Labor Mobility in a Green Transition

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Abstract

Changelog:

- Version 1: Only age in state space. No stochasticity.
- Version 2: Added stochasticity in the form of Type I extreme value sector-specific preference shocks. This implies that the model must be solved in expected value function space.
- Version 3: Implemented switching costs directly into utility. A worker's previously chosen sector enters her state space because switching costs vary across sector-pairs.
- Version 4: Added firms and their production functions which take four inputs: human capital, physical capital, clean energy and dirty energy. Dirty energy generates emissions. I have closed the model by assuming Cobb-Douglas utility from consumption and by specifying the supply of physical capital. Some products are tradeables and which case their output price is exogenous (small country assumption). Other products are non-tradeables and their price is determined endogenously.
- **Version 5:** In terms of additional sections, I have added the following:
  Brief introduction including literature review and contribution (section 1), description of estimation method (section 3) including an estimation routine in appendix D and finally I reordered and rewrote the model section (section 2) and added a description of the human capital equilibrium (section 2.2.7).

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1 Introduction

With climate scientists repeatedly calling for action to mitigate climate change, politicians and governments around the world have committed themselves to reduce the amount of green house gasses emitted into the atmosphere. The US has Build Back Better, the EU has the European Green Deal, the world has the Paris agreement and Denmark, the focus of this paper, has the Climate Law. Governments have set goals that require significant political action. A market-based regulation such as a carbon tax is the solution preferred by most economists.

Politicians often hesitate in following the advice by citing issues such as job loss or inequality. For example the EU's "European Green Deal" explicitly states that the plan must "leave no person behind"[1] However, knowledge about how carbon taxes affect the labor market in the short and medium run is scant.

In this paper, I propose a simple extension of a dynamic discrete choice model framework used in the labor literature to include emissions and labor market frictions important hereto. I do so to answer the question of what happens to labor markets when a carbon tax is implemented. The rich model allows me to understand the heterogeneity of the effects across workers of different educations, genders and ages. The model features workers who solve a dynamic discrete choice problem in order to supply human capital to firms who emit green house gasses. There are multiple barriers to labor mobility, making it suitable for analyzing the labor market effects of various climate regulations, such as a carbon tax. Parameters are estimated using Danish linked worker-firm data. Counterfactual simulations show if particular individuals are hit harder than other from carbon taxes and quantifies the overall costs for workers.

The results have policy implications. By estimating the size of the losses for workers, politicians can make a more informed decision about whether and how to enact carbon taxes. For example, if a government wishes to compensate the losers from a carbon tax, the model can help identify who loses and quantify by how much. The model can also shed light on the importance of the timing of policy implementations e.g. the benefits of announcing taxes in advance or gradual phase ins.

My paper relates to three strands of literature. First, researchers have produced

[1] See `https://ec.europa.eu/info/strategy/priorities-2019-2024/european-green-deal_en`. Visited on April 8 2022.

reduced form evidence of how environmental regulation affects labor markets. Walker (2013) used a difference-in-differences strategy to estimate the employment and earning loss effects of the 1990 Clean Air Act Amendments (CAAA) in the U.S. Although related, the CAAA was a non-market based regulation designed to reduce emissions of non-green house gas emissions such as sulfur dioxide (SO_2). While he controls for various worker characteristics, the differential effects across workers are not a particular focus.

Yip (2018) performs a reduced form study of the carbon tax enacted in British Columbia in Canada also using a difference-in-differences identification strategy. In terms of research question, his study is the one that comes the closest to mine. The estimations show that especially low-educated workers are hit the hardest by the carbon tax. As he himself notes, one might worry that the results are driven by an adverse labor market shock hitting the same province as the carbon tax was enacted in. Corroborating this, the implementation of the carbon tax overlaps quite closely with the beginning of the financial crisis.

The second strand of literature tries to answer the same questions about carbon taxes and employment effects using structural models. There are at least two reasons why structural models add value compared to reduced form studies. First, a frequent shortcoming of reduced form papers in this field is the existence of spillovers through the labor market. The fact that the control group used for identification is often indirectly affected by general equilibrium effects such as price or wage changes means the external validity of a result is less clear (Hafstead and Williams 2018). Second, we are interested in the effects of counterfactual carbon taxes that have not been implemented yet. Examples of structural approaches to these questions include the search and matching model of Hafstead and Williams (2018)[2] and the computable general equilibrium model of Goulder et al. (2019). The former assumes homogeneous households whereas the latter does not model the labor supply decision of households explicitly. By modeling multiple dimensions of heterogeneity across individuals in a model framework ideal for analyzing labor supply decisions, I add to this strand of the literature.

Thirdly, the dynamic discrete choice model framework that I employ has been used in

[2]The model of Hafstead and Williams (2018) features two sectors (clean and dirty) as well as an unemployment sector. It is a search model that does not allow sector-to-sector switches nor on-the-job-search. Their model is simplistic in a variety of dimensions where mine is not: Workers are homogeneous, there is no physical capital, it only involves two sectors and relies on calibration rather than estimation without individual-level data.

multiple earlier papers to analyze the labor market effects of e.g. international trade (Dix-Carneiro 2014; Ashournia 2018; Traiberman 2019) and automation Humlum (2019). To the best of my knowledge, I am the first to extend the framework for use in environmental applications.

2 Model

In each year, a firm representing each sector produces using four inputs: human capital, physical capital, clean energy and dirty energy. Dirty energy represents the firm's use of fossil fuels and generates carbon dioxide emissions. Human capital is supplied by workers of age 30 to 65 after which they retire. In each period, each worker receives a wage offer from each sector that depends on the amount of human capital that she can supply to that sector. With these offers, she chooses the sector which maximizes her expected discounted lifetime utility. Switching sectors has a direct utility cost as well as a wage cost since she loses some of her accumulated human capital by switching.[3] The model equilibrium is found by solving for the values of skill prices that clear the market for human capital.

2.1 Firms and emissions

The set of sectors, denoted $\mathcal{S}$, contains S sectors indexed by s. The discrete time dimension spans T periods indexed by $t = 0, 1, \ldots, T-1$. In each sector s, a representative firm produces its output using four inputs: human capital, H_{st}, physical capital, K_{st}, clean energy E_{st} and fossil fuel based energy, O_{st}. The production technology is given by the Cobb-Douglas function

$$Y_{st} = A_{st} H_{st}^{\alpha_{st}^1} K_{st}^{\alpha_{st}^2} \left(E_{st}^{\theta_s} + O_{st}^{\theta_s} \right)^{\frac{1-\alpha_{st}^1-\alpha_{st}^2}{\theta_s}}. \tag{1}$$

where A_{st} represents TFP and $\left(E_{st}^{\theta_s} + O_{st}^{\theta_s} \right)^{\frac{1}{\theta_s}}$ is a CES aggregate governing the firm's use of energy. The use of fossil fuels generates carbon dioxide emissions Z_{st} with a fixed coefficient η_s,

$$Z_{st} = \eta_s O_{st}. \tag{2}$$

Notice that η_s varies across sectors. This reflects that the mix of dirty energy inputs such as oil, coal and natural gas varies across sectors. Since η_s does not vary across time, I implicitly assume that the optimal mix of dirty inputs is fixed throughout for a given sector.

The representative firm maximizes profits under perfect competition taking input and

[3]NOTE: The accumulation effect has not been implemented yet!

output prices and the emission tax τ_s as given. Denoting the output price p^Y_{st}, the rental price of physical capital r^K_{st}, the skill price of human capital r^H_{st}, the price of clean energy p^E_{st} and the price of dirty energy p^O_{st}, the problem has the four first order conditions

$$\begin{aligned}
r^H_{st} &= p^Y_{st} A_{st} \alpha^1_{st} H^{\alpha^1_{st}-1}_{st} K^{\alpha^2_{st}} \left(E^{\theta_s}_{st} + O^{\theta_s}_{st}\right)^{\frac{1-\alpha^1_{st}-\alpha^2_{st}}{\theta}} \\
&= \alpha^1_{st} p^Y_{st} \frac{Y_{st}}{H_{st}} && (3) \\
r^K_{st} &= p^Y_{st} A_{st} \alpha^2_{st} K^{\alpha^2_{st}-1}_{st} H^{\alpha^1_{st}} \left(E^{\theta_s}_{st} + O^{\theta_s}_{st}\right)^{\frac{1-\alpha^1_{st}-\alpha^2_{st}}{\theta}} \\
&= \alpha^2_{st} p^Y_{st} \frac{Y_{st}}{K_{st}} && (4) \\
p^E_{st} &= p^Y_{st} A_{st} \left(1-\alpha^1_{st}-\alpha^2_{st}\right) \left(E^{\theta_s}_{st} + O^{\theta_s}_{st}\right)^{\frac{1-\alpha^1_{st}-\alpha^2_{st}}{\theta}-1} E^{\theta-1} K^{\alpha^2_{st}}_{st} H^{\alpha^1_{st}} \\
&= \left(1-\alpha^1_{st}-\alpha^2_{st}\right) p^Y_{st} \frac{E^{\theta_s}}{E^{\theta_s}+O^{\theta_s}} \frac{Y_{st}}{E_{st}} && (5) \\
\eta_s \tau_{st} + p^O_{st} &= p^Y_{st} A_{st} \left(1-\alpha^1_{st}-\alpha^2_{st}\right) \left(E^{\theta_s}_{st} + O^{\theta_s}_{st}\right)^{\frac{1-\alpha^1_{st}-\alpha^2_{st}}{\theta}-1} O^{\theta-1} K^{\alpha^2_{st}}_{st} H^{\alpha^1_{st}} \\
&= \left(1-\alpha^1_{st}-\alpha^2_{st}\right) p^Y_{st} \frac{O^{\theta_s}}{E^{\theta_s}+O^{\theta_s}} \frac{Y_{st}}{O_{st}}. && (6)
\end{aligned}$$

We denote sector $s = 0$ as the "unemployment sector". This sector has no production function but simply offers a fixed "wage" corresponding to some relevant average of Danish welfare transfers.

2.2 Workers

In each year, workers active on the labor market choose a sector s to work in to receive the wage w_{ist}. There is no saving or borrowing, and the number of hours worked is fixed, making the choice a discrete one. A worker is active as long as her age a_{it} is between 30 and 65, after which she leaves the labor market. Each worker is characterized by set of state variables collected in Ω_{it}.

The instantaneous utility function for the worker is the sum of wages w, switching costs M and unobserved (to the econometrician) sector-specific preference shocks ε. Each of these elements will be elaborated on below.

Denote by $d_{ist} \in \{0, 1\}$ a dummy variable equal to one when the chosen sector is s in year t for worker i. With this notation the value function at time ι, i.e. the expected present value of the maximum attainable utility, is the solution to the utility maximization problem,

$$V_\iota(\Omega_{i\iota}) = \max_{\{d_{ist}\}_{s\in S, t=\iota,\dots,T-1}} E_\varepsilon \left[\sum_{t=\iota}^{T-1} \rho^t \sum_{s\in S} d_{ist} \left(w_{ist} - M(s, s_{t-1}) + \varepsilon_{ist} \right) \right] \tag{7}$$

where ρ denotes the discount factor and the subscript on the expectations operator reflects that the expectation is with respect to the unobserved preference shock ε.

The maximization of expected utility obeys the Bellman equation

$$V_t(\Omega_{it}) = \max_s V_{st}\left(\Omega_{it}\right). \tag{8}$$

where each of the alternative-specific value functions are given by

$$V_{st}(\Omega_{it}) = \begin{cases} w_{ist} - M(s, s_{it-1}) + \varepsilon_{ist} + \rho E_\varepsilon \left[V_{t+1}\left(\Omega_{it+1}\right) | d_{ist} \right] & \text{for } a_{it} < 65 \\ w_{ist} - M(s, s_{it-1}) + \varepsilon_{ist} & \text{for } a_{it} = 65 \end{cases} \tag{9}$$

for each s and where continuation values are zero when the worker is of age 65 since she will retire in the end of the year. The conditioning of $V_{t+1}\left(\Omega_{it+1}\right)$ on d_{ist} reflects that current choices has implications for next year's state space and therefore continuation values. The state space includes a worker's age, previous sector and her expectation of current and future skill prices:

$$\Omega_{it} = \left\{ a_{it}, s_{it-1}, \{r_{st+\iota}\}_{\iota=0}^{65-a_{it}} \right\}. \tag{10}$$

Strictly speaking, the vector of unobserved preference shocks across sectors $\boldsymbol{\varepsilon_{it}}$ also enters the worker's state space, since she uses the realization of the shock to make her decision.

2.2.1 Wages and human capital accumulation

At the beginning of each period, the worker receives a wage offer from each sector. The wage offer $w_{ist}(\Omega_{it})$ is the product of two parts, the amount of effective human capital that she can supply to sector s, $H_s(\Omega_{it})$, and the unit skill price r_{st}. The worker always takes skill prices as given. For effective human capital, the only relevant part of the state space is a_{it}, and her wage offer is therefore given by

$$w_{ist} = H_s(a_{it}) r_{st} \tag{11}$$

The effective human capital function H_s is given by

$$H_s(a_{it}) \equiv \exp\left(\beta_s^0 a_{it}\right) \tag{12}$$

capturing that old workers on average earn a wage premium.[4]

2.2.2 Switching costs

The worker incurs direct utility costs when switching sectors. These costs could represent searching costs or simply a preference for not moving sector. Similar to Ashournia (2018) I parametrize these switching costs, denoted $M(s_t, s_{t-1})$, in the following way:

$$M(s_t, s_{t-1}) = \exp\left(m(s_t, s_{t-1})\right) \tag{13}$$

where

$$m(s_t, s_{t-1}) = \begin{cases} 0 & \text{if } s_t = s_{t-1} \\ \xi^{out}_{s_{t-1}} + \xi^{in}_{s_t} & \text{if } s_t \neq s_{t-1} \end{cases}. \tag{14}$$

This structure implies that staying in the same sector does not involve additional switching costs and that conditional on switching sectors, the direct utility cost consists of a fixed cost of leaving sector s_{t-1} and entering sector s_t.

Ashournia and Dix-Carneiro normalize one of these to zero. I dont see why this is necessary, they should be identified without

2.2.3 Skill price expectations

A worker's optimal choice today depends on her expectation of future skill prices, $\{r_{st+\iota}\}_{\iota=0}^{65-a_{it}}$. To make this forecast, I follow Lee (2005) and assume that workers have perfect foresight over future unit prices. Since the model terminates in period $T-1$ and workers aged $a < 65$ would take wages in period $t \geq T$ into account, I assume that they act as if skill prices stay fixed for the rest of their time on the labor market, following

[4] Depending on β_s^0, the premium could be negative. The human capital function will be extended in later versions of the paper.

Ashournia (2018). Essentially, the assumption is that workers have static expectations in period $T-1$, i.e. they expect future ($t > T-1$) skill prices to equal current skill prices:

$$\{r_{sT-1+t}\}_{t=1}^{65-a_{iT-1}} = \{r_{sT-1}\}_{t=1}^{65-a_{iT-1}}, \tag{15}$$

where the lack of i subscript on r reflects that all workers have the same expectation about future prices. To summarize, workers always predict skill prices correctly, but when they forecast skill prices that do not exist in the model because they lie later than $T-1$, they simply predict the skill prices observed in $T-1$.

I simply want to say that the last period price is "duplicated" $65 - a_{it}$ times. How can I write that?

2.2.4 Unobserved types

2.2.5 The unemployment sector

2.2.6 Solving the worker's dynamic program

To solve the worker's problem, distributional assumptions on the idiosyncratic preference shocks must be made. I assume that these are independent across sectors and time, and follow the Gumbel, or Type I extreme value, distribution,

$$\varepsilon_{ist} \sim \text{Gumbel}(-0.57721\sigma, \sigma) \tag{16}$$

where 0.57721 is Euler's constant.[5] As explained e.g. in Traiberman (2019), the scale parameter σ determines the worker's sensitivity to wage differentials. When σ is larger, extreme draws are more likely leaving wage differentials with less influence on the optimal choice. As is well known, this particular distributional assumption is computationally convenient because it produces closed-form solutions for continuation values and conditional choice probabilities.

The expected value function, also called the EMAX function or the integrated value function, has the closed form (see e.g. Aguirregabiria 2021):

$$E_\varepsilon\left[V_t(\Omega_{it})\right] = \sigma \log\left(\sum_s \exp\left(\frac{w_{ist} - M(s, s_{it-1}) + \rho E_\varepsilon\left[V_{t+1}(\Omega_{it+1})|d_{ist}\right]}{\sigma}\right)\right). \tag{17}$$

The corresponding conditional (on the state space) choice probabilities also take the

[5]The recentering of the mean implies that shocks have a mean of zero.

common logit-form:

$$P(d_{ist}=1|\Omega_{it}) = \frac{\exp\left(\frac{w_{ist}-M(s,s_{it-1})+\rho E_{\varepsilon}[V_{t+1}(\Omega_{it+1})|d_{ist}]}{\sigma}\right)}{\sum_j \exp\left(\frac{w_{ijt}-M(j,s_{it-1})+\rho E_{\varepsilon}[V_{t+1}(\Omega_{it+1})|d_{ijt}]}{\sigma}\right)}. \tag{18}$$

These probabilities make up the policy function, i.e. the policy function at a particular point in the state space and in time is a vector (bold notation) of probabilities P,

$$\boldsymbol{P}(\Omega_{it}) = \Big(P(d_{i0t}=1|\Omega_{it}),\ldots,P(d_{iSt}=1|\Omega_{it})\Big). \tag{19}$$

The full solution procedure for the worker's problem is stated in appendix A.

2.2.7 Equilibrium on the market for human capital

The unit price of human capital is determined in equilibrium. It is the sequence of skill prices that, in each year and in each sector, equates human capital demand and human capital supply. To measure human capital supply, I introduce a few definitions. MASS_t is the mass of individuals in the economy at time t. $\widetilde{\Omega}$ is the collection of all possible points in the state space, excluding skill prices, and I index each point by ω. $D_t(\omega)$ measures the share of individuals in the population characterized by the point ω at time t such that $\sum_{\omega\in\widetilde{\Omega}} D_t(\omega) = 1$. Finally, define by $P\left(d_{\omega st}=1|\,\omega,\{\boldsymbol{r}_{t+\iota}\}_{\iota=0}^{65-30}\right)$ the probability that an individual, defined by the state space point ω as well as the expectation of future skill prices, chooses to work in sector s at time t. The bold notation $\boldsymbol{r}_t$ refers to the vector of skill prices across sectors $s\in\mathcal{S}$ at time t.

With these definitions, total supply of human capital is given by

$$H_{st}^{supply}(\{\boldsymbol{r}_{t+\iota}\}_{\iota=0}^{65-30}) = \text{MASS}_t \sum_{\omega\in\widetilde{\Omega}} D_t(\omega) P\left(d_{\omega st}=1|\,\omega,\{\boldsymbol{r}_{t+\iota}\}_{\iota=0}^{65-30}\right) \qquad \forall s,t. \tag{20}$$

With human capital demand given by

$$H_{st}^{demand}(r_{st}) = \alpha_{st}^{1} p_{st}^{Y} \frac{Y_{st}}{r_{st}} \tag{21}$$

from (3), the equilibrium conditions which implicitly define the sequence of skill prices are

$$
\begin{aligned}
H^{supply}_{st}(\{\boldsymbol{r}_{t+\iota}\}^{65-30}_{\iota=0}) &= H^{demand}_{st}(r_{st}) \\
\text{MASS}_t \sum_{\omega\in\widetilde{\Omega}_t} D_t(\omega) P\left(d_{\omega st}=1 \middle|\, \omega, \{\boldsymbol{r}_{t+\iota}\}^{65-30}_{\iota=0}\right) &= \alpha^1_{st} p^Y_{st}\frac{Y_{st}}{r_{st}} \qquad \forall s,t. \qquad (22)
\end{aligned}
$$

This concludes the model parts sufficient for estimation.

2.3 Closing the model

To close the model for counterfactual simulations, additional structure and assumptions are necessary. I invoke the small country assumption and assume that the output prices of tradeable sectors are exogenous. Closing the model means endogenizing the output prices in the non-tradeable sectors, p^Y_s for $s \in \mathcal{S}^{NT}$, as well as specifying physical capital supply.

To determine the equilibrium price of non-tradeables, I must specify the, necessarily domestic, demand for these products. To do so, I follow the previous literature (Dix-Carneiro 2014; Ashournia 2018) and assume that consumers' utility over consumption C_{st} is Cobb-Douglas in the $S-1$ consumable products (sector 0 does not produce anything since it represents unemployment):

$$
u(C_{1t},\dots,C_{st},\dots,C_{S-1t}) = \prod_{s=1}^{S-1} C_{st}^{\mu_s} \qquad (23)
$$

implying that the fraction μ_s of income is spent on the good from sector s. I assume that all factors (including energy) being paid consume only in Denmark and that the proceeds from carbon taxes are paid back to consumers in a lump-sum fashions. I also assume that unemployment is financed via lump-sum transfers from employed workers and the other factors. This implies that total income for consumers is given by $\sum_{s=1}^{S-1} p^Y_s Y_s$ (note how the unemployment sector is left out since it represents no net-gain to income in the aggregate).

By definition for nontradeables, what is produced domestically must all be consumed by domestically. Therefore, the equilibrium condition in nominal terms is

$$p_s^Y Y_s = p_s^Y C_s = \mu_s \sum_{j=1}^{S-1} p_j^Y Y_j \tag{24}$$

which must hold for all $s \in \mathcal{S}^{NT}$. This equation system of S^{NT} equations and unknowns can be solved to yield the prices p_s^Y.

To fully close the model, assumptions on physical capital supply are necessary. I will experiment with two different assumptions:

- Assume that physical capital is sector-specific and fixed. This means the return to physical capital varies in simulations, or
- Assume that the return to physical capital is fixed but still sector specific. In the model, the quantity of physical capital in each sector is residually determined to ensure the sector-specific returns are kept fixed. This assumption was made in Ashournia (2018).

3 Estimation

Estimation proceeds in three steps. First, calibration of a number of parameters including the discount factor. Second, a reduced-form estimation of the elasticity governing energy use, θ. Third, a structural estimation of the remaining model parameters.

3.1 Calibration

The calibrated parameters are: αs and the discount factor ρ. I also calibrate the consumption share parameters μ.

3.2 Estimation of the energy elasticity

The energy elasticity is estimated by relying on two model assumptions. First, the functional form of the production function and second, the assumption of exogenous energy prices. Dividing the firm's first order condition for clean energy by the one for dirty energy, taking logs and rearranging gives

$$\log\left(\frac{E_{st}}{O_{st}}\right) = \frac{1}{\theta_s - 1}\log\left(\frac{p_t^E}{\eta_s \tau_{st} + p_s^O}\right) \qquad \forall t \tag{25}$$

This equation will not hold exactly in the data e.g. due to measurement error. Therefore, we can add an error term and run the equivalent regression once for each sector to recover an estimate of θ_s for each sector.

3.3 Estimation of the remaining model parameters

The remaining parameters are estimated by maximum likelihood. The initial period is 1996 and the terminal period is $T - 1 = 2016$. Throughout the estimation, we fix nominal value added deflated to 2016 $p_{st}^Y Y_{st}$ to equal its data counterpart $\overline{p_{st}^Y Y_{st}}$. The value of A_{st} is determined residually to make sure (1) is always upheld. Fixing value added eases estimation considerably, because the equilibrium human capital skill price can be found without regard to the remaining three inputs.

During estimation, I must simulate the economy forward. Rather than simulating distinct individuals, I measure the share of sector employment accounted for by individuals at each point in the state space. Building on the the definitions from section 2.2.7, define

$D_{st}(\omega)$ as the share of total employment in year t that is accounted for by employment in sector s of individuals with state space vector ω.[6] This implies that $\sum_{\omega \in \widetilde{\Omega}} D_{st}(\omega)$ is the employment share of sector s in year t.

By using the policy function, I can measure the share of individuals at each point represented by the share $D_{st}(\omega)$ who move to each of the sectors simply by calculating the vector $D_{st}(\omega)\boldsymbol{P}(\Gamma(\omega, s))$ where Γ is the transformation function which updates the state space in year $t+1$ for an individual who is characterized by ω and the choice of sector s in year t.[7] Using this methodology, we can measure employment shares in all years. To convert this into employment in levels, we simply multiply by MASS_t.

The methodology of calculating employment shares over time sidesteps the need to draw Gumbel shocks for a large number of individuals, saving computational time when simulating. This methodology was also employed in Humlum (2019). The full estimation procedure is outlined as a step-wise procedure in appendix D.

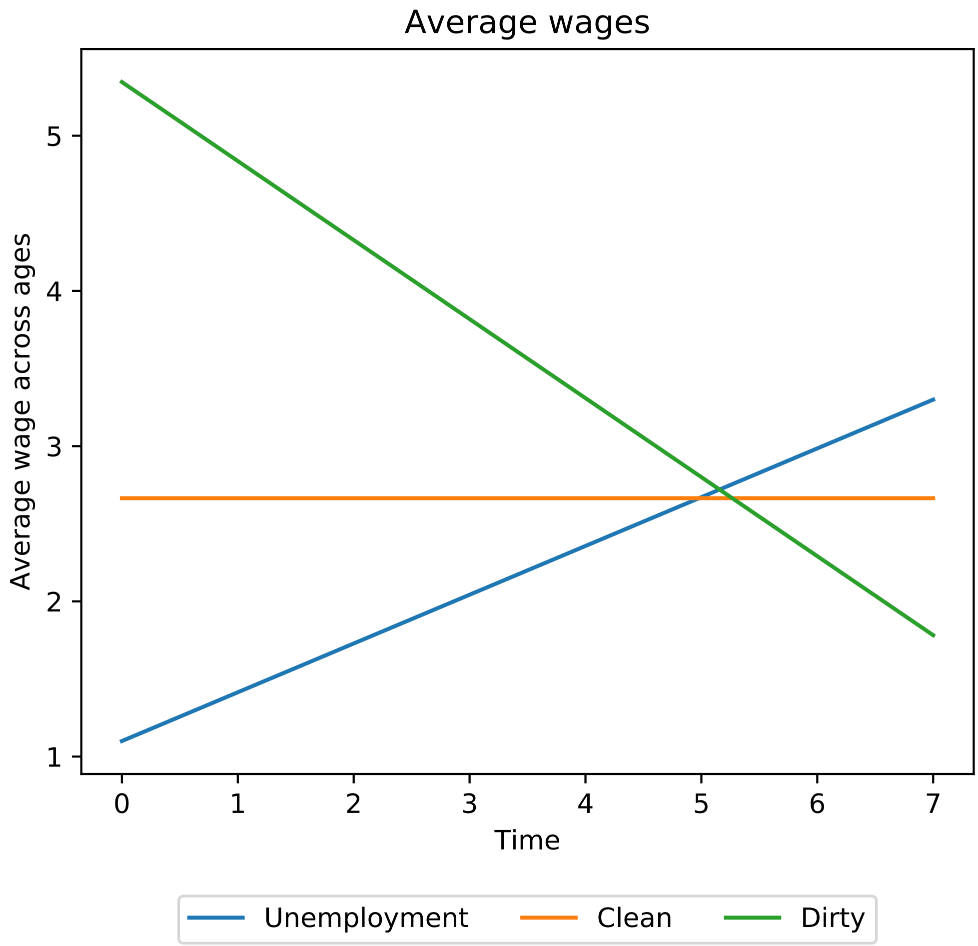
3.4 Estimation results

[6]For example, one of these would measure "the share of total employment accounted for by 37-year old workers, who worked in sector 2 in the previous period, and choose sector $s = 3$ now."

[7]For example, the transformation function adds 1 to age and inserts the choice of sector in year t as next period's lagged sector choice.

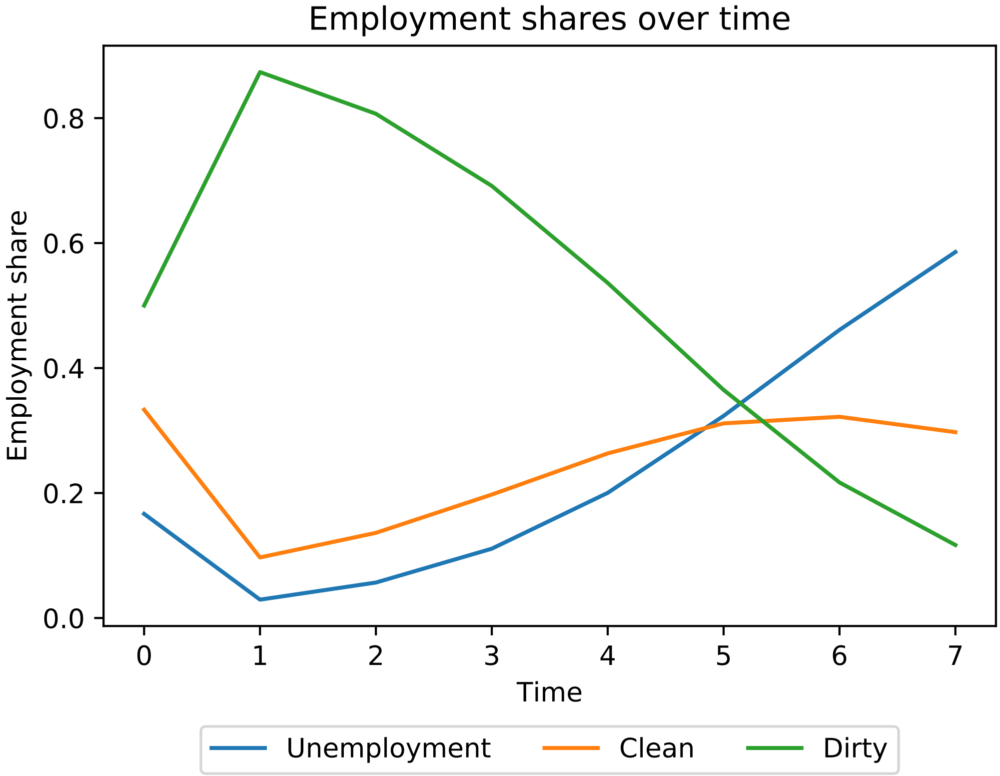
4 Results

Figure 1: Average wages.



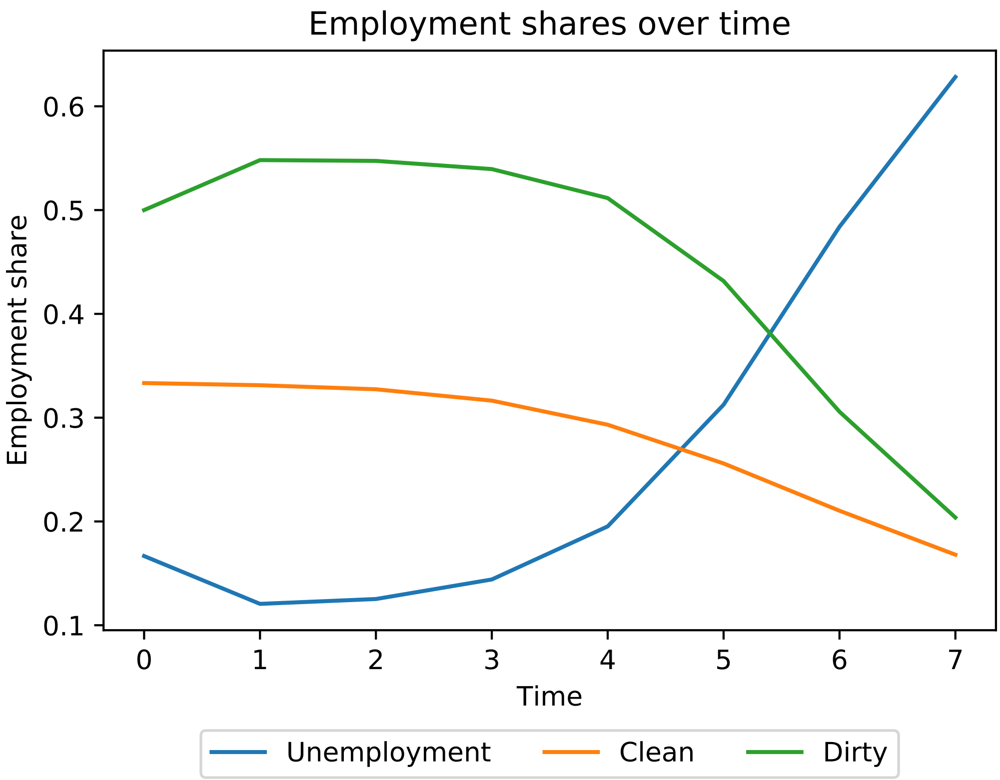
Note: Average wages across ages for each sector over time. At the moment these are completely exogenous.

Figure 2: Employment shares.



Note: Employment shares for each sector over time.

Figure 3: Employment shares, high switching costs.



Note: Employment shares for each sector over time. This simulation has higher switching costs.

Table 1: Unconditional switching probabilities

From ↓ / To →	Unemployment	Clean	Dirty
Unemployment	0.279573	0.240375	0.480052
Clean	0.255692	0.265419	0.478889
Dirty	0.243142	0.221388	0.535470

Note:

Table 2: Unconditional switching probabilities, high switching costs

From ↓ / To →	Unemployment	Clean	Dirty
Unemployment	0.957522	0.002832	0.039646
Clean	0.091361	0.907167	0.001472
Dirty	0.114501	0.000036	0.885463

Note:

5 Counterfactual simulations

When solving the model for counterfactual simulation purposes, the terminal period can be set to be sufficiently into the future, e.g. $T-1=2100$.

5.1 Simulating from the model

5.2 Scenario 1: A gradually increasing carbon tax

A Solution procedure for the worker's problem

The full solution to the problem entails finding all expected value functions, $E_\varepsilon V_t(\Omega_{it})$. In the following I will be explicit about the content of Ω_{it} while suppressing skill prices since these are exogenous to the worker.

Solving the model proceeds in two steps. As explained in the main text, workers have static expectations about future skill prices in the terminal period $T-1$. Since the model is solved with backwards recursion, this is the period where the algorithm starts. What we calculate are the continuation values when the skill prices are constant and equal to $\boldsymbol{r}_{T-1}$ for all the periods that a worker has left on the labor market. To calculate these expected value functions, we perform the following steps:

1. Set skill prices equal to $\boldsymbol{r}_{T-1}$ throughout the following steps 2-4.

2. Start at age $a_i = 65$. Calculate the expected value function, i.e. calculate $E_\varepsilon V_{T-1}(a_i, s_{iT-2})$ using the closed-form expression in (17) with the continuation values equal to zero:

$$E_\varepsilon V_{T-1}(a_i, s_{iT-2}) = \sigma \log\left(\sum_s \exp\left(\frac{w_{ist} - M(s, s_{iT-2})}{\sigma}\right)\right). \tag{26}$$

3. Move to age $a_i = 64$. Calculate $E_\varepsilon V_{T-1}(a_i, s_{iT-2})$ using the continuation value found in the previous step. The closed-form solution is

$$\begin{aligned} &E_\varepsilon\left[V_{T-1}(a_i, s_{iT-2})\right] \\ &= \sigma \log\left(\sum_s \exp\left(\frac{w_{isT-1} - M(s, s_{iT-1}) + \rho E_\varepsilon\left[V_{T-1}(a_i+1, s)\right]}{\sigma}\right)\right). \end{aligned} \tag{27}$$

   The continuation value $\rho E_\varepsilon\left[V_{T-1}(a_i+1, s)\right]$ reflects the expected value of having one year on the labor market when the skill prices equal $\boldsymbol{r}_{T-1}$.

4. Perform the previous step for ages $a_i = 63$ through 30, each time using the continuation value found in the previous step.

This procedure yields $E_\varepsilon V_{T-1}(a_i, s_{iT-2})$ for $a_i = 30, \ldots, 65$, i.e. the values when skill prices are constant for $35, \ldots, 0$ future periods. These, together with corresponding policy functions, represent the solution to the model in period $T-1$.

Having calculated period $T-1$ expected value functions, we can calculate the expected value functions of the remaining periods using backwards recursion. Skill prices across sectors and time $\boldsymbol{r}$ are taken as given throughout and suppressed from the notation. The remaining procedure runs as follows:

1. Start from period $T-2$.

   (a) Start from age $a_i = 65$. Calculate the value function $E_\varepsilon V_{T-2}(a_i, s_{iT-3})$, which has no continuation value, using the closed-form expression (17).

   (b) Set $a_i = 64$. Calculate $E_\varepsilon V_{T-2}(a_i, s_{iT-3})$ by using the continuation value $E_\varepsilon V_{T-1}(a_i+1, s_{iT-2})$, i.e. the value function in the next period when the worker is also one additional year older. The calculation is given by

$$E_\varepsilon \left[V_{T-2}(a_i, s_{iT-3})\right] = \sigma \log \left(\sum_s \exp \left(\frac{w_{isT-2} - M(s, s_{iT-3}) + \rho E_\varepsilon \left[V_{T-1}(a_i+1, s)\right]}{\sigma} \right) \right). \quad (28)$$

   (c) Repeat the previous step recursively for ages $a_i = 63, \ldots, 30$.

2. Perform steps (a) - (c) recursively for periods $t = T-3, \ldots, 0$.

The procedure gives all expected value functions. One can also retrieve the policy function by applying (18).

B Simulating forward

When simulating individuals forward, I treat the initial composition of the state space as exogenous. I also treat the initial composition of entering cohorts as exogenous. When simulating for estimation, these compositions are year-specific and given from the data. As mentioned in the main text, workers have static expectations about skill prices from the terminal period $T-1$ onward.

B.1 Counterfactual simulations

When performing simulations, I wish to produce counterfactual simulations for periods $t > 2016$. To be able to do this, I must solve the model iterating backwards from a future period $t > T-1$, sufficiently into the future, say, year 2100. I still assume that workers with active labor market years left in year 2100 have static expectations over future skill prices r_{st}.

Describe simulation method

B.2 Initial conditions, Retiring and entering cohorts

With each t, I must initialize a new cohort, i.e. a new set of workers of age 30 who enter the model. With an estimation sample spanning years $1996, \dots, 2016$, for these years I have data on all workers of age 30 and use these data directly when simulating. Since the previous sector choice s_{t-1} enters a worker's state space, I use the worker's employment when she was age 29. I treat her sector choice at age 29 as exogenous.

I initialize the simulation with the actual composition of workers as of 1996 and in each consecutive year up until and including 2016, I add a new cohort of age 30 with a composition equal to that observed in the data. For years after 2016, I must make an assumption about the composition of the entering cohort. I assume that the cohort of each year $t > 2016$ has a composition equal to that of year 2016.[8] For example, this means I impose that a certain fraction of the entering 30-year olds worked in sectors 0 to $S-1$ since that information enters their state space.

With these assumptions I can simulate employment from the model in all years 1996 until 2016. It does however require treating the state space of the workers that are in the middle of their careers when I observe them for the first time as exogenous. This might not be ideal, and Wooldridge (2005) has suggested a solution that I might employ later on.

C Counterfactual simulation routine

D Estimation routine

The estimation routine runs in the following steps:

1. Measure value added in real monetary terms, $\overline{p^Y_{st} Y_{st}}$. Set $p^Y_{st} Y_{st} = \overline{p^Y_{st} Y_{st}}$ throughout the estimation routine (similarly to Dix-Carneiro 2014). Previously calibrated and estimated parameters are also held fixed.

[8] I could easily make a different assumption, but this one is simple and transparent.

2. Using some vector of initial parameter values and initial values for skill prices $\boldsymbol{r}^0$, solve for the worker's expected value functions and policy function. With these, simulate choices forward.

3. Calculate H_{st}^{supply} for each s and t. Set $H_{st}^{demand} = H_{st}^{supply}$ and calculate the skill prices r_{st} consistent with the first order conditions from (3). Name the vector of these skill prices $\boldsymbol{r}^1$.

4. Check if all elements of $\boldsymbol{r}^1 - \boldsymbol{r}^0$ are sufficiently close to zero. If not, update to a new proposed skill price vector,

$$\boldsymbol{r}^2 = 0.1\boldsymbol{r}^1 + 0.9\boldsymbol{r}^0. \tag{29}$$

   $\boldsymbol{r}^2$ becomes the next iteration's $\boldsymbol{r}^0$.

5. Repeat the previous three steps until convergence in skill prices is obtained.

6. Calculate the log-likelihood value for the current set of parameters. Using an optimization algorithm, guess a new set of parameters.

7. Repeat the previous 5 steps until the optimizer converges on a set of parameter values.

8. Calculate K_{st}, E_{st} and O_{st} from their respective first order conditions (4), (5) and (6). Then, calculate A_{st} residually by

$$A_{st} = \frac{\overline{p_{st}^{Y} Y_{st}}}{H_{st}^{\alpha_{st}^1} K_{st}^{\alpha_{st}^2} \left(E_{st}^{\theta_s} + O_{st}^{\theta_s}\right)^{\frac{1-\alpha_{st}^1-\alpha_{st}^2}{\theta_s}}}. \tag{30}$$

   where we have normalized the output price of each sector to 1 without loss of generality since it each price ever only enters as a product with A_{st}.

Todo list

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