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Report 6

2. (1 point) You are driving on a long highway with gas stations at distances *d_1*< ...< *d_n* miles from the starting location *S*. Your car can run *M* miles with a full gas tank. You start with a full gas tank and want to reach the final location which is at *d_n* miles from *S*. How would you choose the gas stations to minimize the number of refueling stops? Argue that no other choice can make fewer stops.

The lowest possible number of trips is d_n/M. Use binary search to find the farthest gas station that can be reached. If there is no gas station at d = M+(total distance traveled), then round down to the nearest station. This way you get as close to the optimal d_n/M as possible.

3. (1 point) Let 'maximum spanning tree' be defined as a spanning tree with the maximum total weight. Define the *cut property* for maximum spanning tree as follows. Suppose *X* is a set of edges in a maximum spanning tree. Choose a set of vertices *S* such that no edges in *X* cross from nodes in *S* to nodes in *V-S*. Let *e* be the heaviest edge not in *X* that crosses from *S* to *V-S*. Show that *X union {e}* is a subset of a maximum spanning tree.

If *e* is the heaviest edge not currently in the maximum spanning tree then there is at least one vertex not currently in the maximum spanning tree. If *e* is the heaviest edge to this vertex, then *e* must be part of the maximum spanning tree. Therefore, X union {e} is a subset of a maximum spanning tree.

4. (1 point) A barber shop serves *n* customers in a queue. They have service times *t1,...tn*. Only one customer can be served at any time. The waiting time for any customer is the sum of the service times of all previous customers. How would you order the customers so that the total waiting time for all customers will be minimized? Carefully justify your answer.

The customers with the lowest service time should be put at the front of the queue. The optimal solution would be for each customer to only take a single time slot. The more customers there are, the lower the average wait time. Therefore, by placing the customers with the smallest service times at the front, we can maximize the number of customers in the queue.