Dependently Typed Languages in Statix

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Background: What are Dependent Types?

- Types may depend on values!

Example

```
concat : (A: Set) -> (n m : Nat) -> Vec A n -> Vec A m
         -> Vec A (n + m)
```

- Proof assistants
- For example: Agda, Coq, Lean, ...

Research Question

How suitable is Statix for defining a dependently-typed language?

- Will it be easier than doing it in Haskell?
- Defining system F was a challenge[1], will it even be possible?

[1] Hendrik van Antwerpen et al. Scopes as types.

Why is this important?

Spoofax perspective

Developing a language with a complex type system tests the boundaries of what Spoofax can do.

Dependent Types perspective

Using a language workbench helps with rapid prototyping.

Primary Contribution: Calculus of Constructions in Statix

A lambda calculus with dependent types.

Example 1

```
(\v: Type. v) Type
```

Example 2

```
let f = \T: Type. \x: T. x;
f (T: Type -> Type) (\y: Type. y)
```

Type Checking

Type checking relation

`typeOfExpr : scope * Expr -> Expr`

How do we use scopes?

A scope is used as a replacement for an environment and a context.
One edge `p`. One relation `name` → `NameEntry`, NameEntry is either:

- NType: Stores a type (Corresponds to a context)
- NSubst: Stores a substitution (Corresponds to an environment)

Type Checking: Requires Evaluation

Example 1

```
let T = if false then Int else Bool end;
let b: T = true;
```

Evaluation relation

```
betaHeadReduce : scope * Expr -> scope * Expr
betaReduce : scope * Expr -> Expr
exectBetaEq : (scope * Expr) * (scope * Expr)
```

Type Checking: From inference rules to Statix code

$$\frac{\langle s \mid e \rangle : t_e \qquad \langle \mathsf{sPutSubst}(s, x, (s, e)) \mid b \rangle : t_b}{\langle s \mid \mathsf{Let}(x, e, b) \rangle : t_b}$$

Equivalent Statix code

```
typeOfExpr (s, Let(x, e, b)) = bt :-
  typeOfExpr (s, e) == et,
  typeOfExpr (sPutSubst (s, x, (s, e)), b) == bt
```

Extra contributions

Features

1. Implemented Inference
2. Implemented Inductive Data Types
3. Implemented Universes
4. Interpreter
5. Compiler to Clojure

Evaluation

1. Comparison with implementation in Haskell
2. Comparison with implementation in LambdaPi
3. Evaluation of Spoofax

Conclusions

Spoofax is a great tool for developing dependently typed languages!

- We can use scopes to represent environments and contexts
- Statix can still use improvements