Practical SAT Solving

Lecture 10

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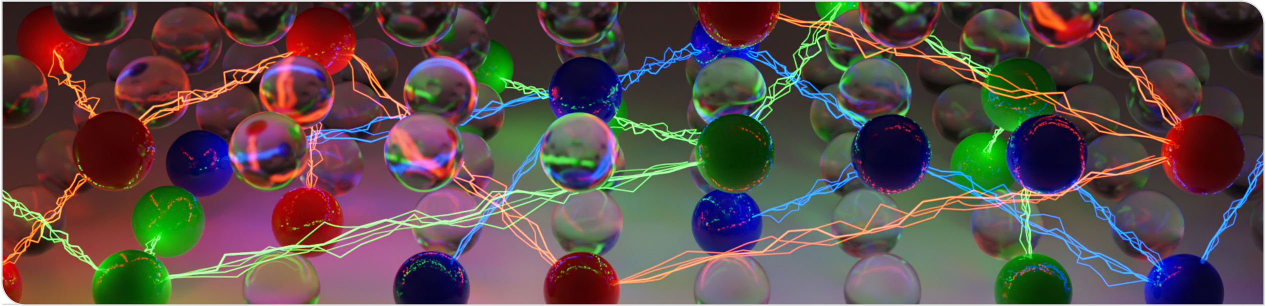


Recap

Lecture 9: MaxSAT

- Motivation for Optimal MaxSAT
- What is MaxSAT (variants) and how to use it
- How do MaxSAT solvers work

Today

SAT Based Planning

What is Planning

Informal Definition

Planning is the process of finding a plan, i.e., a sequence of actions that changes the state of the world from some initial state to a desired (goal) state.

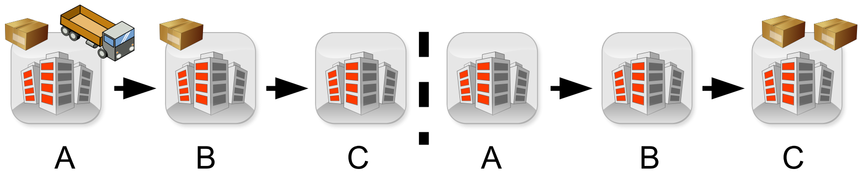
Examples

- Delivering some packages
- Building a submarine
- Robot motion planning
- Fulfilling a scientific goal by an autonomous space probe

Some History

- The Stanford Research Institute Problem Solver, known by its acronym STRIPS, is an automated planner developed by Richard Fikes and Nils Nilsson in 1971 at SRI International.
- The same name was later used to refer to the formal language of the inputs to this planner. This language is the base for most of the languages for expressing automated planning problem instances in use today
- Shakey the robot (on the right) implemented the STRIPS planning algorithm

Trucking Example



- Initial State
  - There is a truck and a package in city A
  - There is a package in city B
- Goal
  - There are two packages in city C
- Possible Actions
  - (Un)loading packages from/on the truck, driving between cities

Formalizing Planning

Planning Problem Definition – SAS+ formalism

A planning problem instance Π is a tuple $(\mathcal{X}, \mathcal{A}, s_I, s_G)$ where

- $\mathcal{X}$ is a set of multivalued variables with finite domains.
  - each variable $x \in \mathcal{X}$ has a finite possible set of values $dom(x)$
- $\mathcal{A}$ is a set actions. Each action $a \in \mathcal{A}$ is a tuple $(pre(a), eff(a))$
  - $pre(a)$ is a set of preconditions of action a
  - $eff(a)$ is a set of effects of action a
  - both are sets of equalities of the form $x = v$ where $x \in \mathcal{X}$ and $v \in dom(x)$
- s_I is the initial state, it is a **full** assignment of the variables in $\mathcal{X}$
- s_G is the set of goal conditions, it is a set of equalities (same as $pre(a)$ and $eff(a)$)

Formalizing Planning II

World State

A state is full assignment of the variables in $\mathcal{X}$ (each variable $x \in \mathcal{X}$ has exactly one value assigned from its domain $dom(x)$. A state can be represented as a set of equalities.

The initial state s_I is a state. A state s is a goal state if $s_G \subseteq s$

Applicable Actions

An action $a \in \mathcal{A}$ is applicable in the state s if $pre(a) \subseteq s$

Applying an Action

When an action $a \in \mathcal{A}$ is applied in the state s it changes to the state s' such that $eff(a) \subseteq s'$ and the difference between s and s' is minimal (only variables used in $eff(a)$ are changed).

Formalizing Planning III

A Plan

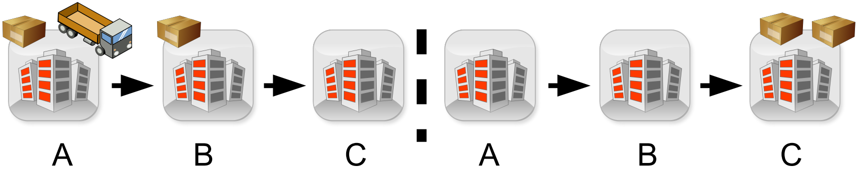
A plan for P for a planning problem $\Pi = (\mathcal{X}, \mathcal{A}, s_I, s_G)$ is sequence of actions $a_1, a_2, \ldots a_n$ such that

- $\forall i \; a_i \in \mathcal{A}$
- let $s_1 = s_I$ and $s_{i+1} = apply(s_i, a_i)$
- a_i is applicable in s_i
- $s_G \subseteq s_{n+1}$

If $P = \{a_1, a_2, \ldots a_n\}$ then n is the lenght of the plan P.

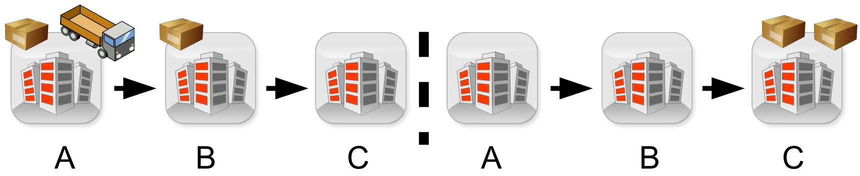
An optimal plan is a plan of shortest length.

Trucking Example



- variables: Truck Location T, $dom(T) = \{A, B, C\}$, Package Locations P_1 and P_2, $dom(P_1) = dom(P_2) = \{A, B, C, T\}$
- Initial state: $\{T = A, P_1 = A, P_2 = B\}$
- Goal: $\{P_1 = C, P_2 = C\}$
- Actions: $load(P_i, L) = (\{T = L, P_i = L\}, \{P_i = T\})$ $unload(P_i, L) = (\{T = L, P_i = T\}, \{P_i = L\})$ $drive(L_1, L_2) = (\{T = L_1\}, \{T = L_2\})$ where $i \in \{1, 2\}$ and $L, L_1, L_2 \in \{A, B, C\}$

Trucking Example



World State

- $T = A, P_1 = A, P_2 = B$
- $T = A, P_1 = T, P_2 = B$
- $T = B, P_1 = T, P_2 = B$
- $T = B, P_1 = T, P_2 = T$
- $T = C, P_1 = T, P_2 = T$
- $T = C, P_1 = C, P_2 = C$

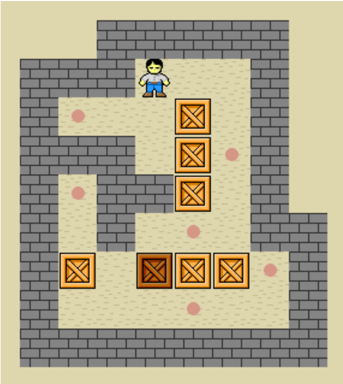
The Plan

- $load(P_1, A)$
- $drive(A, B)$
- $load(P_2, B)$
- $drive(B, C)$
- $unload(P_1, C), unload(P_1, C)$

Sokoban Example

- Initial State
  - There is a worker and a bunch of boxes
- Goal
  - All the boxes must be in goal positions
- Possible Actions
  - moving with the worker
  - pushing a box
- Forbidden
  - to pull boxes
  - move through walls or boxes

http://wki.pe/Sokoban



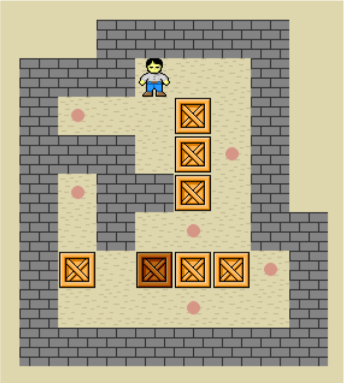
Encoding Sokoban

World State

- Variables – For each location we have variable, the domain is WORKER, BOX, EMPTY
- Initial State – assign values based on the picture
- Goal – goal position variables have value BOX

Actions: move and push for each possible location

- $push(L_1, L_2, L_3) = (\{L_1 = W, L_2 = B, L_3 = E\},$
  $\{L_1 = E, L_2 = W, L_3 = B\})$
- $move(L_1, L_2) = (\{L_1 = W, L_2 = E\}, \{L_1 = E, L_2 = W\})$



Encoding Planning into CNF

Is that even possible?

Encoding Planning into CNF

- We cannot encode the existence of a plan in general
- But we can encode the existence of plan up to some length

Encoding Planning into CNF

- We cannot encode the existence of a plan in general
- But we can encode the existence of plan up to some length

SATPLAN Algorithm

- INPUT: a planning problem Π
- OUTPUT: a plan P

for $m := 1, 2, \ldots$ **do**
  $F = \text{encodePlanExists}(\Pi, m)$
  if solver.isSat(F) **then**
    return extractPlan(Π, m, solver.solution)

Encoding Planning into CNF

The Task

Given a planning problem instance $\Pi = (\mathcal{X}, \mathcal{A}, s_I, s_G)$ and $k \in \mathbb{N}$ construct a CNF formula F such that F is satisfiable if and only if there is plan of length k for Π.

Encoding Planning into CNF

Karlsruher Institut für Technologie

The Task

Given a planning problem instance $\Pi = (\mathcal{X}, \mathcal{A}, s_I, s_G)$ and $k \in \mathbb{N}$ construct a CNF formula F such that F is satisfiable if and only if there is plan of length k for Π.

We will need two kinds of variables

- Variables to encode the actions:
  a_i^t for each $t \in \{1, \ldots, k\}$ and $a_i \in \mathcal{A}$
- Variables to encode the states:
  $b_{x=v}^t$ for each $t \in \{1, \ldots, k+1\}$, $x \in \mathcal{X}$ and $v \in dom(x)$

In total we have $k|\mathcal{A}| + (k+1)\sum_{x \in \mathcal{X}} dom(x)$ variables

Encoding Planning into CNF

We will need 8 kinds of clauses

- The first state is the initial state
- The goal conditions are satisfied in the end
- Each state variable has at least one value
- Each state variable has at most one value
- If an action is applied it must be applicable
- If an action is applied its effects are applied in the next step
- State variables cannot change without an action between steps
- At most one action is used in each step

Encoding Planning into CNF

The first state is the initial state

$$(b^1_{x=v}) \quad \forall (x = v) \in s_I \tag{1}$$

The goal conditions are satisfied in the end

$$(b^{n+1}_{x=v}) \quad \forall (x = v) \in s_G \tag{2}$$

Encoding Planning into CNF

Each state variable has at least one value

$$
\begin{gathered}
(b^t_{x=v_1} \vee b^t_{x=v_2} \vee \cdots \vee b^t_{x=v_d}) \\
\forall x \in X,\ \mathrm{dom}(x) = \{v_1, v_2, \ldots, v_d\},\ \forall t \in \{1, \ldots, k+1\}
\end{gathered}
\tag{3}
$$

Each state variable has at most one value

$$
\begin{gathered}
(\neg b^t_{x=v_i} \vee \neg b^t_{x=v_j}) \\
\forall x \in X,\ v_i \neq v_j, \{v_i, v_j\} \subseteq \mathrm{dom}(x),\ \forall t \in \{1, \ldots, k+1\}
\end{gathered}
\tag{4}
$$

Encoding Planning into CNF

If an action is applied it must be applicable

$$
\begin{gathered}
(\neg a^t \vee b^t_{x=v}) \\
\forall a \in \mathcal{A},\ \forall (x = v) \in \mathrm{pre}(a),\ \forall t \in \{1, \ldots, k\}
\end{gathered} \tag{5}
$$

If an action is applied its effects are applied in the next step

$$
\begin{gathered}
(\neg a^t \vee b^{t+1}_{x=v}) \\
\forall a \in \mathcal{A},\ \forall (x = v) \in \mathrm{eff}(a),\ \forall t \in \{1, \ldots, k\}
\end{gathered} \tag{6}
$$

Encoding Planning into CNF

State variables cannot change without an action between steps

$$\begin{gathered}(\neg b_{x=v}^{t+1} \vee b_{x=v}^{t} \vee a_{s_1}^{t} \vee \cdots \vee a_{s_j}^{t}) \\ \forall x \in X,\ \forall v \in \mathrm{dom}(x),\ \mathrm{support}(x=v) = \{a_{s_1}, \ldots, a_{s_j}\},\ \forall t \in \{1, \ldots, k\}\end{gathered} \tag{7}$$

By $\mathrm{support}(x = v) \subseteq \mathcal{A}$ we mean the set of *supporting actions* of the assignment $x = v$, i.e., the set of actions that have $x = v$ as one of their effects.

Encoding Planning into CNF

At most one action is used in each step

$$
\begin{gathered}
(\neg a_i^t \vee \neg a_j^t) \\
\forall \{a_i, a_j\} \subseteq \mathcal{A},\, a_i \neq a_j \;\forall t \in \{1, \dots, k\}
\end{gathered} \tag{8}
$$

Encoding Planning into CNF

The Task Solved

Given a planning problem instance $\Pi = (\mathcal{X}, \mathcal{A}, s_I, s_G)$ and $k \in \mathbb{N}$ a CNF formula F, which is a conjunction of all the above described clauses is satisfiable if and only if there is plan of length k for Π.

Optimizations

- Better encoding of at-most-one
- Allowing several actions in each step
- Encoding variable transitions instead of variable values

SAT is NP-Hard – proof sketch

- Let M be a non-deterministic Turing machine that accepts an input x in $P(|x|)$ time, where P is a polynomial function.
  - M on x will use at most $P(|x|)$ tape entries
- M on input x as a SAS+ planning problem Π
  - State variables are the state of the TM and the $P(|x|)$ tape entries
  - The transition function table is encoded as actions
  - Initial state: tape contains input, TM state is initial state
  - Goal state: TM state is an accepting state
- Encode Π for plan lenght $k = P(|x|)$ into a CNF formula F_k
- F_k is SAT if and only if M accepts x in $P(|x|)$ time
- F_k has polynomial size w.r.t. to M and x

Planning with incremental SAT

- we are solving a sequence of similar formulas
- how do they differ?
- how to use an incremental solver in this case?

Planning with incremental SAT

- The formula F_k is the subset of F_{k+1} except for the goal clauses.
- The goal clauses will be added as removable (in this case, since they are unit, we can just assume them)

Incremental SATPLAN Algorithm

- INPUT: a planning problem Π
- OUTPUT: a plan P

$S = \mathrm{initSolver}()$
$\mathrm{addInitialStateClauses}(S)$
for $m := 1, 2, \ldots$ **do**
    $\mathrm{addClausesForStep}(m, S)$
    $\mathrm{assumeGoalConditionsAtStep}(m, S)$
    if satisfiable(S) **then return** extractPlan(Π, m, getValues(S))

The DIMSPEC format

- Many other (than planning) problems have a similar structure
  - for example bounded model checking
- They can be specified using the DIMSPEC format
- DIMSPEC is four cnf formulas, where the `"p cnf <n> <m>"` line is replaced by:
  - `i cnf <n> <m>` for the initial state specification (*n* variables)
  - `g cnf <n> <m>` for the goal state specification (*n* variables)
  - `u cnf <n> <m>` for the universal state specification (*n* variables)
  - `t cnf <n> <m>` for the specification of the transition (between two neigbhoring states) (2*n* variables)

The DIMSPEC format example

```
c this is an example of a dimspec file
i cnf 5 3
-1 2 0
2 3 -5 0
4 0
g cnf 5 1
5 0
u cnf 5 2
-1 2 3 0
-3 4 5 0
t cnf 10 2
-2 7 8 0
-4 9 10 0
```

Planning as DIMSPEC

- Initial state specificaion clauses: $(b_{x=v})$ added $\forall(x = v) \in S_I$
- Goal state specificaion clauses: $(b_{x=v})$ added $\forall(x = v) \in S_G$
- Universal state specification clauses:
  - $(b_{x=v_1} \vee b_{x=v_2} \vee \cdots \vee b_{x=v_d})$ added $\forall x \in X$ where $\mathrm{dom}(x) = \{v_1, v_2, \ldots, v_d\}$ – at least one value
  - $(\overline{b_{x=i}} \vee \overline{b_{x=j}})$ added $\forall x \in X\ i \neq j \in \mathrm{dom}(x)$ – at most one value
  - $(\overline{a} \vee b_{x=v})$ added $\forall a \in \mathcal{A},\ \forall(x = v) \in \mathrm{pre}(a)$ – action preconditions
  - $(\overline{a_i} \vee \overline{a_j})$ added $\forall i \neq j$ – at most one action
- Transition specification clauses
  - $(\overline{a} \vee b'_{x=v})$ added $\forall a \in \mathcal{A},\ \forall(x = v) \in \mathrm{eff}(a)$ – action effects
  - $(\overline{b'_{x=v}} \vee b_{x=v} \vee a_{s_1} \vee \cdots \vee a_{s_j})$ added $\forall x \in X,\ \forall v \in \mathrm{dom}(x)$ where $\mathrm{support}(x = v) = \{a_{s_1}, \ldots, a_{s_j}\}$ – values cannot change without a reason

Solving DIMSPEC

- Same as solving planning with incremental SAT

The Basic DISMPEC Solving Algorithm

- INPUT: a DIMSPEC problem
- OUTPUT: a truth assignment

$S = \text{initSolver}()$
$\text{addInitialStateClauses}(S)$
for $m := 1, 2, \ldots$ **do**
  $\text{addUniversalConditionsWithRenaming}(m, S)$
  if $m > 1$ **then** $\text{addTransitionalConditionsWithRenaming}(m, S)$
  $\text{assumeGoalConditionsWithRenaming}(m, S)$
  if satisfiable(S) **then return** getValues(S)