Mobility in a green transition

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Abstract

Changelog:

- Version 1: Only age in state space. No stochasticity.
- Version 2: Added stochasticity in the form of Type I extreme value sector-specific preference shocks. This implies that the model must be solved in expected value function space.
- Version 3: Implemented switching costs directly into utility. A worker's previously chosen sector enters her state space because switching costs vary across sector-pairs.
- **Version 4:** Added firms and their production functions which take four inputs: human capital, physical capital, clean energy and dirty energy. Dirty energy generates emissions. I have closed the model by assuming Cobb-Douglas utility from consumption and by specifying the supply of physical capital. Some products are tradeables and which case their output price is exogenous (small country assumption). Other products are non-tradeables and their price is determined endogenously.

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1 Introduction

Introductory paragraph:

- The climate crisis warrants action. We dont know what happens to the labor market in the short and medium run when climate regulation is enacted. This has political relevance, since: Politicans hesitate, citing issues such as job loss. Find references from state leaders.

Motivation points:

- Main research question: What are the transitory labor market effects and costs of carbon taxes? What are the differential cost for command-and-control or subsidies versus an efficient tax?
- Estimate the extent to which the workers who are central to move away from the carbon intensive sectors also have higher switching costs.
- How long will it take to reach the "long run" steady states that most CGE models etc. find because they do not model mobility costs? How long is the lag from imposing regulation to the effects happen?
- Worker heterogeneity. Who are hit the hardest by carbon taxes? This is of political relevance, because politicians often want to sweeten the deal by compensating losers.
- Policy recommendations? Even though a carbon tax is efficient, many politicians hesitate to enact them, instead turning to command-and-control and subsidies, mostly to preserve "jobs".

Strands of literature

- Labor models of labor mobility: Dix-Carneiro (2014), Ashournia (2018), Traiberman (2019), Humlum (2019), and Lee (2005)
- Papers on the effects of carbon taxes on labor: reduced-form evidence Walker (2013). Search and friction models:

Critique or potential questions:

- Why do we need a structural model to do this?

- Is this externally valid, i.e. is Denmark too special to allow the analysis to say anything about the US economy?
- How do you capture that a carbon tax will increase the PRICE of electricity too?
- How do the sectors affect each other through input-output linkages? The model currently does not feature such links, is that a problem?

2 Model

Write a paragraph that describes the overall structure of the model before turning to specifics.

2.1 Firms

The model has S sectors indexed by s and spans T periods indexed by $t = 0, 1, \ldots, T - 1$. In each sector, a representative firm produces its output using four inputs: human capital, H_{st}, physical capital, K_{st}, clean energy E_{st} and fossil fuel based energy, O_{st}. The production technology is given by the Cobb-Douglas function

$$Y_{st} = A_{st} H_{st}^{\alpha_{st}^1} K_{st}^{\alpha_{st}^2} \left(E_{st}^\theta + O_{st}^\theta\right)^{\frac{1-\alpha_{st}^1-\alpha_{st}^2}{\theta}}. \tag{1}$$

where A_{st} represents TFP and $\left(E_{st}^\theta + O_{st}^\theta\right)^{\frac{1}{\theta}}$ is a CES aggregate governing the firm's use of energy. The use of fossil fuels generates carbon dioxide emissions Z with a fixed coefficient η,

$$Z_{st} = \eta_s O_{st}. \tag{2}$$

Notice that η_s varies across sectors. This reflects that the sectors' mix of dirty energy inputs such as oil, coal and natural gas varies. Since η_s does not vary across time, we implicitly assume that the optimal mix of dirty inputs is fixed throughout for a given sector.

The representative firm maximizes profits under perfect competition taking input and output prices and the emission tax τ_s as given. Denoting the output price as p_{st}^Y, the rental price of physical capital as r_{st}^K, the skill price of human capital as r_{st}^H, the price of clean energy p_{st}^E and the price of dirty energy p_{st}^O, the has the four first order conditions

$$\begin{aligned} r_{st}^H &= p_{st}^Y A_{st} \alpha_{st}^1 H_{st}^{\alpha_{st}^1 - 1} K^{\alpha_{st}^2} \left(E_{st}^\theta + O_{st}^\theta\right)^{\frac{1-\alpha_{st}^1-\alpha_{st}^2}{\theta}} \\ &= \alpha_{st}^1 p_{st}^Y \frac{Y_{st}}{H_{st}} \end{aligned} \tag{3}$$

$$\begin{aligned} r_{st}^K &= p_{st}^Y A_{st} \alpha_{st}^2 K_{st}^{\alpha_{st}^2 - 1} H^{\alpha_{st}^1} \left(E_{st}^\theta + O_{st}^\theta\right)^{\frac{1-\alpha_{st}^1-\alpha_{st}^2}{\theta}} \\ &= \alpha_{st}^2 p_{st}^Y \frac{Y_{st}}{K_{st}} \end{aligned} \tag{4}$$

$$p_{st}^E = p_{st}^Y A_{st} \left(1 - \alpha_{st}^1 - \alpha_{st}^2\right) \left(E_{st}^\theta + O_{st}^\theta\right)^{\frac{1-\alpha_{st}^1-\alpha_{st}^2}{\theta} - 1} E^{\theta - 1} K_{st}^{\alpha_{st}^2} H^{\alpha_{st}^1}$$

$$= \left(1-\alpha_{st}^{1}-\alpha_{st}^{2}\right) p_{st}^{Y} \frac{E^{\theta}}{E^{\theta}+O^{\theta}} \frac{Y_{st}}{E_{st}} \tag{5}$$

$$\eta_s \tau_{st} + p_{st}^{O} = p_{st}^{Y} A_{st} \left(1-\alpha_{st}^{1}-\alpha_{st}^{2}\right) \left(E_{st}^{\theta}+O_{st}^{\theta}\right)^{\frac{1-\alpha_{st}^{1}-\alpha_{st}^{2}}{\theta}-1} O^{\theta-1} K_{st}^{\alpha_{st}^{2}} H^{\alpha_{st}^{1}}$$

$$= \left(1-\alpha_{st}^{1}-\alpha_{st}^{2}\right) p_{st}^{Y} \frac{O^{\theta}}{E^{\theta}+O^{\theta}} \frac{Y_{st}}{O_{st}}. \tag{6}$$

One of the S sectors is an "unemployment sector". This sector has no production function but simply offers a fixed "wage" corresponding to some relevant average of Danish welfare transfers.

2.2 Workers

A worker i chooses a sector, indexed by s, among the set of sectors S. We assume that data on individuals and their choices are available for all T periods which is also the planning horizon of workers. A worker makes this discrete choice once each year as long as her age a_i is between 30 and 65 after which she leaves the labor market. A worker is characterized by a set of state variables collected in Ω_{it}, including her age a_{it}. The remaining content of the state space will be explained below.

2.2.1 Wage offers

At the beginning of each period, she receives a wage offer from each sector. The wage offer w_{ist} is the product of two parts, the amount of effective human capital that she can supply to sector s, $H_s(\Omega_{it})$, and its unit price r_{st}. For effective human capital, the only relevant part of the state space is a_{it}, and her wage offer is therefore given by

$$w_{ist} = H_s(a_{it}) r_{st} \tag{7}$$

The effective human capital function H_s is given by

$$H_s(a_{it}) = \exp\left(\beta_s^0 a_{it}\right) \tag{8}$$

capturing that old workers on average earn a wage premium.[1]

[1] Depending on β_s^0, the premium could be negative. The human capital function will be extended in later versions of the paper.

2.2.2 Switching costs

The worker incurs direct utility costs when switching sectors. These costs could represent searching costs or simply a preference for not moving sector. Similar to Ashournia (2018) I parametrize these switching costs, denoted $M(s_t, s_{t-1})$, in the following way:

$$M(s_t, s_{t-1}) = \exp\left(m(s_t, s_{t-1})\right) \tag{9}$$

where

$$m(s_t, s_{t-1}) = \begin{cases} 0 & \text{if } s_t = s_{t-1} \\ \xi^{out}_{s_{t-1}} + \xi^{in}_{s_t} & \text{if } s_t \neq s_{t-1} \end{cases}. \tag{10}$$

This structure implies that staying in the same sector does not involve additional switching costs and that conditional on switching sectors, the direct utility cost consists of a fixed cost of leaving sector s_{t-1} and entering sector s_t. Since the switching costs depend on the previous sector of employment, we add s_{it-1} to the state space at time t, Ω_{it}.

Ashournia and Dix-Carneiro normalize one of these to zero. I dont see why this is necessary, they should be identified without

2.2.3 Skill price expectations

In each period t the worker chooses the sector s which maximizes expected lifetime utility, as long as she is not retired. To make this forecast, we follow the methodology of Lee (2005) and assume that workers have perfect foresight over future unit prices r_{st}. Since the model terminates in period $T-1$ and workers aged $a < 65$ would take wages in period $t \geq T$ into account, we must make an assumption about how they behave in the terminal period. We assume that they act as if skill prices stay fixed for the rest of their time on the labor market, following Ashournia (2018). Essentially, the assumption is that workers have static expectations in period $T-1$, i.e. they expect future ($t > T-1$) skill prices to equal current skill prices:

$$\{r_s^{T-1+t}\}_{t=1}^{65-a_{iT-1}} = \{r_s^{T-1}\}_{t=1}^{65-a_{iT-1}}, \tag{11}$$

where the lack of i subscript on r reflects that all workers have the same expectation about future prices. To summarize, workers always predict skill prices correctly, but when they forecast skill prices that do not exist in the model because they lie later than $T-1$, they

I simply want to say that the last period price is "duplicated" $65 - a_{it}$ times. How can I write that?

simply predict the skill prices observed in $T-1$.

In solving the model for estimation purposes the terminal period equals the last year where data is available, i.e. 2016. When solving the model for counterfactual simulation purposes, the terminal period can be set to be sufficiently into the future, e.g. $T-1=2100$.

This concludes the specification of the relevant state space.[2] It contains the age, her previous sector and the future expected skill prices:

$$\Omega_{it} = \left\{ a_{it}, s_{it-1}, \{r_s^{t+\tau}\}_{\tau=1}^{65-a_i} \right\}. \tag{12}$$

2.2.4 The worker's problem

The instantaneous utility function for the worker is the sum of wages, switching costs and unobserved (to the econometrician) sector-specific preference shocks, ε. The shocks are independent across sectors and time, and follow the Gumbel, or Type I extreme value, distribution,

$$\varepsilon_{ist} \sim \text{Gumbel}(-0.57721\sigma, \sigma) \tag{13}$$

where 0.57721 is Euler's constant. The recentering of the mean implies that shocks have a mean of zero. As explained e.g. in Traiberman (2019), the scale parameter σ determines the worker's sensitivity to wage differentials. When σ increases extreme draws are more likely which leaves wage differentials with less influence on the optimal choice.

Denote by $d_{ist} \in \{0,1\}$ a dummy variable equal to one when the chosen sector is s in year t for worker i. With this notation the value function at time τ, i.e. the present value of the maximum attainable utility, is the solution to the utility maximization problem,

$$V_\tau(\Omega_{i\tau}) = \max_{\{d_{ist}\}_{s\in S, t=\tau,\ldots,T-1}} E_\varepsilon \left[\sum_{t=\tau}^{T-1} \rho^t \sum_{s\in S} d_{ist} \left(w_{ist}(\Omega_{it}) - M(s, s_{t-1}) + \varepsilon_{ist} \right) \right] \tag{14}$$

where ρ denotes the discount factor, and we write $w_{ist}(\Omega_{it})$ to stress that the wages offered to worker i depend on her state space (age and skill prices). When making her decision, the worker takes skill prices as given. Note also that the subscript on the expectations operator reflects that the expectation is with respect to the unobserved preference shock

[2]The state space consists of all the variables that the worker uses to make her decision.

ε.

The maximization of expected utility obeys the Bellman equation:

$$V_t(\Omega_{it}) = \max_s V_{st}(\Omega_{it}). \tag{15}$$

where each of the alternative-specific value functions are given by

$$V_{st}(\Omega_{it}) = \begin{cases} w_{ist} - M(s, s_{it-1}) + \varepsilon_{ist} + \rho E_\varepsilon \left[V_{t+1}(\Omega_{it+1}) | d_{ist} \right] & \text{for } a_{it} < 65 \\ w_{ist} - M(s, s_{it-1}) + \varepsilon_{ist} & \text{for } a_{it} = 65 \end{cases} \tag{16}$$

for each s and where continuation values are zero when the worker has age 65 since she will retire in the end of the year. The conditioning of Ω_{it+1} on d_{ist} reflects that current choices has implications for next year's state space: Today's choice of sector enters the state space of next period as s_{it-1}. The distributional assumption on ε_{ist} implies that the expected value function, also called the EMAX function or the integrated value function, has a closed form (see e.g. Aguirregabiria 2021), namely:

$$E_\varepsilon \left[V_t(\Omega_{it}) \right] = \sigma \log \left(\sum_s \exp \left(\frac{w_{ist} - M(s, s_{it-1}) + \rho E_\varepsilon \left[V_{t+1}(\Omega_{it+1}) | d_{ist} \right]}{\sigma} \right) \right). \tag{17}$$

The corresponding conditional (on the state space) choice probabilities also take the common logit-formula:

$$P(d_{ist} = 1 | \Omega_{it}) = \frac{\exp \left(\frac{w_{ist} - M(s, s_{it-1}) + \rho E_\varepsilon [V_{t+1}(\Omega_{it+1}) | d_{ist}]}{\sigma} \right)}{\sum_j \exp \left(\frac{w_{ijt} - M(j, s_{it-1}) + \rho E_\varepsilon [V_{t+1}(\Omega_{it+1}) | d_{ijt}]}{\sigma} \right)}. \tag{18}$$

These probabilities make up the policy function, i.e. the policy function at a particular point in the state space is a vector of probabilities,

$$\boldsymbol{P}(\Omega_{it}) = \left(P(d_{i0t} = 1 | \Omega_{it}), \ldots, P(d_{iSt} = 1 | \Omega_{it}) \right). \tag{19}$$

Is this true? When we simulate we draw epsilons, so the "policy function" takes these into account when simulating right? The worker does not choose randomly because they observe epsilon!

The full solution procedure is stated in appendix A.

2.3 Closing the model

To close the model for counterfactual simulations, additional assumptions are necessary. First, we invoke the small country assumption and assume that world prices of clean and dirty energy exist and are exogenous. Similarly, the output prices of tradeable sectors are also held fixed. Assuming that we have estimated all parameters of the model as well as each sector's TFP level, closing the model means endogenizing the output prices in the non-tradeable sectors, S^{NT}, as well as specifying physical capital supply.

To determine the equilibrium price of non-tradeables, we must specify the, necessarily domestic, demand for these products. To do so, I follow the previous literature (Dix-Carneiro 2014; Ashournia 2018) and assume that consumers' utility over consumption C_{st} is Cobb-Douglas in the $S-1$ consumable products (sector 0 does not produce anything since it represents unemployment):

$$u(C_{1t}, \ldots, C_{st}, \ldots, C_{S-1t}) = \prod_{s=1}^{S-1} C_{st}^{\mu_s} \tag{20}$$

implying that the fraction μ_s of income is spent on the good from sector s. By definition for nontradeables, what is produced domestically must all be consumed by domestically. I assume that all factors (including energy) being paid consume only in Denmark and that the proceeds from carbon taxes are paid back to consumers in a lump-sum fashions. I also assume that unemployment is financed via lump-sum transfers from employed workers and the other factors. This implies that total income for consumers is given by $\sum_{s=1}^{S-1} p_s^Y Y_s$ (note how the unemployment sector is left out since it represents no net-gain to income in the aggregate). Then, the equilibrium condition in nominal terms is

$$p_s^Y Y_s = p_s^Y C_s = \mu_s \sum_{j=1}^{S-1} p_j^Y Y_j \tag{21}$$

which must hold for all $s \in S^{NT}$. This equation system of s^{NT} equations and unknowns can be solved to yield the prices p_s^Y.

To fully close the model, assumptions on physical capital supply are necessary. Two of such assumptions are:

- Assume that physical capital is sector-specific and fixed. This means the return to physical capital varies in simulations, or

- assume that the return to physical capital is fixed but still sector specific. In the model, the quantity of physical capital in each sector is residually determined to ensure the sector-specific returns are kept fixed. This assumption was made in Ashournia (2018).

2.4 Estimation issues

Two of the previous papers (Dix-Carneiro 2014; Ashournia 2018) have performed a solve-and-simulate structural estimation using the simulated method of moments technique. I have planned on using maximum likelihood instead. The two mentioned papers assume, when estimating, that the nominal output of a sector, $p^Y_{st}Y_{st}$, is exactly equal to its data counterpart (call it $\overline{p^Y_{st}Y_{st}}$). This "identifies" the term $B_{st} = p^Y_{st}A_{st}$ as the residual from (1) with p^Y_{st} multiplied onto each side of the equation. The authors then simply set $A_{st} = B_{st}$ and $p^Y_{st} = 1$. They also calculate income shares for the different inputs and calibrate the share parameters α to these. This methodology of fixing $p^Y_{st}Y_{st}$ to $\overline{p^Y_{st}Y_{st}}$ implies that the equilibrium skill prices r_{st} can be found by solving their counterpart to my equation (3), the first order condition for human capital, and none of the other first order conditions. In other words, we do not need to know the quantities of O and E to estimate the parameters related to human capital. While this makes things easy, it also means that the parameter θ, the driver of the elasticity between clean and dirty energy, is **not identified**. This makes sense: θ drives changes to the produced quantity of a sector when the price of either E or O changes. But since output is kept fixed throughout their estimation procedure, such effects would be ruled out. This leads me to conclude that I have one of two possible ways out:

1. Fix nominal output to $\overline{p^Y_{st}Y_{st}}$ and make θ unidentified. Calibrate or estimate θ by other means. I do not currently know which, but the strands of literature on production function estimation are vast.

2. Do not fix output but instead keep it endogenous. This raises the question of how to estimate A_{st}. In theory, this could maybe just be done as part of the structural estimation of the other parameters and θ. I am not sure this is feasible and clearly we would need to take care of prices p^Y_{st}. Before, output prices were found as the residual, as explained above. With nominal output no longer exogenized, this is

no longer the case. One option would be to measure these prices in the data and exogenize their values throughout the estimation procedure (for non-tradeables AND tradeables).

2.5 Simulating from the model

When performing simulations, we wish to produce counterfactual simulations for periods $t > 2016$. To be able to do this, we must solve the model iterating backwards from a future period $t > T - 1$, sufficiently into the future, say, year 2200. Since this is not estimation, all parameters stay fixed. We still assume that workers with active labor market years left in year 2200 have static expectations over future skill prices r_{st}. Since the model is (at the current stage) partial, we simply postulate values for future skill prices. Then, the solution procedure is identical to that for estimation.

Note that since the model is not intended to capture aggregate trends in the size of the population, we will fix the number of people active in the labor market.

With the solution to the model in hand, there are two ways we can proceed. The first option is to simulate a large number of individuals including drawing sector-specific shocks for each. With these draws, each individual chooses optimal sector taking the continuation values into account. Post simulation, we can measure the share of the simulated individuals who work in different sectors and then multiply that share by the mass of workers. The second option, which is more straightforward computationally, implicitly invokes a "law of large numbers" by simply calculating the share of employment of a sector at each point in the state space. To do this, we initialize the model with employment shares given from the data. For example, one employment share (of the starting year, whichever is chosen) could be "the share of employment in sector 1 accounted for by 37-year old workers, who worked in sector 2 in the previous period".

Denote the collection of these shares by $D_{st}(\Omega_t)$ where the D refers to density. Then, to get the "density" of period $t+1$ employment for some particular sector s, we calculate the share of workers, at each point in the state space at period t, Ω_t, who switch into sector s. This share is simply given by the conditional choice probabilities found from solving the model since the two coincide under the "law of large numbers" assumption. The share of workers moving into sector s at point Ω_t in the state space is then multiplied by the period t density of that same point. These choices also determine the share of workers who move into particular points in the state space, however as of now where the only individual-specific part of the state space is age, this is not necessary. The methodology of simply updating the "density" of the model over time sidesteps the need to draw Gumbel shocks for a large number of individuals, saving computational time when simulating. The method follows that of Humlum (2019).

2.5.1 Initial conditions problem

With each t, we must initialize a new cohort, i.e. a new set of workers of age 30 who enter the model. With an estimation sample spanning years $1996, \dots, 2016$, for these years we have data on all workers of age 30 and use these data directly when simulating. Since the previous sector choice s_{t-1} enters a worker's state space, we use the worker's employment when she was age 29. We treat her sector choice at age 29 as exogenous.

We initialize the simulation with the actual composition of workers as of 1996 and in each consecutive year up until and including 2016, we add a new cohort of age 30 with a composition equal to that observed in the data. For years after 2016, we must make an assumption about the composition of the entering cohort. We assume that the cohort of each year $t > 2016$ has a composition equal to that of year 2016.[3] For example, this means we impose that a certain fraction of the entering 30-year olds worked in sectors 1 to S since that information enters their state space.

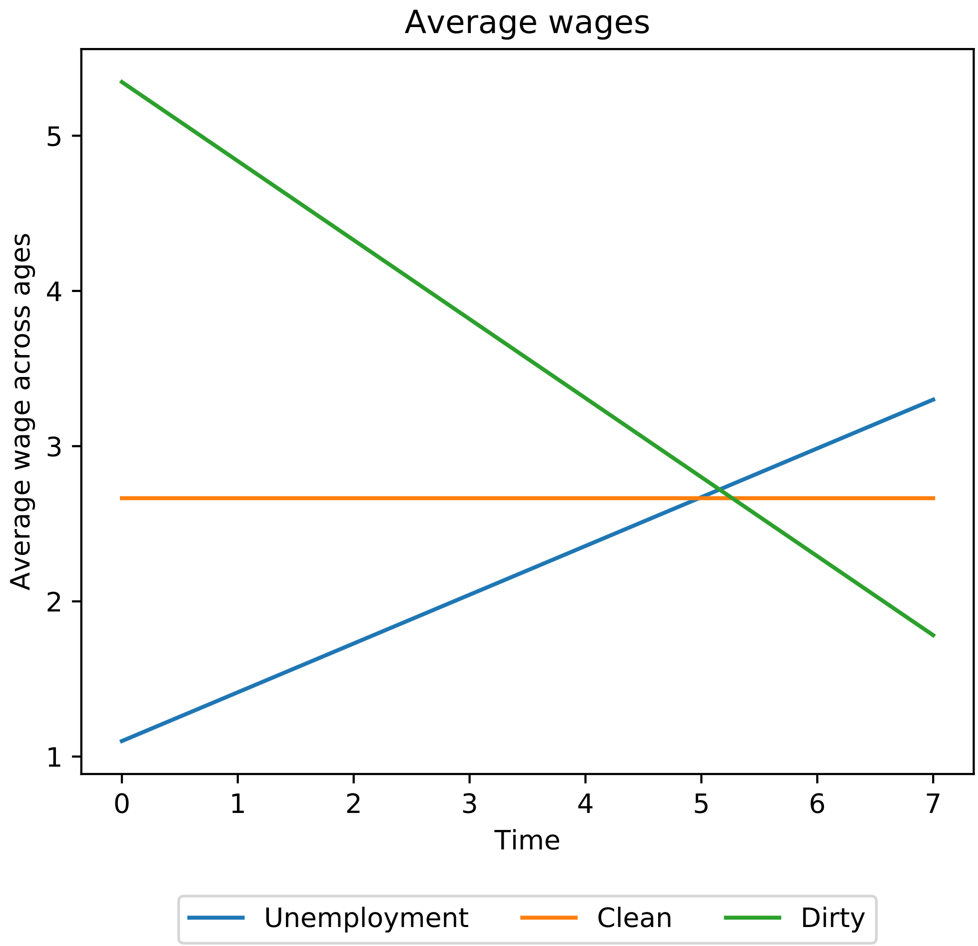
With these assumptions we can simulate employment from the model in all years 1996 until 2200.

The procedure outlined above essentially treats the state space of the workers that are in the middle of their careers when we observe them for the first time as exogenous. This might not be ideal, and Wooldridge (2005) has suggested a solution that we might employ later on.

[3] We could easily make a different assumption, but this one is simple and transparent.

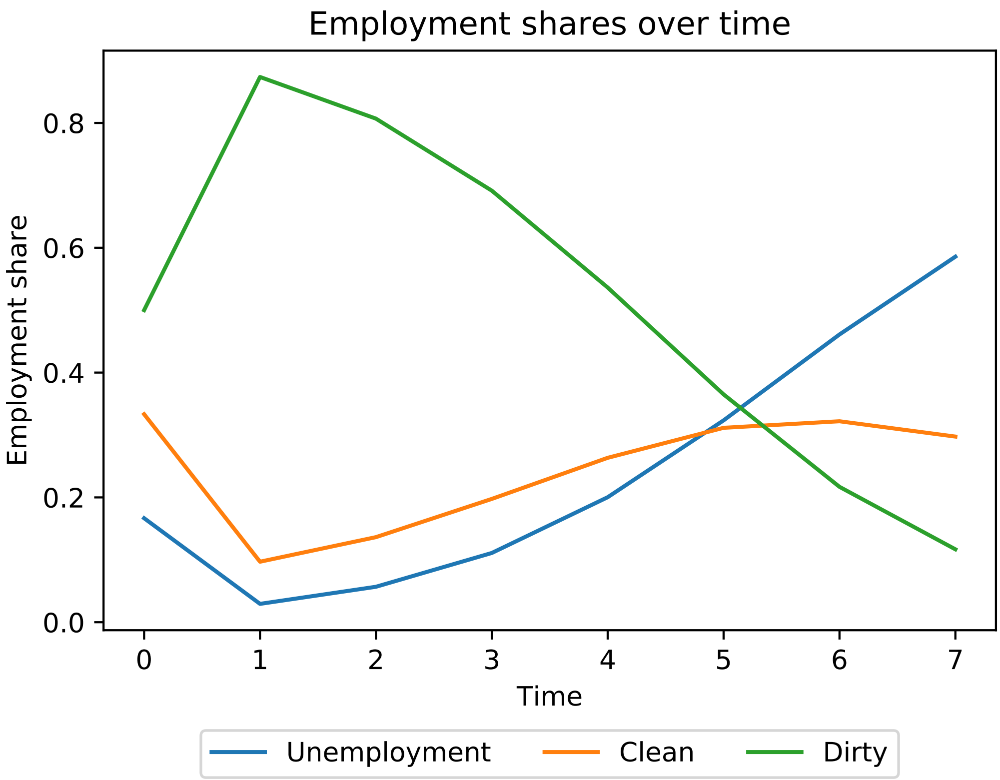
3 Results

Figure 1: Average wages.



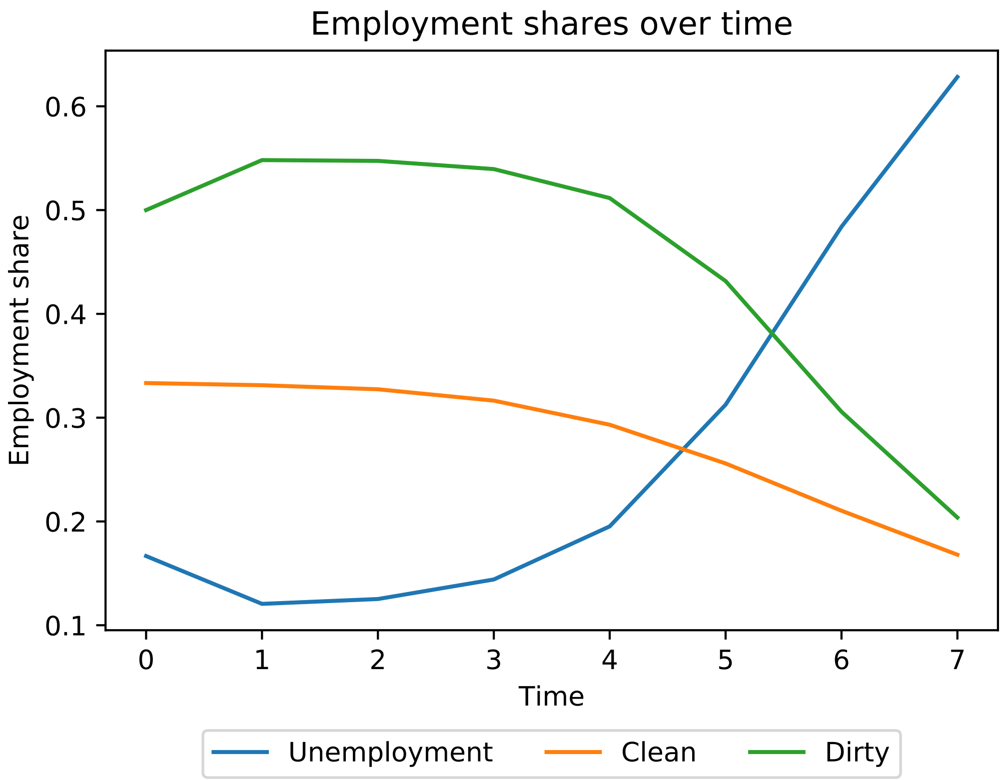
Note: Average wages across ages for each sector over time. At the moment these are completely exogenous.

Figure 2: Employment shares.



Note: Employment shares for each sector over time.

Figure 3: Employment shares, high switching costs.



Note: Employment shares for each sector over time. This simulation has higher switching costs.

Table 1: Unconditional switching probabilities

From ↓ / To →	Unemployment	Clean	Dirty
Unemployment	0.279573	0.240375	0.480052
Clean	0.255692	0.265419	0.478889
Dirty	0.243142	0.221388	0.535470

Note:

Table 2: Unconditional switching probabilities, high switching costs

From ↓ / To →	Unemployment	Clean	Dirty
Unemployment	0.957522	0.002832	0.039646
Clean	0.091361	0.907167	0.001472
Dirty	0.114501	0.000036	0.885463

Note:

A Solution procedure

The full solution to the problem entails finding all expected value functions, $E_\varepsilon V_t(\Omega_{it})$. In the following I will be explicit about the content of Ω_{it} while suppressing skill prices since these are exogenous to the worker. If the model is solved for estimation purposes, the policy function is also retrieved (for maximum likelihood estimation).

Solving the model proceeds in two steps. First, since the model does not exist after period $T-1$, we must make some assumption about how workers who would still be active in the labor market in periods $t > T-1$ behave. To do so, I assume static expectations in the sense that workers expect skill prices to stay unchanged in the future (Ashournia 2018). The only period in which static expectations are relevant is in period $T-1$, which is where the solution procedure starts. Skill prices r_s^{T-1} are hence taken as given. Then, what we need are the continuation values when the skill prices are constant and equal to r_s^{T-1} for all the periods that a worker has left on the labor market. To do calculate these expected value functions, we perform the following steps:

1. Set skill prices equal to r_s^{T-1} throughout the following steps 2-4.

2. Start at age $a_i = 65$. Calculate the expected value function, i.e. calculate $E_\varepsilon V_{T-1}(a_i, s_{iT-2})$ using the closed-form expression in (17) with the continuation values equal to zero:

$$E_\varepsilon V_{T-1}(a_i, s_{iT-2}) = \sigma \log \left(\sum_s \exp \left(\frac{w_{ist} - C(s, s_{iT-2})}{\sigma} \right) \right). \tag{22}$$

3. Move to age $a_i = 64$. Calculate $E_\varepsilon V_{T-1}(a_i, s_{iT-2})$ using the continuation value found in the previous step. The closed-form solution is

$$\begin{aligned} &E_\varepsilon \left[V_{T-1}(a_i, s_{iT-2}) \right] \\ &= \sigma \log \left(\sum_s \exp \left(\frac{w_{isT-1} - C(s, s_{iT-1}) + \rho E_\varepsilon \left[V_{T-1}(a_i + 1, s) \right]}{\sigma} \right) \right). \end{aligned} \tag{23}$$

   The continuation value $\rho E_\varepsilon \left[V_{T-1}(a_i + 1, s) \right]$ reflects the expected value of having one year on the labor market when the skill prices equal r_s^{T-1}.

4. Perform the previous step for ages $a_i = 63$ through 30, each time using the contin-

uation value found in the previous step.

This procedure yields $E_\varepsilon V_{T-1}(a_i, s_{iT-2})$ for $a_i = 30, \ldots, 65$, i.e. the values when skill prices are constant for $35, \ldots, 0$ future periods. These, together with corresponding policy functions, represent the solution to the model in period $T-1$.

Having calculated period $T-1$ expected value functions, we can calculate the expected value functions of the remaining periods using backwards recursion. Skill prices r_{st} are taken as given throughout and suppressed from the notation. The remaining procedure runs as follows:

1. Start from period $T-2$.

    (a) Start from age $a_i = 65$. Calculate the value function $E_\varepsilon V_{T-2}(a_i, s_{iT-3})$, which has no continuation value, using the closed-form expression (17).

    (b) Set $a_i = 64$. Calculate $E_\varepsilon V_{T-2}(a_i, s_{iT-3})$ by using the continuation value $E_\varepsilon V_{T-1}(a_i + 1, s_{iT-2})$, i.e. the value function in the next period when the worker is also one additional year older. The calculation is given by

$$
\begin{aligned}
&E_\varepsilon \left[V_{T-2}(a_i, s_{iT-3})\right] \\
&= \sigma \log \left(\sum_s \exp \left(\frac{w_{isT-2} - C(s, s_{iT-3}) + \rho E_\varepsilon \left[V_{T-1}(a_i + 1, s)\right]}{\sigma} \right) \right). \qquad (24)
\end{aligned}
$$

    (c) Repeat the previous step recursively for ages $a_i = 63, \ldots, 30$.

2. Perform steps (a) - (c) recursively for periods $t = T-3, \ldots, 0$.

The procedure gives all expected value functions. One can also retrieve the policy function by applying (18).

Todo list

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