The basic outline/plan

- Lefschetz condition for "nice" DM stack, following Hartshorn ASAV chapter 4, begin a definition here
- something

DM Stack basicially means that the stablizers groups are all finite.

1 Introduction

Definition 1. this is where the definition of **Lef** for weighted projective stacks will go

Definition 2. this is where the definition of **Leff** for weighted projective stacks will go

2 Formal Geometry on Stacks

See this source: https://math.stanford.edu/ conrad/papers/formalgaga.pdf

See this source: https://www.math.uchicago.edu/ emerton/pdffiles/formal-stacks.pdf

Comment: Probably the most technical part (and the most useful) is going to be here.

Definition 3. **Formal completion of a stack along a substack**

Proposition 1. Following 1.1 of Hartshorn ASAV, equivalence of cd(X-Y) $< n-1$ is equivalent to Lef(X,Y) and Y meets every effective Cartier divisor on X

Definition 4. Give the correct definition of a **complete intersection** on a stack, probably intersection of divisors

Theorem 2. Following theorem 1.5 of Hartshorn ASAV, give the correct statement of this theorem. As a first attempt take X to be equal to the weighted projective stack. Something about fake weighted projective stacks (maybe?)

A complete intersection, Y, has Leff(X,Y) assuming that that Y is dimension at least 2.

3 Application to the Picard Group

Theorem 3. Following theorem 3.1 in Hartshorn ASAV

Proof. What we need for the proof is:

- Deformation theory argument
- third point of Hartshorn's version deals with $H^i(Y, I^n/I^{n+1})$ this is where the deformation theory comes in
- We have a line bundle L on Y, we want to lift it to X
- There are a series of "natural" maps $Pic(X) \to Pic(U) \to Pic(\widehat{X}) \to Pic(Y)$. Need to show that they are all isomorphisms and that their composition is the "natural" map $Pix(X) \to Pic(Y)$ that comes from the pull back $\mathcal{L} \mapsto i^*(\mathcal{L})$ where $i : Y \hookrightarrow X$ is the inclusion of Y into X.

□

4 Cohomological properties of DM Stacks

Serre duality for tame DM stacks. (Tame basically means that if characteristic = p, then stablizer groups do not have order p^n for any n)

See: https://link.springer.com/article/10.1007/s40687-022-00367-7

Serre's theorem on global generation on Stacks

See: https://arxiv.org/abs/1306.5418

Theorem 4. Generalizing theorem 5.18 in Hartshorn's AG

5 Calculate the cohomology of line bundles on weighted projective stack

Theorem 5. Generalizing theorem 5.1 in Hartshorn's AG