Mobility in a green transition

Jonathan Leisner*

March 21, 2022

Abstract

Changelog:

- Version 1: Only age in state space. No stochasticity.
- Version 2: Added stochasticity in the form of Type I extreme value sector-specific preference shocks. This implies that the model must be solved in expected value function space.

*University of Copenhagen. Oster Farimagsgade 5, 1353 Copenhagen K, Denmark. E-mail: jl@econ.ku.dk.

1 Model

A worker i chooses a sector, indexed by s, among the set of sectors S. The measure of sectors includes an "unemployment" sector. We assume that data on individuals and their choices are available for a period of T periods indexed by $t = 0, 1, \ldots, T-1$ which is also the planning horizon of workers. A worker makes this discrete choice once each year as long as her age a_i is between 30 and 65 after which she leaves the labor market. A worker is characterized by a set of state variables collected in Ω_{it}, including her age a_{it}. The remaining content of the state space will be explained below.

At the beginning of each period, she receives a wage offer from each sector. The wage offer w_{ist} is the product of two parts, the amount of effective human capital that she can supply to sector s, $H_s(\Omega_{it})$, and its unit price r_{st}. For effective human capital, the only relevant part of the state space is a_{it}, and her wage offer is therefore given by

$$w_{ist} = H_s(a_{it}) r_{st} \tag{1}$$

The effective human capital function H_s is given by

$$H_s(a_{it}) = \exp\left(\beta_s^0 a_{it}\right) \tag{2}$$

capturing that old workers on average earn a wage premium.[1]

In each period t the worker chooses the sector s which maximizes expected lifetime utility, as long as she is not retired. To make this forecast, we follow the methodology of Lee (2005) and assume that workers have perfect foresight over future unit prices r_{st}. Since the model terminates in period $T-1$ and workers aged $a < 65$ would take wages in period $t \geq T$ into account, we must make an assumption about how they behave in the terminal period. We assume that they act as if skill prices stay fixed for the rest of their time on the labor market, following Ashournia (2018). Essentially, the assumption is that workers have static expectations in period $T-1$, i.e. they expect future $(t > T-1)$ skill prices to equal current skill prices:

$$\{r_s^{T-1+t}\}_{t=1}^{65-a_{iT-1}} = \{r_s^{T-1}\}_{t=1}^{65-a_{iT-1}}, \tag{3}$$

[1]Depending on β_s^0, the premium could be negative. The human capital function will be extended in later versions of the paper.

where the lack of i subscript on r reflects that all workers have the same expectation about future prices. To summarize, workers always predict skill prices correctly, but when they forecast skill prices that do not exist in the model because they lie later than $T-1$, they simply predict the skill prices observed in $T-1$.

I simply want to say that the last period price is "duplicated" $65-a_{it}$ times. How can I write that?

In solving the model for estimation purposes the terminal period equals the last year where data is available, i.e. 2016. When solving the model for counterfactual simulation purposes, the terminal period can be set to be sufficiently into the future, e.g. $T-1 = 2100$.

This concludes the specification of the relevant state space.[2] It contains the age and the future expected skill prices:

$$\Omega_{it} = \left\{ a_{it}, \{r_s^{t+\tau}\}_{\tau=1}^{65-a_i} \right\}. \tag{4}$$

The instantaneous utility function for the worker is the sum of wages and unobserved (to the econometrician) sector-specific preference shocks, ε. The shocks are independent across sectors and time, and follow the Gumbel, or Type I extreme value, distribution,

$$\varepsilon_{ist} \sim \text{Gumbel}(-0.57721\sigma, \sigma) \tag{5}$$

where 0.57721 is Euler's constant. The recentering of the mean implies that shocks have a mean of zero.

Denote by $d_{ist} \in \{0, 1\}$ a dummy variable equal to one when the chosen sector is s in year t for worker i. With this notation the value function at time τ, i.e. the present value of the maximum attainable utility, is the solution to the utility maximization problem,

$$V_\tau(\Omega_{i\tau}) = \max_{\{d_{ist}\}_{s\in S, t=\tau,\ldots,T-1}} E_\varepsilon \left[\sum_{t=\tau}^{T-1} \rho^t \sum_{s\in S} d_{ist} \left(w_{ist}(\Omega_{it}) + \varepsilon_{ist} \right) \right] \tag{6}$$

where ρ denotes the discount factor, and we write $w_{ist}(\Omega_{it})$ to stress that the wages offered to worker i depend on her state space (age and skill prices). When making her decision, the worker takes skill prices as given. Note also that the subscript on the expectations operator reflects that the expectation is with respect to the unobserved preference shock ε.

[2]The state space consists of all the variables that the worker uses to make her decision.

The maximization of expected utility obeys the Bellman equation:

$$V_t(\Omega_{it}) = \max_s V_{st}(\Omega_{it}) \,. \tag{7}$$

where each of the alternative-specific value functions are given by

$$V_{st}(\Omega_{it}) = \begin{cases} w_{ist} + \varepsilon_{ist} + \rho E_{\varepsilon}\left[V_{t+1}(\Omega_{it+1})\,|d_{ist}\right] & \text{for } a_{it} < 65 \\ w_{ist} + \varepsilon_{ist} & \text{for } a_{it} = 65 \end{cases} \tag{8}$$

for each s and where continuation values are zero when the worker has age 65 since she will retire in the end of the year.[3] The distributional assumption on ε_{ist} implies that the expected value function, also called the EMAX function or the integrated value function, has a closed form (see e.g. Aguirregabiria 2021), namely:

$$E_{\varepsilon}\left[V_t(\Omega_{it})\right] = \sigma \log\left(\sum_s \exp\left(\frac{w_{ist} + \rho E_{\varepsilon}\left[V_{t+1}(\Omega_{it+1})|d_{ist}\right]}{\sigma}\right)\right). \tag{9}$$

The corresponding conditional (on the state space) choice probabilities also take the common logit-formula:

$$P(d_{ist} = 1|\Omega_{it}) = \frac{\exp\left(\frac{w_{ist}+\rho E_{\varepsilon}[V_{t+1}(\Omega_{it+1})|d_{ist}]}{\sigma}\right)}{\sum_j \exp\left(\frac{w_{ijt}+\rho E_{\varepsilon}[V_{t+1}(\Omega_{it+1})|d_{ijt}]}{\sigma}\right)}. \tag{10}$$

These probabilities make up the policy function, i.e. the policy function at a particular point in the state space is a vector of probabilities,

$$\boldsymbol{P}\left(\Omega_{it}\right) = \left(P(d_{i0t} = 1|\Omega_{it}), \ldots, P(d_{iSt} = 1|\Omega_{it})\right). \tag{11}$$

Is this true? When we simulate we draw epsilons, so the "policy function" takes these into account when simulating right? The worker does not choose randomly in the real world version of the model!

We are now ready to state the solution procedure.

[3] The conditioning of Ω_{it+1} on d_{ist} reflects that current choices can have implications for next year's state space. With the current version of the model that is not the case though, since the choice of sector has no implication for either age or future skill prices. With a more complex model however, this conditioning will be important.

1.1 Solving the model

The full solution to the problem entails finding all expected value functions, $E_\varepsilon V_t(\Omega_{it})$. If the model is solved for estimation purposes, the policy function is also retrieved (for maximum likelihood estimation).

Solving the model proceeds in two steps. First, since the model does not exist after period $T-1$, we must make some assumption about how workers who would still be active in the labor market in periods $t > T-1$ behave. To do so, I assume static expectations in the sense that workers expect skill prices to stay unchanged in the future. The only period in which static expectations are relevant is in period $T-1$, which is where the solution procedure starts. Skill prices r_s^{T-1} are hence taken as given. Then, what we need are the continuation values when the skill prices are constant and equal to r_s^{T-1} for all the periods that a worker has left on the labor market. To do calculate these expected value functions, we perform the following steps:

1. Set skill prices equal to r_s^{T-1} throughout the following steps 2-4.

2. Start at age $a_i = 65$. Calculate the expected value function, i.e. calculate $E_\varepsilon V_{T-1}(a_i)$ using the closed-form expression in (9) with the continuation values equal to zero:

$$E_\varepsilon V_{T-1}(a_i) = \sigma \log \left(\sum_s \exp\left(\frac{w_{ist}}{\sigma}\right)\right). \tag{12}$$

3. Move to age $a_i = 64$. Calculate $E_\varepsilon V_{T-1}(a_i)$ using the continuation value found in the previous step. The closed-form solution is

$$E_\varepsilon \left[V_{T-1}(a_i)\right] = \sigma \log \left(\sum_s \exp\left(\frac{w_{isT-1} + \rho E_\varepsilon \left[V_{T-1}(a_i+1)|d_{ist}\right]}{\sigma}\right)\right). \tag{13}$$

   The continuation value $\rho E_\varepsilon \left[V_{T-1}(a_i+1)|d_{ist}\right]$ reflects the expected value of having one year on the labor market when the skill prices equal r_s^{T-1}.

4. Perform the previous step for ages $a_i = 63$ through 30, each time using the continuation value found in the previous step.

This procedure yields $E_\varepsilon V_{T-1}(a_i)$ for $a_i = 30, \ldots, 65$, i.e. the values when skill prices are constant for $35, \ldots, 0$ future periods. These, together with corresponding policy functions, represent the solution to the model in period $T-1$.

Having calculated period $T-1$ expected value functions, we can calculate the expected value functions of the remaining periods using backwards recursion. Skill prices r_{st} are taken as given throughout. The remaining procedure runs as follows:

1. Start from period $T-2$.
   (a) Start from age $a_i = 65$. Calculate the value function $E_\varepsilon V_{T-2}(a_i)$, which has no continuation value, using the closed-form expression (9).
   (b) Set $a_i = 64$. Calculate $E_\varepsilon V_{T-2}(a_i)$ by using the continuation value $E_\varepsilon V_{T-1}(a_i + 1)$, i.e. the value function in the next period when the worker is also one additional year older. The calculation is given by

$$E_\varepsilon \left[V_{T-2}(a_i)\right] = \sigma \log \left(\sum_s \exp \left(\frac{w_{isT-2} + \rho E_\varepsilon \left[V_{T-1}(a_i+1)|d_{ist}\right]}{\sigma} \right) \right). \tag{14}$$

   (c) Repeat the previous step recursively for ages $a_i = 63, \ldots, 30$.
2. Perform steps (a) - (c) recursively for periods $t = T-3, \ldots, 0$.

The procedure gives all expected value functions. One can also retrieve the policy function by applying (10).

1.2 Simulating from the model

1.2.1 Solving the model (for simulation)

When performing simulations, we wish to produce counterfactual simulations for periods $t > 2016$. To be able to do this, we must solve the model iterating backwards from a future period $t > T-1$, sufficiently into the future, say, year 2200. Since this is not estimation, all parameters stay fixed. We still assume that workers with active labor market years left in year 2200 have static expectations over future skill prices r_{st}. Since the model is (at the current stage) partial, we simply postulate values for future skill prices. Then, the solution procedure is identical to that for estimation.

With the solution to the model in hand, we can simulate individuals of different ages and state spaces. The policy function tells us, which sector s an individual of age a in time period t will choose.

With each t, we must initialize a new cohort, i.e. a new set of workers of age 30 who enter the model. With an estimation sample spanning years $1996, \ldots, 2016$, for these years we have data on all workers of age 30 and use these data directly when simulating. We initialize the simulation with the actual composition of workers as of 1996 and in each consecutive year up until and including 2016, we add a new cohort of age 30 with a composition equal to that observed in the data. For years after 2016, we must make an assumption about the composition of the entering cohort. We assume that the cohort of each year $t > 2016$ has a composition equal to that of year 2016.[4] At the moment the state space only includes age and so the "composition" is trivial. However, this methodology will become important as soon as we add worker heterogeneity in the form of e.g. education.

With these assumptions we can simulate workers from the model in all years 1996 until 2200.

[4] We could easily make a different assumption, but this one is simple and transparent.

2 Ideas and notes

Notes for stochasticity:

I assume that each epsilon is randomly drawn from a GEV1(-gamma sigma, sigma)-distribution. This implies closed-form solution for the choice probabilities and the integrated value function, or the EMAX. (?).

The recentering of the mean has no implications for the choice probabilities since my optimal choice only depends on utility DIFFERENCES. However, the absolute level of the value function, or utility, will be affected. Since the mean of the gumpel is generally $m + sigmagamma$, the choice above means the distribution is centered around zero.

The choice probabilities have the usual form except the deterministic part includes the next period's expected value, emax. however each of these deterministic parts must be divided by the sigma-factor multiplied onto the shock (or alternatively assumed about its distribution).

When the transportation function is stochastic, we will need to do simulated integration here on top. For now we do not. But we still need that continuation value... There is a closed form solution one can use.

Without the closed form, we would have to perform numerical integration I think. I guess I could make a small example and try to implement the numerical integration in the setting where I can compare with the closed form solution. THat would make good sense hehe.

There is a good derivation of the closed form stuff at the end of Aguirregabiria's book. However it does not derive the "closed form continuation value" directly, as far as I see it. And to me it is unclear how that is affected by a recentering. It should definitely alter it.

Papers:

- Modellen:
  - Traiberman (2019, AER)
  - Dix-Carneiro (2014, Econometrica)
  - Ashournia (2017, EJ)
  - Humlum (2019, WP)

  - Roy (1951)

- Labor og generel ligevægt:

  - Hafstead and Williams (2018, JoPE)

- Equity effects of green regulation:

  - Yip (2018, JEEM)
  - Bento (2013, ARRE)
  - Curtis (2018, RoEaS)
  - Walker (2013, QJE)

- Dynamics:

  - Marin Vona (2019, JEEM)

- Other:

  - Vona Marin Consoli Popp (2018, AERE)

- Books:

  - miranda and faugner
  - adda and cooper

References

Aguirregabiria, Victor (2021). "Empirical Industrial Organization: Models, Methods, and Applications". *University of Toronto*.

Ashournia, Damoun (2018). "Labour market effects of international trade when mobility is costly". *The Economic Journal* 128.616, pp. 3008–3038.

Lee, Donghoon (2005). "An estimable dynamic general equilibrium model of work, schooling, and occupational choice". *International Economic Review* 46.1, pp. 1–34.

Non-cited sources

Hafstead, Marc A.C. and Roberton C. Williams (2018). "Unemployment and environmental regulation in general equilibrium". *Journal of Public Economics* 160, pp. 50–65. ISSN: 0047-2727.