3D Simulation of Planetary Atmospheric Circulation

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Simulation of planetary atmospheric circulation requires immense computational power due to the complexity of the processes that govern the balance of these circulations. In this study we attempt to simulate Earth's 3 cell atmospheric circulation using simpler methods from Stam [2003], and approximations.

Code: https://github.com/JonathanMcIntosh0/COMP559FinalProj
Video: https://youtu.be/oU3UyVfLUnE

Additional Key Words and Phrases: physics simulation, computer animation, 3D Fluid animation

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1 INTRODUCTION

Simulation of planetary scale atmospheric circulation is a rapid emerging field (e.g. Imamura et al. [2020]; Read et al. [2018]). With the discovery of more and more planets outside of our solar-system, understanding the weather and circulation of these atmospheres will allow us to potentially quantify the habitability of these planets. Utilising our understanding and data of Earth's atmospheric circulation, and of nearby similar Earth-like planets (or planet-like) like Mars, Venus and Triton, will allow us to attempt to model circulation on planets for which we have less data. Within this paper we will attempt to model Earth's atmospheric circulation by expanding upon Stam [2003]'s method of fluid simulation.

2 RELATED WORK

As mentioned above, our simulation solver works in a manner identical to Stam [2003] but with modification to allow for 3D fluid simulation and conditions that resemble those found in Earth's atmosphere. To model convective buoyancy and external heating from the sun, we use Boussinesq approximation as seen in chapter 13 of Tritton [2012]. We additionally reference Tritton [2012], as it contains in depth explanations of various fluid simulation techniques and the dimensionless parameters that govern what types of flow we encounter. For an understanding of the physics and processes that govern our Earth's general planetary circulation, we refer to Wallace and Hobbs [2006]. This also contains a description on how Earth's circulation in usually modeled using the primitive equations.

3 METHODS

We reuse the code from assignment 3 but modify the methods to allow for 3D fluid simulation. However, unlike simulations that use the primitive equations mentioned in Wallace and Hobbs [2006] (e.g. Li and Titi [2018]), we use a uniform grid size of dx in all 3 directions instead of the usual anisotropic dimensions. This will be further discussed in section 4.

3.1 Boussinesq approximation

To deal with the variation in density from the change in temperature we use Boussinesq approximation which assumes constant density except when dealing with buoyancy force. This gives the following equation of motion

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho_0}\nabla(p - \rho_0 \vec{g} \cdot \vec{z}) + \nu \nabla^2 \vec{u} - \vec{g}\alpha(T - T_0)$$

where α is the coefficient of thermal expansion, ρ_0 is the fixed density at reference temperature T_0. For the density we have

$$\rho = \rho_0 + \Delta\rho = \rho_0 - \alpha\rho_0(T - T_0)$$

thus giving us the buoyancy term related to temperature instead of density, as seen above. Note that since we do not explicitly require the exact pressure we may instead just solve for

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho_0}\nabla P + \nu \nabla^2 \vec{u} - \vec{g}\alpha(T - T_0)$$

using our project method.

Along with the equation for motion we also have an equation for temperature:

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \kappa \nabla^2 T + \frac{J}{\rho C_p}$$

where J is the rate of internal heat generation (solar heating and adiabatic heating from condensation), and we suppose we have a constant heat capacity per unit volume, ρC_p. Note that in our model we only take into account solar heating when specifying J thus giving a fully North-South added heat gradient. It may be more accurate to include the adiabatic heating from latent heat generated by condensation which would add a vertical gradient.

3.2 Boundary conditions and Earth's physical features

We simplify Earth's spherical geometry down to a grid by imposing a certain set of boundary conditions. Since Earth's circulation (and most other planets we are interested in) are thermally driven by the difference in solar heating between the poles and the equator [Read et al. 2018], this causes little to no circulation to occur horizontally across the poles. Therefore, we will simulate the poles as solid boundaries that do not allow the passing of wind or temperature. Then to simulate the continuous nature of a sphere's topology, we impose a "wrap around" boundary in the East-West directions. In other words, when wind exits through the East boundary, it enters through the West boundary and vice versa. Finally, for the vertical boundaries, we model the Earth's surface and the tropopause by a solid boundary for wind but for temperature, we constrain it to specific temperature values to emulate heating and cooling through thermal conduction from Earth's surface and space respectively.

In addition to the imposed boundary conditions, we make use of the fact that, due to the sheer size of the Earth versus the height of the

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atmosphere, we may ignore the differences caused by the curvature of the Earth. In other words, our horizontal wind speeds are in fact all tangential to Earth's surface but when doing calculations, we use cartesian coordinates instead of spherical.

3.3 Forces

The 3 main forces that affect planetary atmospheric circulation are: coriolis force, centrifugal force and frictional force. Although the former 2 are so called 'fictitious' forces since we are describing motion within a rotating reference frame.

When it comes to Earth's circulation, we observe what is called geo-strophic balance where the pressure gradient force is counter balanced by the coriolis force. When we are in geo-strophic balance, centrifugal force is negligible and can be ignored. However, on slower rotating planets like Venus and Triton, we either observe cyclo-strophic or gradient wind balance where the pressure gradient force is in balance with the centrifugal force or both, centrifugal and coriolis force. Thus we have not implemented centrifugal force but this causes issues with accuracy when the rotation speed is too low. Coriolis force has the following equation:

$$\vec{C} = -2\Omega \sin(\phi)\vec{k} \times \vec{V}$$

where Ω is the planetary rotation speed, ϕ is latitude, $\vec{k}$ is the local vertical unit vector and $\vec{V}$ is horizontal wind. This gives the component wise horizontal acceleration

$$\frac{du}{dt}_c = 2\Omega \sin(\phi) v \quad \text{and} \quad \frac{dv}{dt}_c = -2\Omega \sin(\phi) u$$

Now when it comes to friction due to Earth's surface, this becomes negligible at the height of the free atmosphere as the pressure gradient and coriolis force are much stronger in comparison. However, in the boundary layer, the force of friction should be taken into account. As our simulation is not extremely accurate and does not make use of the primitive equations, we have not implemented friction. However in the future we could implement a simple horizontal frictional force below a certain height, proportional to the horizontal wind speed.

3.4 3D visualisation

To allow us to visualise the circulation in 3D, we implemented Arc Ball from Shoemake [1992] to allow rotation of our model. Additionally, since the visualisation of the wind speed vector field and temperature can be noisy and overwhelming when displayed all at once, we implemented controls that allow the user to display 'slices' along all 3 axes of the wind speeds and temperature.

4 RESULTS

Unfortunately, we were not able to replicate Earth's 3 cell circulation model. We suspect this is due to multiple factors pertaining to the complexity of Earths general circulation such as the absence of hydrostatic equilibrium in our model. When simulating atmospheric circulation, the usual method is to use pressure (or geo-potential height) as the vertical dimension and solve the equations of motion and thermodynamics at each vertical layer separately, then use the equation of conservation to obtain the vertical motion from the previously calculated horizontal motion. This is most accurate as the Earth's atmosphere is usually at equilibrium vertically and thus vertical motion tends to be almost entirely dictated by the horizontal motion.

However, we were able to create distinct Hadley cells (with some distortion and interesting circulations depending on parameters). We believe that we failed to create the 3 cells as Earth's general circulation is quite complex and we attempted to model it using a small scale fluid simulation model. Whereas, when dealing with dimensions as large as a planet, fluid simulation becomes much more complex due to the many processes that govern and balance the motion. One of such processes that our simulation fails to capture is baroclinic instability which is the main driving process that creates the Ferrel cell. This is due to the use of Boussinesq's approximation as it fails when the difference between the fixed density and the actual density gets too large. Many papers have tried to rectify this pitfall of the approximation by modifying the buoyancy term [Zhang and Xia 2022].

5 CONCLUSIONS

When dealing with such a large scale circulations, small scale approximations fail and cause inaccuracies. However, our simulation could prove useful to simulate specific portions of the circulation such as an isolated hadley cell. In the future, we would like to attempt to incorporate pressure as the vertical dimension instead of distance, as this is much more accurate since the height of the troposphere is minuscule in comparison to the radius of the Earth. Additionally, as mentioned previously, we would like to add frictional forces and adiabatic heating among other processes that would make the simulation more accurate.

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