Practical Verification of QuadTrees

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1. Introduction

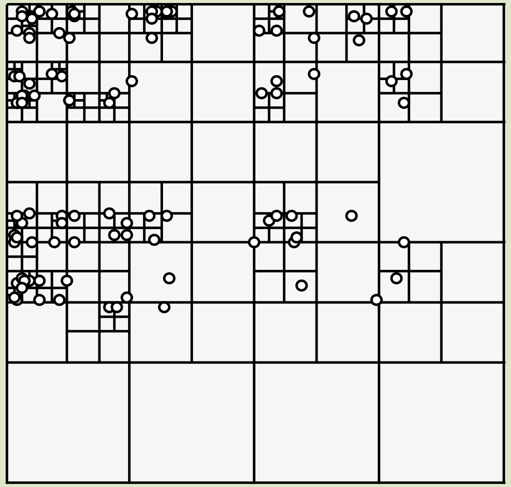
QuadTree is a Haskell library. This paper aims to rewrite this in Agda, so it can be formally verified using Curry Howard, which encodes proofs as types. It can then be compiled back to Haskell using Agda2hs.

```
takesGtFive : (n : Nat)
            -> IsTrue (n > 5) -> ?
```

Can agda2hs be used to produce a verified implementation of the QuadTree library?

2. QuadTrees

QuadTrees are used for storing two-dimensional information in a functional style. They consist of a size and a root quadrant. Each quadrant is either a Leaf, or a Node consisting of 4 sub-quadrants.



3. Implementation

How was the QuadTree library re-implemented?
Translate the code from Haskell to Agda (trivial)
Issues: Agda does not allow non-termination
Agda does not have escape latches
Agda2hs needed some modifications

4. Verification

How was the QuadTree library verified?

- Find properties to prove, using techniques from the "Ready, set, verify!" paper [1]
- Depending on the type of property, verify them in a certain way [follow arrows]
- To reduce the time needed:
  - Postulate theorems about libraries
  - Use automatic proof search
  - First prove invariants and preconditions, then post-conditions.

5. Conclusions

The library was successfully verified!
Verified 2 invariants, 3 preconditions and 4 post-conditions. Still, quite a lot of effort! Whether it is worth it depends on the situation.

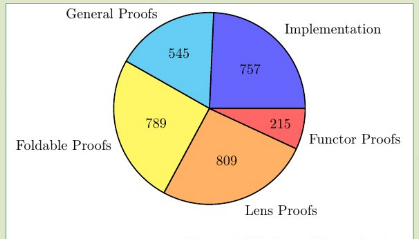


Figure 2: Division of lines of code

Invariants

Invariants are proven by adding the proof as an implicit constructor argument. To verify that a quadrant is compressed (no identical leafs) and has a certain depth, we can use:

```
data VQuadrant (t : Set) {dep : Nat} : Set where
  CVQuadrant : (qd : Quadrant t)
             -> {.(IsTrue (depth qd <= dep && isCompressed qd))}
             -> VQuadrant t {dep}
```

Preconditions

Preconditions are proven by adding the proofs as implicit arguments to the function. To verify that the location given to getLocation is in the QuadTree, one can use:

```
getLocation : (loc : Nat × Nat) -> {dep : Nat}
    -> (qt : QuadTree t)
    -> {.( IsTrue (isInsideQuadTree loc qt) )} -> t
```

Alternatively, we can pass in a datatype with an invariant. This getLeaf function only takes a leaf as input

```
getLeaf : (VQuadrant t {0}) -> t
```

Postconditions

Postconditions are proven as separate functions. For example, this is a proof that this function returns a number greater than 5.

```
gt5 : Bool -> Nat
gt5 _ = 42
```

```
gt5-is-gt5 : (b : Bool)
  -> IsTrue (gt5 b > 5)
```

[1] Joachim Breitnet, Antal Spector-Zabusky, Yao Li, Christine Rizkallah, John Wiegley, Joshua Cohen, and Stephanie Weirich. Ready, set, verify! applying hs-to-coq to real-world haskell code. Journal of Functional Programming, 31, 2021.