

1 Minimal Perfect Hashing (20 Points)

1.1 Multiplication Competition (10 Points)

Write a program that takes as arguments two integers a and b . If either a or b is not in the range $[0, 2^{64})$, the program should print an error message and exit. Otherwise, the program should output the multiplier circuit encoding in CNF that computes the product $a \cdot b$. The first 64 variables of the CNF should be the bits of the product starting with the least significant bit. Variable 65 should indicate whether an overflow happened during multiplication. The program should output the CNF in DIMACS format (including a header with the number of variables and clauses). The fastest encoding wins, the winner will receive 10 bonus points, the second 5 bonus points, and the third 3 bonus points.

1.2 Hashing (10 Points)

Given a bound B and a set of n 64-bit integers keys $K = \{k_1, k_2, \dots, k_n\}$, write a program which determines a seed s for the hash function depicted in Algorithm 1 in such a way, that the function maps all the keys $k \in K$ to a set of n integers in the range $[0, B)$. shows the finalization step of the Murmur3 hash function.^a

^a<http://zimbry.blogspot.com/2011/09/better-bit-mixing-improving-on.html>

Algorithm 1: Murmur3Final

Data: Key k: uint64

Data: Seed s: uint64

$k \leftarrow k + s$

$k \leftarrow k \otimes (k \gg 33)$

$k \leftarrow k \times 0\text{xff}51\text{afd}7\text{ed}558\text{ccd}$

$k \leftarrow k \otimes (k \gg 33)$

$k \leftarrow k \times 0\text{xc}4\text{ceb}9\text{fe}1\text{a}85\text{ec}53$

$k \leftarrow k \otimes (k \gg 33)$

return k;

1.3 Perfect Hashing (10 Points)

Extend your solution in such a way, that the function maps all the keys $k \in K$ *uniquely* to n integers in the range $[0, B)$.

1.4 Minimal Perfect Hashing (10 Points)

Extend your solution in such a way, that the function maps all the keys $k \in K$ *uniquely* to B integers in the range $[0, B)$.