

Report 8

2. (1 point) Suppose you are running a conference and want to assign 40 papers to 12 reviewers. Each reviewer bids for 20 papers. You want each paper to be reviewed by 3 reviewers. Formulate this problem as a max-flow problem, i.e., describe the architecture of the network and the capacities of the edges. Assume a reviewer cannot review more than 11 papers. What is the maximum flow you can expect in your network?

- s = node 0 (you)
- t = node 13 (back to you)
- Nodes 1-12 are reviewers
- For any given node, the summed capacity of all outgoing edges cannot exceed 11 because that is the maximum number of papers any given reviewer can review.
- The shortest length of any given path between s - t should be 4 edges long.
 $s \rightarrow \text{reviewer1} \rightarrow \text{reviewer2} \rightarrow \text{reviewer3} \rightarrow t$
- The maximum flow should be 44 because 4 groups of 3 reviewers, each group reviewing 11 papers will have a max flow of 44 papers

3. (1 point: Hall's theorem) Consider a bipartite matching problem of matching N boys to N girls. Show that there is a perfect match **if and only if** every subset of S of boys is connected to at least $|S|$ girls. Hint: Consider applying the Max-flow Min-cut theorem.

4. (1 point) Suppose you want to find the shortest path from node s to t in a directed graph where edge (u,v) has length $l[u,v] > 0$. Write the shortest path problem as a linear program. Show that the dual of the program can be written as $\text{Max } X[s] - X[t]$, where $X[u] - X[v] \leq l[u,v]$ for all $(u,v) \in E$. [An interpretation of this dual is given in page 290 of DPV.]