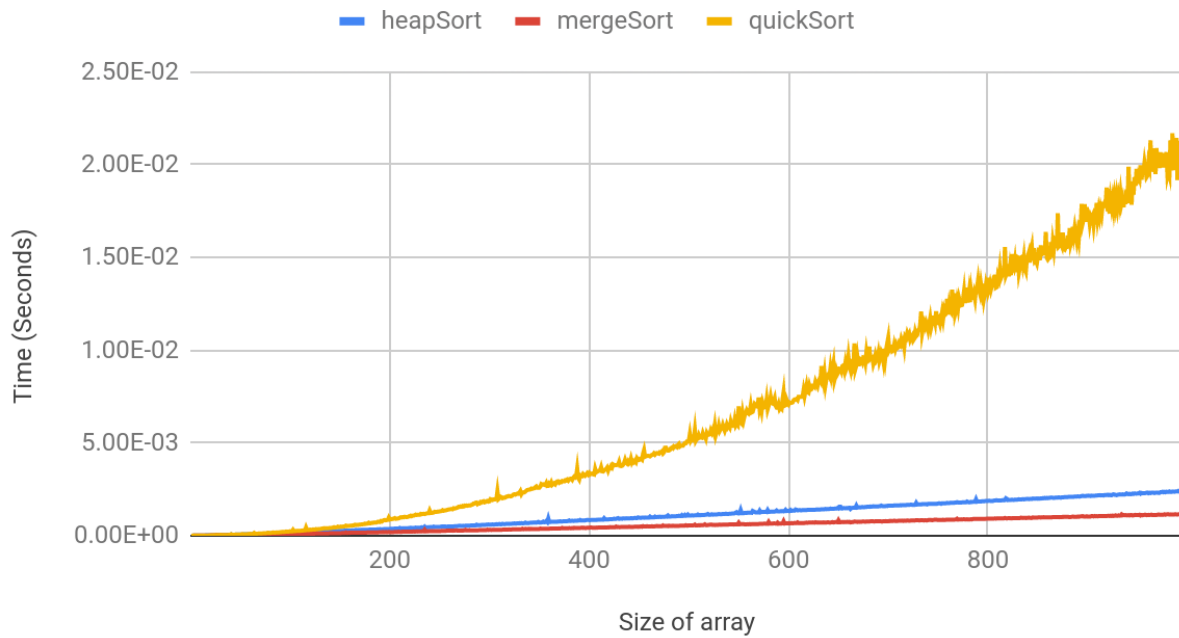
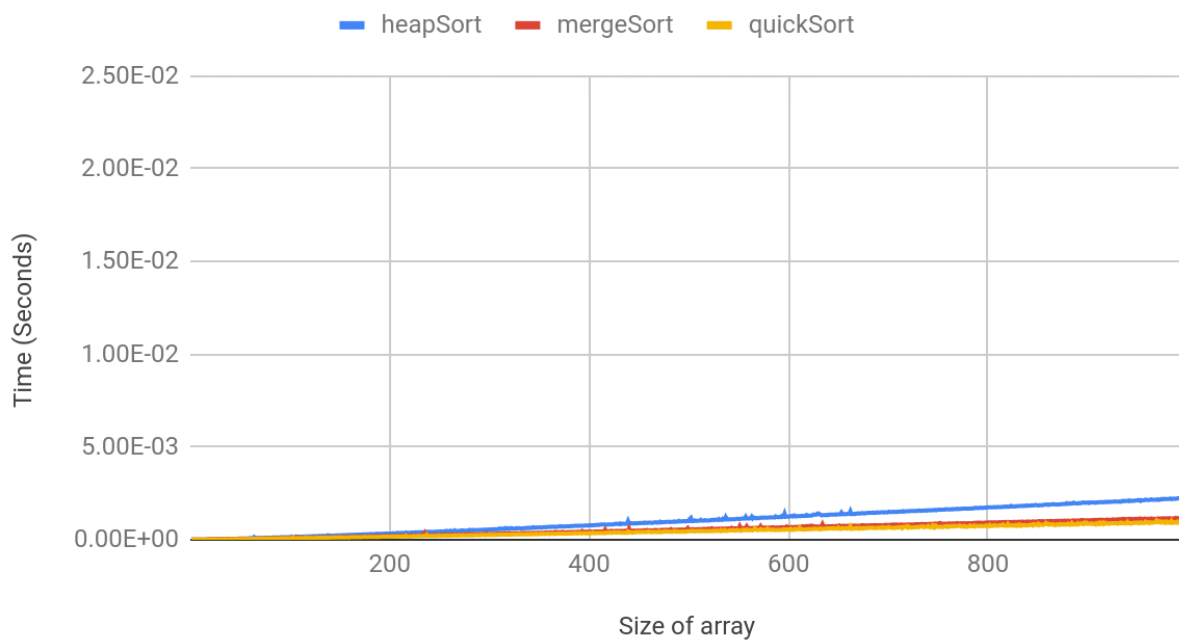


### HW3

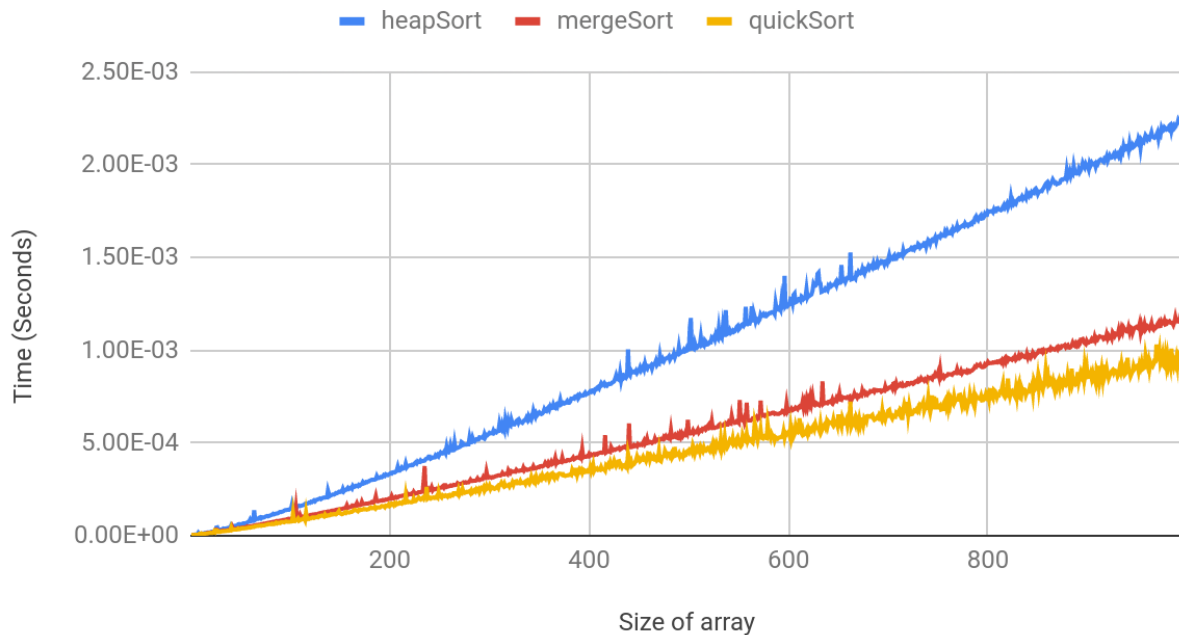
#### Sorted Data



#### Random Data



## Random Data



The only difference between the two Random Data graphs is the scale. Both are representing the same set of data.

Consider two *sorted* arrays  $A$  of size  $m$  and  $B$  of size  $n$ .

a) Design an efficient ( $O(\log m + \log n)$ ) Divide-and-Conquer algorithm to find the  $k$ 'th element in the merged array. To be efficient, you should do this without actually merging the two arrays.

Def kelement( $A, B, k$ ):

- 1) Check if the lengths of either of the arrays is 1.
  - a) If so, determine the larger array. Let's assume  $A$  is the larger array.
  - b) If  $k == 1$ , return  $\min(A[0], B[0])$ .
  - c) If  $k == 2$ , return  $\max(A[0], B[0])$ .
  - d) Else if  $A[k-1] < B[0]$ , return  $A[k-1]$
  - e) Else return  $\max(A[k-2], B[0])$
- 2) Find mid indices,  $mid1$  and  $mid2$ , of arrays  $A$  and  $B$  respectively
- 3) If  $mid1 + mid2 + 1 < k$ , we know that we will be dropping the first half of one of the arrays
  - a) If  $A[mid1] < B[mid2]$ , return  $\text{kelement}(A[mid1+1:], B, k-mid1-1)$ . We drop the first half of  $A$  because it has the most small numbers before the  $k$ th element
  - b) Else, return  $\text{kelement}(A, B[mid2+1:], k-mid2-1)$  for the same reasoning
- 4) If  $mid1+mid2+1 > k$ , we know that we will be dropping the second half of one of the arrays
  - a) If  $A[mid1] < B[mid2]$ , return  $\text{kelement}(A, B[:mid2+1], k)$ . We drop the second half of  $B$  because it has the most large numbers after the  $k$ th element

b) Else return kelement( $A[:mid1+1]$ , B, k). We drop the second half of A for the same reasoning

(b) Prove that the time complexity is  $O(\log n + \log m)$ .

The algorithm above is  $O(\log n + \log m)$  because with each iteration, one of the arrays is divided in half. We stop dividing in half once one of the arrays is down to size 1. The maximum amount of times we can divide each array is  $\log(\text{size of array})$ . So the worst case scenario is dividing both arrays until they are both of size 1. Because the size of one array is independent of the other, we must add the complexities together making the total complexity  $O(\log n + \log m)$