Mobility in a green transition

Jonathan Leisner*

March 12, 2022

Abstract

Abstract text

*University of Copenhagen. Oster Farimagsgade 5, 1353 Copenhagen K, Denmark. E-mail: jl@econ.ku.dk.

1 Model

A worker i chooses a sector, indexed by s, among the set of sectors S. The measure of sectors includes an "unemployment" sector. The worker makes this choice once each year as long as her age is between 30 and 65 years old after which she leaves the labor market. We assume that data on individuals and their choices are available for a period of T periods indexed by $t = 0, 1, \ldots, T-1$. At the beginning of each period, she receives a wage offer from each sector. The wage offer w_{ist} is the product of two parts, the amount of effective human capital that she can supply to sector s times its unit price:

$$w_{ist} = H_s(\Omega_{it}) r_{st} \tag{1}$$

where Ω_{it} collects the state space variables relevant for the worker. For effective human capital, the relevant variable is the worker's age, a_{it}. The effective human capital function H_s is given by

$$H_s(a_{it}) = \exp\left(\beta_s^0 a_{it}\right). \tag{2}$$

In each period t the worker chooses the sector s which maximizes expected lifetime utility, as long as she is not retired. To make this forecast, we follow the methodology of Lee (2005) and assume that workers have perfect foresight over future unit prices r_{st} whenever t lies in the years where we have data, i.e. inside the estimation sample. Suppose the last year of our estimation sample is 2016. Then, workers of age 65 do not care about future wages when deciding where to work in year 2016, since they retire anyways. Workers of age $a < 65$ however, take these outside-sample wages into account when making their choice. To make this feasible, I assume that workers have static expectations of future wages in the final sample year. In other words when $t = 2016$, workers assume that the unit prices of human capital, $\{r_s^t + \tau\}_{\tau=1}^{65-a}$ in the following years relevant to them, stay fixed so that

$$\{r_s^{t+\tau}\}_{\tau=1}^{65-a_i} = \{r_s^t\}_{\tau=1}^{65-a_i}. \tag{3}$$

In all periods but that last, workers have rational expectations. However, "intermediate" cases exist. For example, a 60 year-old worker with 5 future work years left, will in 2015 correctly predict r_s^{2016} but then assume that the wages in the following 4 years simply equal r_s^{2016}. This methodology follows Ashournia (2018).

Denote by $d_{ist} \in \{0,1\}$ a dummy variable equal to one when the chosen sector is s in year t for worker i. With this notation the value function at time τ, i.e. the maximum attainable utility is the solution to a utility maximization problem,

$$V_\tau(\Omega_{i\tau}) = \max_{\{d_{ist}\}_{s\in S, t=\tau,\dots,T-1}} \sum_{t=\tau}^{T-1} \rho^t \sum_{s\in S} d_{st} w_{ist}(a_{it}) \tag{4}$$

where ρ denotes the discount factor. The state space contains the age and the future expected skill prices:

$$\Omega_{it} = \{a_{it}, \{r_s^{t+\tau}\}_{\tau=1}^{65-a_i}\}. \tag{5}$$

where the lack of i subscript on r reflects that all workers have the same expectation about future prices. The utility maximization problem obeys the Bellman equation:

$$V_t(\Omega_{it}) = \max_s V_{st}\left(\Omega_{it}\right). \tag{6}$$

where each of the alternative-specific value functions are given by

$$V_{st}(\Omega_{it}) = \begin{cases} w_{ist} + \rho V_{t+1}\left(\Omega_{it+1} | d_{st}\right) & \text{for } a < 65 \\ w_{ist} & \text{for } a = 65 \end{cases} \tag{7}$$

for each s and where continuation values are zero when the worker has age 65 since she will retire in the end of the year.

1.1 Solving the model (for estimation)

Solving the model proceeds in two steps. Because workers at the terminal period $T-1$ plan using static expectations, we must solve the model first using static expectations. The only period in which static expectations are relevant is in period $T-1$, which is where the solution procedure starts. Suppose that r_s^{T-1} are observed for all s. Then, what we need are the continuation values when the skill prices are constant and equal to r_s^{T-1} for all the periods that a worker has left on the labor market. To do calculate these value functions, we perform the following steps:

1. Set skill prices equal to r_s^{T-1} throughout the following steps 2-4.

2. Start at age $a_i = 65$. Calculate the maximum attainable value, i.e. calculate $V_{T-1}(a_i = 65)$ using (6) and (7). Each alternative-specific value function is given by

$$V_{sT-1}(a_i = 65) = w_{isT-1} \tag{8}$$

3. Move to age $a = 64$. Calculate $V_{T-1}(a = 64)$ using the continuation value found in the previous step. Each alternative-specific value is given by

$$V_{sT-1}(a_i = 64) = w_{isT-1} + \rho V_{T-1}(a_i = a_i + 1) \tag{9}$$

   The continuation value, $V_{T-1}(a_i = 65)$, reflects the value of having one year on the labor market when the skill prices equal r_s^{T-1}.

4. Perform the previous step for ages $a_i = 63$ through 30, each time using the continuation value found in the previous step.

This procedure yields $V_{T-1}(a_i)$ for $a_i = 30, \ldots, 65$, i.e. the value of wages when unit skill prices are constant for $35, \ldots, 0$ future periods. This represents the solution to the model in period $T-1$. Note, since skill prices are constant and the state space only contains age, each of the found value functions are scalars. When the model is extended, they will become functions of the state space.

Having calculated period $T-1$ value functions, we can calculate the value functions of the remaining periods using backwards recursion. The remaining procedure runs as follows:

1. Start from period $T-2$.

   (a) Start from age $a_i = 65$. Calculate the value function $V_{T-2}(a_i = 65)$.

   (b) Set $a_i = 64$. Calculate $V_{T-2}(a_i = 64)$ by using the continuation value $V_{T-1}(a_i = 65)$, i.e. the value function in the next period when the worker is also one additional year older. Each alternative-specific value function is given by

$$V_{sT-2}(a_i = 64) = w_{isT-2} + \rho V_{sT-1}(a_i = 64 + 1) \tag{10}$$

(c) Repeat the previous step recursively for ages $a_i = 63, \ldots, 30$.

2. Perform steps (a) - (c) recursively for periods $t = T - 3, \ldots, 0$.

The procedure gives all value functions as well as the policy function, i.e. the optimal sector choice at each point in time for all points in the state space (argmax).

1.2 Simulating from the model

1.3 Solving the model (for simulation)

When performing simulations, we wish to produce counterfactual simulations for periods $t > 2016$. To be able to do this, we must solve the model iterating backwards from a future period $t > T - 1$, say, year 2200. Since this is not estimation, all parameters stay fixed. We still assume that workers with active labor market years left in year 2200 have static expectations over future skill prices r_{st}. Since the model is (at the current stage) partial, we simply postulate value for future skill prices. Then, the solution is identical to that for estimation.

With the solution to the model in hand, we can simulate individuals of different ages and state spaces. The policy function tells us, which sector s an individual of age a in time period t will choose.

With each t, we must initialize a new cohort, i.e. a new set of workers of age 30 who enter the model. With an estimation sample spanning years $1996, \ldots, 2016$, for these years we have data on all workers of age 30 and use these data directly when simulating. We initialize the simulation with the actual composition of workers as of 1996 and in each consecutive year up until and including 2016, we add a new cohort of age 30 with a composition equal to that observed in the data. For years after 2016, we must make an assumption about the composition of the entering cohort. We assume that the cohort of each year $t > 2016$ has a composition equal to that of year 2016. At the moment the state space only includes age and so the "composition" is trivial. However, this methodology will become important as soon as we add worker heterogeneity in the form of e.g. education.

With these assumptions we can simulate workers from the model in all years 1996 until 2200.

2 Ideas

Papers:

- Modellen:
  - Traiberman (2019, AER)
  - Dix-Carneiro (2014, Econometrica)
  - Ashournia (2017, EJ)
  - Humlum (2019, WP)
  - Roy (1951)
- Labor og generel ligevægt:
  - Hafstead and Williams (2018, JoPE)
- Equity effects of green regulation:
  - Yip (2018, JEEM)
  - Bento (2013, ARRE)
  - Curtis (2018, RoEaS)
  - Walker (2013, QJE)
- Dynamics:
  - Marin Vona (2019, JEEM)
- Other:
  - Vona Marin Consoli Popp (2018, AERE)

References

Ashournia, Damoun (2018). "Labour market effects of international trade when mobility is costly". *The Economic Journal* 128.616, pp. 3008–3038.

Lee, Donghoon (2005). "An estimable dynamic general equilibrium model of work, schooling, and occupational choice". *International Economic Review* 46.1, pp. 1–34.

Non-cited sources

Hafstead, Marc A.C. and Roberton C. Williams (2018). "Unemployment and environmental regulation in general equilibrium". *Journal of Public Economics* 160, pp. 50–65. ISSN: 0047-2727.