

Practical SAT Solving

Exercise 1

Markus Iser, Dominik Schreiber, Tomáš Balyo | April 23, 2024



SLUR Formulas

Single Look-ahead Unit Resolution - SLUR algorithm

```

01 if  $\perp \in \text{UnitPropagation}(F)$  then return UNSAT else return SLUR( $F$ )
02 function SLUR( $F$ )
03   if all variables appear in a unit clause then return SAT
04    $v = \text{SelectVariable}(F)$ 
05    $F_1 = \text{UnitPropagation}(F \wedge (v))$ 
06    $F_2 = \text{UnitPropagation}(F \wedge (\bar{v}))$ 
07   if  $\perp \in F_1$  and  $\perp \in F_2$  then return GIVE-UP
08   if  $\perp \in F_1$  and  $\perp \notin F_2$  then return SLUR( $F_2$ )
09   if  $\perp \notin F_1$  and  $\perp \in F_2$  then return SLUR( $F_1$ )
10   if  $\perp \notin F_1$  and  $\perp \notin F_2$  then return SLUR( $F_1$ ) or SLUR( $F_2$ )

```

A CNF formula F is **SLUR** if the SLUR algorithm **never gives up** on F (regardless of the choices in lines 04 and 10).

SLUR Formulas

Algorithm 1: Single-lookahead Unit Resolution (SLUR)

```
1 if all variables appear in a unit clause then return SAT;  
2  $v \leftarrow \text{SelectVariable}(F)$  ;  
3  $F_1 \leftarrow \text{UnitResolution}(F \wedge (v))$  ;  
4  $F_2 \leftarrow \text{UnitResolution}(F \wedge (\bar{v}))$  ;  
5 if  $\perp \in F_1$  and  $\perp \in F_2$  then  
6   | return GIVE-UP;  
7 if  $\perp \in F_1$  and  $\perp \notin F_2$  then  
8   | return SLUR( $F_2$ ) ;  
9 if  $\perp \notin F_1$  and  $\perp \in F_2$  then  
10  | return SLUR( $F_1$ ) ;  
11 return SLUR( $F_1$ ) or SLUR( $F_2$ ) ;
```

SLUR Formulas

Properties of SLUR Formulas [?]:

- Solvable in **polynomial time** (using the SLUR algorithm)
- SLUR is an **umbrella class for polynomially solvable classes**
 - All **Horn and Hidden Horn** formulas are SLUR formulas
 - Also true for Extended Horn, CC-balanced, and Propagation Complete formulas

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Properties of SLUR Formulas [?]:

- Solvable in **polynomial time** (using the SLUR algorithm)
- SLUR is an **umbrella class for polynomially solvable classes**
 - All **Horn and Hidden Horn** formulas are SLUR formulas
 - Also true for Extended Horn, CC-balanced, and Propagation Complete formulas
- It is **co-NP-complete** to recognize whether a given CNF is a SLUR formula or not

Integer Multiplication / Factorization

