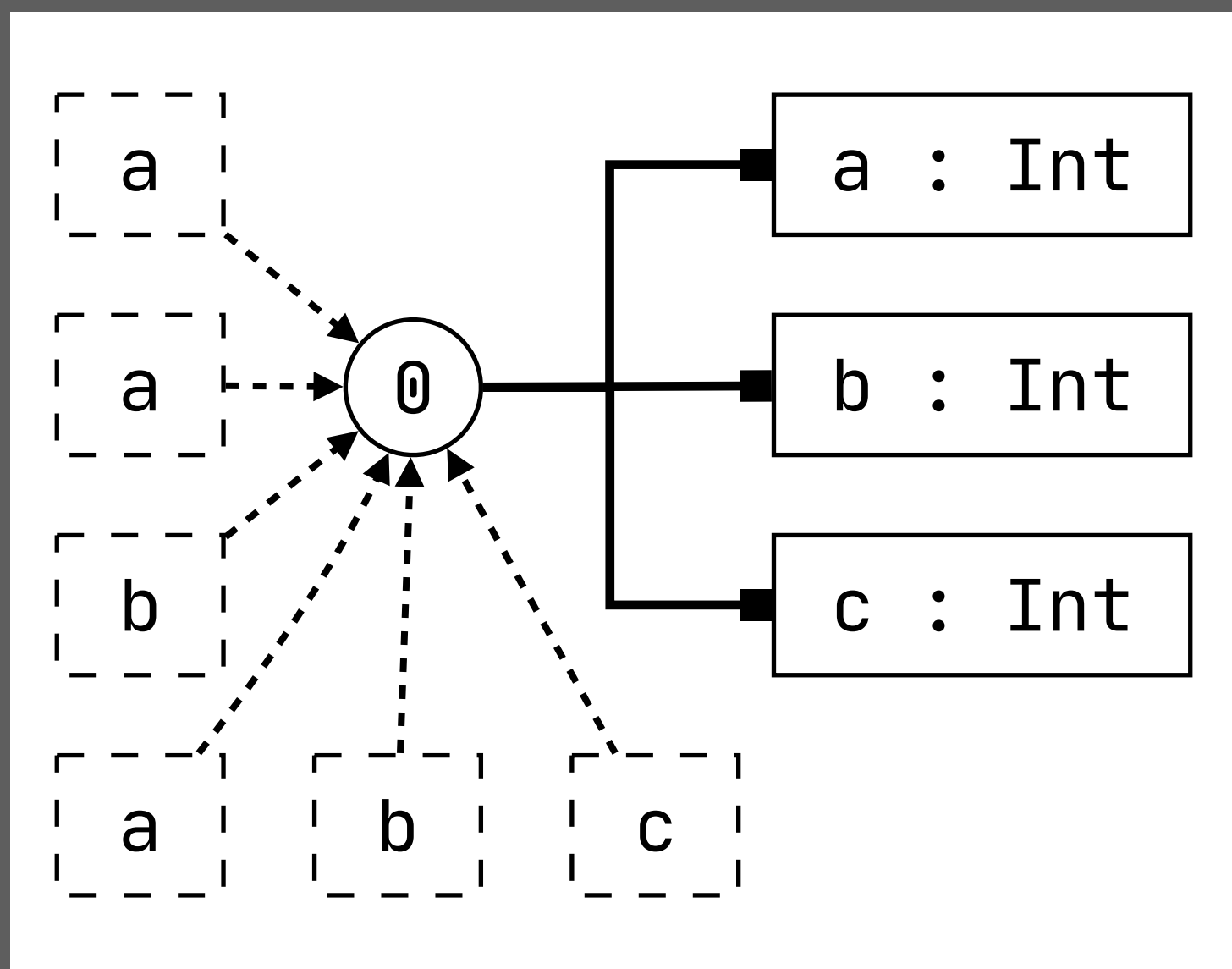
variable x is declared in scope s  
with type T

variable x in scope s resolves to list  
of declarations

variable x in scope s resolves to  
declaration x' with type T

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference



# Undefined Variable

signature

relations

var : ID  $\rightarrow$  TYPE

rules

declareVar : scope \* ID \* TYPE

resolveVar : scope \* ID  $\rightarrow$  list((path \* (ID \* TYPE)))

typeOfVar : scope \* ID  $\rightarrow$  TYPE

declareVar(s, x, T) :-

!var[x, T] in s.

resolveVar(s, x) = ps :-

query var

filter e and { x' :- x' = x }

min and true

in s  $\mapsto$  ps.

typeOfVar(s, x) = T :- {x'}

resolveVar(s, x) = [(\_, (x', T))].

declaration relation

variable x is declared in scope s  
with type T

variable x in scope s resolves to list  
of declarations

variable x in scope s resolves to  
declaration x' with type T

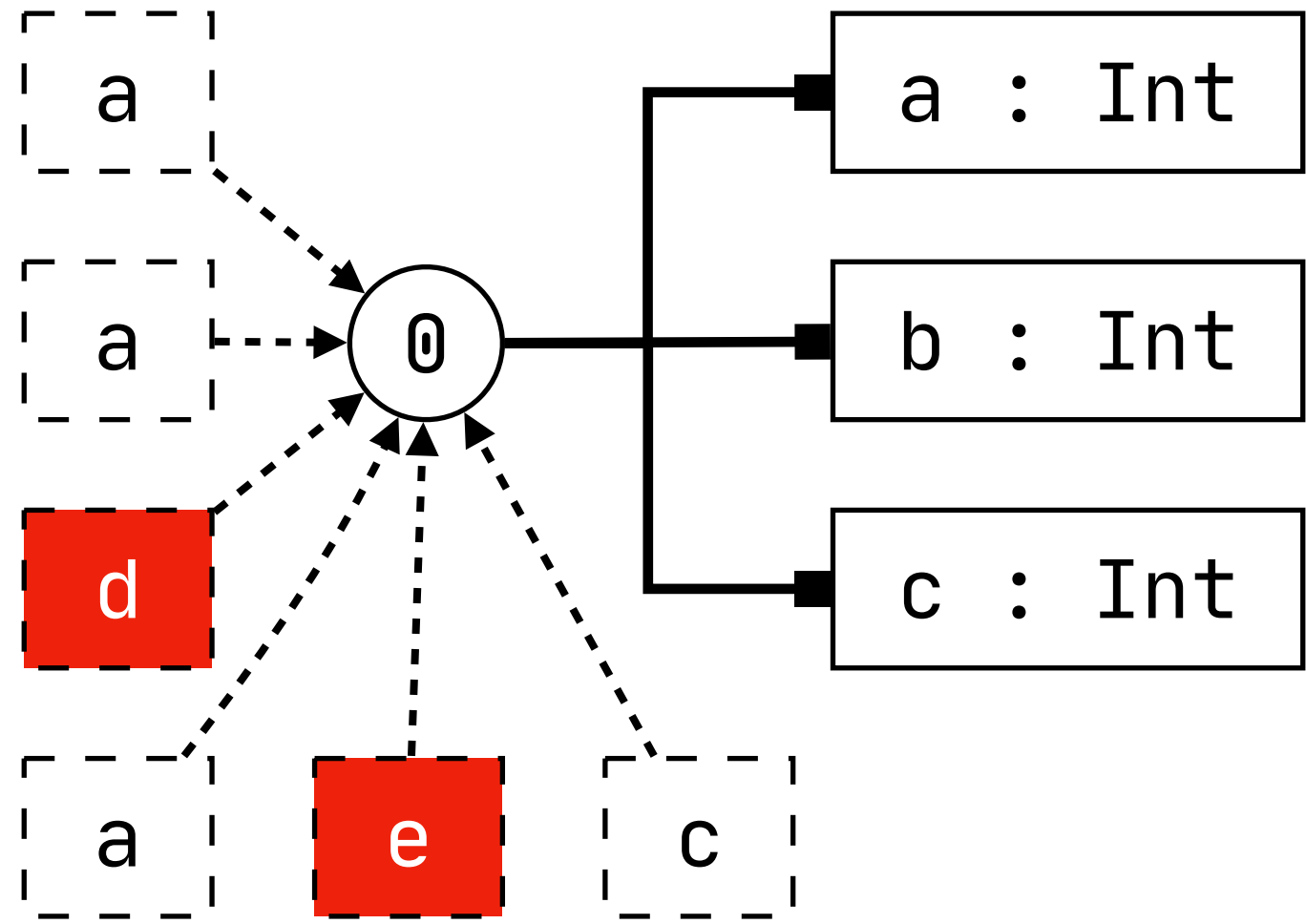
def a = 0

def b = a + 1

def c = a + d

> a + e + c

undefined variable



resolveVar returns empty list of declarations

# Duplicate Definition

**signature**  
**relations**  
var : ID → TYPE  
**rules**

declareVar : scope \* ID \* TYPE  
resolveVar : scope \* ID → list((path \* (ID \* TYPE)))  
typeOfVar : scope \* ID → TYPE

declareVar(s, x, T) :-  
!var[x, T] in s.

resolveVar(s, x) = ps :-  
query var  
filter e and { x' :- x' = x }  
min and true  
in s → ps.

typeOfVar(s, x) = T :- {x'}  
resolveVar(s, x) = [(\_, (x', T))].

declaration relation

variable x is declared in scope s  
with type T

variable x in scope s resolves to list  
of declarations

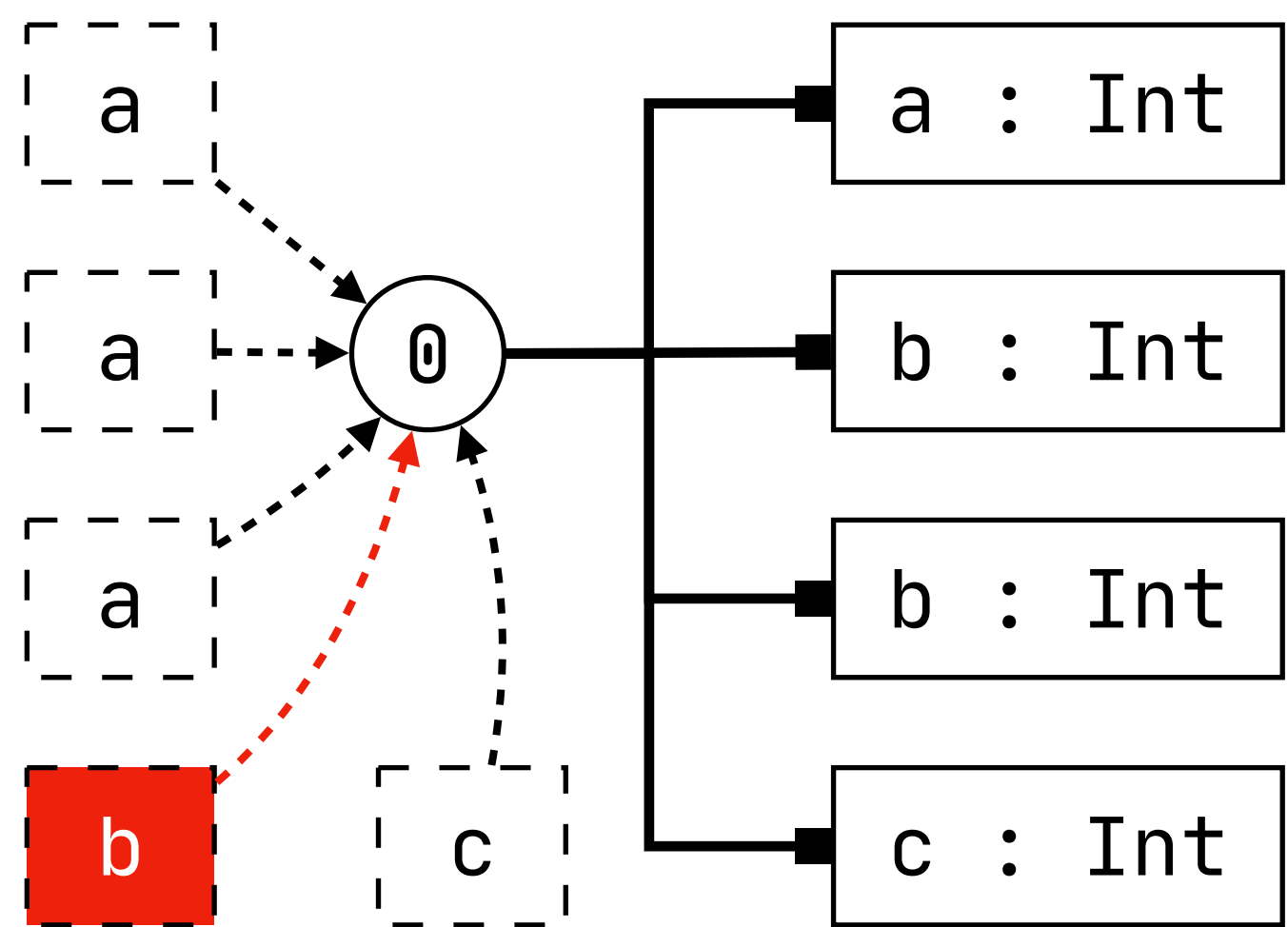
variable x in scope s resolves to  
declaration x' with type T

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

what we want

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

what we get



# Duplicate Definition: Permissive Resolution

## signature

### relations

var : ID  $\rightarrow$  TYPE

### rules

declareVar : scope \* ID \* TYPE

resolveVar : scope \* ID  $\rightarrow$  list((path \* (ID \* TYPE)))

typeOfVar : scope \* ID  $\rightarrow$  TYPE

declareVar(s, x, T) :-

!var[x, T] in s,

resolveVar(s, x) = [(\_, (\_, \_))]

| error \$[Duplicate definition of variable [x]].

resolveVar(s, x) = ps :-

query var

filter e and { x' :- x' = x }

min and true

in s  $\mapsto$  ps.

typeOfVar(s, x) = T :- {x'}

resolveVar(s, x) = [(\_, (x', T))|\_]

| error \$[Variable [x] not defined].

declaration relation

variable x is declared in scope s  
with type T

there should only be one declaration

variable x in scope s resolves to list  
of declarations

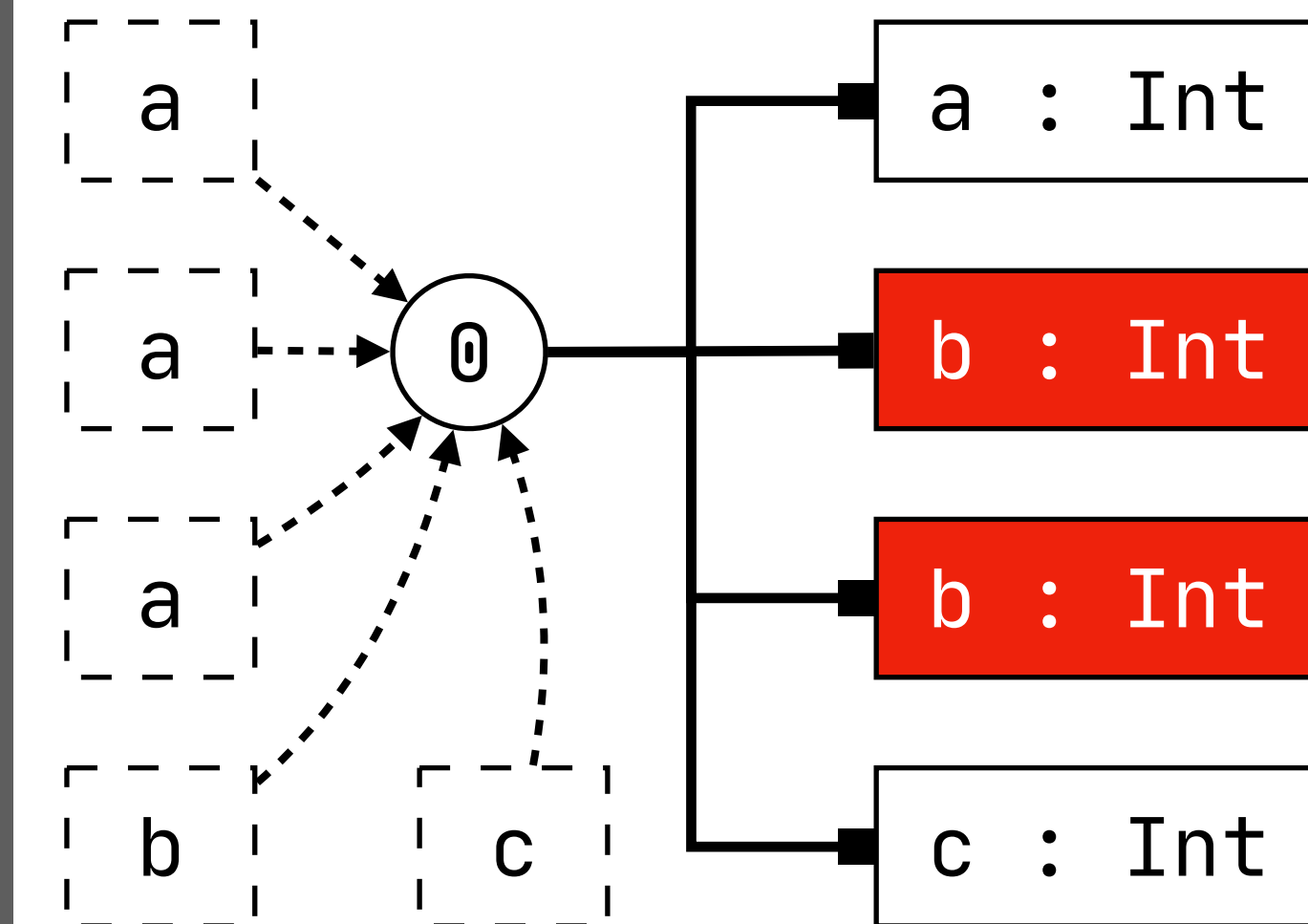
variable x in scope s resolves to  
declaration x' with type T

there should be at least one  
matching declaration

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```

duplicate definition

```
def a = 0
def b = a + 1
def b = 2 + a
def c = 3
> a + b + c
```



# Reference and Type Attributes

**signature**  
**relations**  
var : ID → TYPE  
**rules**

declareVar : scope \* ID \* TYPE  
resolveVar : scope \* ID → list((path \* (ID \* TYPE)))  
typeOfVar : scope \* ID → TYPE

declareVar(s, x, T) :-  
!var[x, T] in s,  
resolveVar(s, x) = [(\_, (\_, \_))]  
| error \$[Duplicate definition of variable [x]],  
@x.type := T.

resolveVar(s, x) = ps :-  
query var  
filter e and { x' :- x' = x }  
min and true  
in s → ps.

typeOfVar(s, x) = T :- {x'}  
resolveVar(s, x) = [(\_, (x', T))|\_]  
| error \$[Variable [x] not defined],  
@x.ref := x'.

declaration relation

variable x is declared in scope s  
with type T

there should only be one declaration

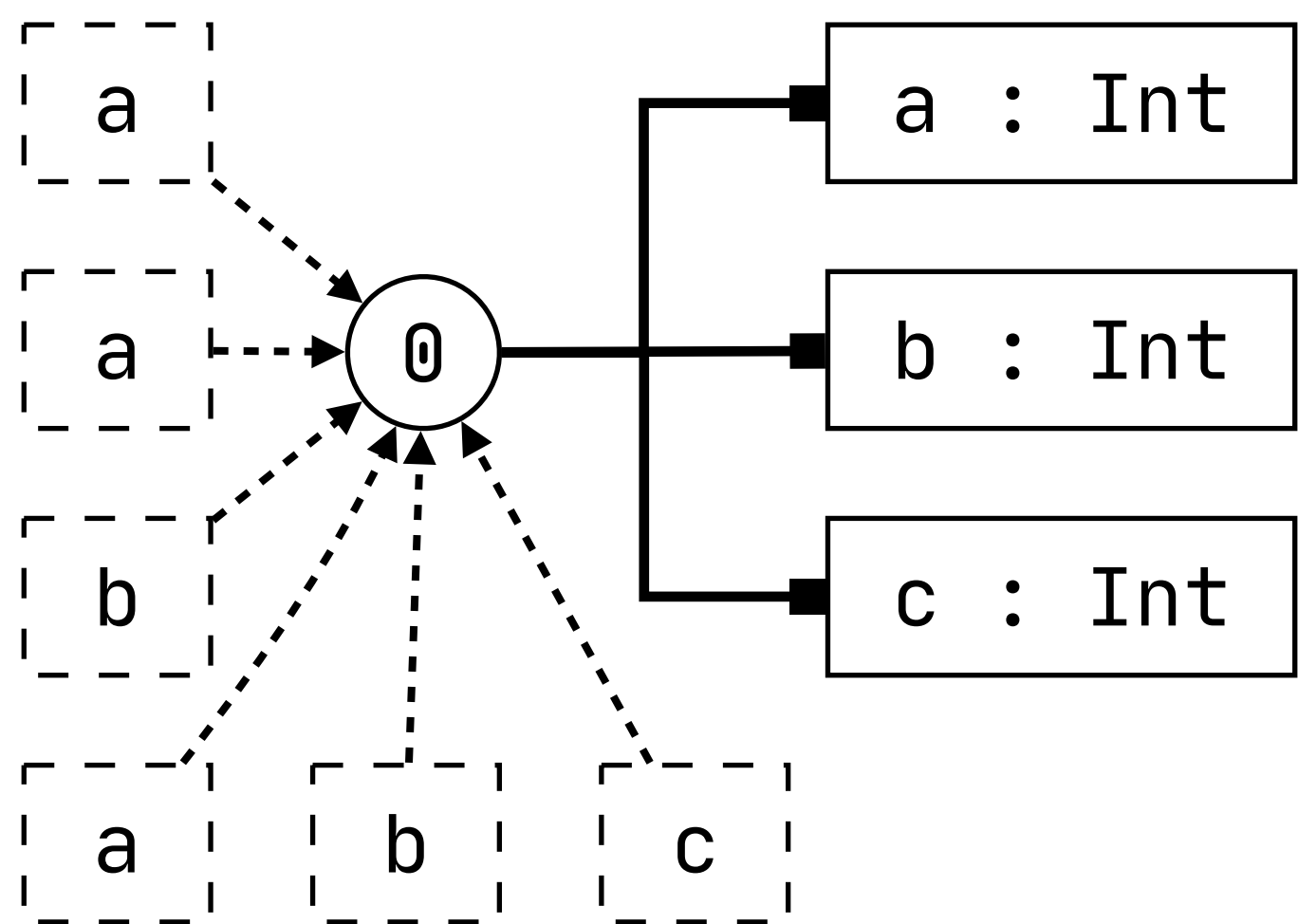
variable x in scope s resolves to list  
of declarations

variable x in scope s resolves to  
declaration x' with type T

there should be at least one  
matching declaration

```
def a = 0
def b = a + 1
def c = a + b
> a + b + c
```

declaration and reference



# Type Annotation Indirection (for Parallel Type Checking)

**signature**

**relations**

`var` : ID  $\rightarrow$  **scope**

`typeOf` :  $\rightarrow$  TYPE

**rules**

`declareVar(s, x, T) :-`  
  `!var[x, withType(T)] in s.`

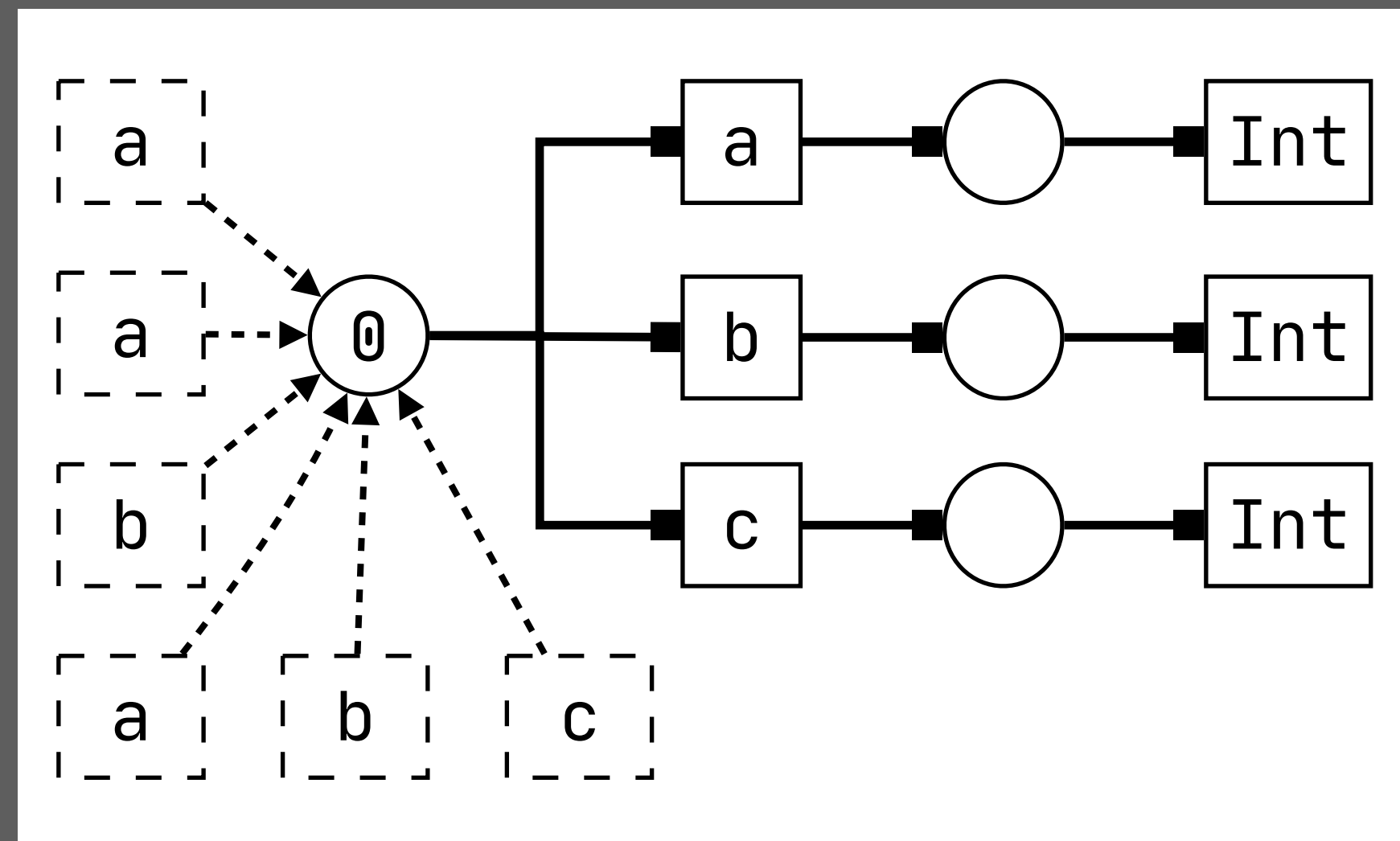
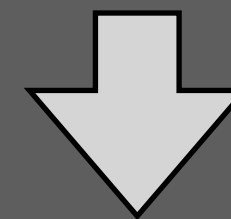
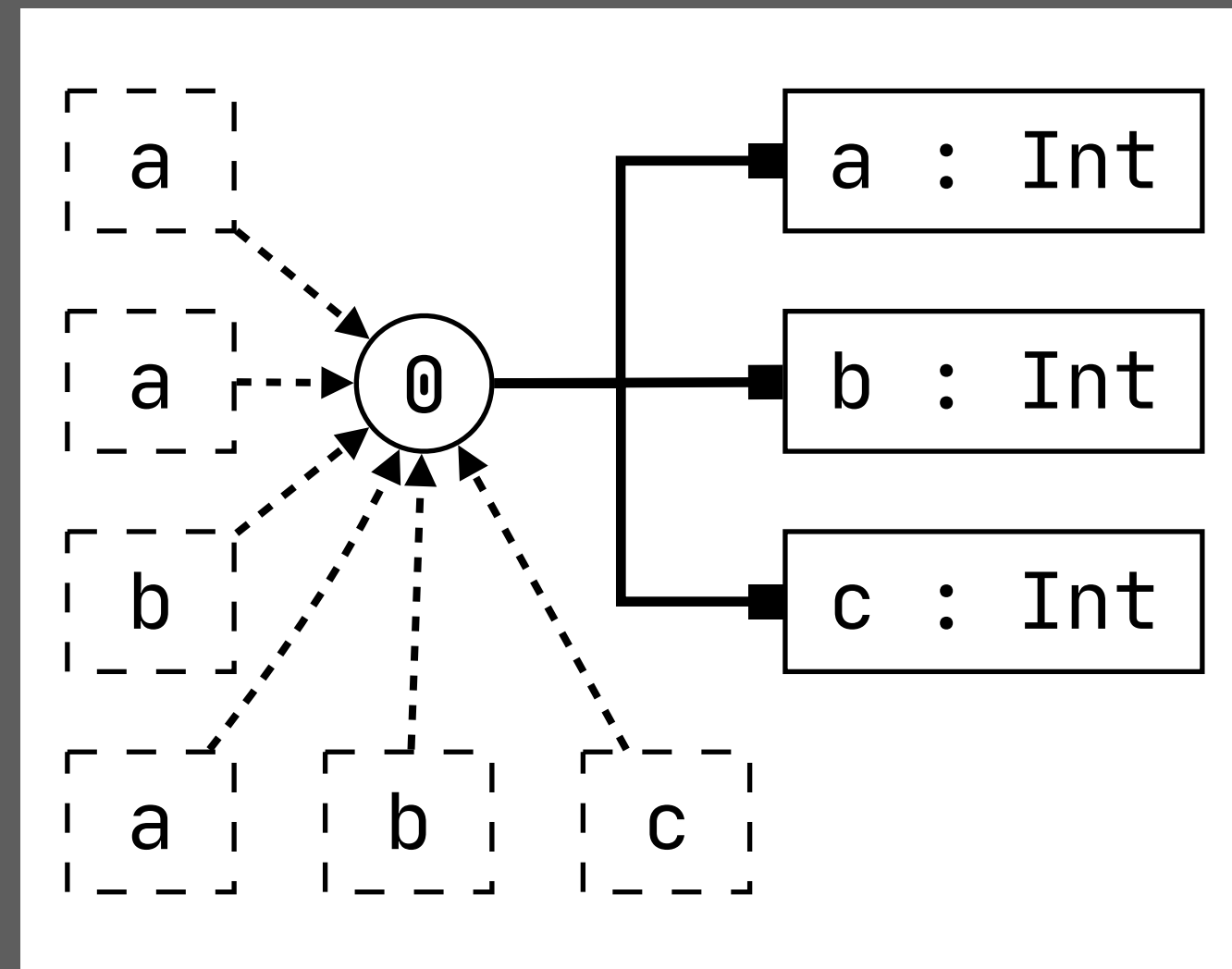
`typeOfVar(s, x) = typeOf(T) :- {x'}`  
  `resolveVar(s, x) = [(_, (x', T))].`

`typeOf` : **scope**  $\rightarrow$  TYPE

`typeOf(s) = T :-`  
  `query typeOf`  
    `filter e and true`  
    `min /* */ and true`  
  `in s  $\mapsto$  [(_, T)].`

`withType` : TYPE  $\rightarrow$  **scope**

`withType(T) = s :-`  
  `new s, !typeOf[T] in s.`



How about shadowing?

# Lexical Scope



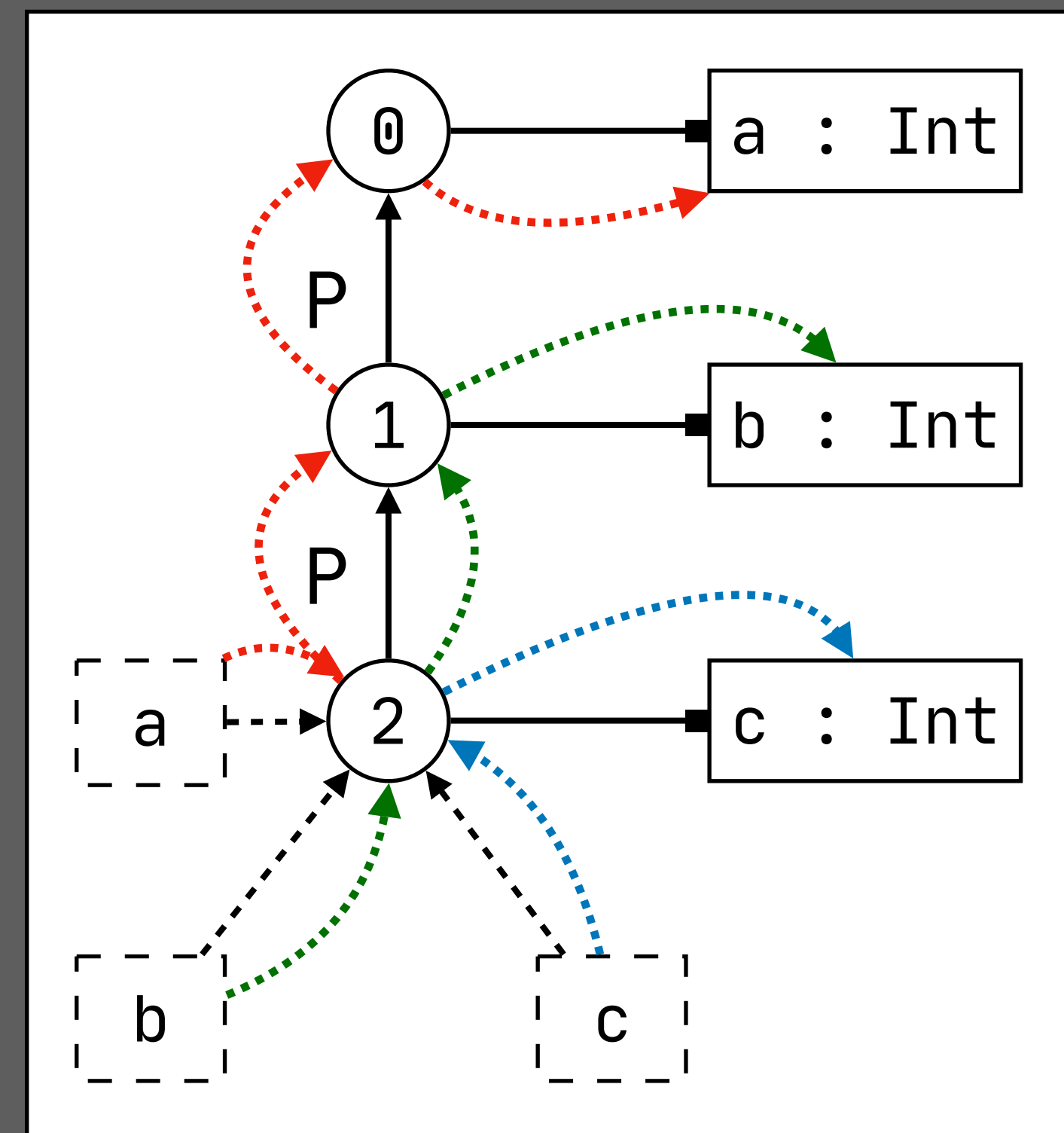
# Labeled Scope Edges: What Scopes are Reachable?

signature

constructors

Let : ID \* Exp \* Exp  $\rightarrow$  Exp

```
let a = 1 in
let b = 2 in
let c = 3 in
a + b + c
```



# New Scope and Scope Edge Constraints

signature

constructors

**Let** : ID \* Exp \* Exp  $\rightarrow$  Exp

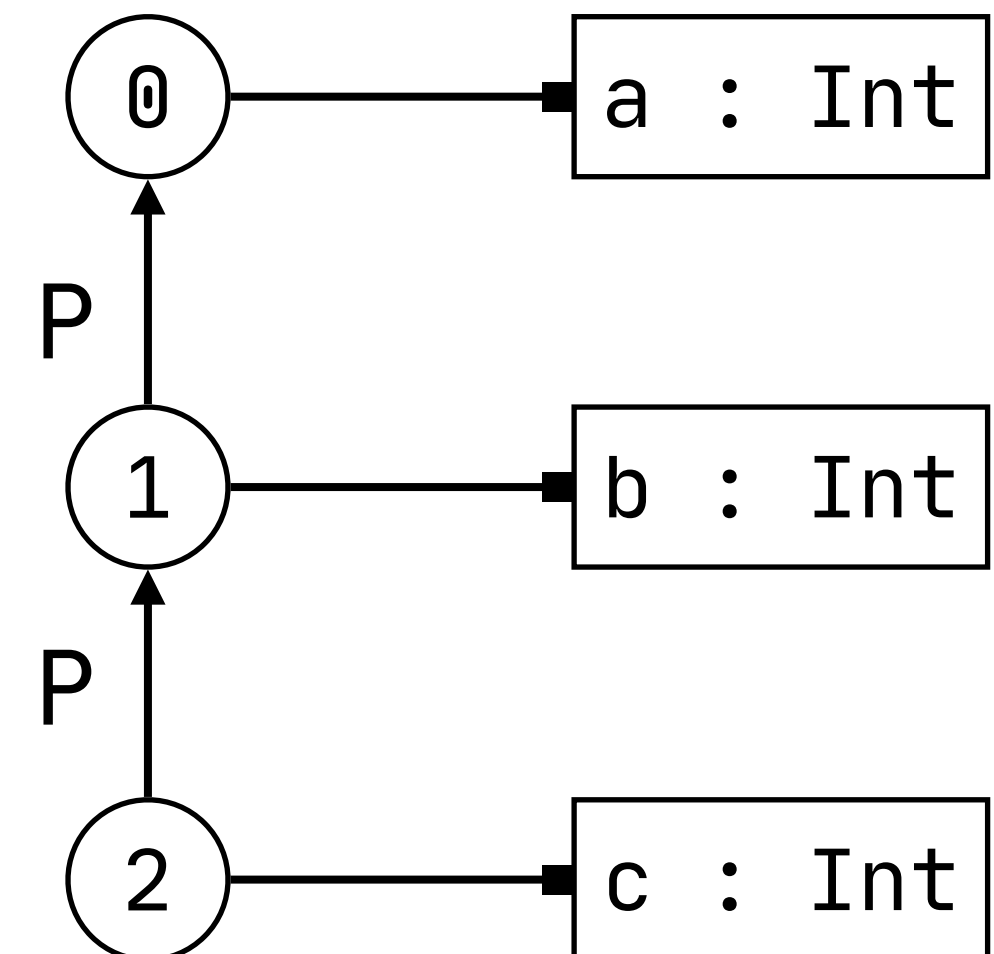
rules

```
typeOfExp(s, Let(x, e1, e2)) = T :- {S s_let}  
  typeOfExp(s, e1) = S,  
  new s_let, s_let -P $\rightarrow$  s,  
  declareVar(s_let, x, S),  
  typeOfExp(s_let, e2) = T.
```

new scope

scope edge

```
let a = 1 in  
let b = 2 in  
let c = 3 in  
  a + b + c
```



# Path Wellformedness: Reachability

## signature

### constructors

Let : ID \* Exp \* Exp  $\rightarrow$  Exp

## rules

```
typeOfExp(s, Let(x, e1, e2)) = T :- {S s_let}
typeOfExp(s, e1) = S,
new s_let, s_let -P $\rightarrow$  s,
declareVar(s_let, x, S),
typeOfExp(s_let, e2) = T.
```

new scope

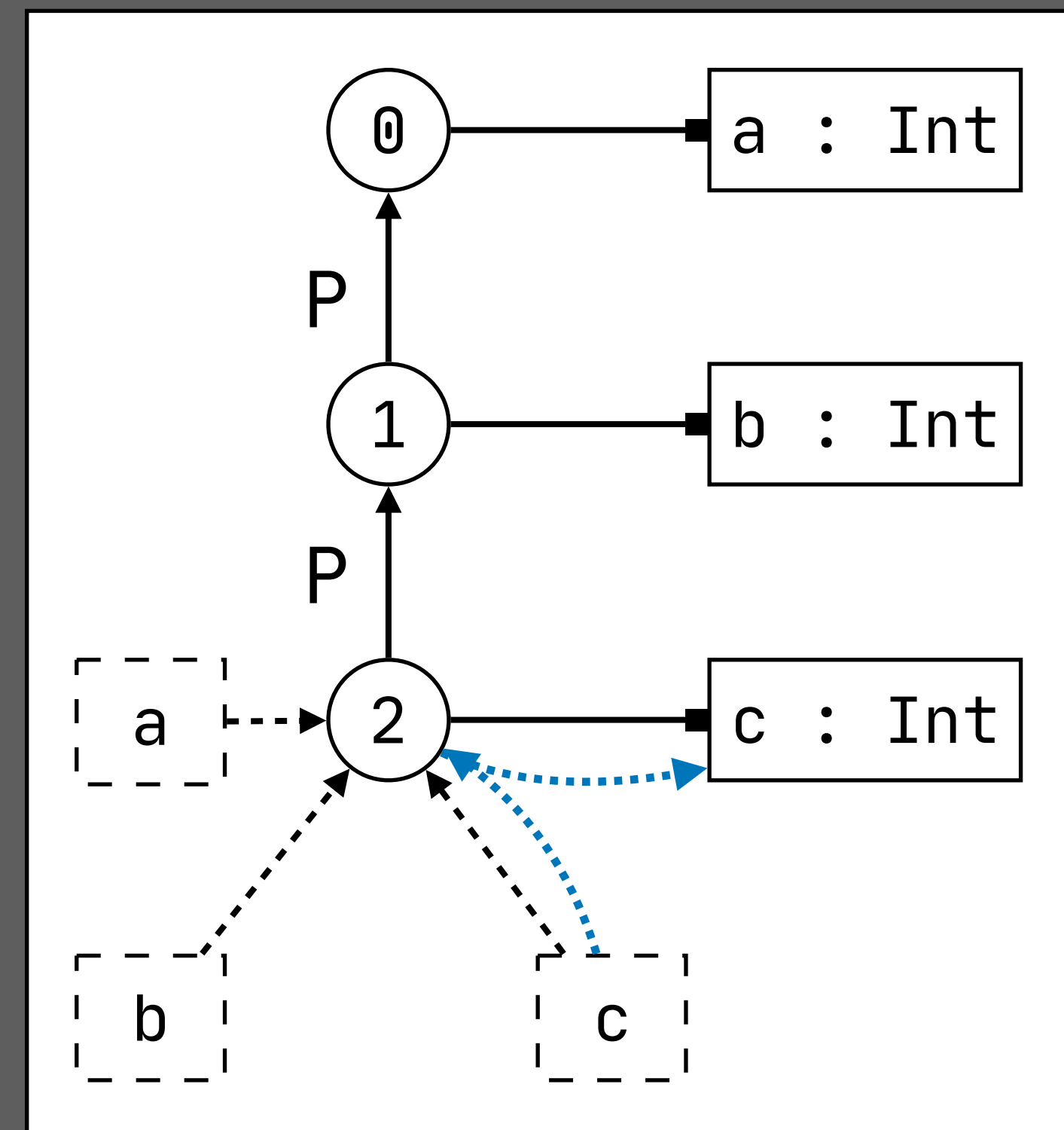
scope edge

```
resolveVar(s, x) = ps :-
  query var
  filter e and { x' :- x' = x }
  min /* */ and true
  in s  $\mapsto$  ps.
```

empty path **e** only allows resolution in 'this' scope

```
let a = 1 in
let b = 2 in
let c = 3 in
  a + b + c
```

```
let a = 1 in
let b = 2 in
let c = 3 in
  a + b + c
```



# Path Wellformedness: Reachability

## signature

### constructors

Let : ID \* Exp \* Exp  $\rightarrow$  Exp

### name-resolution

labels P

## rules

```
typeOfExp(s, Let(x, e1, e2)) = T :- {S s_let}
typeOfExp(s, e1) = S,
new s_let, s_let -P $\rightarrow$  s,
declareVar(s_let, x, S),
typeOfExp(s_let, e2) = T.
```

new scope

scope edge

```
resolveVar(s, x) = ps :-
```

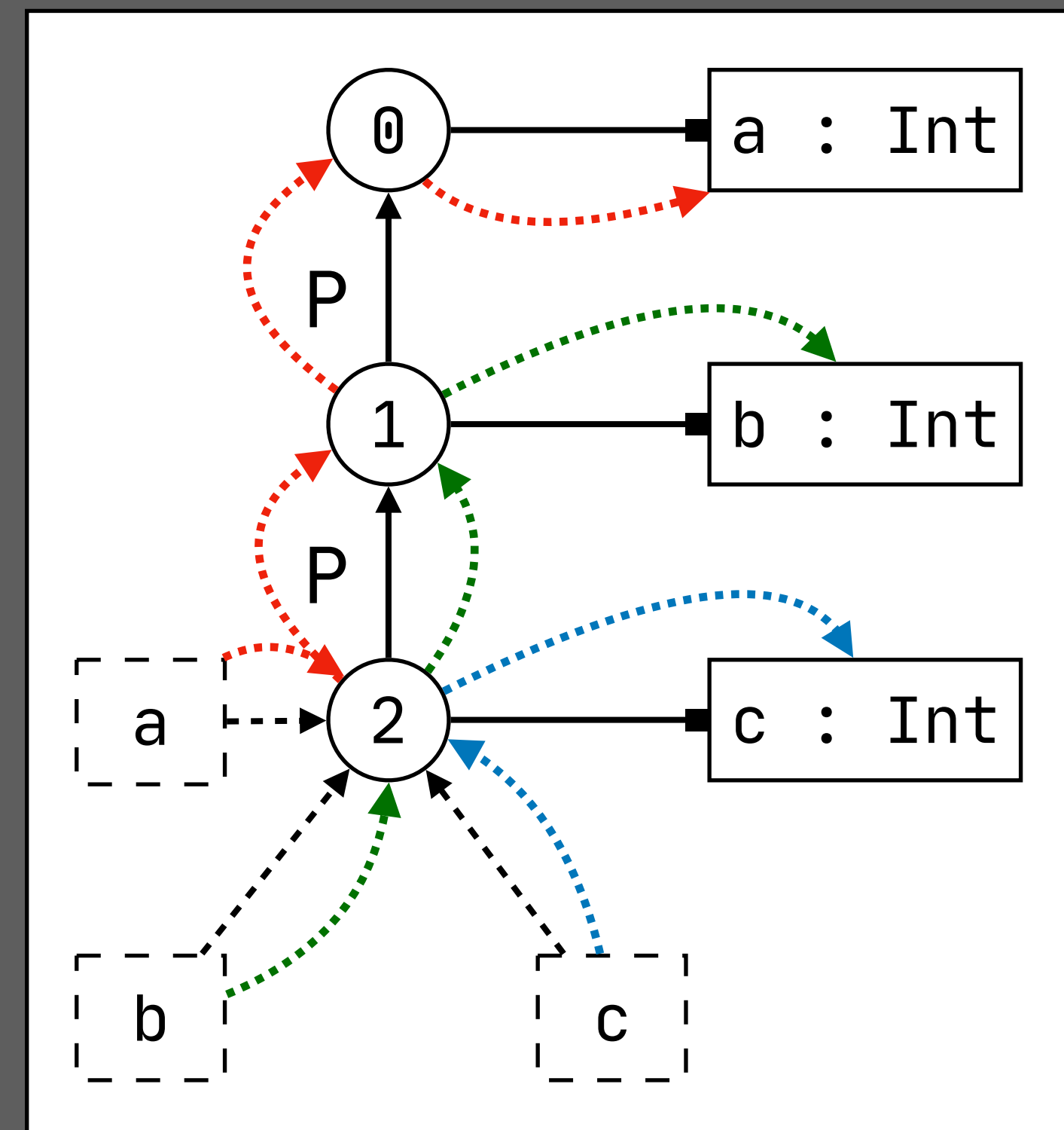
```
  query var
```

```
    filter P* and { x' :- x' = x }
```

```
      min /* */ and true
```

```
    in s  $\mapsto$  ps.
```

```
let a = 1 in
let b = 2 in
let c = 3 in
  a + b + c
```



path  $P^*$  allows resolution through zero or more  $P$  edges

# Duplicate Definitions Revisited

## signature

### constructors

Let : ID \* Exp \* Exp  $\rightarrow$  Exp

### name-resolution

labels P

## rules

```
typeOfExp(s, Let(x, e1, e2)) = T :- {S s_let}
typeOfExp(s, e1) = S,
new s_let, s_let  $\xrightarrow{P}$  s,
declareVar(s_let, x, S),
typeOfExp(s_let, e2) = T.
```

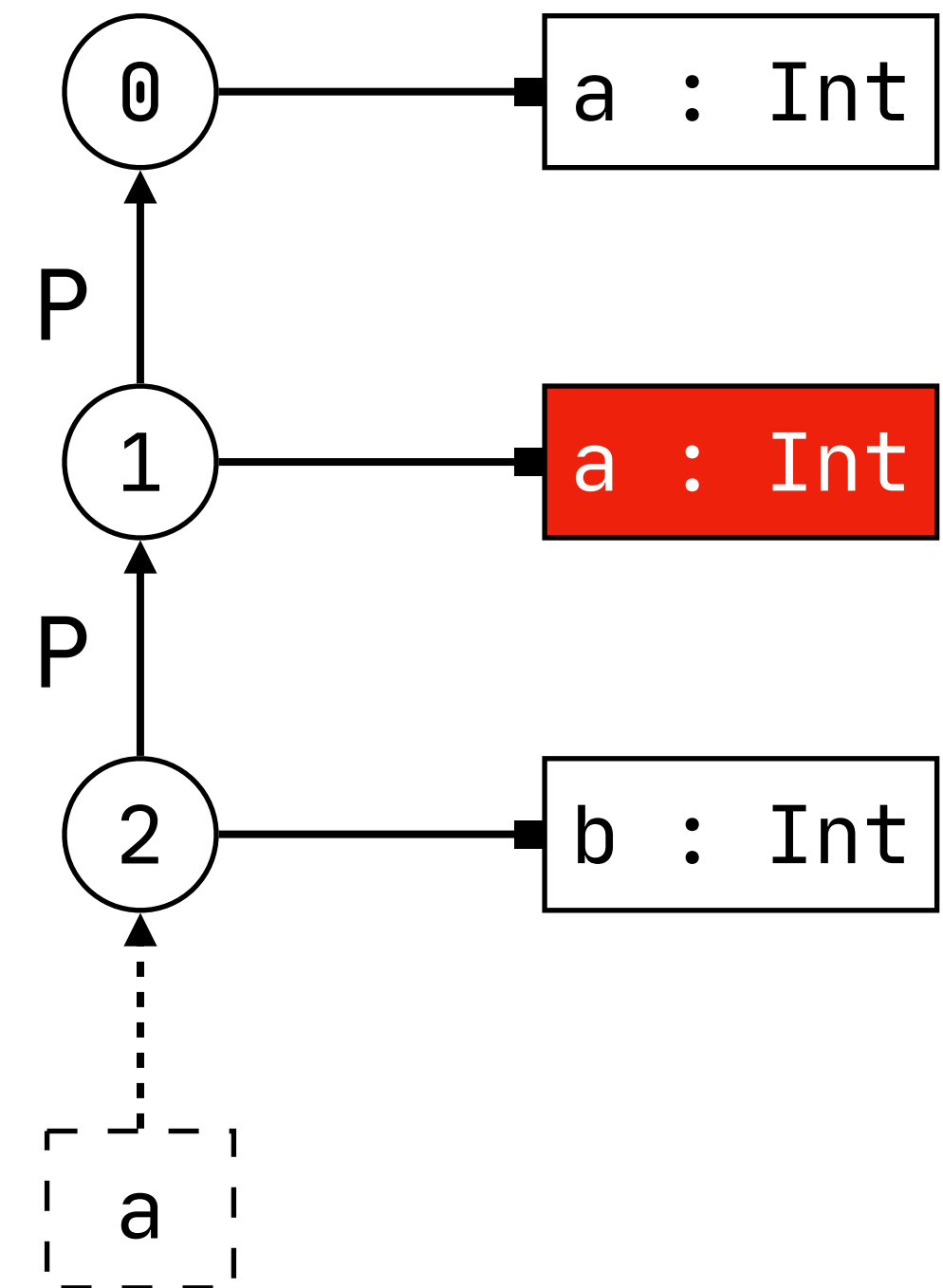
new scope

scope edge

```
resolveVar(s, x) = ps :-
  query var
  filter P* and { x' :- x' = x }
  min /* */ and true
  in s  $\mapsto$  ps.
```

```
let a = 1 in
let a = 2 in
let b = 3 in
a
```

duplicate definition



path  $P^*$  allows resolution through zero or more  $P$  edges

# Duplicate Definitions Revisited

## signature

### constructors

Let : ID \* Exp \* Exp  $\rightarrow$  Exp

### name-resolution

labels P

## rules

```
typeOfExp(s, Let(x, e1, e2)) = T :- {S s_let}
typeOfExp(s, e1) = S,
new s_let, s_let  $\xrightarrow{P}$  s,
declareVar(s_let, x, S),
typeOfExp(s_let, e2) = T.
```

new scope

scope edge

```
resolveVar(s, x) = ps :-
```

```
  query var
```

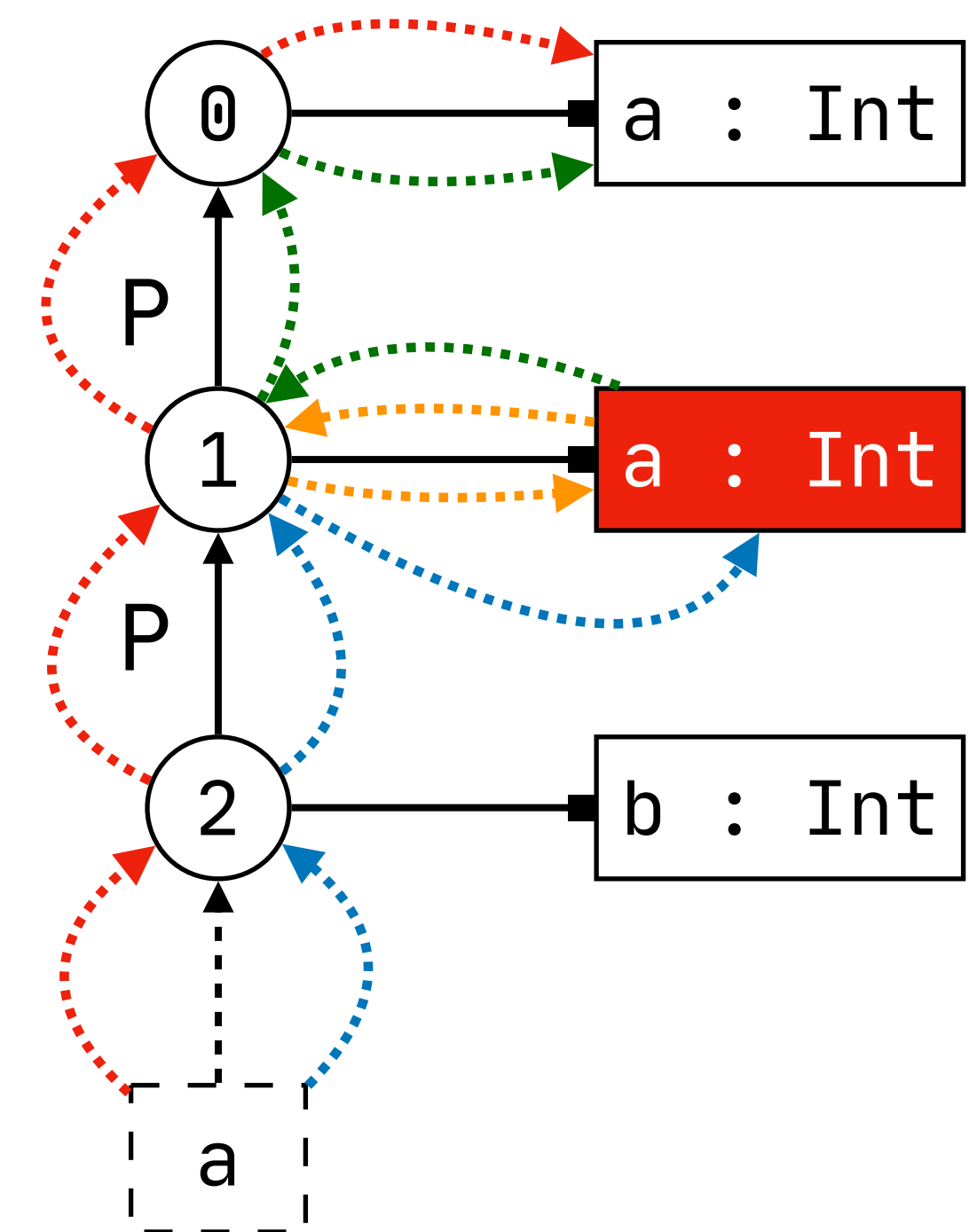
```
    filter P* and { x' :- x' = x }
```

```
      min /* */ and true
```

```
    in s  $\mapsto$  ps.
```

```
let a = 1 in
let a = 2 in
let b = 3 in
a
```

duplicate definition



path  $P^*$  allows resolution through zero or more  $P$  edges

# Path Specificity: Visibility (Shadowing)

**signature**  
**constructors**  
Let : ID \* Exp \* Exp → Exp  
**name-resolution**  
**labels** P

**rules**

```
typeOfExp(s, Let(x, e1, e2)) = T :- {S s_let}
typeOfExp(s, e1) = S,
new s_let, s_let -P→ s,
declareVar(s_let, x, S),
typeOfExp(s_let, e2) = T.
```

new scope

scope edge

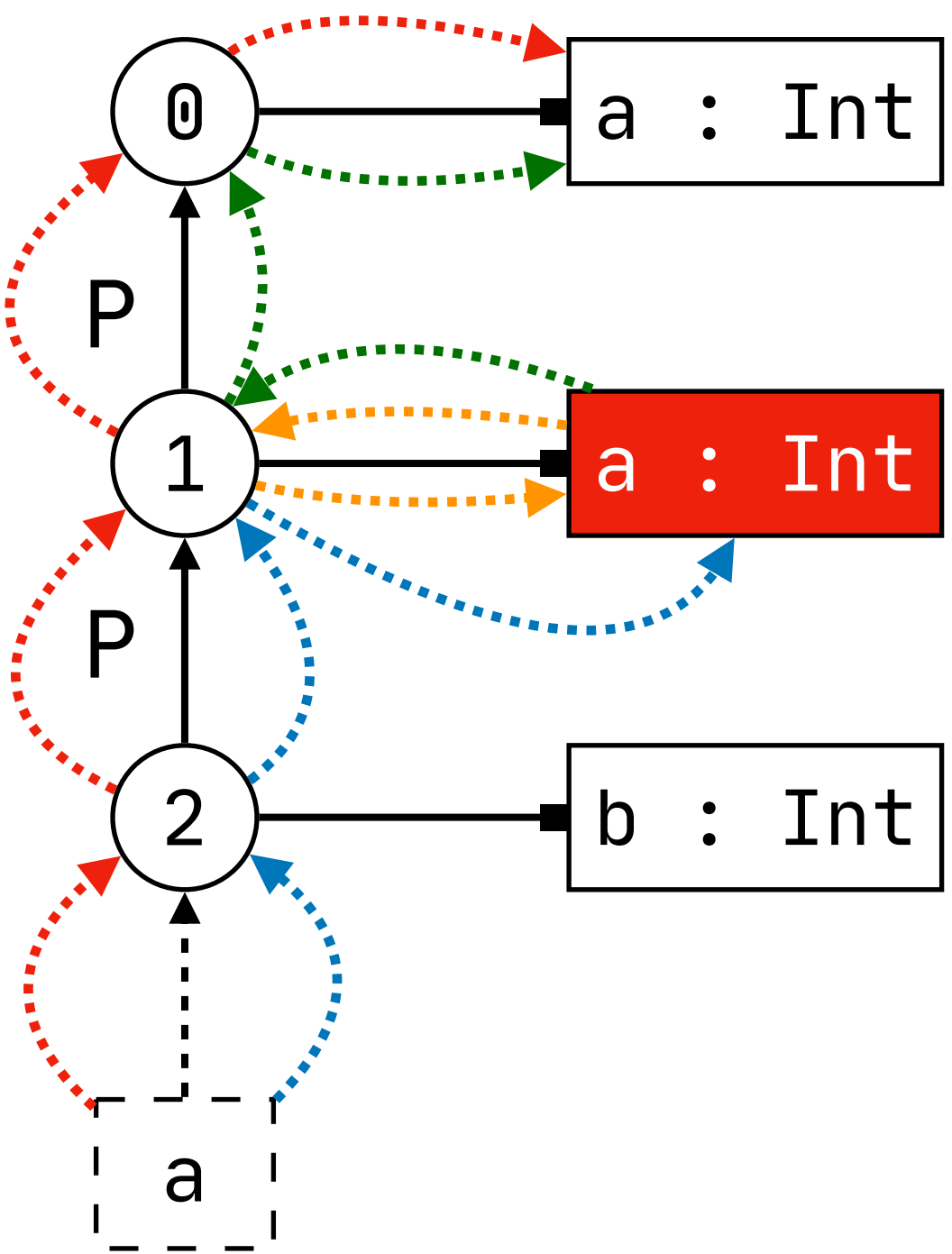
```
resolveVar(s, x) = ps :-
  query var
  filter P* and { x' :- x' = x }
  min /* */ and true
  in s ↦ ps.
```

path  $P^*$  allows resolution through zero or more  $P$  edges

prefer local scope (\$) over parent scope ( $P$ )

```
let a = 1 in
let a = 2 in
let b = 3 in
a
```

duplicate definition



prefer blue path over red path

prefer orange path over green path

# Path Specificity: Visibility (Shadowing)

**signature**  
**constructors**  
Let : ID \* Exp \* Exp  $\rightarrow$  Exp  
**name-resolution**  
**labels** P

## rules

```
typeOfExp(s, Let(x, e1, e2)) = T :- {S s_let}  
typeOfExp(s, e1) = S,  
new s_let, s_let -P $\rightarrow$  s,  
declareVar(s_let, x, S),  
typeOfExp(s_let, e2) = T.
```

new scope

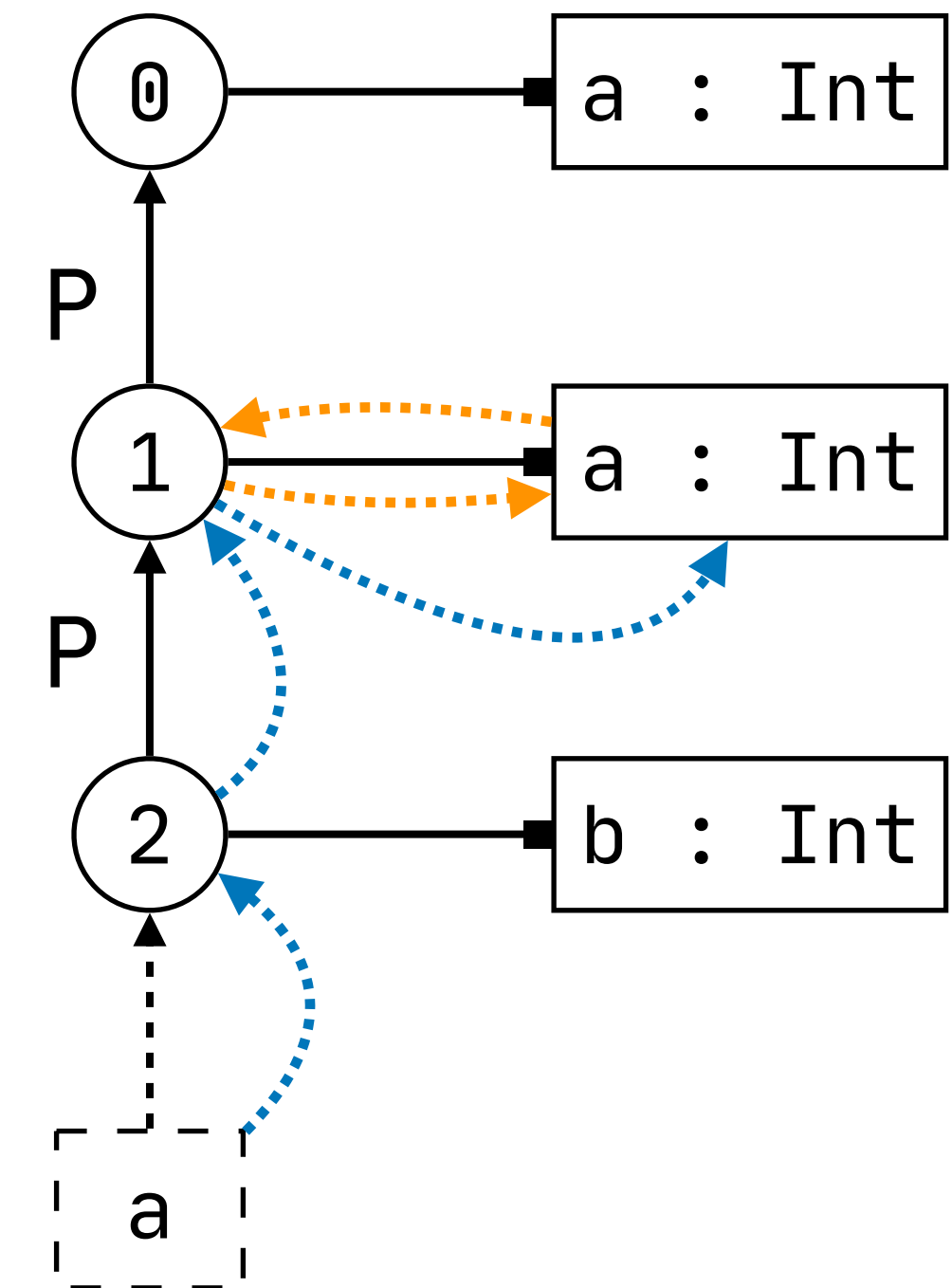
scope edge

```
resolveVar(s, x) = ps :-  
  query var  
  filter P* and { x' :- x' = x }  
  min $ < P and true  
  in s  $\mapsto$  ps.
```

path  $P^*$  allows resolution through zero or more  $P$  edges

prefer local scope (\$) over parent scope (P)

```
let a = 1 in  
let a = 2 in  
let b = 3 in  
a
```





# Statix Queries

query RELATION filter REGEX and MATCH min LABELORD and SHADOW in SCOPE  $\mapsto$  RESULT

## RELATION

- the relation we are querying

## filter

- filter applied to individual paths to rule out ‘non-wellformed’ paths

## min

- a filter applied to pairs of paths to select the ‘minimal’ paths from a set of paths

## SCOPE

- the source scope of the query

## RESULT

- the list of query results

# Statix Queries

query **RELATION** *filter* **REGEX** and **MATCH** *min* **LABELORD** and **SHADOW** *in* **SCOPE**  $\mapsto$  **RESULT**

## **RELATION**

- Can be a custom relation name, `decl`, or `()` which only looks at the reachable scopes

## **REGEX**

- Specifies path well-formedness using a regular expression (e.g. `P* I*`) over edge labels

## **MATCH**

- Specifies which data in the relation to match
- To match on a name `x`, the match is an anonymous rule  $\{x' \text{ :- } x' = x\}$  which is tested against all reachable declarations

## **LABELORD**

- Determines the edge label order using inequalities (e.g. `$ < P`, `I < P`) over edge labels

## **SHADOW**

- Determines which declarations shadow each other
- `true`: all declarations shadow each other and we only get the declarations reached via the shortest path
- `false`: none of the declarations shadow (which could be used to find all reachable declarations)
- Anonymous rule (e.g.  $\{x, x' \text{ :- } x' = x\}$ ): only shadow between things with the same name

How about non-lexical bindings?

# Scopes as Types / Modules

# Modules: Scopes as Types

## signature

### constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

scope as type

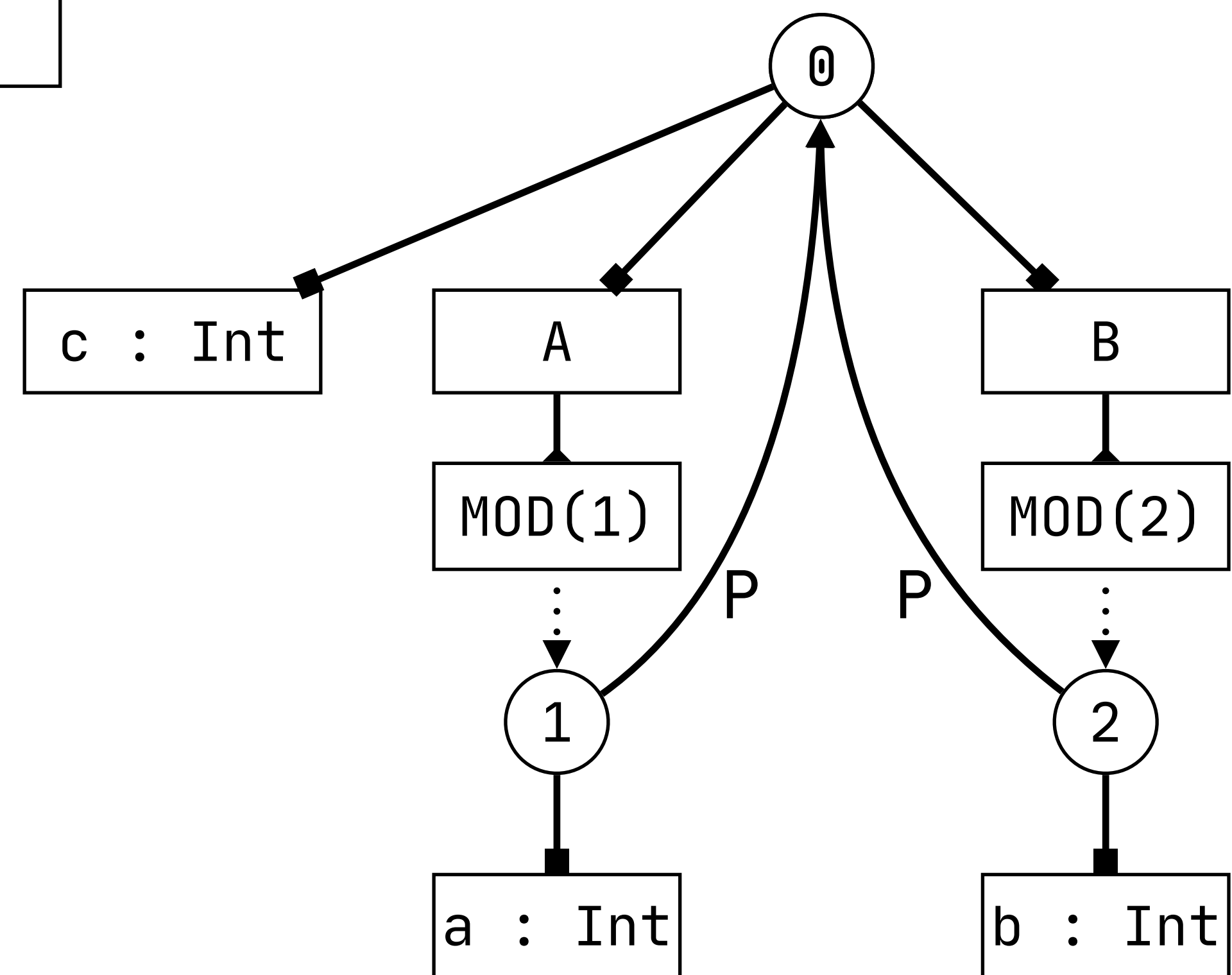
## rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

lexical scope

scope as type

```
def c = 0
module A {
  import B
  def a = b + c
}
module B {
  def b = 2
}
```



# Resolving Import

## signature

### constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

scope as type

```
def c = 0
module A {
  import B
  def a = b + c
}
module B {
  def b = 2
}
```

## rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

lexical scope

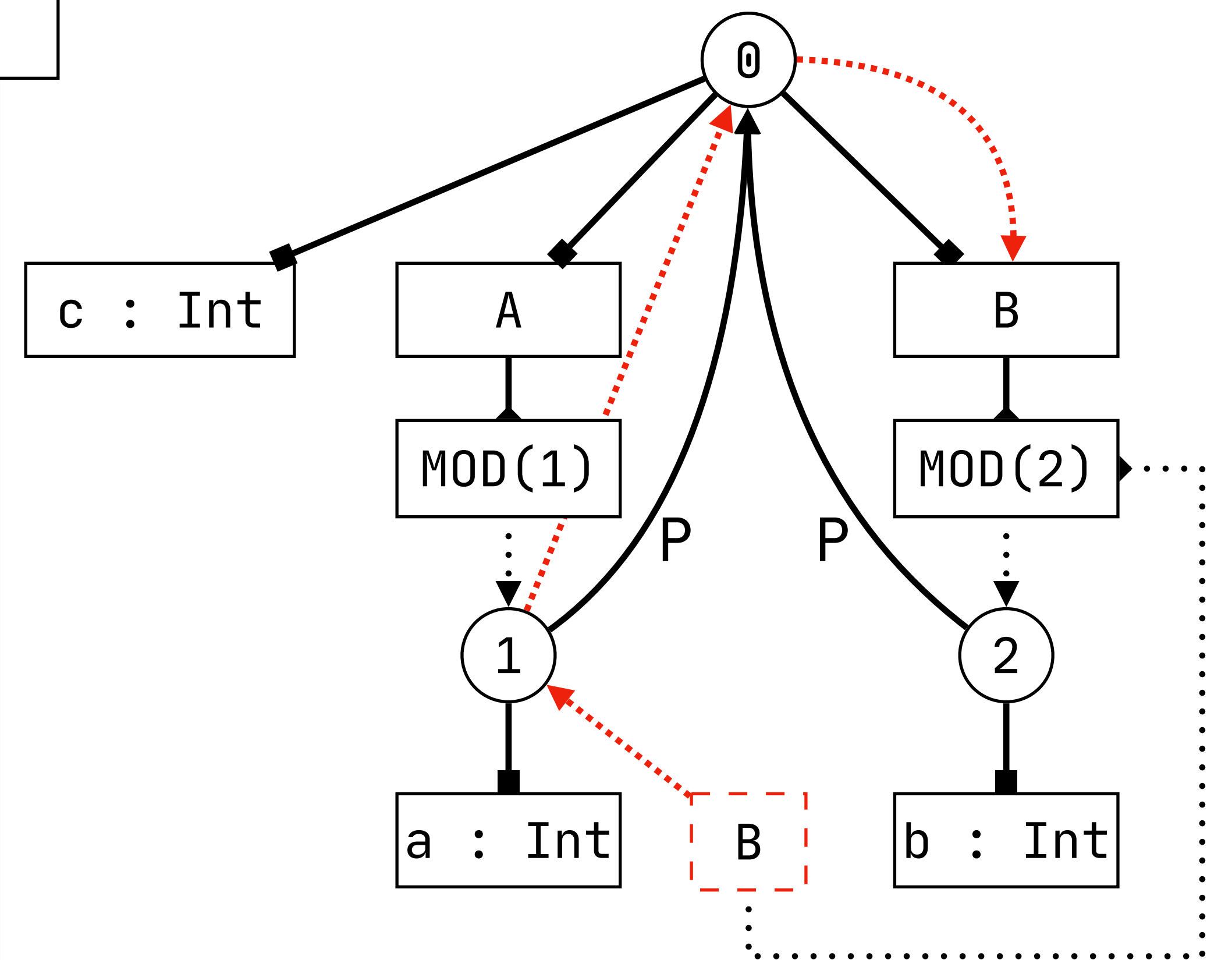
scope as type

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

resolve import

```
resolveMod(s, x) = ps :-
  query mod
  filter P* and { x' :- x' = x }
  min $ < P and true
  in s ↦ ps.
```

resolve  
module name



# Import Edge

## signature

### constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

scope as type

```
def c = 0
module A {
  import B
  def a = b + c
}
module B {
  def b = 2
}
```

## rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

lexical scope

scope as type

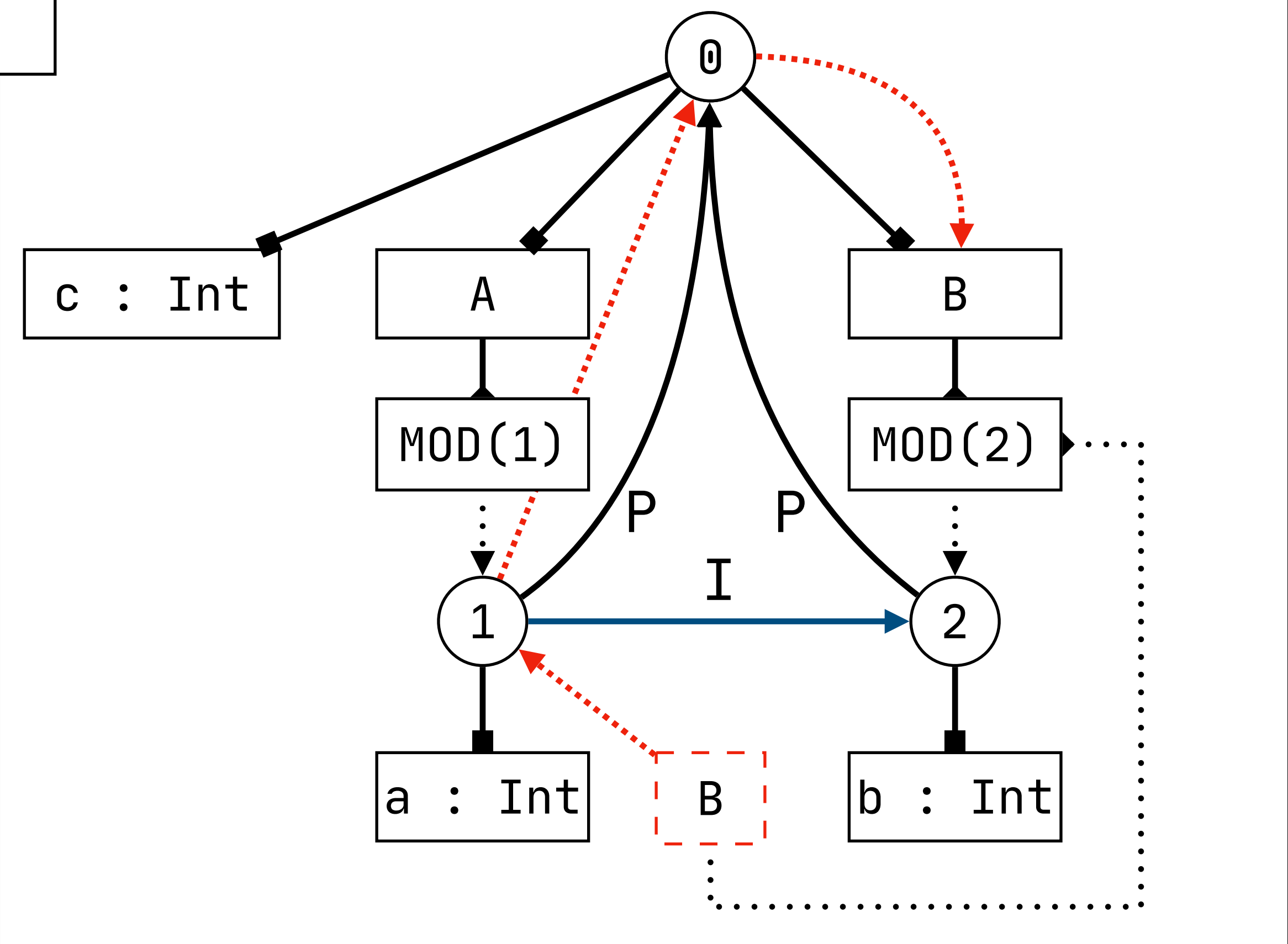
```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

resolve import

import edge

```
resolveMod(s, x) = ps :-
  query mod
  filter P* and { x' :- x' = x }
  min $ < P and true
  in s ↦ ps.
```

resolve  
module name



# Resolving Variable through Import Edge

## signature

### constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

scope as type

```
def c = 0
module A {
  import B
  def a = b + c
}
module B {
  def b = 2
}
```

## rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

lexical scope

scope as type

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

resolve import

import edge

```
resolveVar(s, x) = ps :-
```

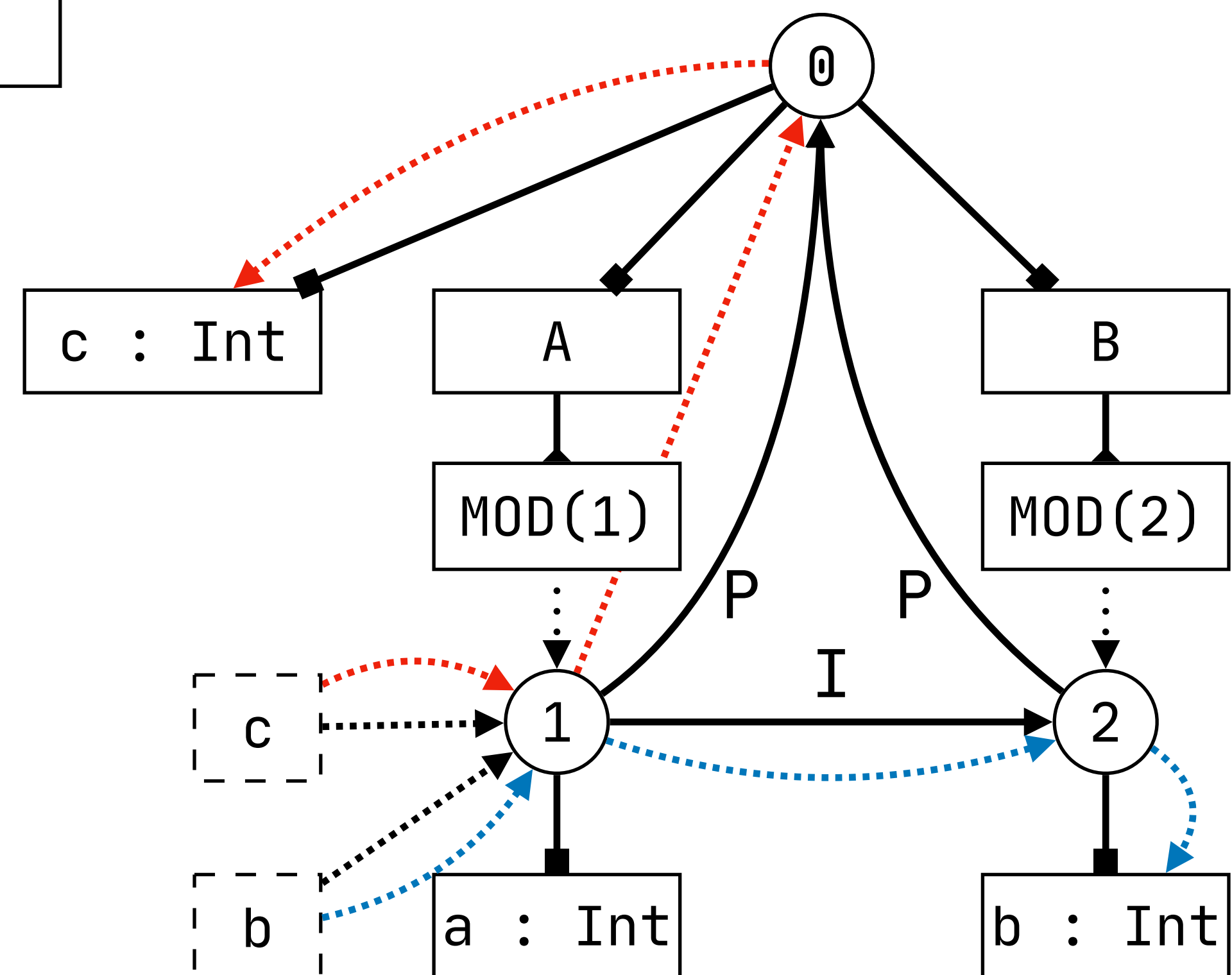
```
  query var
```

```
    filter P* I* and { x' :- x' = x }
```

```
    min $ < P, $ < I, I < P and true
```

```
    in s ↦ ps.
```

resolve variable through  
import edges





# Import Shadows Parent (Lexical Context)

## signature

### constructors

MOD : scope  $\rightarrow$  TYPE  
Module : ID \* list(Decl)  $\rightarrow$  Decl  
Import : ID  $\rightarrow$  Decl

scope as type

```
def b = 0
module A {
  import B
  def a = b
}
module B {
  def b = 2
}
```

## rules

declOk(s, Module(m, decls)) :- {s\_mod}  
new s\_mod, s\_mod  $\rightarrow$  s,  
declareMod(s, m, MOD(s\_mod)),  
declsOk(s\_mod, decls).

lexical scope

scope as type

declOk(s, Import(p)) :- {s\_mod s\_end}  
typeOfModRef(s, p) = MOD(s\_mod),  
s  $\rightarrow$  s\_mod.

resolve import

import edge

resolveVar(s, x) = ps :-

query var

filter P\* I\* and { x' :- x' = x }

import after parent

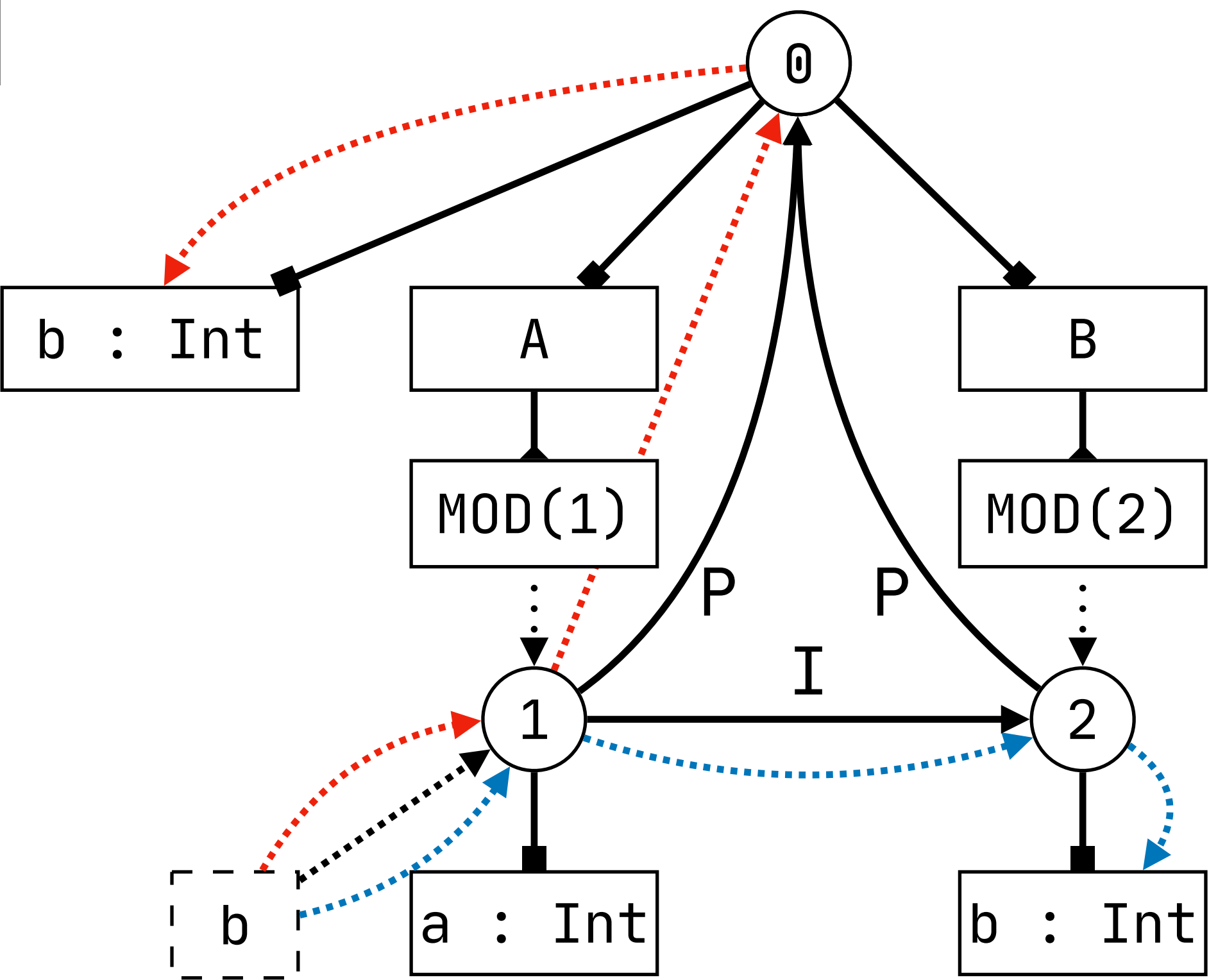
min \$ < P, \$ < I, I < P and true

prefer import

in s  $\rightarrow$  ps.

resolve variable through  
import edges

prefer blue path over red path



# Mutually Recursive Imports

## signature

### constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

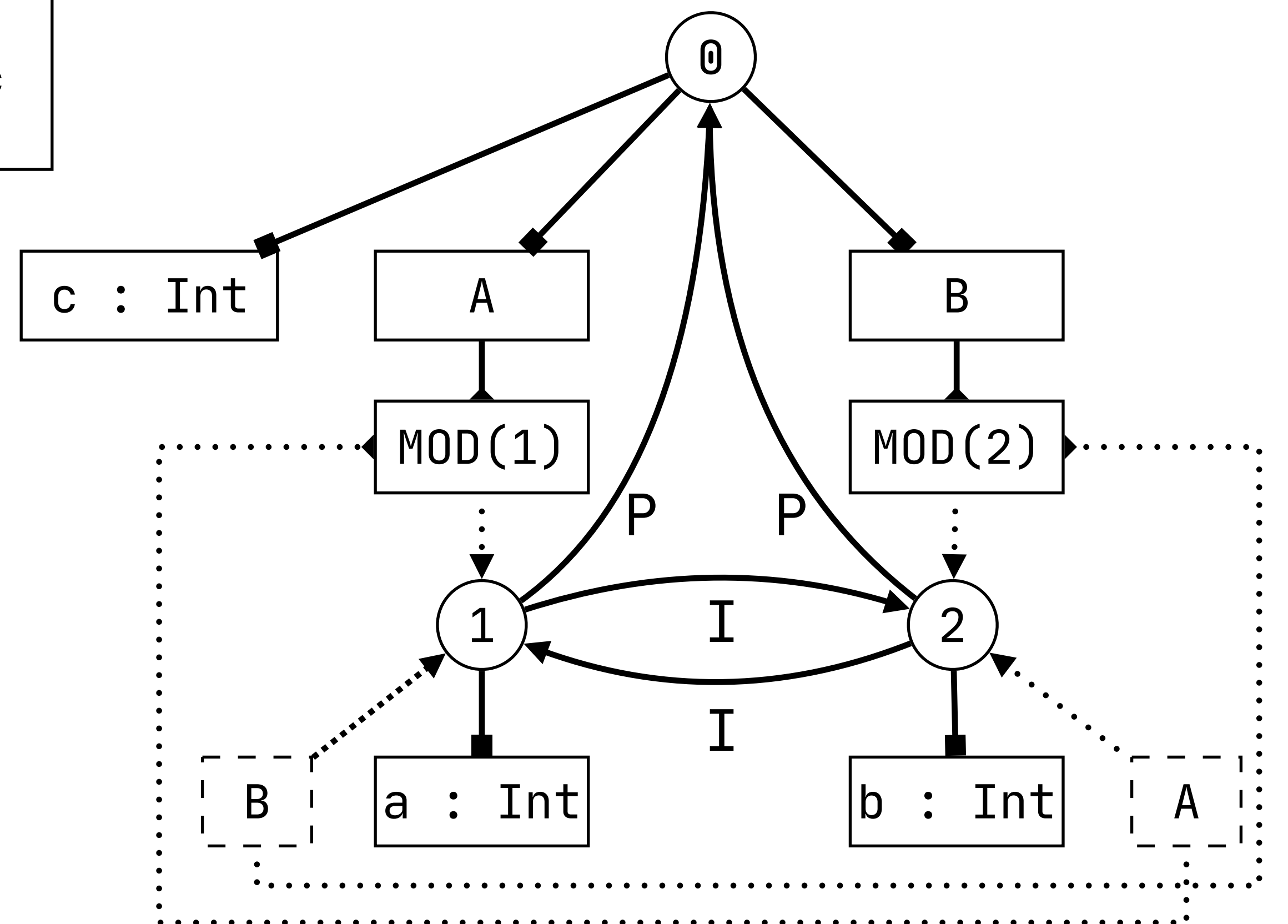
## rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
def c = 0
module A {
  import B
  def a = b + c
}
module B {
  import A
  def b = 2
  def d = a + c
}
```

```
resolveVar(s, x) = ps :-
  query var
  filter P* I* and { x' :- x' = x }
  min $ < P, $ < I, I < P and true
  in s ↦ ps.
```



# Mutually Recursive Imports

signature

## constructors

MOD : scope  $\rightarrow$  TYPE

```
Module : ID * list(Decl) → Decl
```

Import :  $ID \rightarrow Decl$

scope as type

```
def c = 0
```

```
module A {
```

```
import B
```

```
def a = b + c
```

}

```
module B {
```

```
import A
```

```
def b = 2
```

```
def d = a + c
```

}

## rules

```
dec10k(s, Module(m, dec1s)) :- {s_mod}
```

**new** s\_mod, s\_mod  $-P \rightarrow$  s,

```
declareMod(s, m, MOD(s_mod)),
```

```
    dec1s0k(s_mod, dec1s).
```

scope as type

```
declOk(s, Import(p)) :- {s_mod s_end}
```

```
typeOfModRef(s, p) == MOD(s_mod),
```

$$s - I \rightarrow s_{\text{mod.}}$$

```
resolve import
```

```
import edge
```

```
resolveVar(s, x) = ps :-
```

query var

**filter**  $P^* \ I^*$  and  $\{ x' \ :- \ x' = x \}$

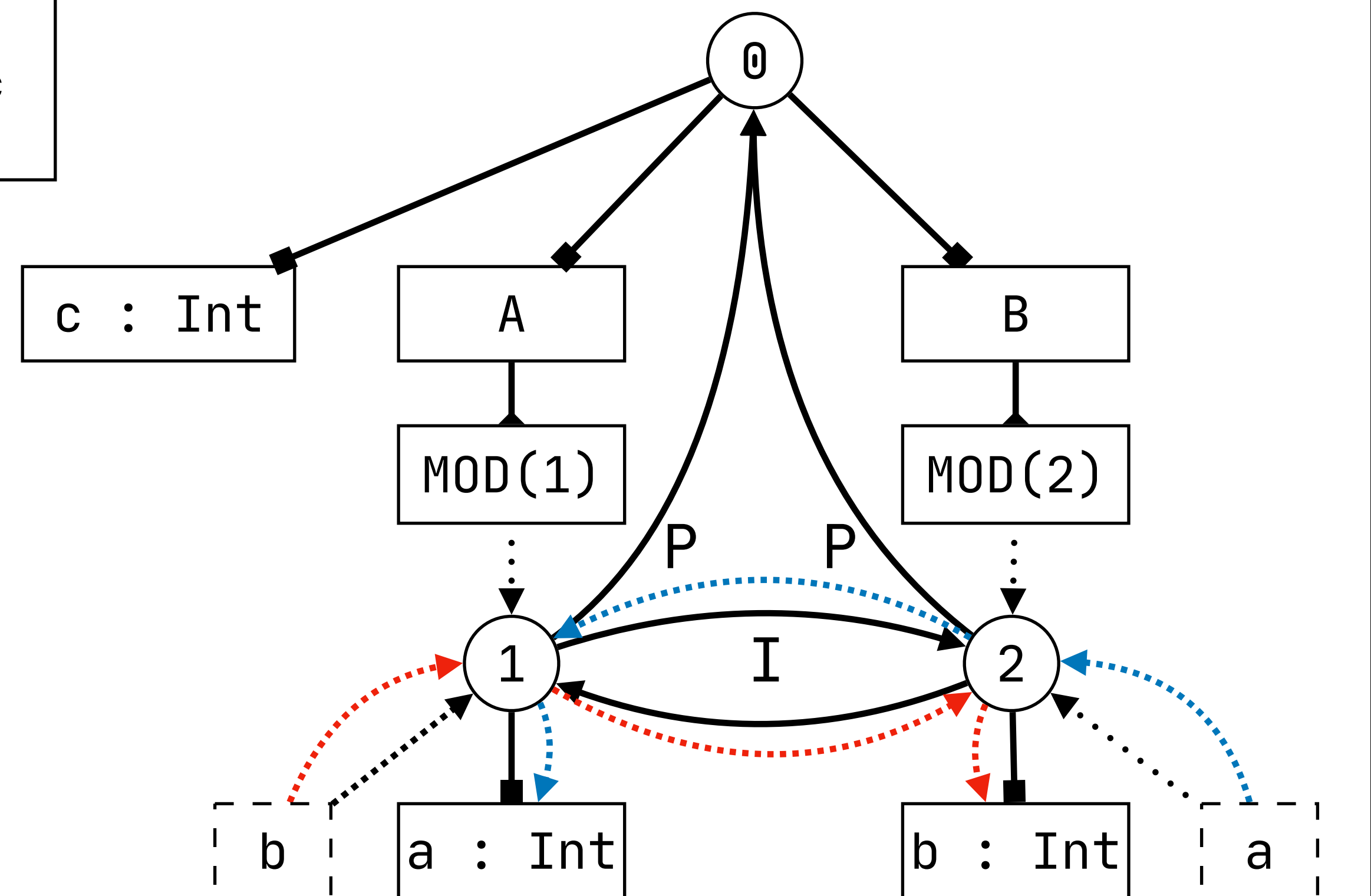
$\min \$ < P, \$ < I, I < P$  and true

in  $s \mapsto ps$ .

import after parent

prefer import

resolve variable through  
import edges



# Transitive Import

## signature

### constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

## rules

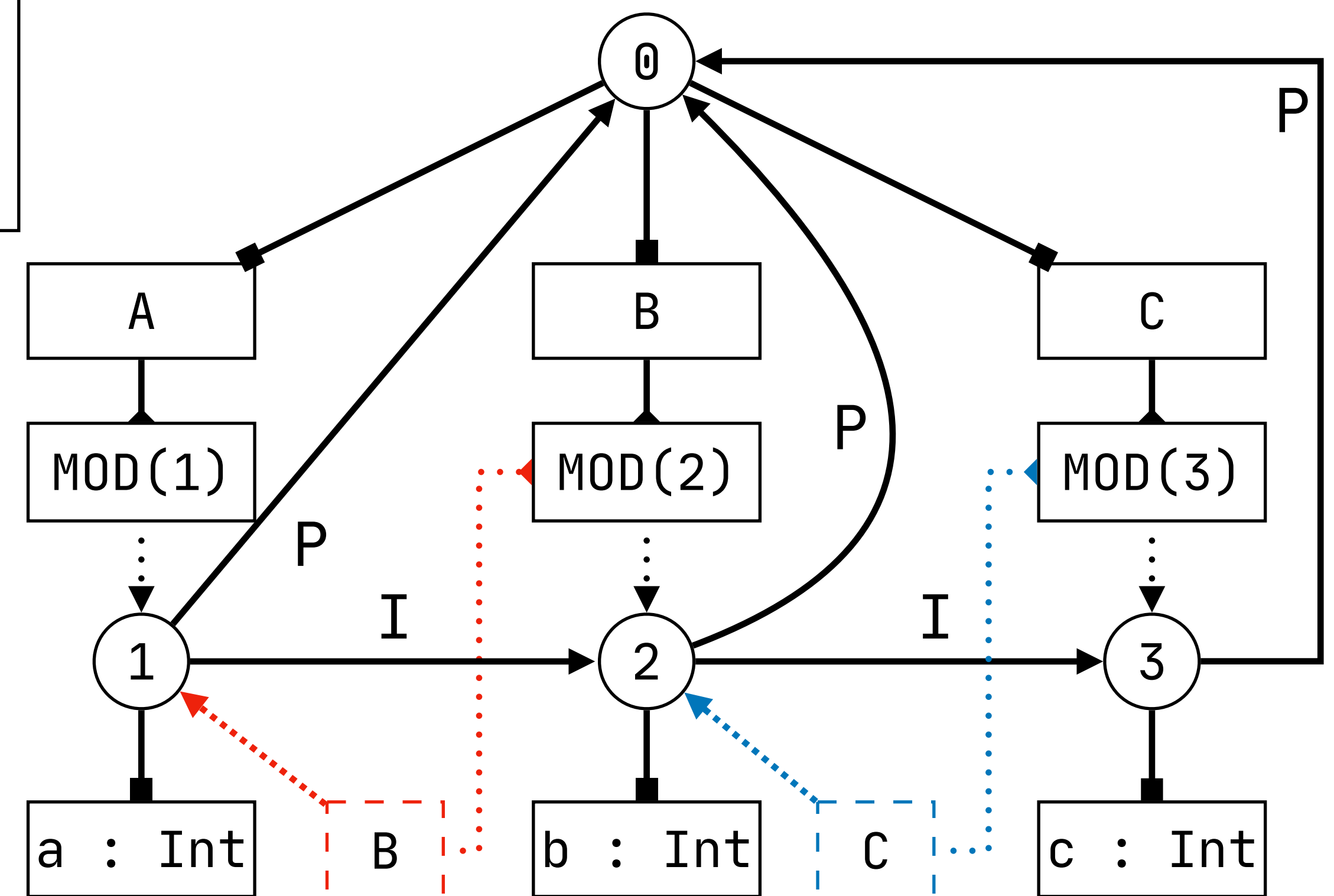
```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
resolveVar(s, x) = ps :-
```

```
  query var
    filter P* I* and { x' :- x' = x }
    min $ < P, $ < I, I < P and true
    in s ↦ ps.
```

```
module A {
  import B
  def a = b + c
}
module B {
  import C
  def b = c + 2
}
module C {
  def c = 1
}
```



# Transitive Import

## signature

### constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

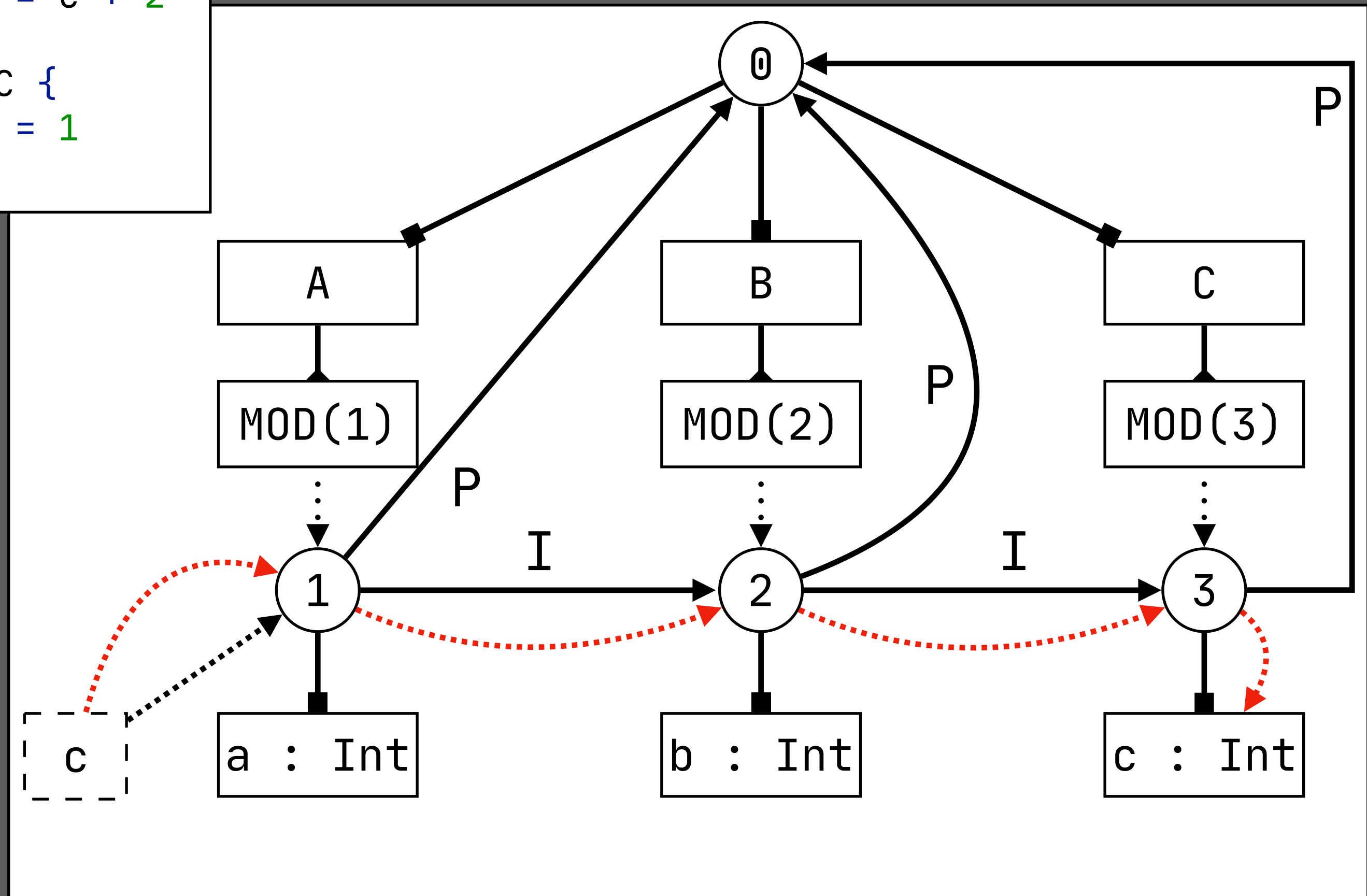
## rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
resolveVar(s, x) = ps :-
  query var
    filter P* I* and { x' :- x' = x }
    min $ < P, $ < I, I < P and true
    in s ↦ ps.
```

```
module A {
  import B
  def a = b + c
}
module B {
  import C
  def b = c + 2
}
module C {
  def c = 1
}
```



# Changing Query Outcomes

(is not allowed)

# Nested Modules

signature

## constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

## rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

```

declOk(s, Import(p)) :- {s_mod s_end}
    typeOfModRef(s, p) = MOD(s_mod),
    s -I→ s_mod.

```

```
resolveVar(s, x) = ps :-
```

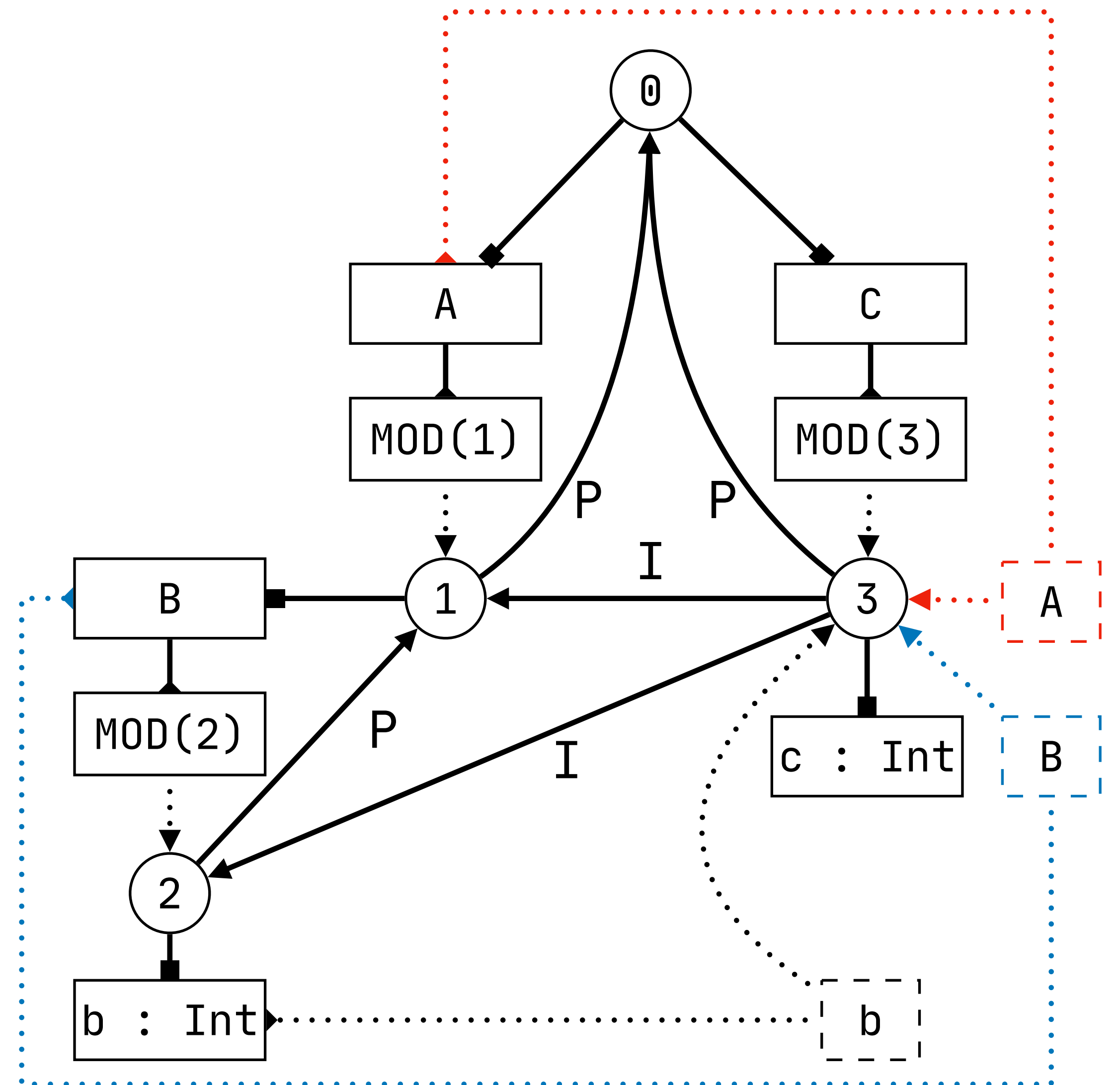
```

query var
  filter P* I* and { x' :- x' = x }
  min $ < P, $ < I, I < P and true
  in s  $\mapsto$  ps.

```

```
module A {
  module B {
    def b = 1
  }
}
```

```
module C {
    import A
    import B
    def c = b
}
```





# Changing Result of Query

```
signature
constructors
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

## rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

```

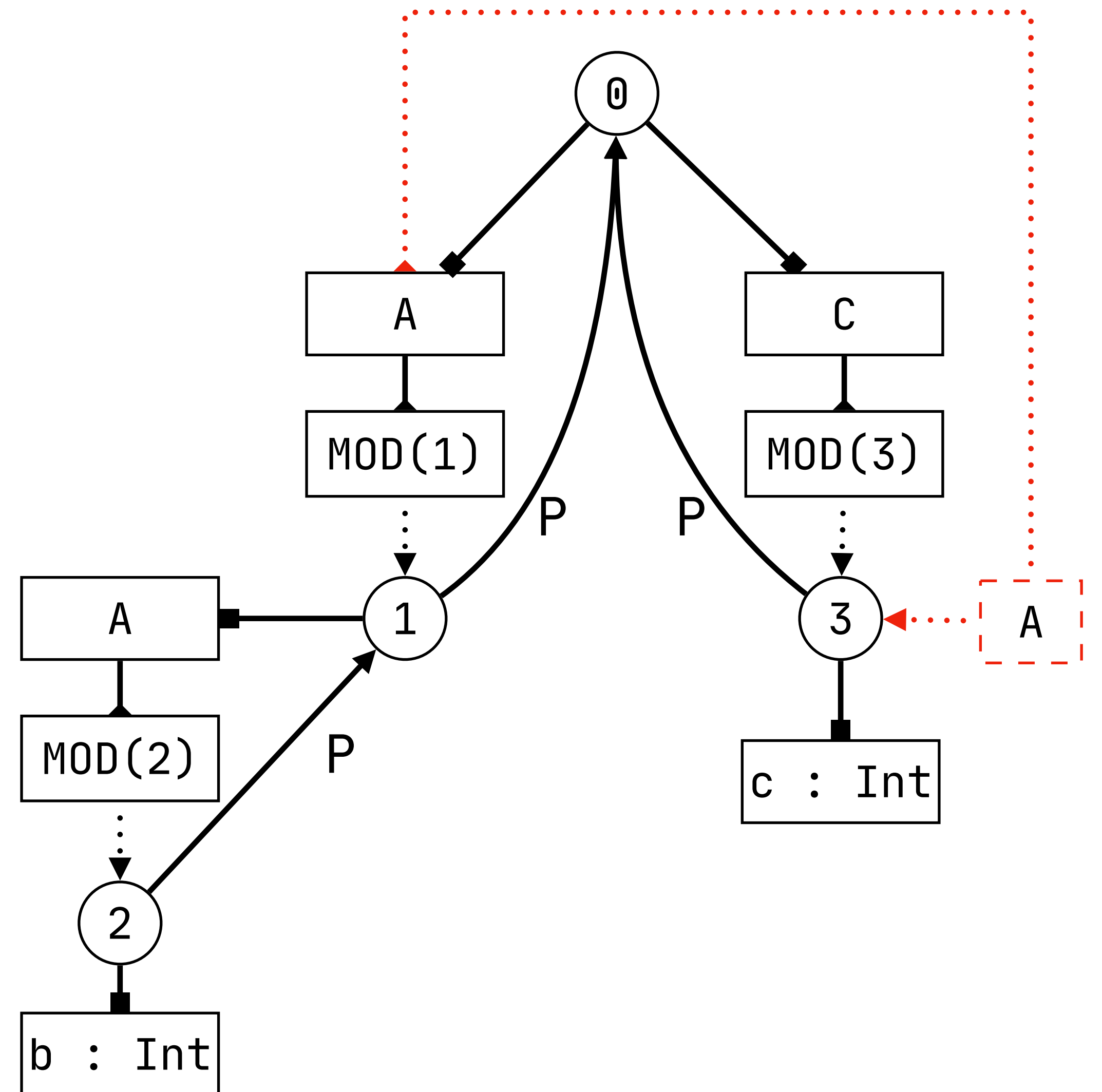
declOk(s, Import(p)) :- {s_mod s_end}
    typeOfModRef(s, p) = MOD(s_mod),
    s -I→ s_mod.

```

```
resolveVar(s, x) = ps :-  
  query var  
    filter P* I* and { x' :- x' = x }  
    min $ < P, $ < I, I < P and true  
    in s  $\mapsto$  ps.
```

```
module A {
  module A {
    def b = 1
  }
}
```

```
module C {
    import A
    import A
    def c = b
}
```





# Changing Result of Query

## signature constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

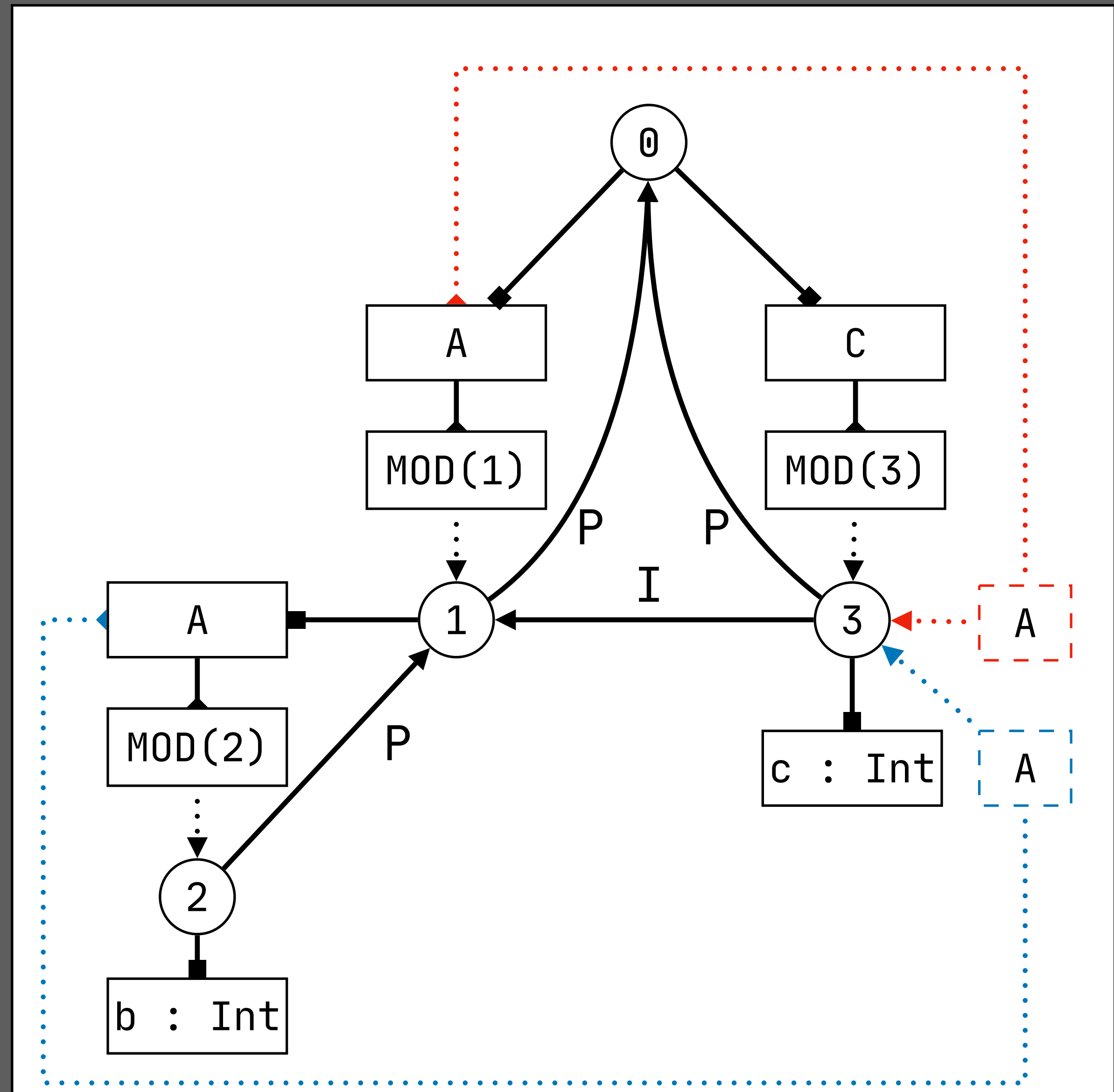
## rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
module A {
  module A {
    def b = 1
  }
}
```

```
module C {
  import A
  import A
  def c = b
}
```



```
resolveVar(s, x) = ps :-
  query var
  filter P* I* and { x' :- x' = x }
  min $ < P, $ < I, I < P and true
  in s ↦ ps.
```

# Changing Result of Query

## signature constructors

```
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
```

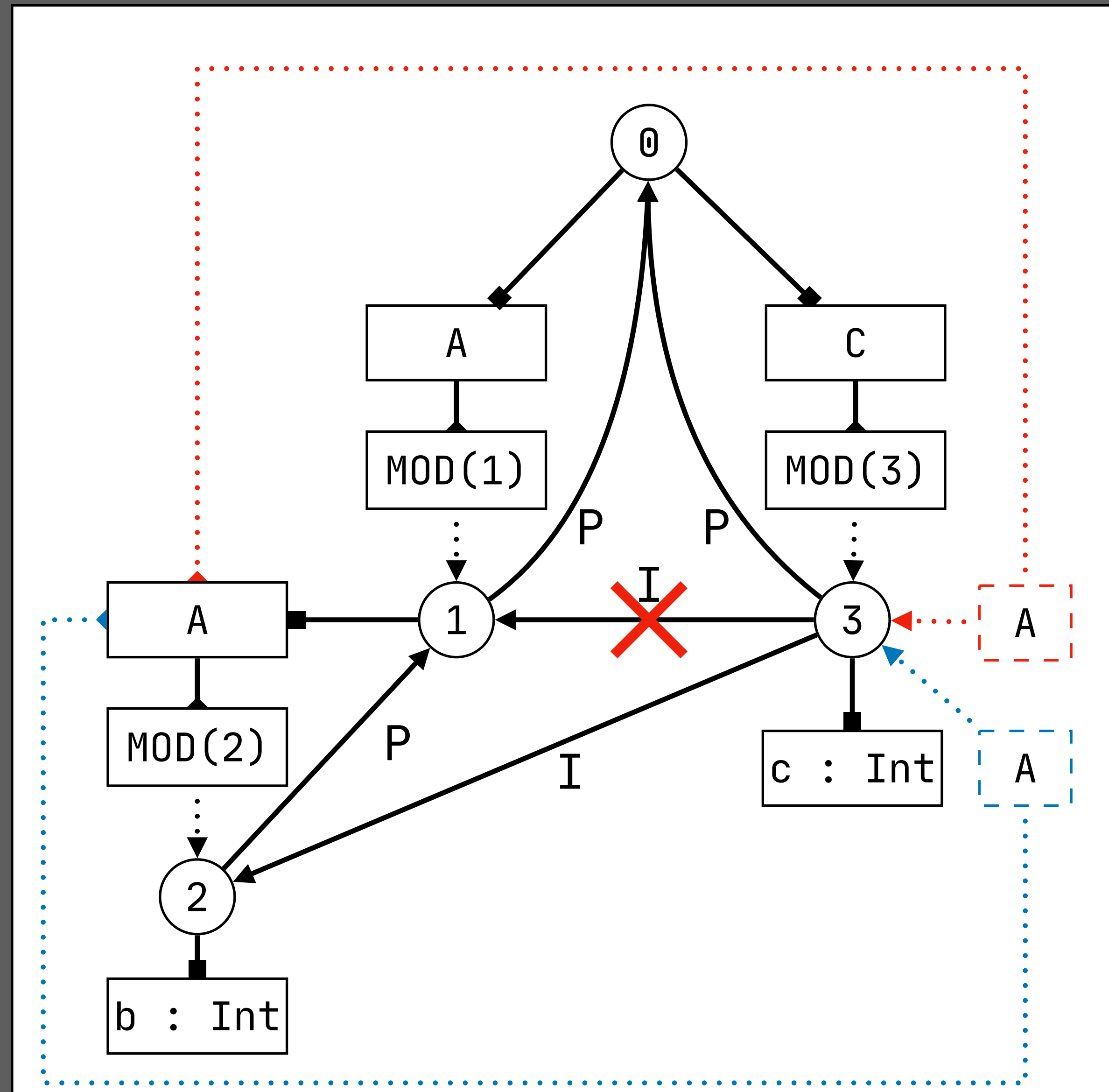
## rules

```
declOk(s, Module(m, decls)) :- {s_mod}
  new s_mod, s_mod -P→ s,
  declareMod(s, m, MOD(s_mod)),
  declsOk(s_mod, decls).
```

```
declOk(s, Import(p)) :- {s_mod s_end}
  typeOfModRef(s, p) = MOD(s_mod),
  s -I→ s_mod.
```

```
module A {
  module A {
    def b = 1
  }
}
```

```
module C {
  import A
  import A
  def c = b
}
```



```
resolveVar(s, x) = ps :-
  query var
  filter P* I* and { x' :- x' = x }
  min $ < P, $ < P, I < P and true
  in s ↦ ps.
```

# Alternative Encoding: Scoped Imports

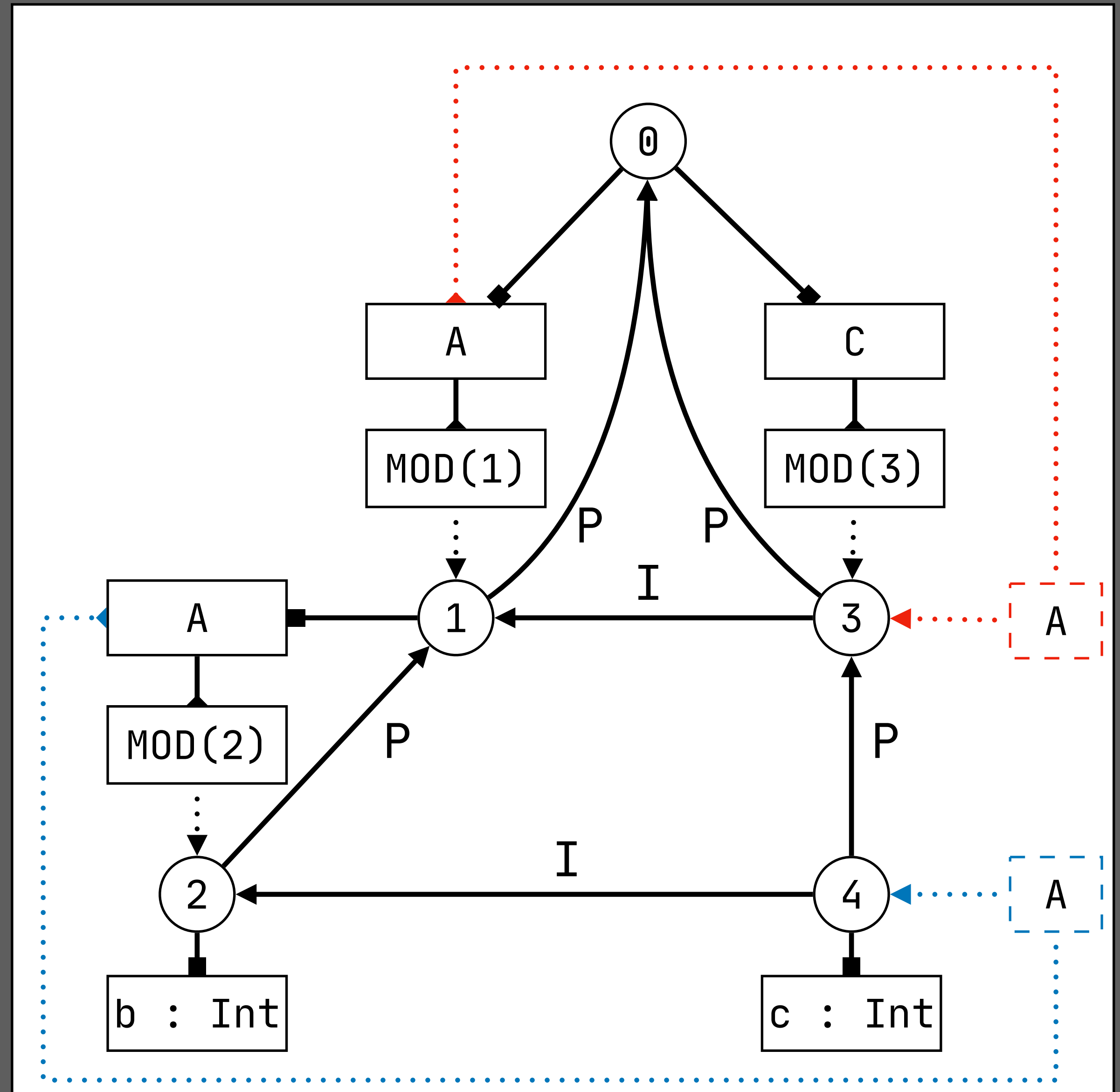
```
signature
  sorts DecGroups
  constructors
    MOD      : scope → TYPE
    Module   : ID * DecGroups → Decl
    Import   : ID → Decl
    ModRef   : ID * ID → Exp

    Decs     : list(Decl) → DecGroups
    Seq      : list(Decl) * DecGroups
              → DecGroups
```

```
module A {
    module A {
        def b = 1
    }
}

module C {
    import A;
    import A
    def c = b
}
```

```
resolveVar(s, x) = ps :-  
  query var  
    filter P+ I* and { x' :- x' = x }  
    min $ < P, $ < I, I < P and true  
    in s  $\mapsto$  ps.
```



# Alternative Encoding: Scoped Imports — M Edge Label

## signature

sorts DecGroups

## constructors

MOD : scope  $\rightarrow$  TYPE

Module : ID \* DecGroups  $\rightarrow$  Decl

Import : ID  $\rightarrow$  Decl

ModRef : ID \* ID  $\rightarrow$  Exp

Decs : list(Decl)  $\rightarrow$  DecGroups

Seq : list(Decl) \* DecGroups  
 $\rightarrow$  DecGroups

```
module A {  
  module A {  
    def b = 1  
  }  
}
```

```
module C {  
  import A;  
  import A  
  def c = b  
}
```

resolveVar(s, x) = ps :-

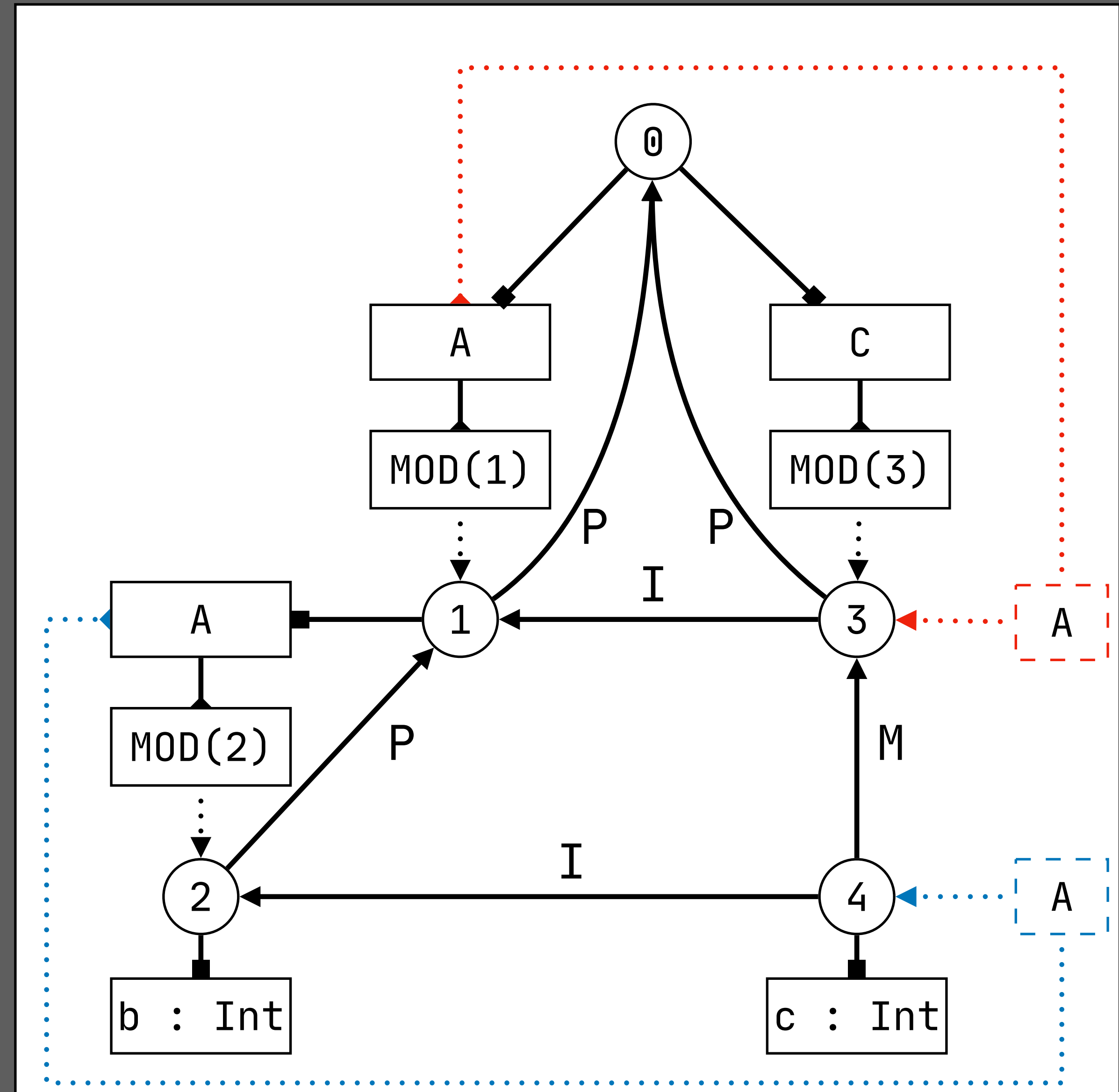
query var

filter (P | M) P\* (I | M)\*

and { x' :- x' = x }

min \$ < P, \$ < I, I < P and true

in s  $\mapsto$  ps.



# Permission to Extend

# Permission to Extend

```
signature
constructors
MOD      : scope → TYPE
Module   : ID * list(Decl) → Decl
Import   : ID → Decl
ExtendRemote : ID * ID * Exp → Decl
```

```
module A {  
  def a = b  
}  
  
module B {  
  def A.b := 2  
}
```

```

    extend remote: def M.x := e
  extend module M with declaration of x

```

```
rules // extend remote
```

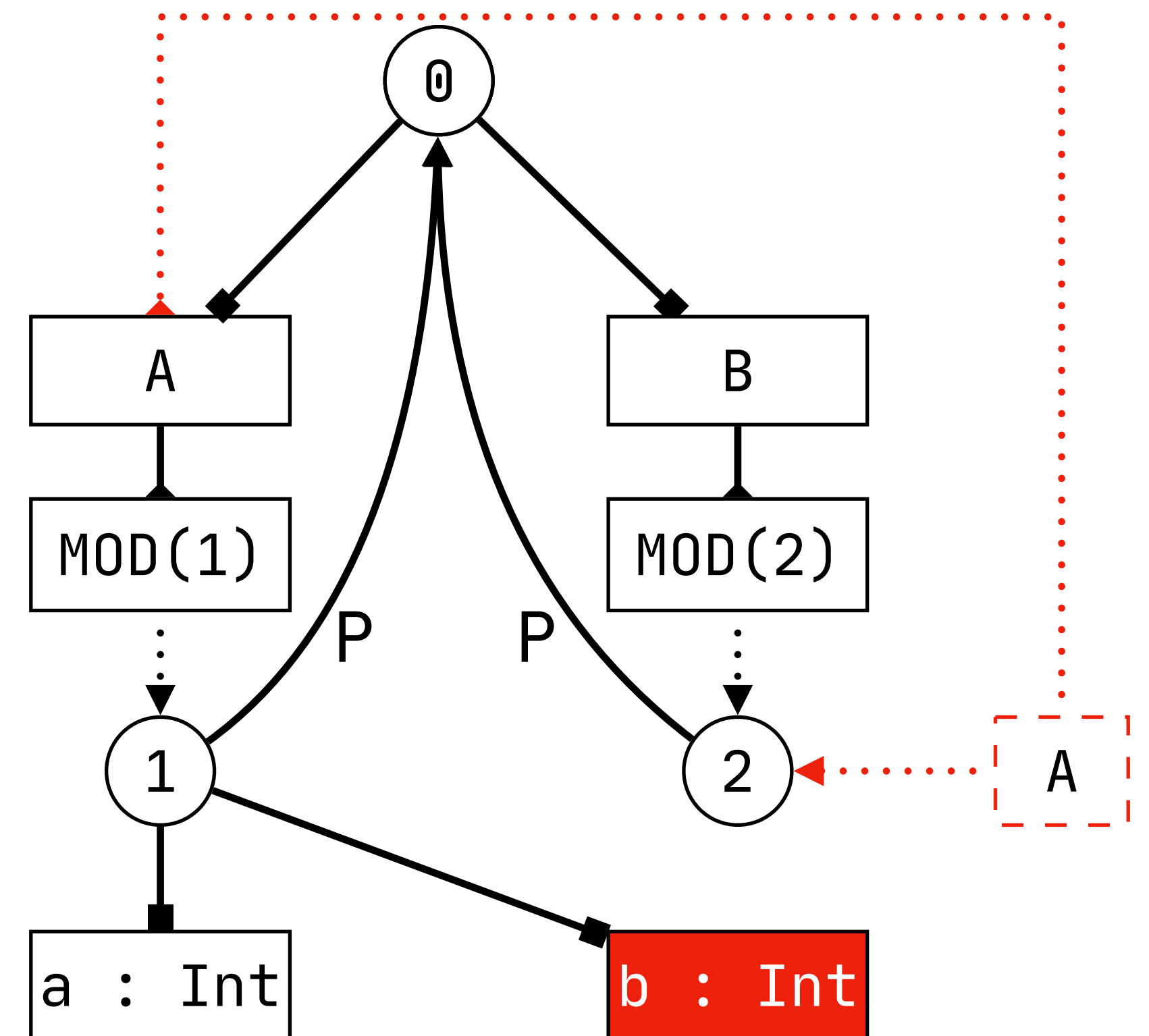
```
declOk(s, ExtendRemote(m, x, e)) :- {s_mod T}
    typeOfModRef(s, m) = MOD(s_mod),
    typeOfExp(s, e) = T,
    declareVar(s_mod, x, T).
// no permission to extend
```

Add  
obta

## Add declaration to scope obtained through a query

## This is not allowed in Statix

A predicate can only extend scopes over which it has ownership, i.e. that it creates or gets passed down as an argument



# Type-Dependent Name Resolution / Records

# Records

## signature

### constructors

```
REC      : scope → TYPE
Record   : ID * list(FDecl) → Decl
FDecl    : ID * Type → FDecl
New      : ID * list(FBind) → Exp
FBind    : ID * Exp → FBind
Proj     : Exp * ID → Exp
```

```
record Point { x : Int, y : Int }
```

```
def p = Point{ x = 1, y = 2 }
```

```
> p.y
```



# Record Type: Scope as Type

## signature

### constructors

```
REC      : scope → TYPE
Record   : ID * list(FDecl) → Decl
FDecl    : ID * Type → FDecl
New      : ID * list(FBind) → Exp
FBind    : ID * Exp → FBind
Proj     : Exp * ID → Exp
```

scope as type

## rules // record type

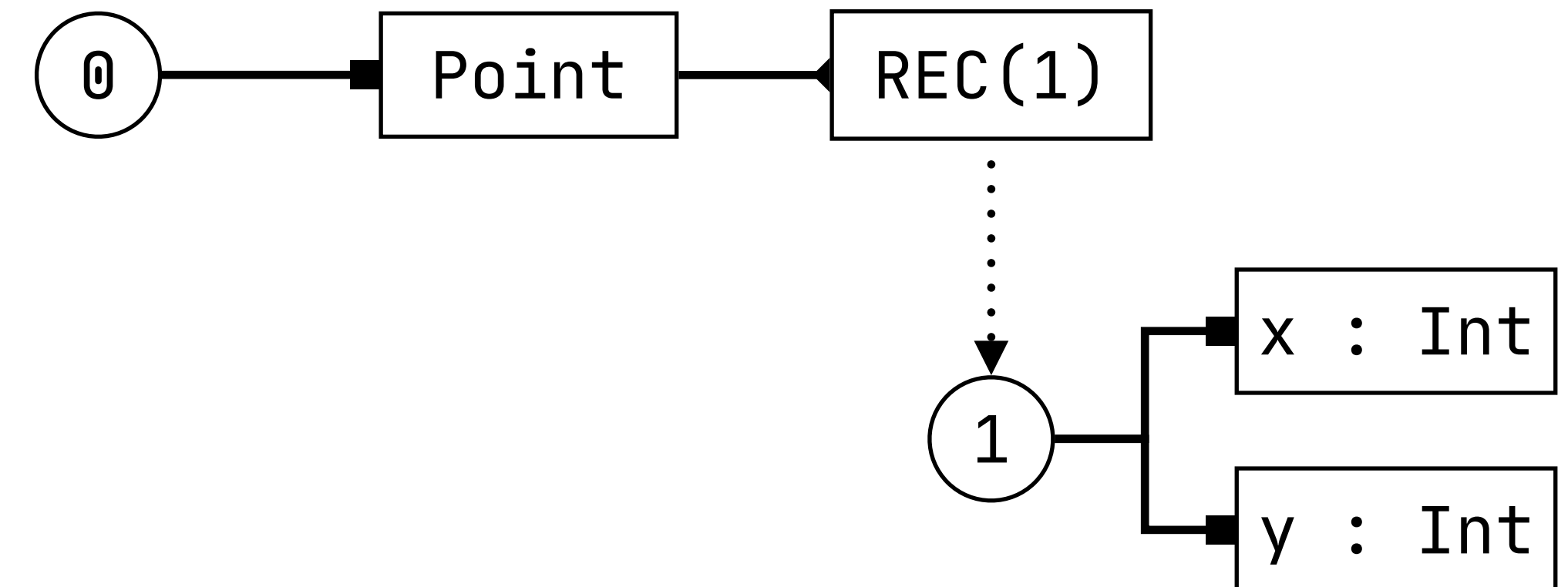
```
declOk(s, Record(x, fdecls)) :- {s_rec}
  new s_rec,
  fdeclsOk(s_rec, s, fdecls),
  declareType(s, x, REC(s_rec)).
```

```
fdeclOk(s_bnd, s_ctx, FDecl(x, t)) :- {T}
  typeOfType(s_ctx, t) = T,
  declareVar(s_bnd, x, T).
```

```
record Point { x : Int, y : Int }
```

```
def p = Point{ x = 1, y = 2 }
```

```
> p.y
```



# Record Construction & Initialization

## signature

### constructors

```
REC      : scope → TYPE
Record   : ID * list(FDecl) → Decl
FDecl    : ID * Type → FDecl
New      : ID * list(FBind) → Exp
FBind    : ID * Exp → FBind
Proj     : Exp * ID → Exp
```

## rules // record construction

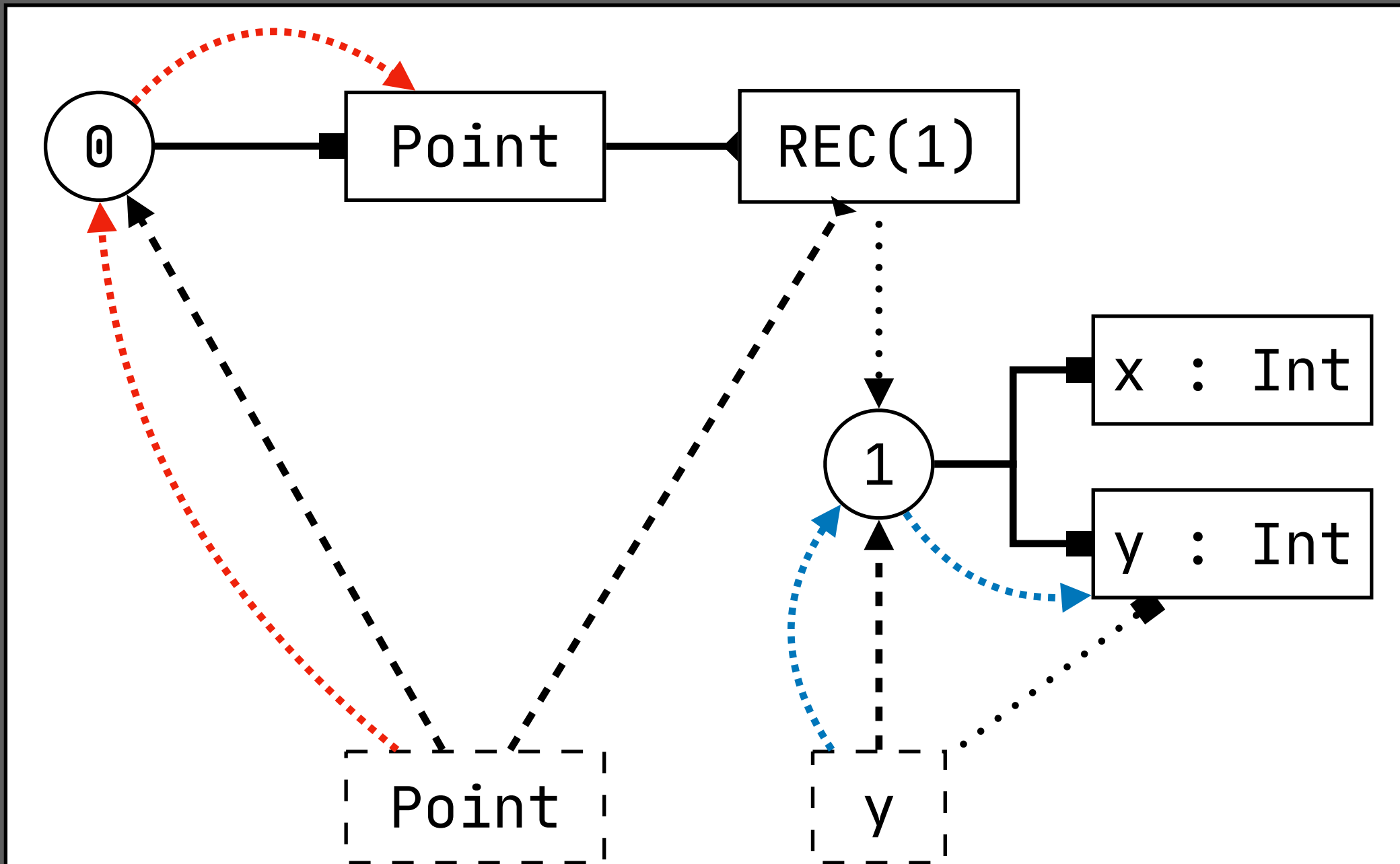
```
typeOfExp(s, New(x, fbinds)) = REC(s_rec) :- {p d}
  typeOfTypeRef(s, x) = REC(s_rec),
  fbindsOk(s, REC(s_rec), fbinds).
```

```
fbindOk(s, T_rec, FBind(x, e)) :- {T1 T2}
  typeOfExp(s, e) = T1,
  proj(T_rec, x) = T2,
  subtype(e, T1, T2).
```

```
record Point { x : Int, y : Int }
```

```
def p = Point{ x = 1, y = 2 }
```

```
> p.y
```



# Type-Dependent Name Resolution

## signature

### constructors

```
REC      : scope → TYPE
Record   : ID * list(FDecl) → Decl
FDecl    : ID * Type → FDecl
New      : ID * list(FBind) → Exp
FBind    : ID * Exp → FBind
Proj     : Exp * ID → Exp
```

## rules // record construction

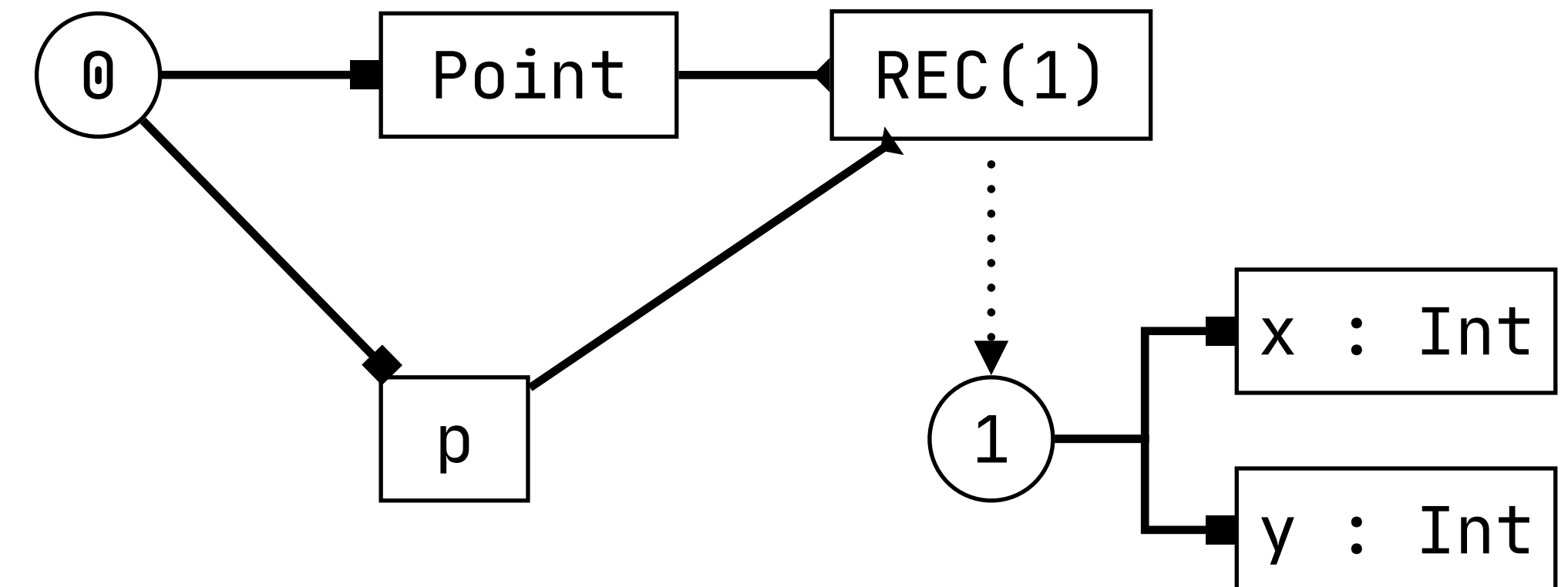
```
typeOfExp(s, New(x, fbinds)) = REC(s_rec) :- {p d}
  typeOfTypeRef(s, x) = REC(s_rec),
  fbindsOk(s, REC(s_rec), fbinds).
```

```
fbindOk(s, T_rec, FBind(x, e)) :- {T1 T2}
  typeOfExp(s, e) = T1,
  proj(T_rec, x) = T2,
  subtype(e, T1, T2).
```

```
record Point { x : Int, y : Int }
```

```
def p = Point{ x = 1, y = 2 }
```

```
> p.y
```



# Type-Dependent Name Resolution

## signature

### constructors

```
REC      : scope → TYPE
Record   : ID * list(FDecl) → Decl
FDecl    : ID * Type → FDecl
New      : ID * list(FBind) → Exp
FBind    : ID * Exp → FBind
Proj     : Exp * ID → Exp
```

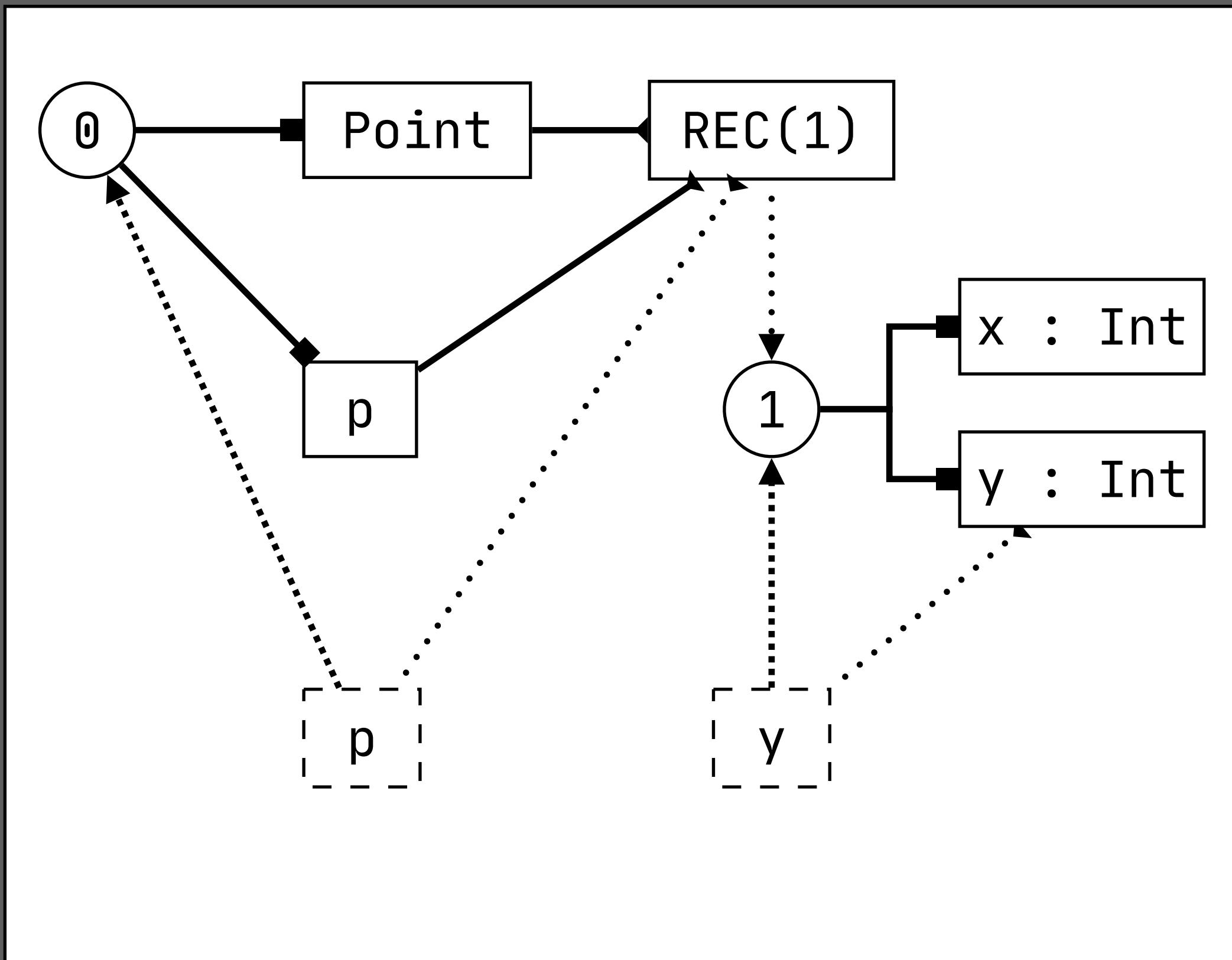
## rules // record projection

```
typeOfExp(s, Proj(e, x)) = T :- {p d s_rec S}
  typeOfExp(s, e) = REC(s_rec),
  typeOfVar(s_rec, x) = T.
```

```
record Point { x : Int, y : Int }
```

```
def p = Point{ x = 1, y = 2 }
```

```
> p.y
```



# With

## signature

### constructors

```
REC      : scope → TYPE
Record   : ID * list(FDecl) → Decl
FDecl    : ID * Type → FDecl
New      : ID * list(FBind) → Exp
FBind    : ID * Exp → FBind
Proj     : Exp * ID → Exp
```

## rules // with record value

```
typeOfExp(s, With(e1, e2)) = T :- {s_with s_rec}
typeOfExp(s, e1) = REC(s_rec),
new s_with, s_with -P→ s, s_with -R→ s_rec,
typeOfExp(s_with, e2) = T.
```

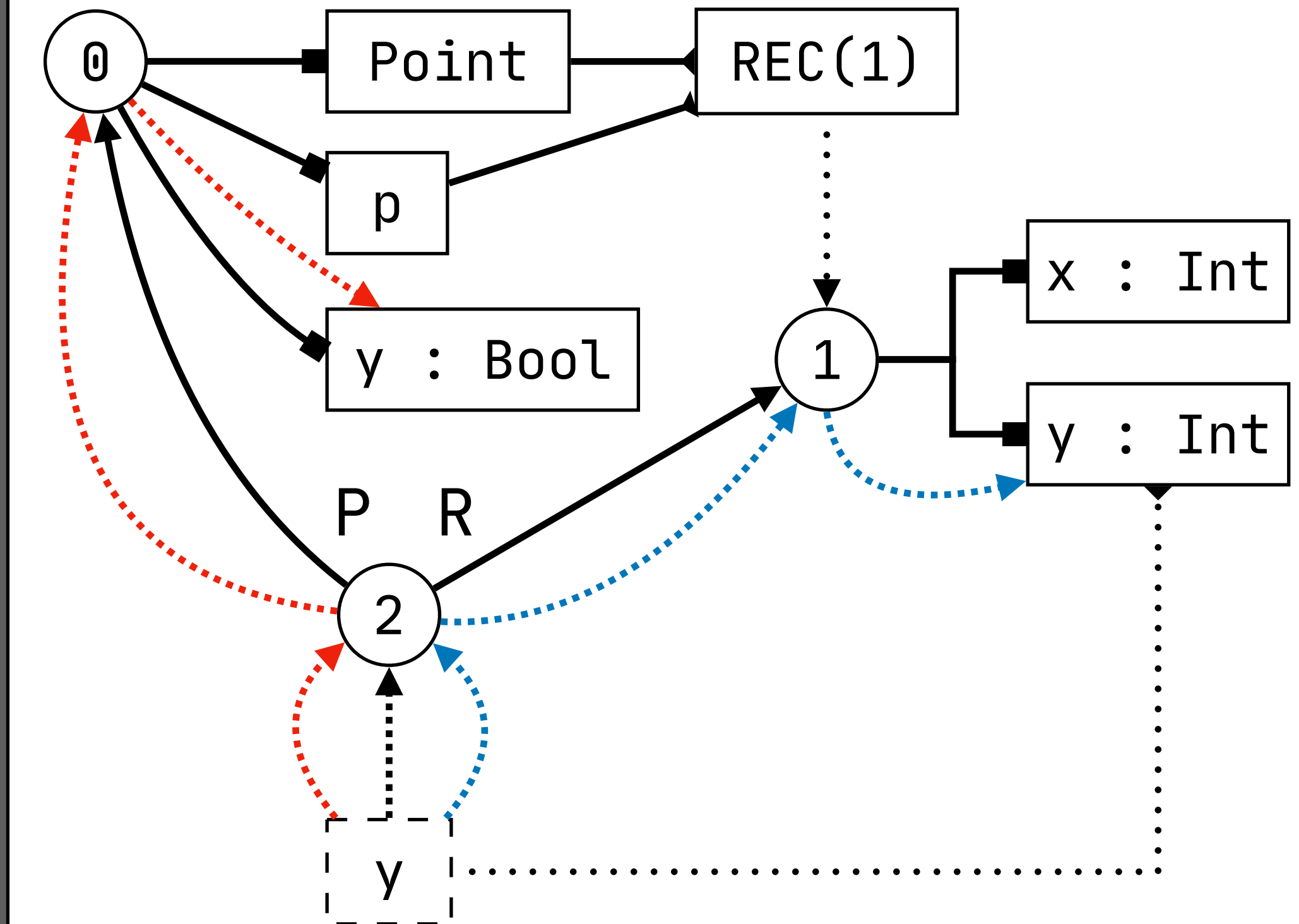
```
resolveVar(s, x) = ps :-
  query var
  filter P* R* and { x' :- x' = x }
  min $ < P, R < P and true
  in s ↦ ps.
```

```
record Point { x : Int, y : Int }
```

```
def p = Point{x = 1, y = 2}
```

```
def y = true
```

```
> with p do y
```



# Scheduling Constraint Resolution

# Scheduling in Type Checkers

## Type checker constructs scope graph

- Module, variable declarations
- Module imports
- Scopes

## Type checker queries scope graph

- Type of variable reference

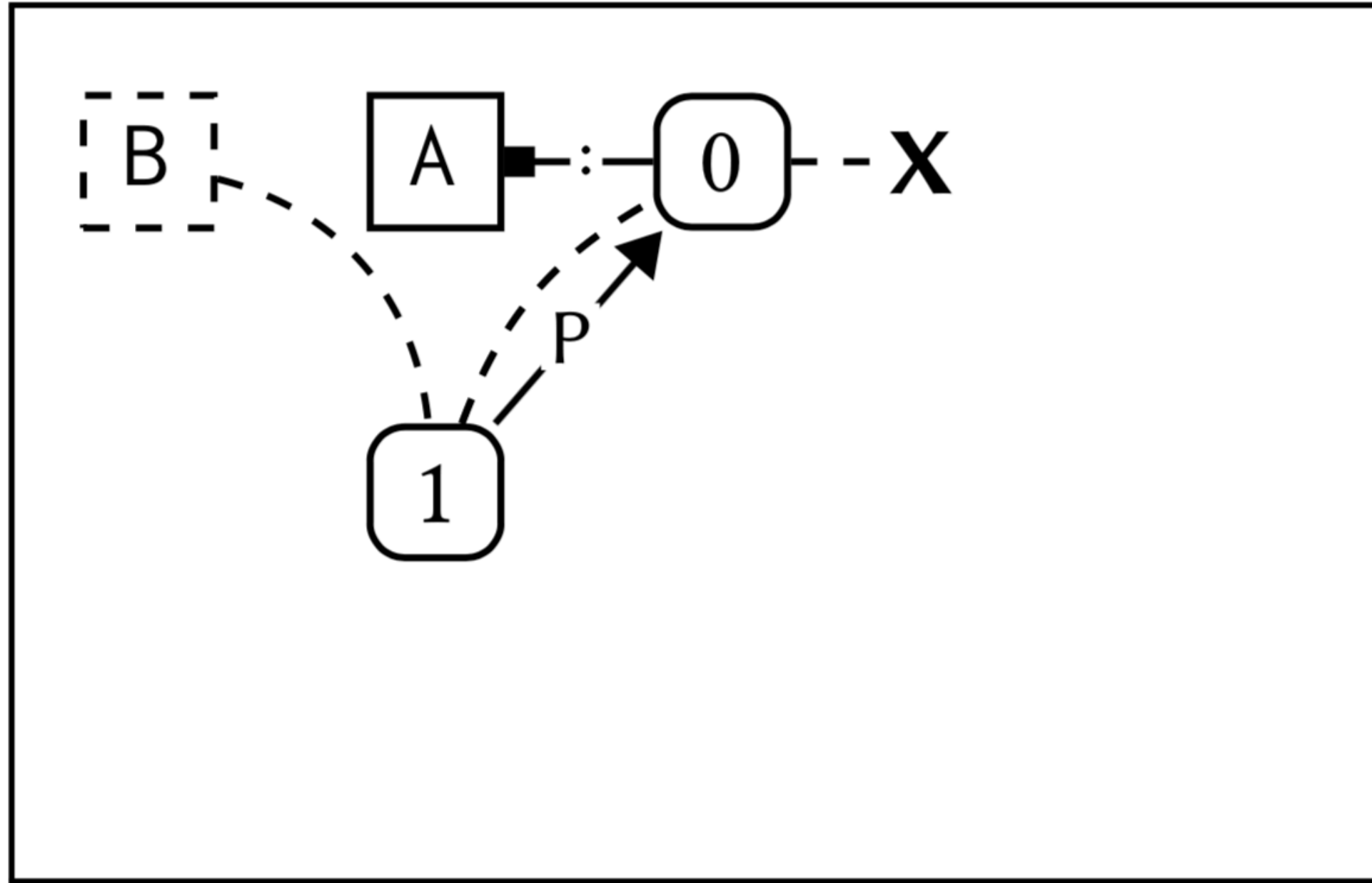
## Scope graph construction depends on queries

- Imports require name resolution of module name

## When is it safe to query the scope graph?

- In what order should type checker perform construction, querying?

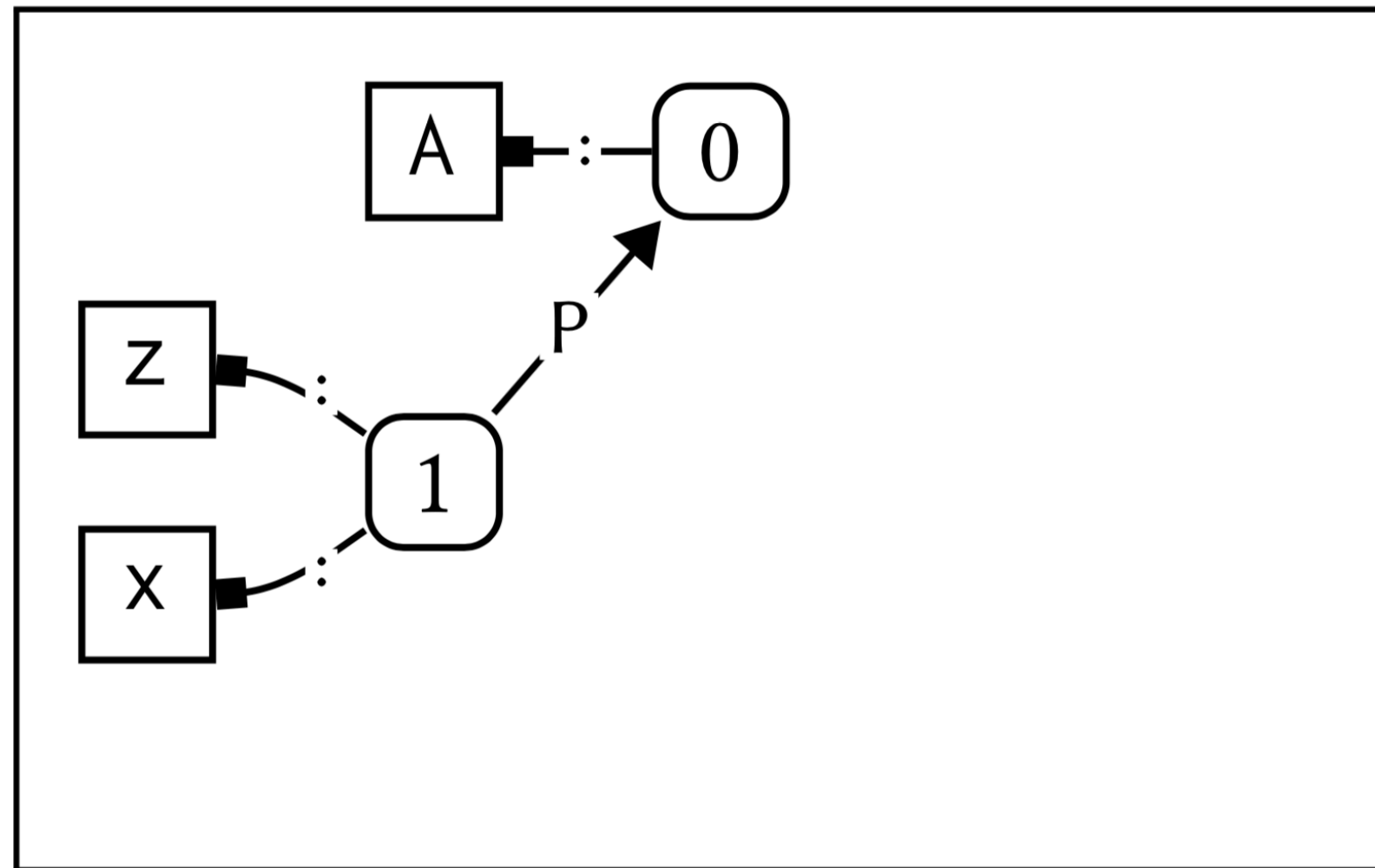
# A Single Stage Type Checker (Fails)



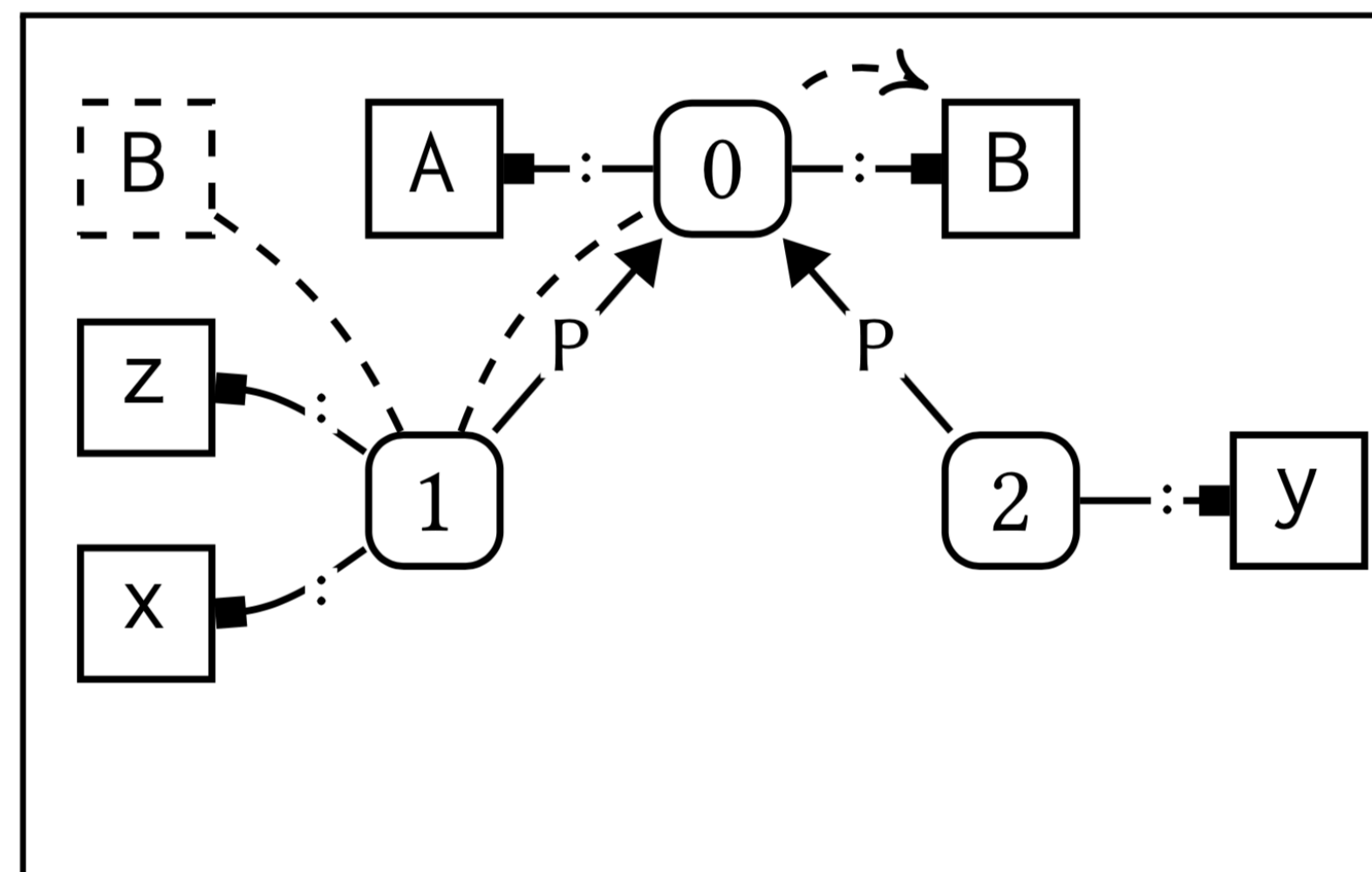
```
module A {  
  import B  
  def z:int = 3  
  def x:int = y + z  
}  
module B {  
  import A  
  def y:int = z * 2  
}
```



# A Two Stage Type Checker: Stage 1 (Build Module Table)



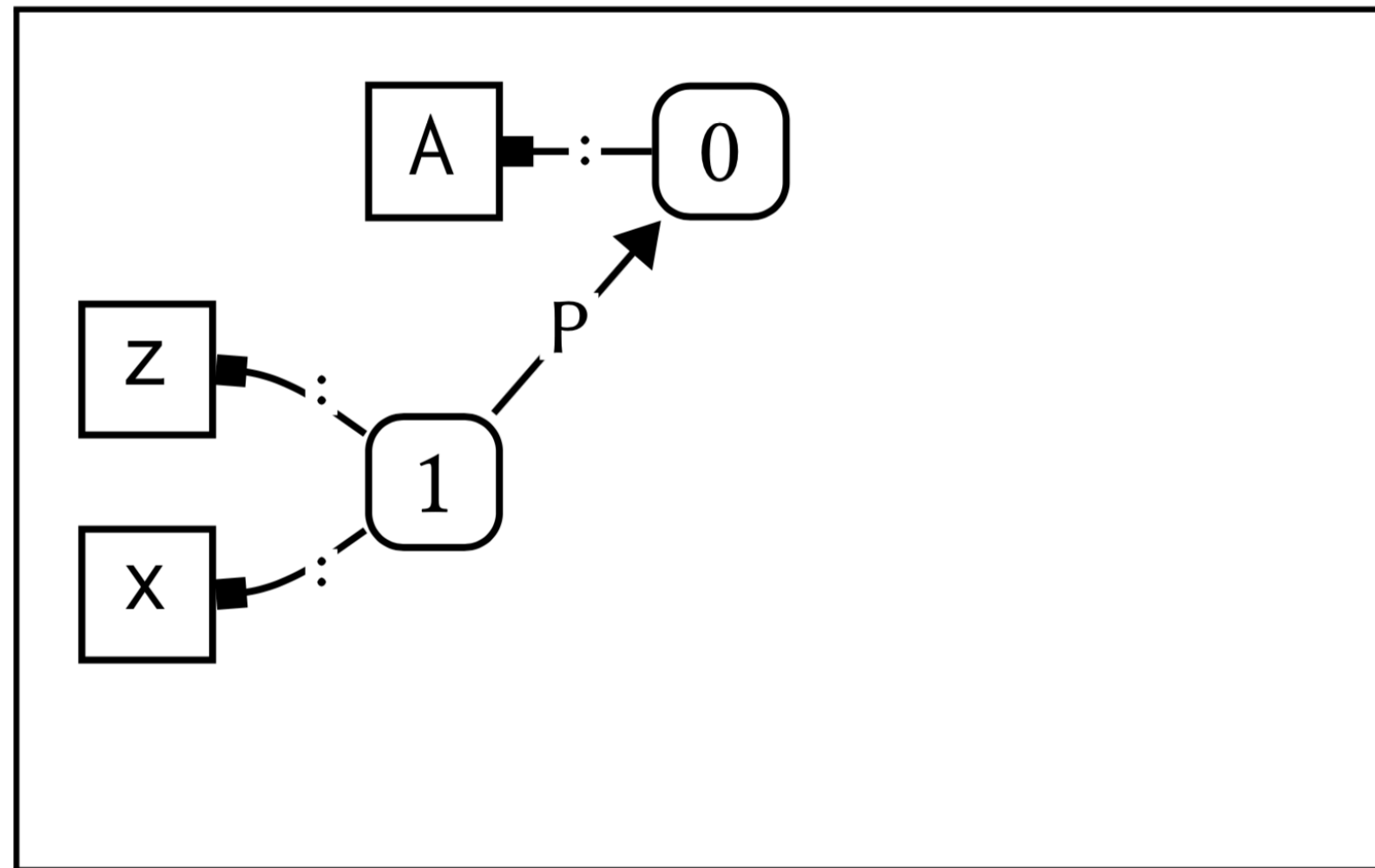
(1)



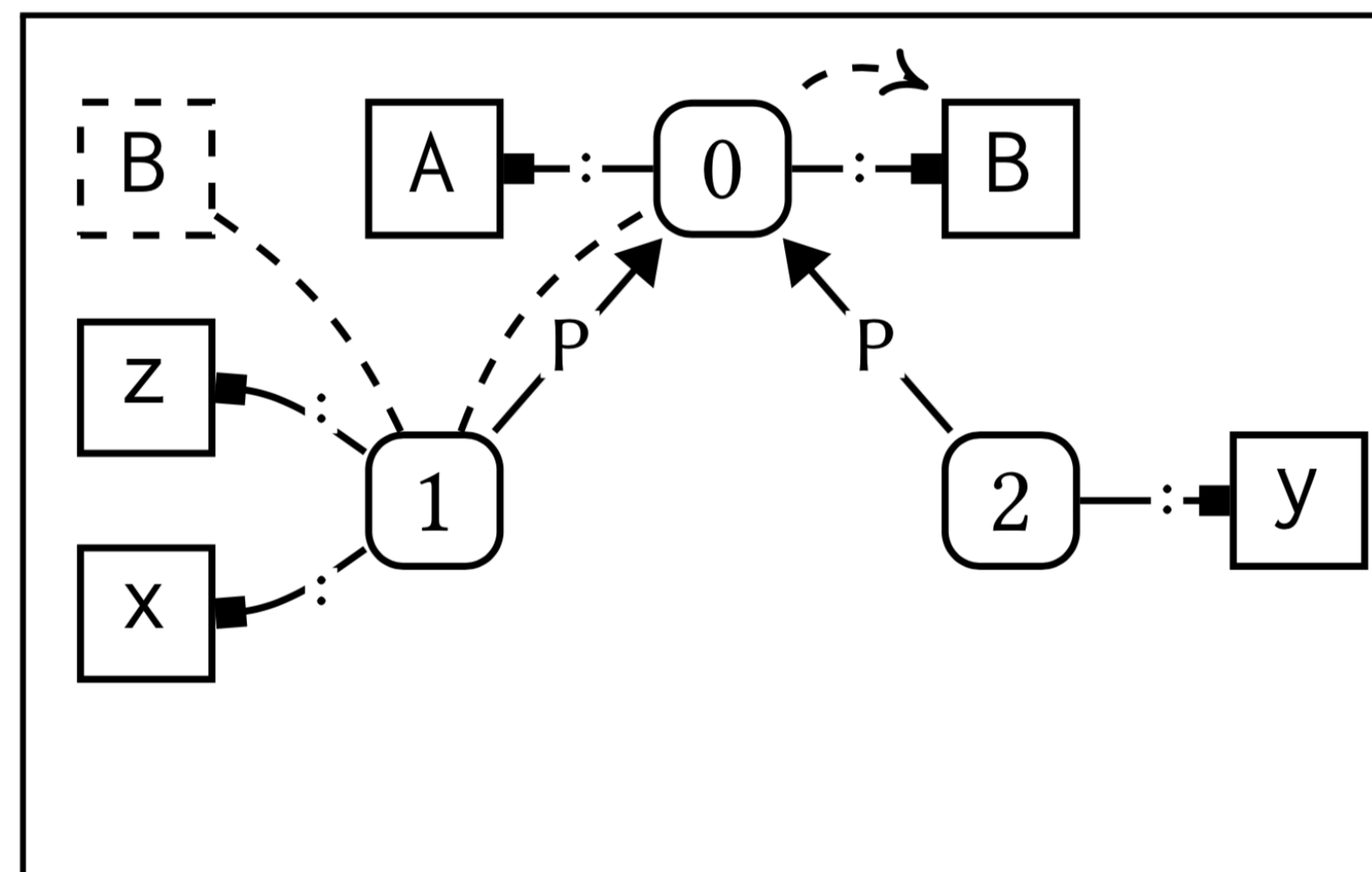
(2)

```
module A {  
    import B  
    def z:int = 3  
    def x:int = y + z  
}  
module B {  
    import A  
    def y:int = z * 2  
}
```

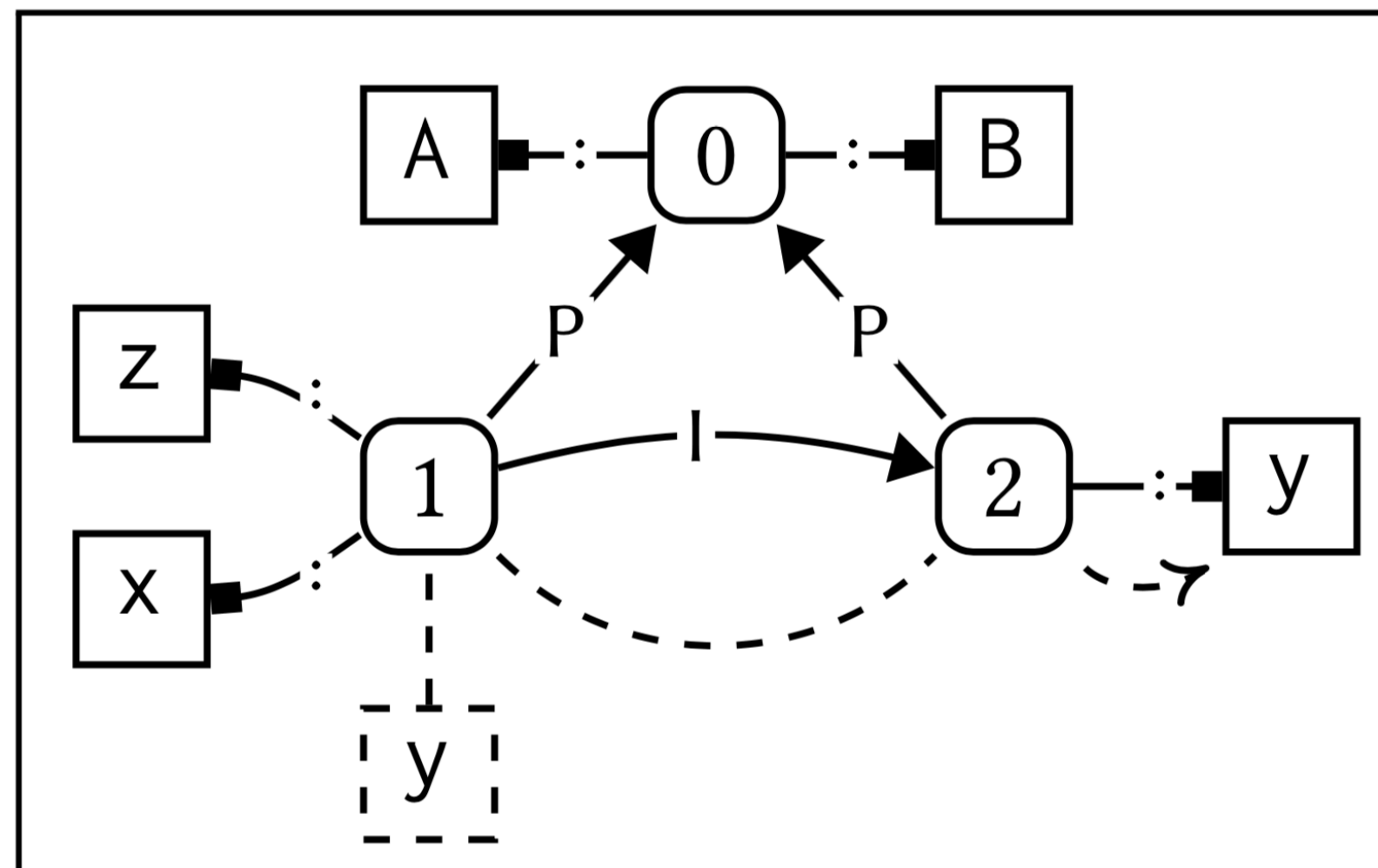
# A Two Stage Type Checker: Stage 2 (Check Modules)



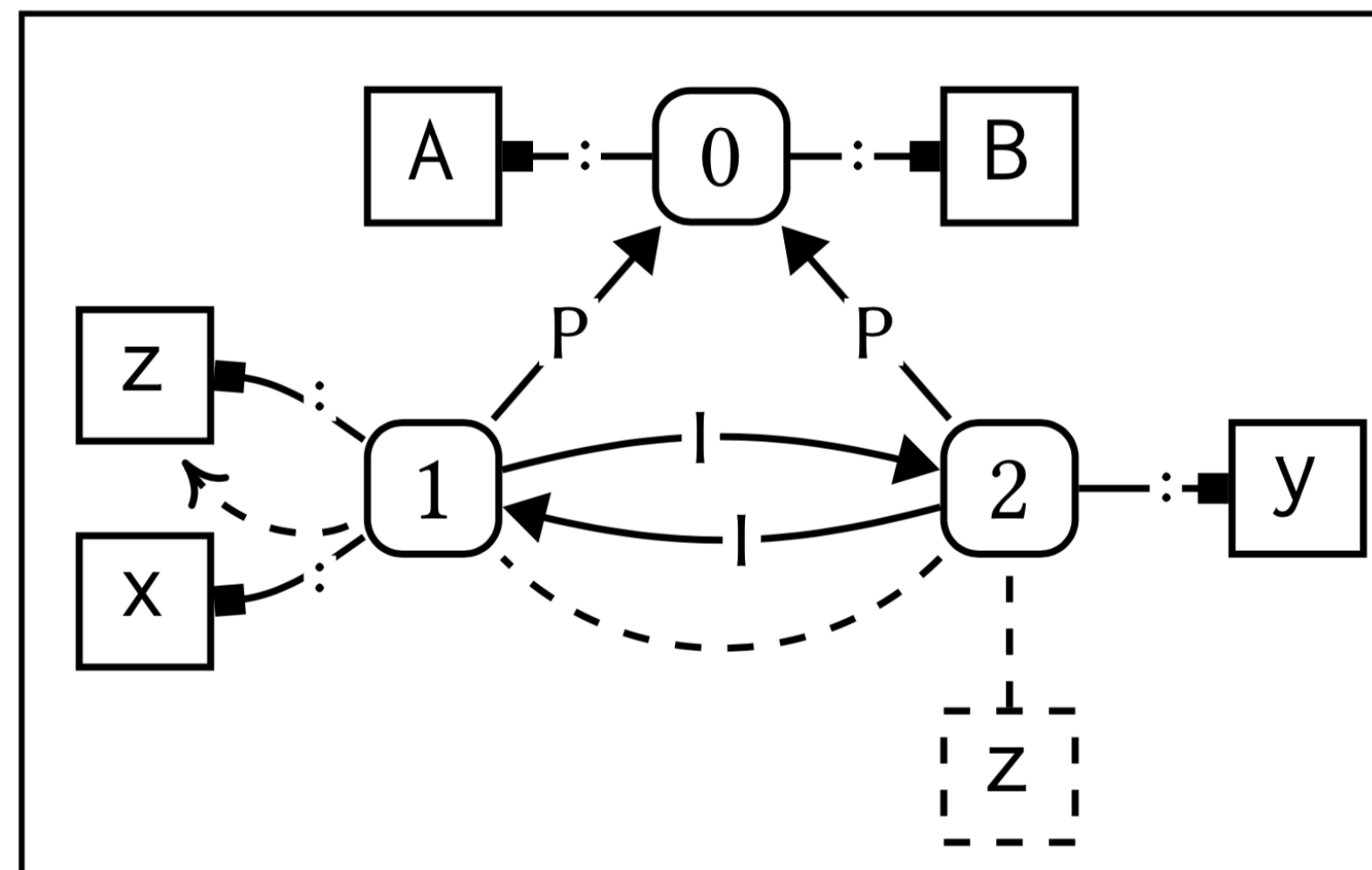
(1)



(2)



(3)

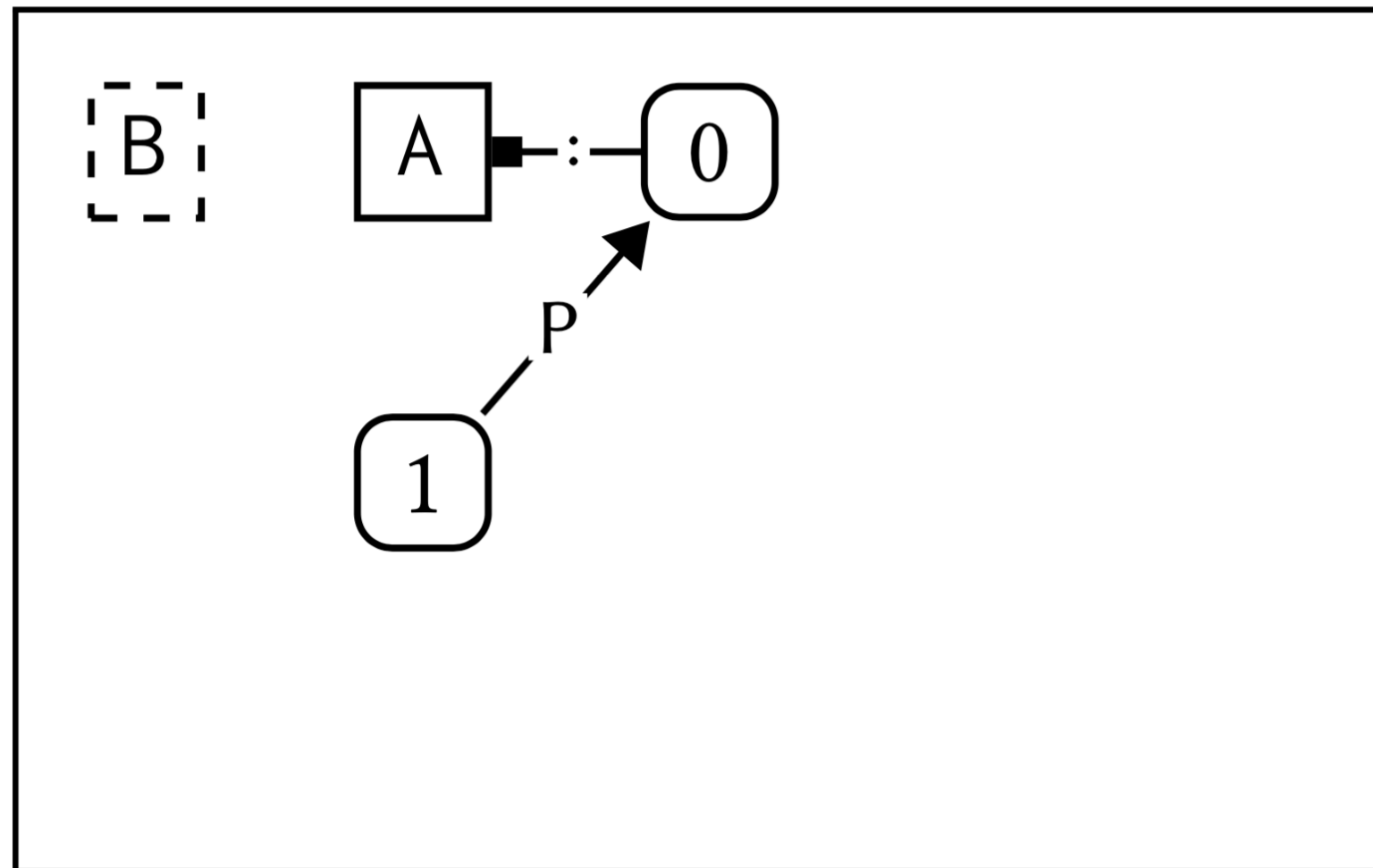


(4)

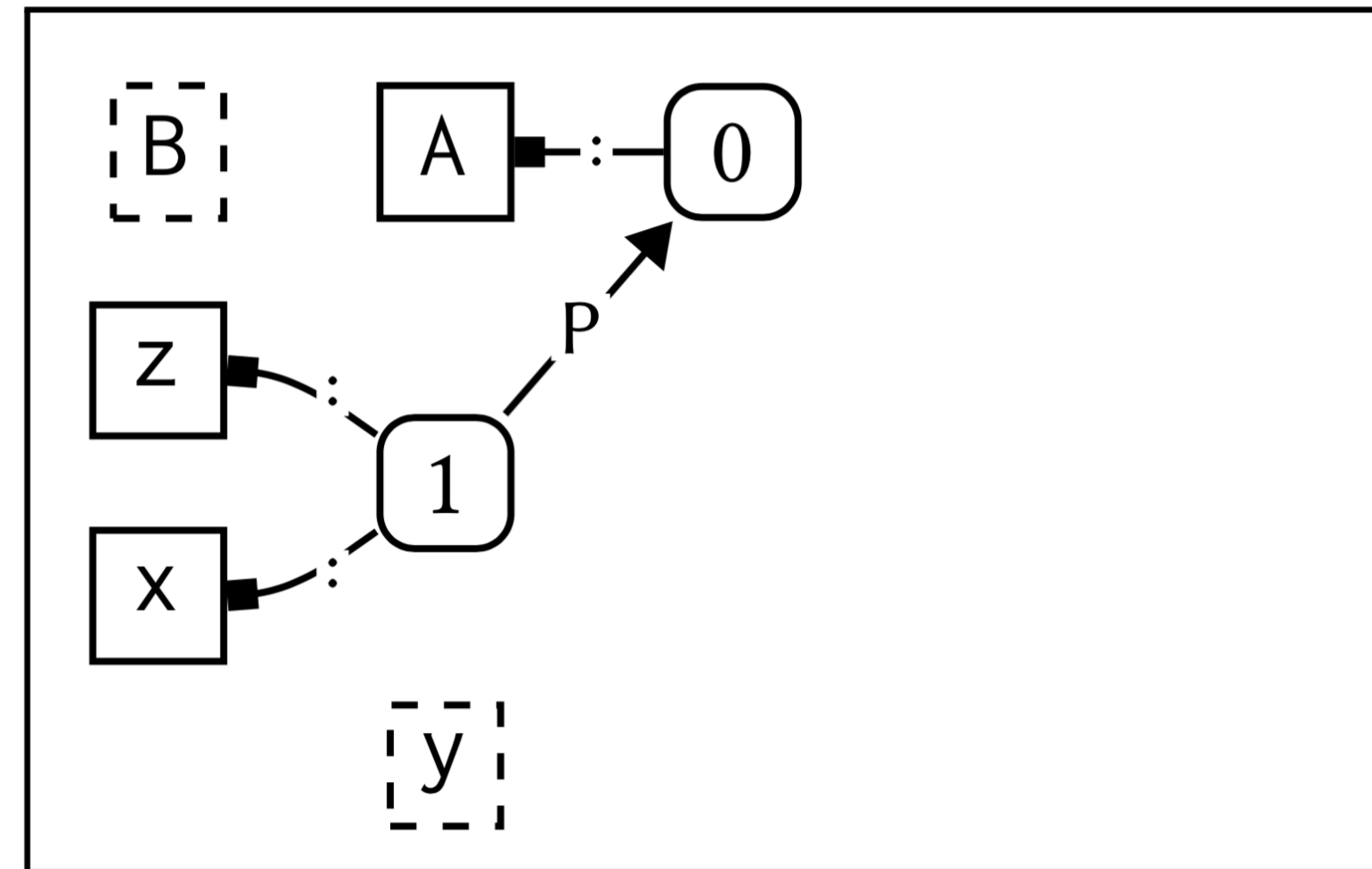
Requires that imports  
are resolved before  
variable references

```
module A {  
  import B  
  def z:int = 3  
  def x:int = y + z  
}  
module B {  
  import A  
  def y:int = z * 2  
}
```

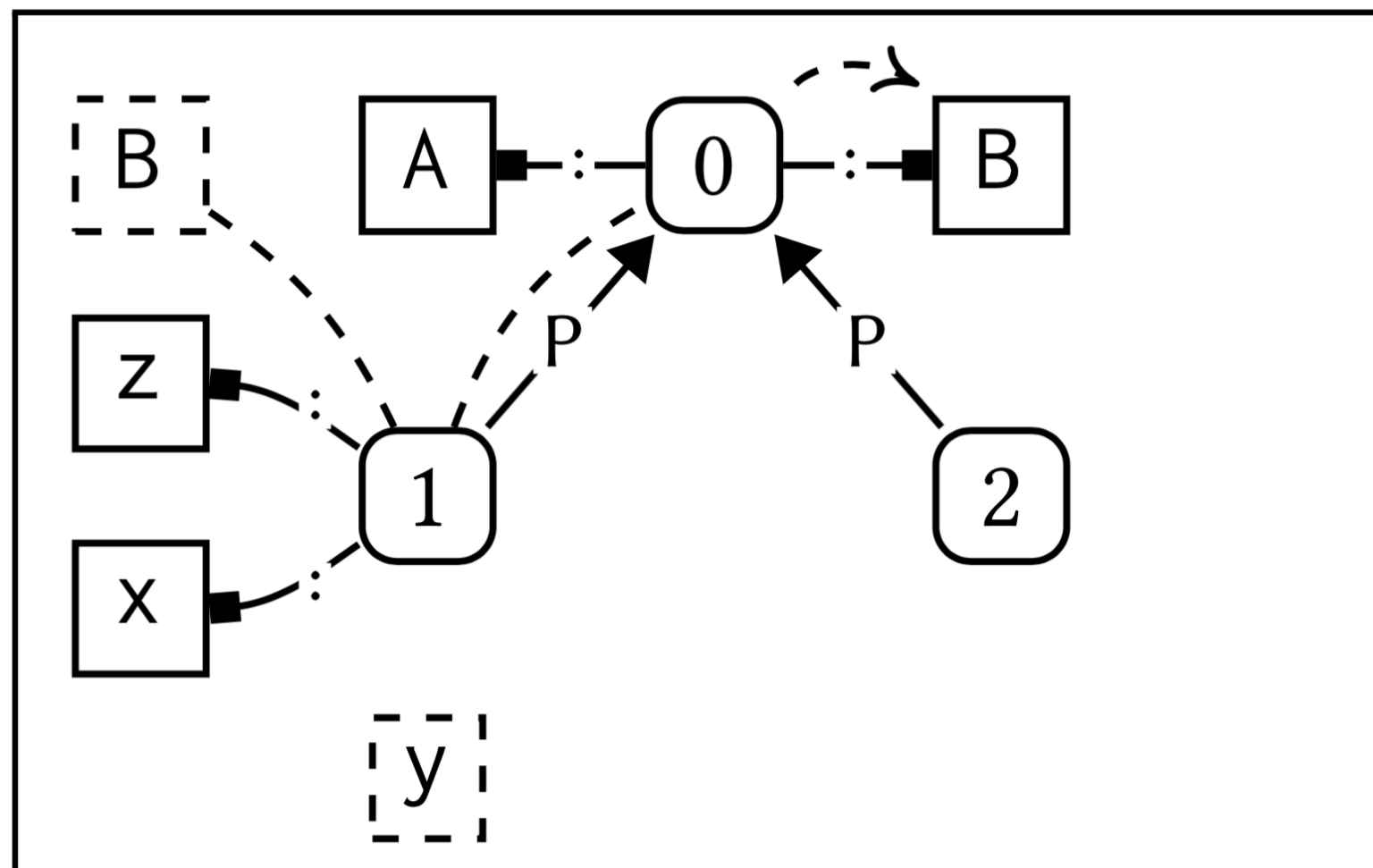
# Dynamic



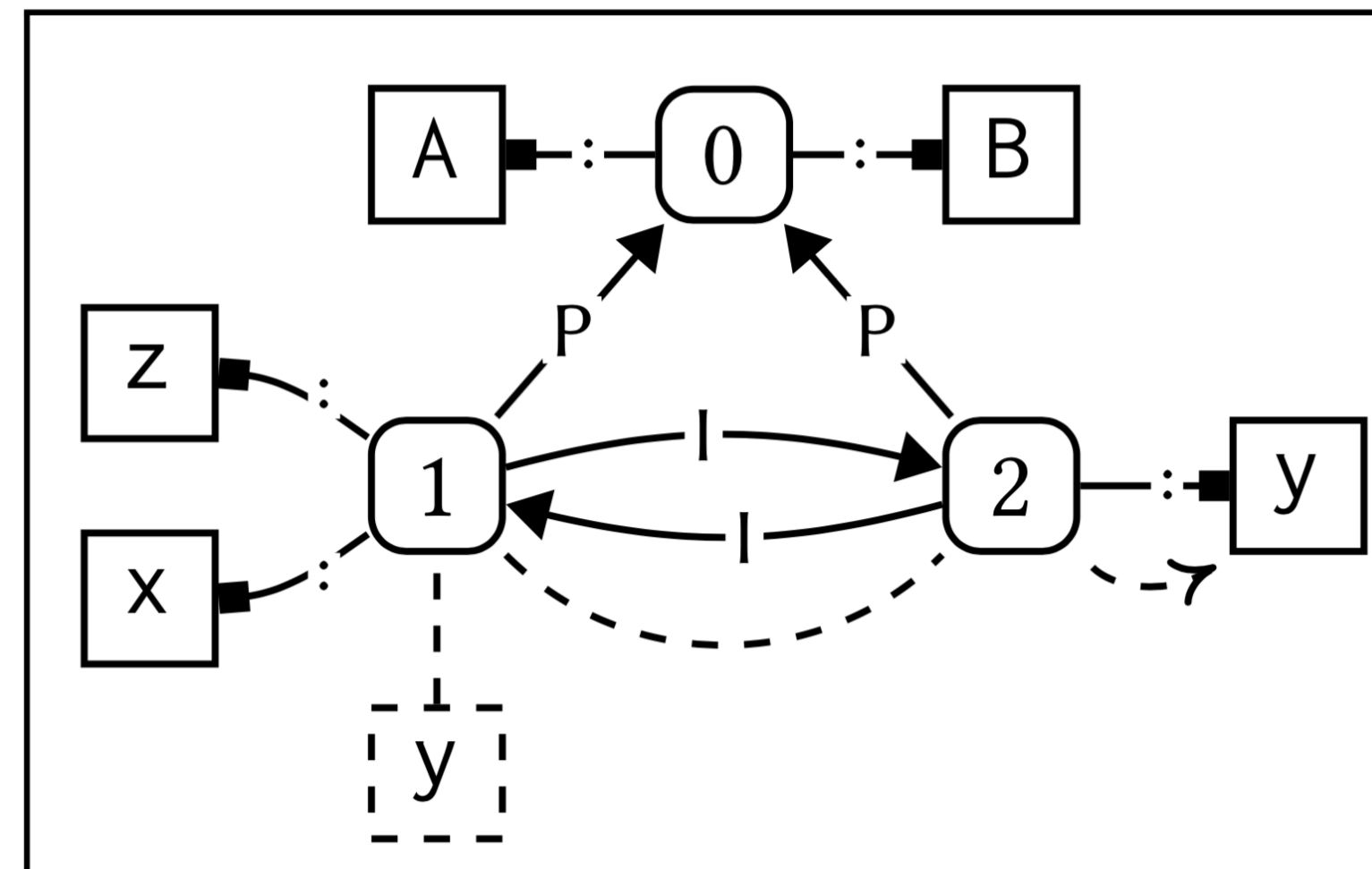
(1)



(2)



(3)

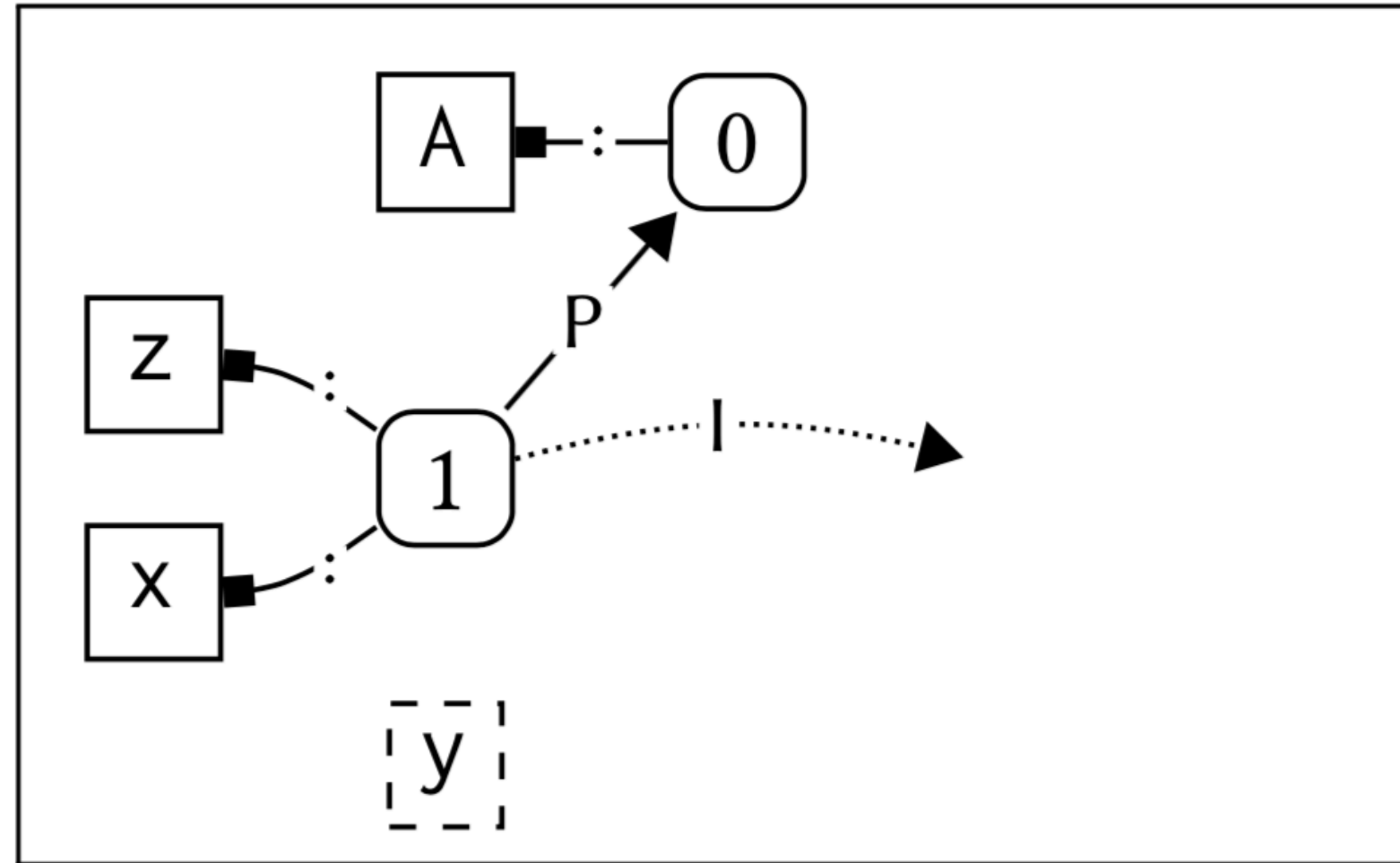


(4)

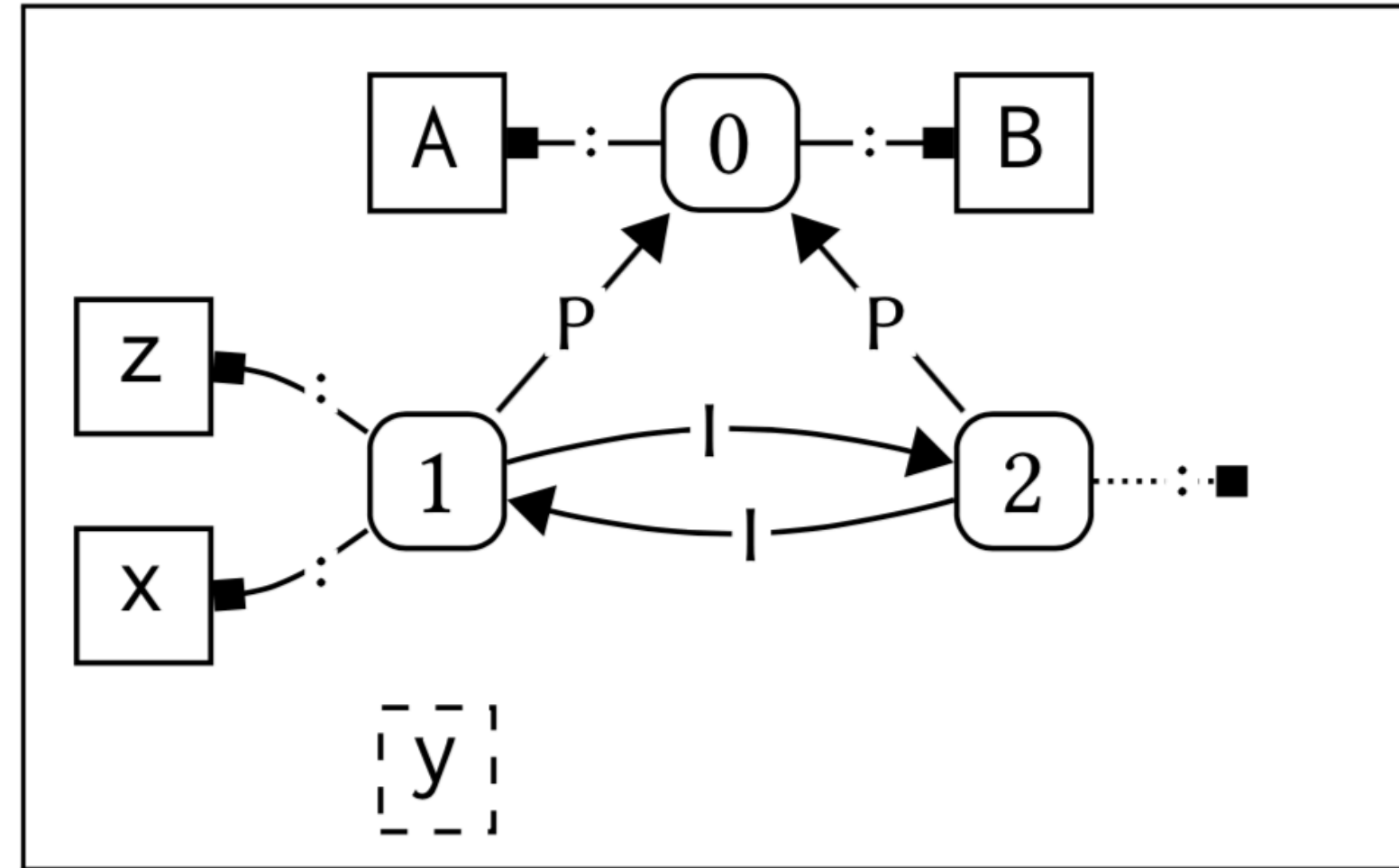
When do we have sufficient information to answer a query?

```
module A {  
  import B  
  def z:int = 3  
  def x:int = y + z  
}  
module B {  
  import A  
  def y:int = z * 2  
}
```

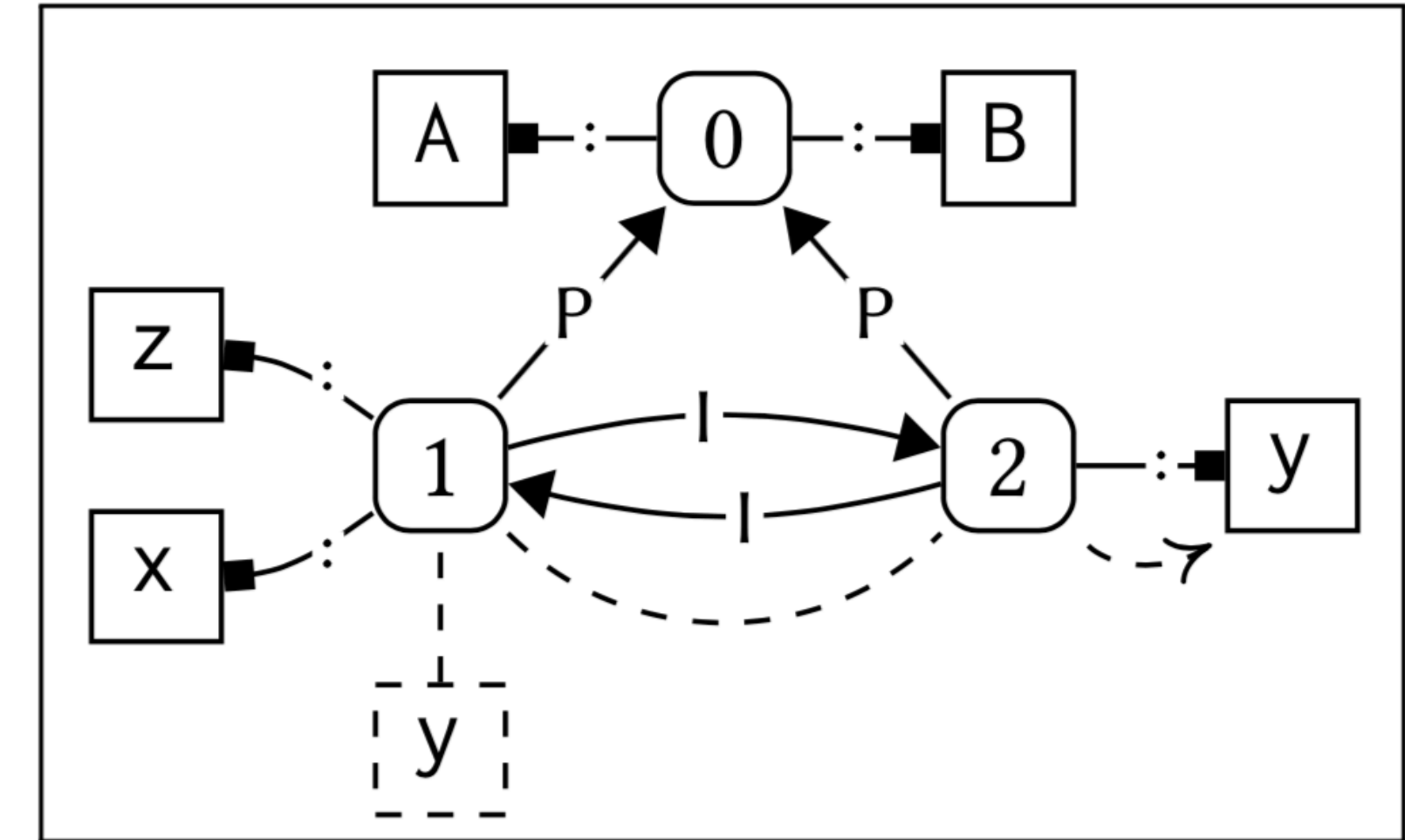
# Critical Edges



(a) Intermediate scope graph



(b) Intermediate scope graph



(c) Final scope graph

```

module A {
  import B
  def z:int = 3
  def x:int = y + z
}
module B {
  import A
  def y:int = z * 2
}
    
```

# (Weakly) Critical Edges

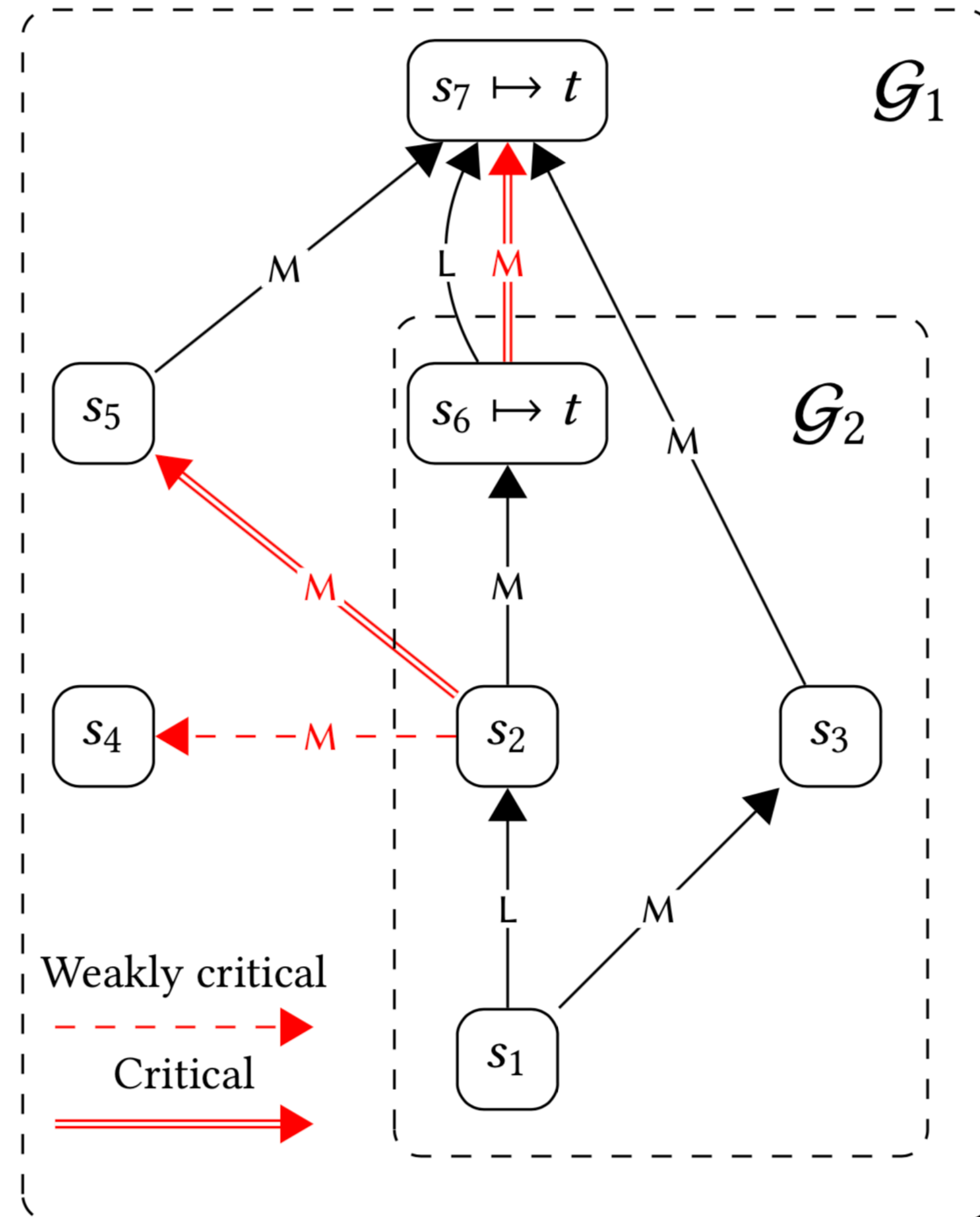


Fig. 11. (Weakly) critical edges for the query  $s_1 \xrightarrow{LM^*} \gg_R D$ , assuming  $t \in D$

## Scope graph represents context information

- Type checker constructs scope graph
- Type checker queries scope graph
- Scope graph construction depends on queries

## When is it safe to query the scope graph?

- When there are no more critical edges *for this query*

# Conclusion



## Modeling Name Binding with Scope Graphs

- Scopes + declarations + edges (reachability)
- Queries to resolve references
- Visibility policies = path disambiguation
  - path well-formedness + path specificity
- Model wide range of name binding policies

## Scheduling Constraint Resolution [OOPSLA'20]

- Declarative: no explicit scheduling / staging / stratification of traversal
- Only perform queries when outcome will not be changed (capture)
- Don't extend scopes 'remotely' (permission to extend)

**Examples in this lecture: [ESOP'15] + [PEPM'16] in Statix**



## Scopes as Types [OOPSLA'18]

### Applications

- Structural (sub)typing (records)
- Parametric polymorphism (System F)
- Nominal subtyping (FJ)
- Generic classes (FGJ)

### Under investigation

- Make those encodings less clunky
- Hindley-Milner: inference supported, but how to generalize?

# Ongoing Work

## Incremental multi-file analysis

- Given a change, which files need to be reanalyzed?

## Code completion [vision: ECOOP 2019]

- Given a hole, what can be filled in?
- Expressions, but also declarations, ...

## Refactoring

- Renaming, inlining, ...

## Other editor services

- Quick fixes, ...

## Random term generation

- Generate program that is well-typed and well-bound