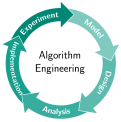
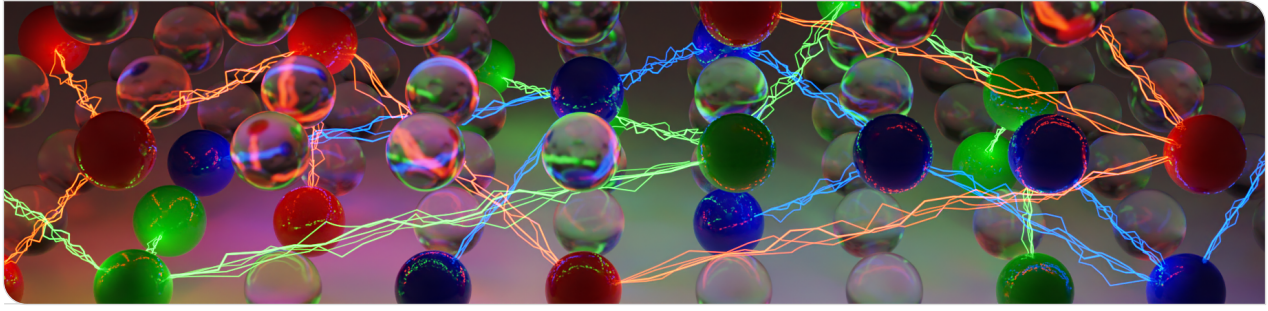
Karlsruher Institut für Technologie



Practical SAT Solving

Exercise 1

Markus Iser, Dominik Schreiber, Tomáš Balyo | April 23, 2024



Pigeon Hole Principle

The pigeonhole principle asserts that there is no injective mapping from m pigeons to n holes as long as $m > n$.

Image by https://jineralknowledge.com/

SLUR Formulas (Čepek et al., 2012)

A CNF formula is SLUR if the SLUR algorithm never gives up on it regardless of the non-deterministic choices in Lines 2 and 8.

Algorithm: Single-lookahead Unit Resolution (SLUR)

$F \leftarrow$ `UnitResolution(`F`)`
if $\bot \in F$ **then return** `UNSAT`
else return `SLURSAT(`F`)`

Function: SLURSAT

1 **if** *all variables appear in a unit clause* **then return** `SAT`
2 $v \leftarrow$ `SelectVariable(`F`)` `// non-det. choice`
3 $F_1 \leftarrow$ `UnitResolution(`$F \wedge (v)$`)`
4 $F_2 \leftarrow$ `UnitResolution(`$F \wedge (\overline{v})$`)`
5 **if** $\bot \in F_1$ ***and*** $\bot \in F_2$ **then return** `GIVE-UP`
6 **if** $\bot \in F_1$ ***and*** $\bot \notin F_2$ **then return** `SLUR(`F_2`)`
7 **if** $\bot \notin F_1$ ***and*** $\bot \in F_2$ **then return** `SLUR(`F_1`)`
8 **return** `SLUR(`F_1`)` **non-det. or** `SLUR(`F_2`)` `// non-det. choice`

Properties of SLUR Formulas

- Solvable in polynomial time (using the SLUR algorithm)
- SLUR is an umbrella class for polynomially solvable classes
  - All Horn and Hidden Horn formulas are SLUR formulas
  - Also true for Extended Horn, Balanced, and Propagation Complete formulas
- It is co-NP-complete to recognize whether a given CNF is a SLUR formula or not