Mobility in a green transition

Jonathan Leisner*

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Abstract

Abstract text

*University of Copenhagen. Oster Farimagsgade 5, 1353 Copenhagen K, Denmark. E-mail: jl@econ.ku.dk.

1 Model

A worker i chooses a sector, indexed by s, among the set of sectors S. The measure of sectors includes an "unemployment" sector. We assume that data on individuals and their choices are available for a period of T periods indexed by $t = 0, 1, \dots, T-1$ which is also the planning horizon of workers. A worker makes this discrete choice once each year as long as her age a_i is between 30 and 65 after which she leaves the labor market. A worker is characterized by a set of state variables collected in $(\Omega_{it}$, including her age a_{it}. The remaining content of the state space will be explained below.

At the beginning of each period, she receives a wage offer from each sector. The wage offer w_{ist} is the product of two parts, the amount of effective human capital that she can supply to sector s, $H_s(\Omega_{it})$, and its unit price r_{st}. For effective human capital, the only relevant part of the state space is a_{it}, and her wage offer is therefore given by

$$w_{ist} = H_s(a_{it}) r_{st} \tag{1}$$

The effective human capital function H_s is given by

$$H_s(a_{it}) = \exp\left(\beta_s^0 a_{it}\right) \tag{2}$$

capturing that old workers on average earn a wage premium.[1]

In each period t the worker chooses the sector s which maximizes expected lifetime utility, as long as she is not retired. To make this forecast, we follow the methodology of Lee (2005) and assume that workers have perfect foresight over future unit prices r_{st}. Since the model terminates in period $T-1$ and workers aged $a < 65$ would take wages in period $t \geq T$ into account, we must make an assumption about how they behave in the terminal period. We assume that they act as if skill prices stay fixed for the rest of their time on the labor market, following Ashournia (2018). Essentially, the assumption is that workers have static expectations in period $T-1$, i.e. they expect future $(t > T-1)$ skill prices to equal current skill prices:

$$\{r_s^{T-1+t}\}_{t=1}^{65-a_{iT-1}} = \{r_s^{T-1}\}_{t=1}^{65-a_{iT-1}}, \tag{3}$$

[1]Depending on β_s^0, the premium could be negative. The human capital function will be extended in later versions of the paper.

where the lack of i subscript on r reflects that all workers have the same expectation about future prices. To summarize, workers always predict skill prices correctly, but when they forecast skill prices that do not exist in the model because they lie later than $T-1$, they simply predict the skill prices observed in $T-1$.

I simply want to say that the last period price is "duplicated" $65 - a_{it}$ times. How can I write that?

In solving the model for estimation purposes the terminal period equals the last year where data is available, i.e. 2016. When solving the model for counterfactual simulation purposes, the terminal period can be set to be sufficiently into the future, e.g. $T-1 = 2200$.

This concludes the specification of the relevant state space.[2] It contains the age and the future expected skill prices:

$$\Omega_{it} = \left\{a_{it}, \{r_s^{t+\tau}\}_{\tau=1}^{65-a_i}\right\}. \tag{4}$$

Denote by $d_{ist} \in \{0,1\}$ a dummy variable equal to one when the chosen sector is s in year t for worker i. With this notation the value function at time τ, i.e. the maximum attainable utility is the solution to a utility maximization problem,

$$V_\tau(\Omega_{i\tau}) = \max_{\{d_{ist}\}_{s\in S, t=\tau,\ldots,T-1}} \sum_{t=\tau}^{T-1} \rho^t \sum_{s\in S} d_{st} w_{ist}(\Omega_{it}) \tag{5}$$

where ρ denotes the discount factor and we write $w_{ist}(\Omega_{it})$ to stress that the wages offered to worker i depend on her state space (age and skill prices). When making her decision, the worker considers skill prices as given. The utility maximization problem obeys the Bellman equation:

$$V_t(\Omega_{it}) = \max_s V_{st}(\Omega_{it}). \tag{6}$$

where each of the alternative-specific value functions are given by

$$V_{st}(\Omega_{it}) = \begin{cases} w_{ist} + \rho V_{t+1}(\Omega_{it+1} | d_{st}) & \text{for } a < 65 \\ w_{ist} & \text{for } a = 65 \end{cases} \tag{7}$$

for each s and where continuation values are zero when the worker has age 65 since she

[2] The state space consists of all the variables that the worker uses to make her decision.

will retire in the end of the year. The conditioning of Ω_{it+1} on d_{st} reflects that current choices can have implications for next year's state space. With the current version of the model that is not the case though, since the choice of sector has no implication for either age or future skill prices. With a more complex model however, this conditioning will be important.

1.1 Solving the model (for estimation)

Solving the model proceeds in two steps. First, since the model does not exist after period $T-1$, we must make some assumption about how workers who would still be active in the labor market in periods $t > T-1$ behave. To do so, we have assumed static expectations, and therefore we must solve the model first using static expectations. The only period in which static expectations are relevant is in period $T-1$, which is where the solution procedure starts. Skill prices r_s^{T-1} are taken as given. Then, what we need are the continuation values when the skill prices are constant and equal to r_s^{T-1} for all the periods that a worker has left on the labor market. To do calculate these value functions, we perform the following steps:

1. Set skill prices equal to r_s^{T-1} throughout the following steps 2-4.

2. Start at age $a_i = 65$. Calculate the maximum attainable value, i.e. calculate $V_{T-1}(a_i)$ using (6) and (7). Each alternative-specific value function is given by

$$V_{sT-1}(a_i) = w_{isT-1}(a_i) \tag{8}$$

3. Move to age $a_i = 64$. Calculate $V_{T-1}(a_i)$ using the continuation value found in the previous step. Each alternative-specific value is given by

$$V_{sT-1}(a_i) = w_{isT-1} + \rho V_{T-1}(a_i + 1) \tag{9}$$

   The continuation value $\rho V_{T-1}(a_i + 1)$ reflects the value of having one year on the labor market when the skill prices equal r_s^{T-1}.

4. Perform the previous step for ages $a_i = 63$ through 30, each time using the continuation value found in the previous step.

This procedure yields $V_{T-1}(a_i)$ for $a_i = 30, \ldots, 65$, i.e. the values when skill prices are constant for $35, \ldots, 0$ future periods. These, together with corresponding policy functions, represent the solution to the model in period $T-1$.

Having calculated period $T-1$ value functions, we can calculate the value functions of the remaining periods using backwards recursion. Skill prices r_{st} are taken as given throughout. The remaining procedure runs as follows:

1. Start from period $T-2$.

   (a) Start from age $a_i = 65$. Calculate the value function $V_{T-2}(a_i)$ which has no continuation value.

   (b) Set $a_i = 64$. Calculate $V_{T-2}(a_i)$ by using the continuation value $V_{T-1}(a_i+1)$, i.e. the value function in the next period when the worker is also one additional year older. Each alternative-specific value function is given by

   $$V_{sT-2}(a_i) = w_{isT-2}(a_i) + \rho V_{sT-1}(a_i+1) \tag{10}$$

   (c) Repeat the previous step recursively for ages $a_i = 63, \ldots, 30$.

2. Perform steps (a) - (c) recursively for periods $t = T-3, \ldots, 0$.

The procedure gives all value functions as well as the policy function, i.e. the optimal sector choice at each point in time for all points in the state space.

1.2 Simulating from the model

1.2.1 Solving the model (for simulation)

When performing simulations, we wish to produce counterfactual simulations for periods $t > 2016$. To be able to do this, we must solve the model iterating backwards from a future period $t > T-1$, sufficiently into the future, say, year 2200. Since this is not estimation, all parameters stay fixed. We still assume that workers with active labor market years left in year 2200 have static expectations over future skill prices r_{st}. Since the model is (at the current stage) partial, we simply postulate values for future skill prices. Then, the solution procedure is identical to that for estimation.

With the solution to the model in hand, we can simulate individuals of different ages and state spaces. The policy function tells us, which sector s an individual of age a in time period t will choose.

With each t, we must initialize a new cohort, i.e. a new set of workers of age 30 who enter the model. With an estimation sample spanning years $1996, \ldots, 2016$, for these years we have data on all workers of age 30 and use these data directly when simulating. We initialize the simulation with the actual composition of workers as of 1996 and in each consecutive year up until and including 2016, we add a new cohort of age 30 with a composition equal to that observed in the data. For years after 2016, we must make an assumption about the composition of the entering cohort. We assume that the cohort of each year $t > 2016$ has a composition equal to that of year 2016.[3] At the moment the state space only includes age and so the "composition" is trivial. However, this methodology will become important as soon as we add worker heterogeneity in the form of e.g. education.

With these assumptions we can simulate workers from the model in all years 1996 until 2200.

[3]We could easily make a different assumption, but this one is simple and transparent.

2 Ideas

Papers:

- Modellen:
  - Traiberman (2019, AER)
  - Dix-Carneiro (2014, Econometrica)
  - Ashournia (2017, EJ)
  - Humlum (2019, WP)
  - Roy (1951)
- Labor og generel ligevægt:
  - Hafstead and Williams (2018, JoPE)
- Equity effects of green regulation:
  - Yip (2018, JEEM)
  - Bento (2013, ARRE)
  - Curtis (2018, RoEaS)
  - Walker (2013, QJE)
- Dynamics:
  - Marin Vona (2019, JEEM)
- Other:
  - Vona Marin Consoli Popp (2018, AERE)

References

Ashournia, Damoun (2018). "Labour market effects of international trade when mobility is costly". *The Economic Journal* 128.616, pp. 3008–3038.

Lee, Donghoon (2005). "An estimable dynamic general equilibrium model of work, schooling, and occupational choice". *International Economic Review* 46.1, pp. 1–34.

Non-cited sources

Hafstead, Marc A.C. and Roberton C. Williams (2018). "Unemployment and environmental regulation in general equilibrium". *Journal of Public Economics* 160, pp. 50–65. ISSN: 0047-2727.