

Causality in Hypergraph/Volfram Model

Rewriting Systems:

$H = (V, E) \sim$ hypergraph,

$V \sim$ vertex (multi)set; $E \sim$ hyperedge (multi)set

Graph: $E \subseteq V \times V$

Hypergraph: $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$ "event"

Loosely (very): VM rule: $H_1 = (V_1, E_1) \xrightarrow{e} H_2 = (V_2, E_2)$
 $\begin{matrix} \text{input hypergraph} & & \text{output hypergraph} \\ \text{"pattern"} & & \text{"pattern"} \end{matrix}$

Causality: $In(e) = E_1 \setminus E_2$; $Out(e) = E_2 \setminus E_1$

$(e_1, e_2) \sim$ causally related $\iff Out(e_1) \cap In(e_2) \neq \emptyset$

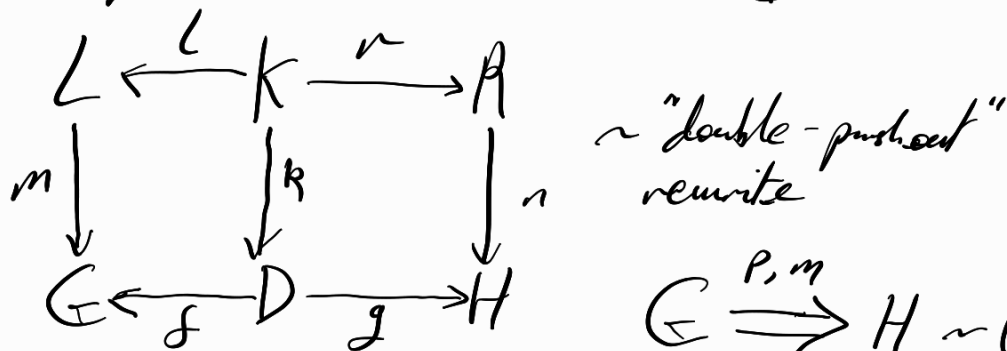
$\mathcal{G}_{can} = (V_{can}, E_{can})$

(Hyper)graph Production:

$(e_1 \rightarrow e_2) \in E_{can}$

$P = (L \xleftarrow{\ell} K \xrightarrow{\hat{\ell}} R) \sim$ span of monomorphisms.

(Hyper)graph "Match": $m: L \rightarrow G$



$G \xRightarrow{P, m} H \sim$ (direct)

$\ell \circ k = m \circ \ell$ $g \circ k = n \circ g$

hypergraph derivation.

$P_1 = (L_1 \xleftarrow{\ell_1} K_1 \xrightarrow{\hat{\ell}_1} R_1)$

$(G_0 \xRightarrow{*} G_n) \sim$ "multistep system" /

$P_2 = (L_2 \xleftarrow{\ell_2} K_2 \xrightarrow{\hat{\ell}_2} R_2)$

$In(P_2) = L_2 \setminus K_2$

$Out(P_1) = R_1 \setminus K_1$

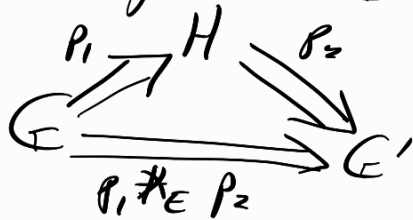
$(L_2 \setminus K_2) \cap (R_1 \setminus K_1) \neq \emptyset$ "abstract rewriting system".

Concurrency Theorem: $P_1, P_2 \sim \text{"E-related"}$

hypergraph productions

E-related transformation sequence: $G \xRightarrow{P_1} H \xRightarrow{P_2} G'$

"E-concurrent" production: $G \Rightarrow G'$



Parallelism Theorem: $P_1, P_2 \sim \text{"sequentially-independent"}$:

$$G \xRightarrow{P_1} H_1 \xRightarrow{P_2} G'$$

$$G \xRightarrow{P_2} H_2 \xRightarrow{P_1} G'$$

$P_1 + P_2 \sim \text{"parallel production"}$:

$$G \xRightarrow{P_1 + P_2} G'$$

