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CS 514

HW1

```
def factors(num):
  if(num == 1):
    return []
  if(num%2 == 0):
    return [2] + factors(int(num/2))

  # find all prime numbers that could be factors
  # Only have to search up to sqrt(num) because any number higher will not be a factor
  i = 3
  while(i*i <= num):                        #sqrt(num)/2 times
    isprime = 0
    #check if i is prime
    # To determine if a number is prime, we only have to have to search
    # until the sqrt of the number because all other numbers will be a multiple
    # of a value already checked
    j = 3
    while (j*j <= i):                       #sqrt(sqrt(n))/2 * sqrt(n)/2 times
      #Check if i is a multiple of j
      if(i%j == 0):                         #sqrt(sqrt(n))/2 * sqrt(n)/2 times
        isprime = 1
        #breaks when i is not prime
        break
      j+=2                                  #sqrt(sqrt(n))/2 * sqrt(n)/2 times
    #if i is prime, check if it is a factor of num
    if(isprime == 0):
      if(num%i == 0):                       #sqrt(num)/2 times
        return [i] + factors(int(num/i))
    i+=2                                    #sqrt(num)/2 times

  #if num is prime, return
  return [int(num)]
```

2)

2. a) Derivation of the running time assuming that multiplications and additions take constant time

T(n) = $\sum_{x=0}^{log_2 n} c * \sqrt{n}/2 + c * \left(\sqrt{\sqrt{n}}\right)/2 * \sqrt{n}/2$

T(n) = $c * \left(\sqrt{n}/2 + \sqrt{\sqrt{n}}/2 * \sqrt{n}/2\right) * log_2 n$

O(n) = $\left(\sqrt{n}/2 + \sqrt{\sqrt{n}}/2 * \sqrt{n}/2\right) * log_2 n$

2. b) Derivation of the running time assuming multiplication and division of n-bit numbers take O(n^2) time and additions and subtractions take O(n) time

T(n) = T(n/2) + O(n^2) + O(n^2) + O(n^2) + O(n^2) + O(n^2) + O(n^2) + O(n) + O(n)

T(n) = T(n/2) + 5O(n^2) + 2O(n)

T(n) = $(5n^2 + 2n)log_2 n$

3) Give a table T(n) vs n

| n | T(n) |
|---|---|
| 1 | 0:00:00.000002 |
| 11 | 0:00:00.000002 |
| 111 | 0:00:00.000006 |
| 1111 | 0:00:00.000004 |
| 11111 | 0:00:00.000007 |
| 111111 | 0:00:00.000005 |
| 1111111 | 0:00:00.000034 |
| 11111111 | 0:00:00.000018 |
| 111111111 | 0:00:00.000078 |
| 1111111111 | 0:00:00.000045 |
| 11111111111 | 0:00:00.009810 |
| 111111111111 | 0:00:00.000026 |
| 1111111111111 | 0:00:00.006192 |
| 11111111111111 | 0:00:00.001302 |
| 111111111111111 | 0:00:00.000342 |
| 1111111111111111 | 0:00:00.000515 |
| 11111111111111111 | 0:00:05.288088 |
| 111111111111111111 | 0:00:00.127748 |
| 1111111111111111110 | 0:00:00.165751 |
| 1111111111111111111 | Over 16 minutes before I gave up |
| 1111111111111111112 | 0:00:00.166283 |
| 22222222222222222222 | 0:00:00.000167 |
| 1 | 0:00:00.000001 |
| 2 | 0:00:00.000003 |
| 1369 | 0:00:00.000004 |
| 2209 | 0:00:00.000005 |
| 2614 | 0:00:00.000006 |
| 4946 | 0:00:00.000008 |

| | |
|---|---|
| 9769 | 0:00:00.000009 |
| 10609 | 0:00:00.000010 |
| 10623 | 0:00:00.000012 |
| 18144 | 0:00:00.000016 |
| 19543 | 0:00:00.000017 |
| 20147 | 0:00:00.000019 |
| 36241 | 0:00:00.000020 |
| 36749 | 0:00:00.000021 |
| 37253 | 0:00:00.000022 |
| 44563 | 0:00:00.000031 |
| 77317 | 0:00:00.000032 |
| 85831 | 0:00:00.000033 |
| 85849 | 0:00:00.000034 |
| 86171 | 0:00:00.000035 |
| 86857 | 0:00:00.000036 |
| 94201 | 0:00:00.000047 |
| 94249 | 0:00:00.000049 |
| 94513 | 0:00:00.000052 |
| 97117 | 0:00:00.000053 |
| 101009 | 0:00:00.000054 |
| 102259 | 0:00:00.000055 |
| 108541 | 0:00:00.000056 |
| 109741 | 0:00:00.000057 |
| 145441 | 0:00:00.000065 |
| 146519 | 0:00:00.000066 |
| 146521 | 0:00:00.000068 |
| 166043 | 0:00:00.000071 |
| 167311 | 0:00:00.000073 |
| 184153 | 0:00:00.000076 |
| 185069 | 0:00:00.000080 |
| 202567 | 0:00:00.000082 |
| 203761 | 0:00:00.000085 |
| 214603 | 0:00:00.000086 |
| 216217 | 0:00:00.000089 |
| 232381 | 0:00:00.000095 |
| 243517 | 0:00:00.000096 |
| 250073 | 0:00:00.000097 |
| 347981 | 0:00:00.000104 |
| 348709 | 0:00:00.000119 |
| 513829 | 0:00:00.000147 |
| 513833 | 0:00:00.000151 |
| 513839 | 0:00:00.000157 |
| 624607 | 0:00:00.000160 |
| 864757 | 0:00:00.000181 |
| 864855 | 0:00:00.000219 |
| 880699 | 0:00:00.000456 |

| | |
|---|---|
| 881357 | 0:00:00.000504 |
| 896669 | 0:00:00.000584 |
| 1019399 | 0:00:00.000697 |
| 1188287 | 0:00:00.000721 |
| 3347768 | 0:00:00.000750 |
| 3386191 | 0:00:00.000943 |
| 3431933 | 0:00:00.000960 |
| 3632054 | 0:00:00.001019 |
| 3777601 | 0:00:00.001151 |
| 3925541 | 0:00:00.001154 |
| 7216151 | 0:00:00.001185 |
| 8160043 | 0:00:00.001319 |
| 8891093 | 0:00:00.001332 |
| 9426383 | 0:00:00.001393 |
| 9492827 | 0:00:00.001395 |
| 9859693 | 0:00:00.002396 |
| 20503429 | 0:00:00.002439 |
| 21652513 | 0:00:00.002553 |
| 23210263 | 0:00:00.002567 |

T(n) = $c * \left(\sqrt{n}/2 + \sqrt{\sqrt{n}}/2 * \sqrt{n}/2\right) * log_2 n$

T(377601) = 0.001151 = $c * \left(\sqrt{377601}/2 + \sqrt{\sqrt{377601}}/2 * \sqrt{377601}/2\right) * log_2 377601$

= $c * (307 + 12 * 307) * 18.5 = c * 73833.5$

C = $\frac{0.001151}{73833.5}$

For 5 seconds T(n) = 5

5 = $\frac{0.001151}{73833.5} * \left(\sqrt{n}/2 + \sqrt{\sqrt{n}}/2 * \sqrt{n}/2\right) * log_2 n$

321015217.4 = $\left(\sqrt{n}/2 + \sqrt{\sqrt{n}}/2 * \sqrt{n}/2\right) * log_2 n$

n = 5.474

For 5 minutes T(n) = 300

300 = $\frac{0.001151}{73833.5} * \left(\sqrt{n}/2 + \sqrt{\sqrt{n}}/2 * \sqrt{n}/2\right) * log_2 n$

321015217.4 = $\left(\sqrt{n}/2 + \sqrt{\sqrt{n}}/2 * \sqrt{n}/2\right) * log_2 n$

n = 2.3086 * 10^12

For 5 hours T(n) = 18000

18000 = $\frac{0.001151}{73833.5} * \left(\sqrt{n}/2 + \sqrt{\sqrt{n}}/2 * \sqrt{n}/2\right) * log_2 n$

n = 4.3365*10^14

For 5 days T(n) = 432000

n = 2.5783*10^16

For > 10 years T(n) = 2.024707179*10^16

n = 1.2925 * 10^20