Jonathan Koning
koningj@oregonstate.edu

Report 8

2. (1 point) Suppose you are running a conference and want to assign 40 papers to 12 reviewers. Each reviewer bids for 20 papers. You want each paper to be reviewed by 3 reviewers. Formulate this problem as a max-flow problem, i.e., describe the architecture of the network and the capacities of the edges. Assume a reviewer cannot review more than 11 papers. What is the maximum flow you can expect in your network?

- s = node 0 (you)
- t = node 13 (back to you)
- Nodes 1-12 are reviewers
- For any given node, the summed capacity of all outgoing edges cannot exceed 11 because that is the maximum number of papers any given reviewer can review.
- The shortest length of any given path between s-t should be 4 edges long.
  s-> reviewer1 -> reviewer2 -> reviewer3 -> t
- The maximum flow should be 44 because 4 groups of 3 reviewers, each group reviewing 11 papers will have a max flow of 44 papers

3. (1 point: Hall's theorem) Consider a bipartite matching problem of matching *N* boys to *N* girls. Show that there is a perfect match **if and only if** every subset of S of boys is connected to at least |S| girls. Hint: Consider applying the Max-flow Min-cut theorem.

By definition of a perfect matching, every node has exactly 1 edge. Therefore, given a bipartite graph with a group A, the boys, and a group B, the girls, for there to be a perfect matching every subset S of A is matched with a different node of B. This means that |S| <= |not S|.

4. (1 point) Suppose you want to find the shortest path from node *s* to *t* in a directed graph where edge *(u,v)* has length *l[u,v] > 0*. Write the shortest path problem as a linear program. Show that the dual of the program can be written as *Max X[s]-X[t]*, where *X[u]-X[v] <= l[u,v]* for all *(u,v) in E.* [An interpretation of this dual is given in page 290 of DPV.]

Minimize L[u,v] from s to t
Subject to L[u,v] > 0 for all (u,v), s <= u < v <= t