

Report 6

2. (1 point) You are driving on a long highway with gas stations at distances  $d_1 < \dots < d_n$  miles from the starting location  $S$ . Your car can run  $M$  miles with a full gas tank. You start with a full gas tank and want to reach the final location which is at  $d_n$  miles from  $S$ . How would you choose the gas stations to minimize the number of refueling stops? Argue that no other choice can make fewer stops.

3. (1 point) Let 'maximum spanning tree' be defined as a spanning tree with the maximum total weight. Define the *cut property* for maximum spanning tree as follows. Suppose  $X$  is a set of edges in a maximum spanning tree. Choose a set of vertices  $S$  such that no edges in  $X$  cross from nodes in  $S$  to nodes in  $V-S$ . Let  $e$  be the heaviest edge not in  $X$  that crosses from  $S$  to  $V-S$ . Show that  $X \cup \{e\}$  is a subset of a maximum spanning tree.

4. (1 point) A barber shop serves  $n$  customers in a queue. They have service times  $t_1, \dots, t_n$ . Only one customer can be served at any time. The waiting time for any customer is the sum of the service times of all previous customers. How would you order the customers so that the total waiting time for all customers will be minimized? Carefully justify your answer.