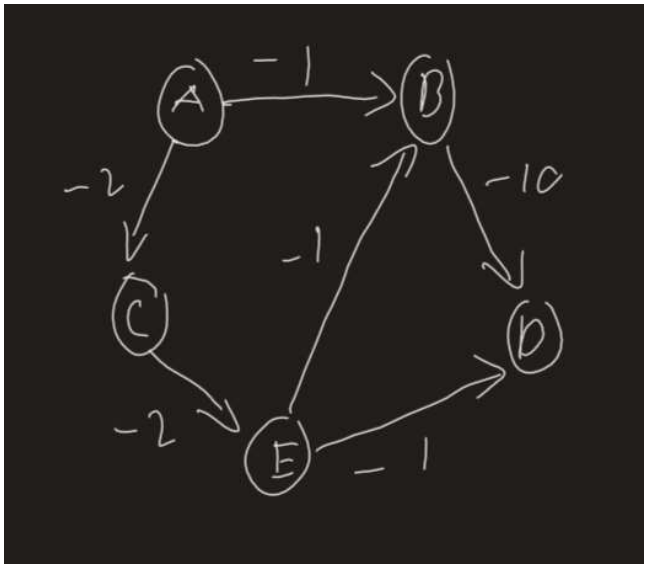
Jonathan Koning

koningj@oregonstate.edu

HW 5

2) Suppose that the only negative edges are those that leave the starting node s. Does Dijkstra's algorithm find the shortest path from s to every other node in this case? Justify your answer carefully.

No, Dijkstra's algorithm does not work with only negative numbers because a node is removed from the heap when all input the smallest distance is found to it. The algorithm assumes that the distances will increase and therefore will not consider that a value will get smaller. Consider the following example:



The algorithm will remove B before it checks the distance from B to D because the distance from ACED is just as long as the distance from ACEB. The algorithm will assume that the distance from B to D will only increase the total and will therefore remove B from contention.

3) Give an O(n^3) algorithm that takes a directed graph as input and returns the length of the shortest cycle in the graph where n is the number of nodes.

- Perform A dfs search that also keeps track of every back edge. This will let you know which nodes are connected via a cycle.
- Run Dijkstra's algorithm on for each pair of nodes connected to via a back edge, starting at the destination node of the back edge so that it must go through then entire cycle.

4) You are given a strongly connected directed graph G = (V,E) with positive edge weights. Give an efficient algorithm for finding the shortest paths between all pairs of nodes with the restriction that they all must pass through the node A.

For each pair of nodes, use Dijkstra's algorithm to go from node u to node A to node v. This way you find the shortest path from node u to node A, then find the shortest path from node A to node u.