Two-Body Elastic Collisions

Before specializing to two-body Coulomb collisions, it is convenient to develop a general theory of two-body elastic collisions. Consider an elastic collision between a particle of type 1 and a particle of type 2. Let the mass and instantaneous velocity of the former particle be m_1 and $\mathbf{v}_1$, respectively. Likewise, let the mass and instantaneous velocity of the latter particle be m_2 and $\mathbf{v}_2$, respectively. The velocity of the center of mass is given by

$$\mathbf{U} = \frac{m_1\,\mathbf{v}_1 + m_2\,\mathbf{v}_2}{m_1+m_2}. \tag{3.10}$$

Moreover, conservation of momentum implies that $\mathbf{U}$ is a constant of the motion. The relative velocity is defined

$$\mathbf{u} = \mathbf{v}_1 - \mathbf{v}_2. \tag{3.11}$$

We can express $\mathbf{v}_1$ and $\mathbf{v}_2$ in terms of $\mathbf{U}$ and $\mathbf{u}$ as follows:

$$\mathbf{v}_1 = \mathbf{U} + \frac{\mu_{12}}{m_1}\,\mathbf{u}, \tag{3.12}$$

$$\mathbf{v}_2 = \mathbf{U} - \frac{\mu_{12}}{m_2}\,\mathbf{u}. \tag{3.13}$$

Here,

$$\mu_{12} = \frac{m_1\,m_2}{m_1+m_2} \tag{3.14}$$

is the *reduced mass*. The total kinetic energy of the system is written

$$K = \frac{1}{2}\,m_1\,v_1^2 + \frac{1}{2}\,m_2\,v_2^2 = \frac{1}{2}\,(m_1+m_2)\,U^{\,2} + \frac{1}{2}\,\mu_{12}\,u^2. \tag{3.15}$$

Now, the kinetic energy is the same before and after an elastic collision. Hence, given that U is constant, we deduce that the magnitude of the relative velocity, u, is also the same before and after such a collision.

Thus, it is only the direction of the relative velocity vector, rather than its length, that changes during an elastic collision.

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