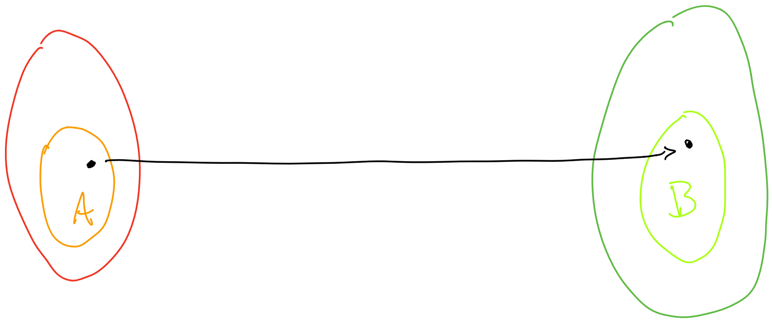
internal picture

$\forall r \in A, \exists f \in B : \phi(a,b)$ "$\hat{\phi}(A) \subseteq B$"



external picture

$\boxed{\text{if } T \text{ gets any } a \in A\text{, it can provide some } b \in B}$

$\Phi(A,B) = \text{true}$

$\phi : R \to F$

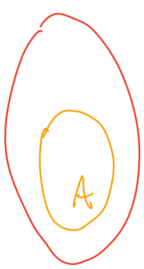
$\hat{\phi}$

$/\!/ ?$

$\Phi : \mathcal{P}(R) \to \mathcal{P}(F)$

more generally:

$\langle r \rangle$



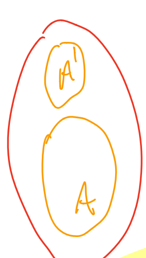
$A \otimes B$
$= \{ a \otimes b \mid a \in A, b \in B \}$

$\Phi(A,B)$

If T get guaranteed A, it can guarantee B

possibility of transforming A into B

$E_{a \in A'}$



ϕ

$x, y \in A'$

$z, w \in A$

$M_{b \in B'}$

$M_{\in A}$

$zMw \otimes xEy$

ϕ

$E_{b \in B}$

$a \in A$ $b \in B$

$\Psi(A,B)$

$\forall f \in B\ \exists r \in A:\ \psi(r,f) = \text{true}$

more generally:

~~$\Psi(A,B)$ = true means: if T is given any $b \in B$, it can provide an $a \in A$~~

$\Psi(A,B)$ = true means: if T has no choice on B, it can provide any $b \in B$ if it has free choice of elements of A

keep (mistake here) for $R \nrightarrow F$

$\phi : R \nrightarrow F$

① $\hat{\phi} : \mathcal{L}(R) \nrightarrow \mathcal{L}(F)$

$\hat{\phi}(A,B) = \text{true}$ iff $\forall a \in A\ \exists b \in B:\ \phi(a,b) = \text{true}$

② $\check{\phi} : \mathcal{L}(F)^{op} \nrightarrow \mathcal{L}(R)^{op}$

$\check{\phi}(A,B) = \text{true}$ iff $\forall a \in A\ \exists b \in B:\ \phi(a,b) = \text{true}$

$$\check{\phi} : (\mathcal{L}(F)^{op})^{op} \times \mathcal{L}(R)^{op} \longrightarrow \mathrm{Bool}$$

$$\parallel$$

$$\mathcal{L}(R)^{op} \times \mathcal{L}(F) \longrightarrow \mathrm{Bool}$$

<u>So $\hat{\phi} = \check{\phi}$</u>

(3) $\tilde{\phi} : \mathcal{U}(R)^{op} \nrightarrow \mathcal{U}(F)^{op}$

$\tilde{\phi}(A, B) = \text{true}$ iff $\forall f \in B \; \exists r \in A : \phi(r, f) = \text{true}$.

$B \subseteq B'$ $\quad \forall f \in B' \; \exists r \in A : \phi(r, f)$?

(4) $\breve{\phi} : \mathcal{U}(F) \nrightarrow \mathcal{U}(R)$ with "$\breve{\phi} = \tilde{\phi}$"

$\phi : R \nrightarrow F$

$\tilde{\phi} : R^{op} \longrightarrow \mathcal{U}(F)$

$\tilde{\phi}(r) = \{ f \in F \mid \phi(r,f) = T$

$\tilde{\tilde{\phi}} : \wp(R^{op}) \longrightarrow \mathcal{U}(F)$?

$A \longmapsto \bigcup_{a \in A} \tilde{\phi}(a)$

$= \{ f \in F \; \exists a \in A : \phi(a,f) = T \}$

$\tilde{\tilde{\phi}} : \mathcal{U}(R^{op}) \longrightarrow \mathcal{U}(F)$

$\mathcal{U}(R) \longrightarrow \mathcal{U}(F)$

$A \longmapsto \{ f \in F : \exists a \in A : \phi(a,f) = T \}$

$\tilde{\phi} : \mathcal{U}(R)^{op} \nrightarrow \mathcal{U}(F)^{op}$

$\tilde{\phi}(A,B) = T$ iff $\forall b \in B \; \exists a \in A : \phi(a,b) = T$

$\tilde{\phi}(A,X) = T$
&
$\tilde{\phi}(A,Y) = T$
$\Rightarrow \tilde{\phi}(A, X \cup Y) = T$

largest B
s.t. $\tilde{\phi}(A,B) = T$

$$\begin{array}{c} \phi : X^{op} \not\longrightarrow Y \\ \Big\updownarrow \\ F : \hat{X} \longrightarrow \hat{Y} \end{array}$$

cocontinuous (?)

$$\hat{X} = [X^{op}, \mathrm{Set}]$$

$$\hat{X} = [X^{op}, \mathrm{Bool}] = \text{lower sets}(X)$$

$$\otimes : \mathcal{U}(\mathcal{L}(\mathbb{R})) \times \mathcal{U}(\mathcal{L}(\mathbb{R})) \longrightarrow \mathcal{U}(\mathcal{L}(\mathbb{R}))$$

$$(\quad , \quad)$$