

Practical SAT Solving

Exercise 1

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Pigeon Hole Principle

The pigeonhole principle asserts that there is no injective mapping from m pigeons to n holes as long as $m > n$.



Image by <https://jineralknowledge.com/>

SLUR Formulas (Čepek et al., 2012)

A CNF formula is SLUR if the SLUR algorithm **never gives up** on it **regardless of the non-deterministic choices** in Lines 2 and 8.

Algorithm: Single-lookahead Unit Resolution (SLUR)

```

 $F \leftarrow \text{UnitResolution}(F)$ 
if  $\perp \in F$  then return UNSAT
else return SLURSAT}(F)

```

Function: SLURSAT

```

1 if all variables appear in a unit clause then return SAT
2  $v \leftarrow \text{SelectVariable}(F)$  // non-det. choice
3  $F_1 \leftarrow \text{UnitResolution}(F \wedge (v))$ 
4  $F_2 \leftarrow \text{UnitResolution}(F \wedge (\bar{v}))$ 
5 if  $\perp \in F_1$  and  $\perp \in F_2$  then return GIVE-UP
6 if  $\perp \in F_1$  and  $\perp \notin F_2$  then return SLUR}(F_2)
7 if  $\perp \notin F_1$  and  $\perp \in F_2$  then return SLUR}(F_1)
8 return SLUR}(F_1) non-det. or SLUR}(F_2) // non-det. choice

```

Properties of SLUR Formulas

- Solvable in **polynomial time** (using the SLUR algorithm)
- SLUR is an **umbrella class for polynomially solvable classes**
 - All **Horn and Hidden Horn** formulas are SLUR formulas
 - Also true for Extended Horn, Balanced, and Propagation Complete formulas
- It is **co-NP-complete** to recognize whether a given CNF is a SLUR formula or not