

## 2

## Transformations

Set the wheel in motion!

**R. Mukundan** ([mukundan@canterbury.ac.nz](mailto:mukundan@canterbury.ac.nz))

Department of Computer Science and Software Engineering  
University of Canterbury, New Zealand.



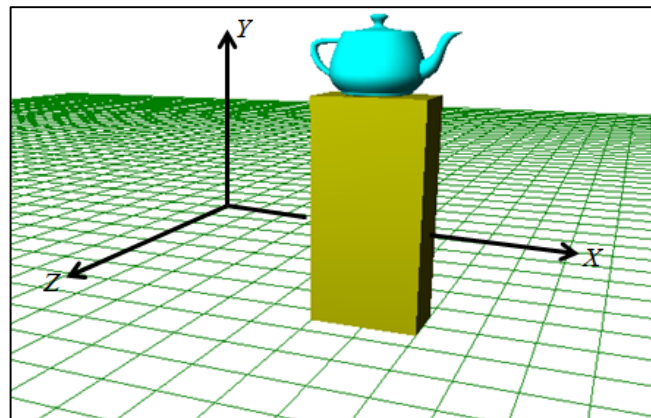
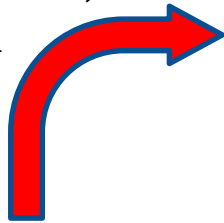
# Transformations

- A transformation changes the coordinates of a vertex or the components of a vector.
- Several types of transformations are commonly used in a graphics application:
  - Transformations of objects within the same frame: Egs. **Translations, rotations and scale transformations**. These transformations are called **Model Transformations**.
  - Transformation of an object from one reference frame to another. Eg. The transformation from world space to the camera space (**View Transformation**)
  - Projection transformations. These are based on the camera's frustum parameters.
  - In this section, we will consider model transformations.

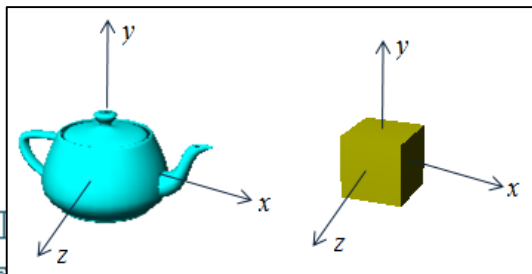
# Model-View Transformation

- Objects are created in their own local coordinate space and then transformed into the world coordinate space.
- They are transformed again into the coordinate space of the camera to generate the view as seen by the camera.

Model  
Transformation. Eg.  
**Scale, Rotation,  
Translation**



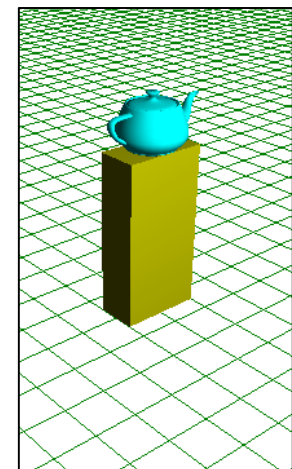
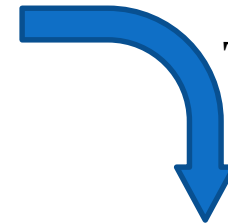
World Space



Modelling Space

`glutSolidCube(...)`  
`glutSolidTeapot(...)`

View  
Transformation  
`gluLookAt(...)`



Camera Space

# Transformations in OpenGL

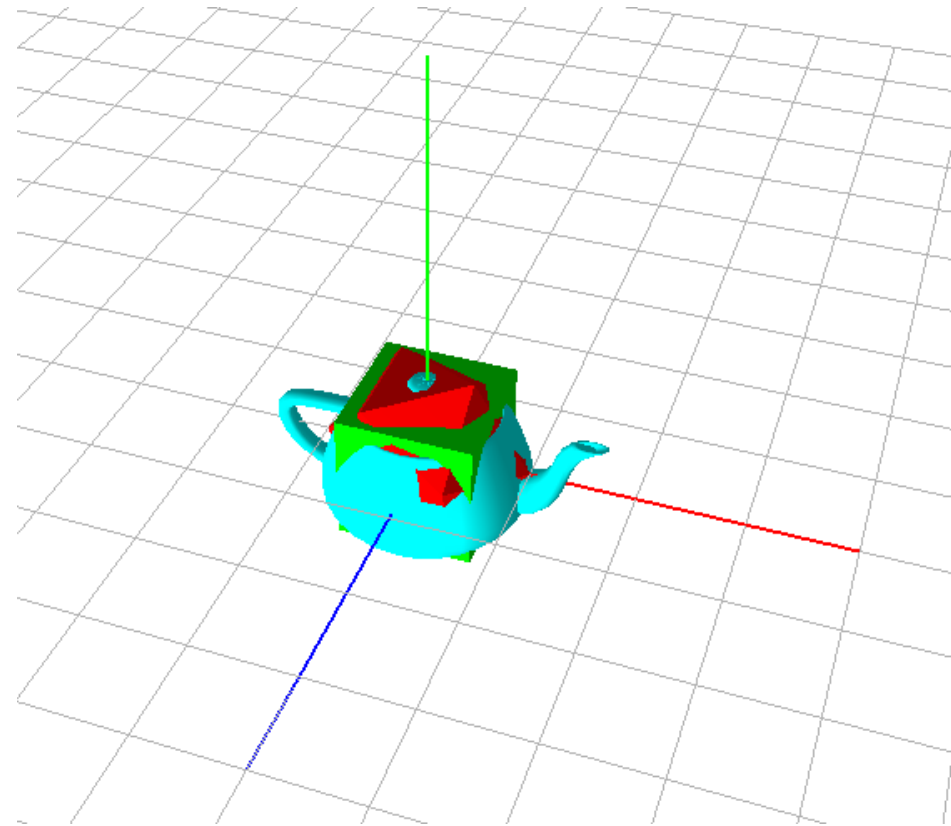
- OpenGL supports the following types of three-dimensional model transformations:
  - Translations: `glTranslatef(a, b, c);`
  - Rotations: `glRotatef(angle, l, m, n);`
  - Scale Transformations: `glScalef(sx, sy, sz);`
  - Generalized transformation: `glMultMatrixf(mat);`
- *All* transformations are stored as 4x4 matrices (discussed later in the course).
- Model transformations form part of the **model-view matrix**.

```
glMatrixMode(GL_MODELVIEW);  
glLoadIdentity();
```

# Model Creation

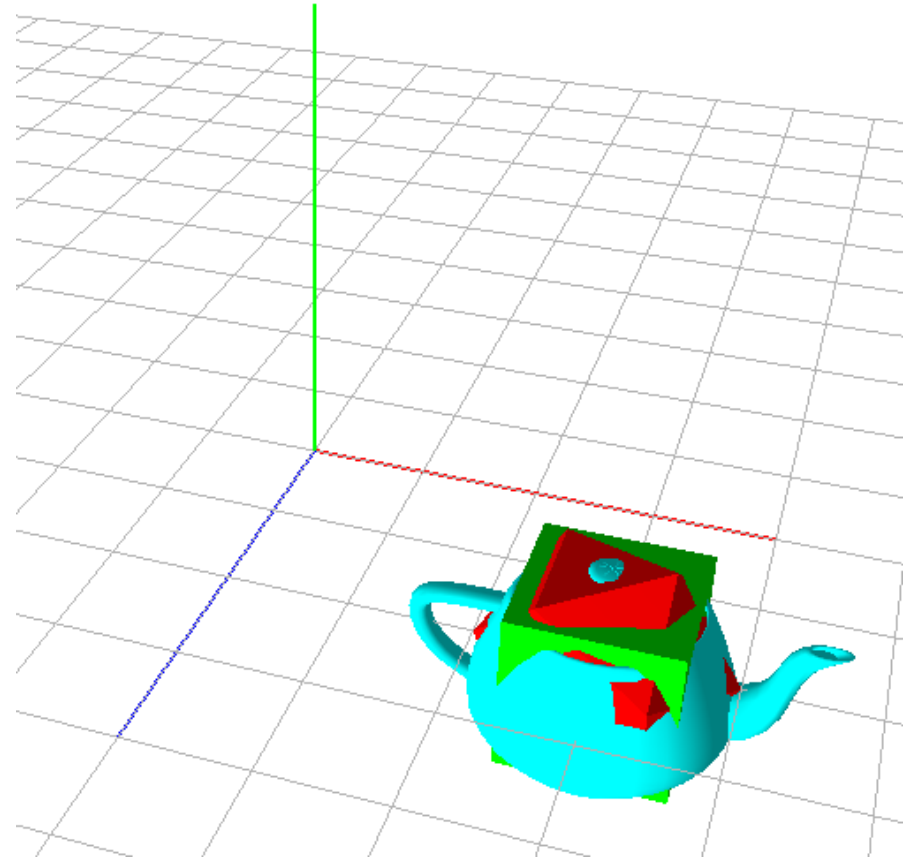
- A model's vertex coordinates are *usually* defined with the origin at its centre.
- The display of a model, without any transformation, therefore appears at or near the origin.

```
void display()
{
    ...
    ...
    glColor3f(0.,1.,1.);
    glutSolidTeapot(1);
    glColor3f(0., 1., 0.);
    glutSolidCube(1.3);
    glColor3f(1, 0, 0);
    glutSolidIcosahedron();
    glFlush();
}
```



# Model Transformation

```
void display()
{
    ...
    ...
    glTranslatef(3, 0, 2);
    glColor3f(0.,1.,1.);
    glutSolidTeapot(1);
    glColor3f(0., 1., 0.);
    glutSolidCube(1.3);
    glColor3f(1, 0, 0);
    glutSolidIcosahedron();
    glFlush();
}
```



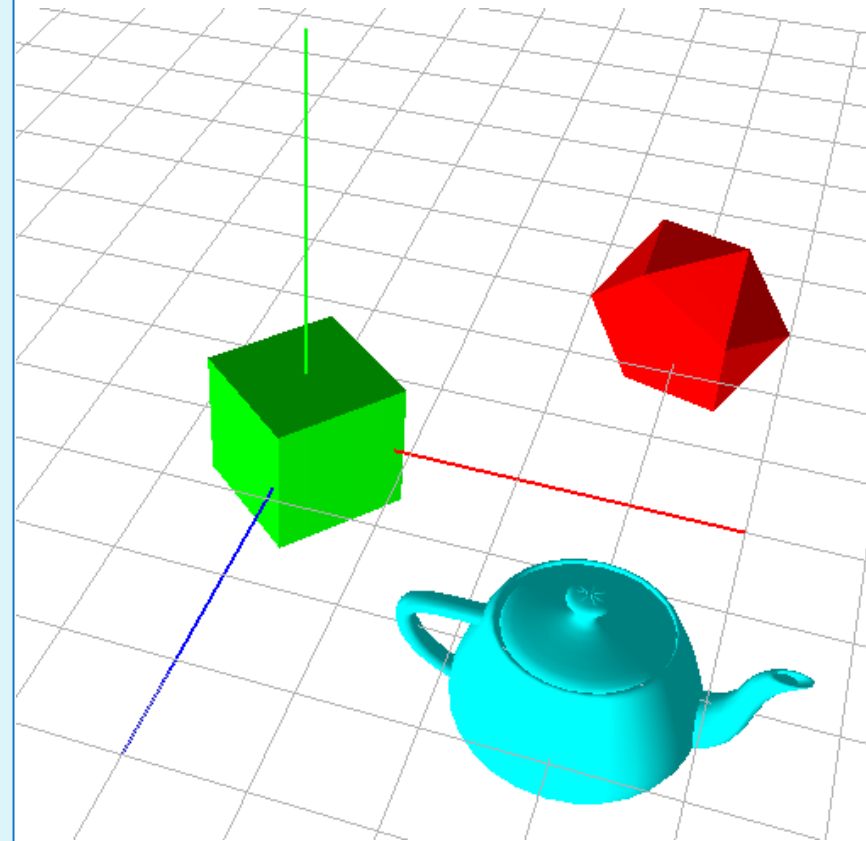
- A transformation function to translate an object by 3 units along x-direction, and 2 units along z-direction is used.
- In the above example, the transformation is applied to all three objects.

# Independent Transformations

```
void display()
{
    ...
    glPushMatrix();
        glTranslatef(3, 0, 2);
        glColor3f(0.,1.,1.);
        glutSolidTeapot(1);
    glPopMatrix();

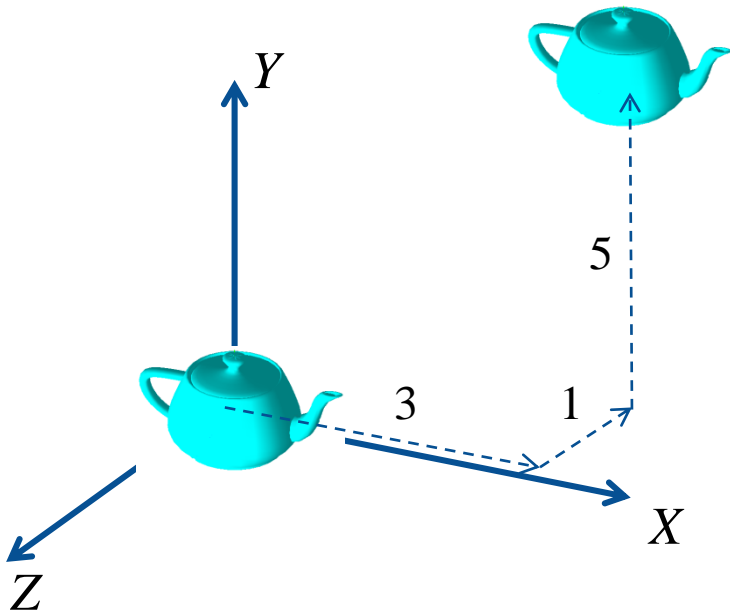
    glPushMatrix();
        glRotatef(50, 0, 1, 0);
        glColor3f(0., 1., 0.);
        glutSolidCube(1.3);
    glPopMatrix();

    glPushMatrix();
        glTranslatef(3, 0, -3);
        glColor3f(1, 0, 0);
        glutSolidIcosahedron();
    glPopMatrix();
    glFlush();
}
```



# Translation

OpenGL function: **glTranslatef**(a, b, c);



## Example:

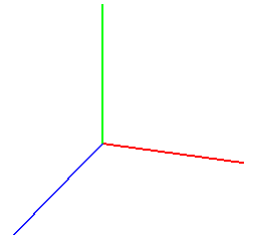
```
void display()
{
    ...
    glMatrixMode(GL_MODELVIEW);
    glLoadIdentity();
    gluLookAt(...)
    glLightfv(...)
    glTranslatef(3, 5, -1);
    glutSolidTeapot(1.0);
    glFlush();
}
```

$$(x, y, z) \xrightarrow{T(a, b, c)} (x + a, y + b, z + c)$$

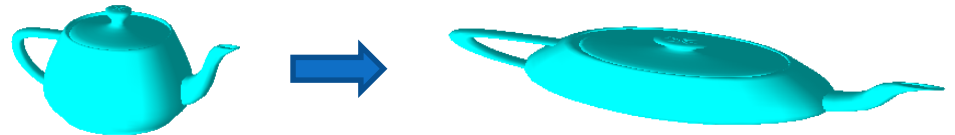


# Scaling

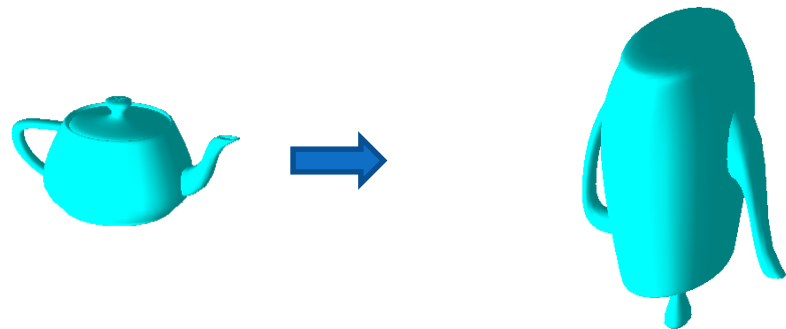
- OpenGL function: **glScalef** (a, b, c)
- A negative scale factor corresponds to a reflection.
- A zero scale factor corresponds to a projection



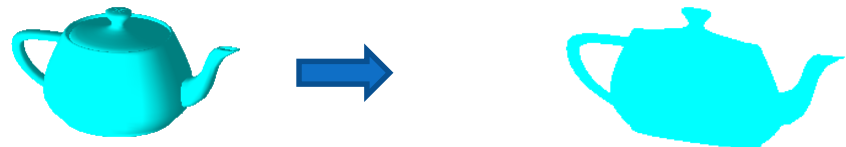
```
glScalef(2.0, 0.5, 1);  
glutSolidTeapot(1);
```



```
glScalef(1, -3, 2);  
glutSolidTeapot(1);
```



```
glScalef(1, 1, 0);  
glutSolidTeapot(1);
```

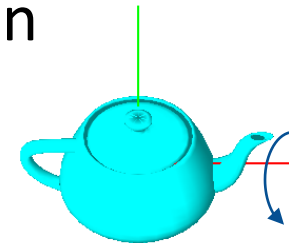


$$(x, y, z) \xrightarrow{S(a, b, c)} (xa, yb, zc)$$

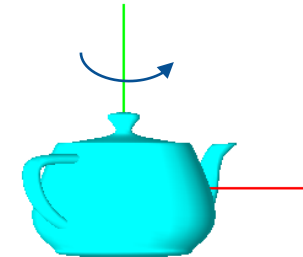
# Rotations

- OpenGL function: **glRotatef**(theta, l, m, n)
- A positive angle corresponds to a rotation in the anti-clockwise sense about the axis of rotation

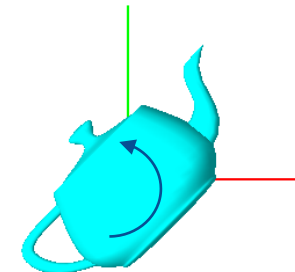
```
glRotatef(45, 1, 0, 0);  
glutSolidTeapot(1);
```



```
glRotatef(45, 0, 1, 0);  
glutSolidTeapot(1);
```

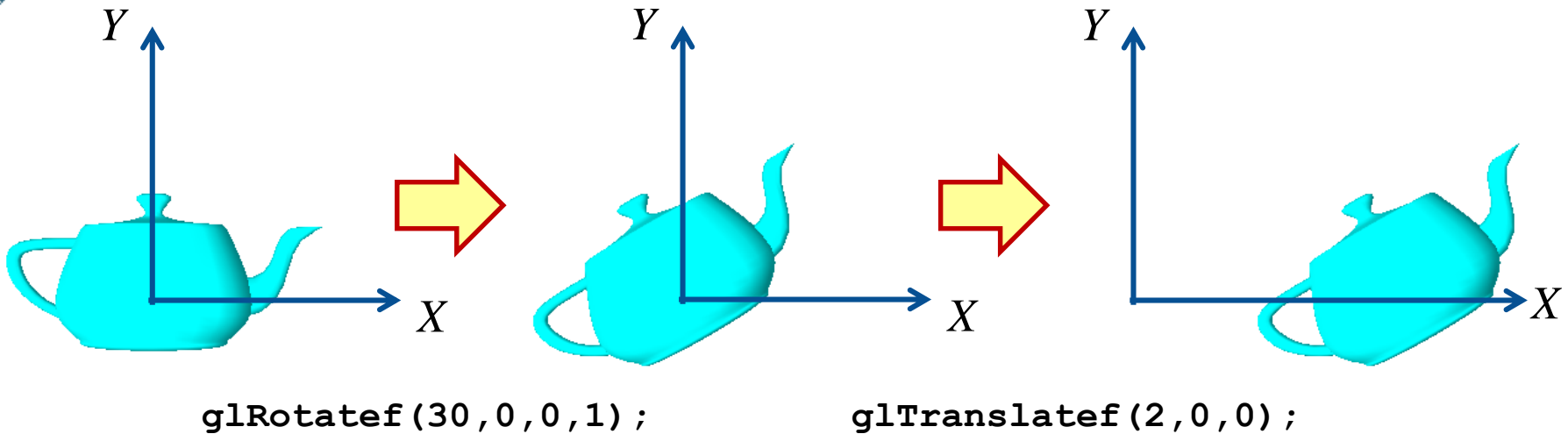


```
glRotatef(45, 0, 0, 1);  
glutSolidTeapot(1);
```



# Composite Transformations

Example: A rotation followed by a translation

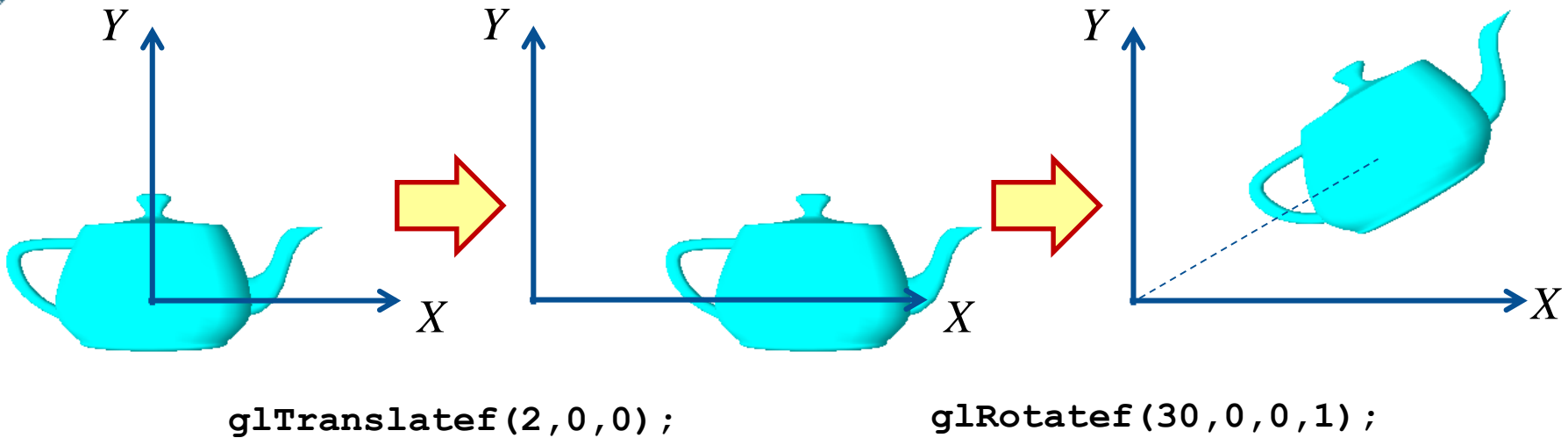


```
glTranslatef(2., 0., 0.);  
glRotatef(30., 0., 0., 1.);  
glutSolidTeapot(1);
```

Order of  
Transformations

# Composite Transformations

Example: A translation followed by a rotation

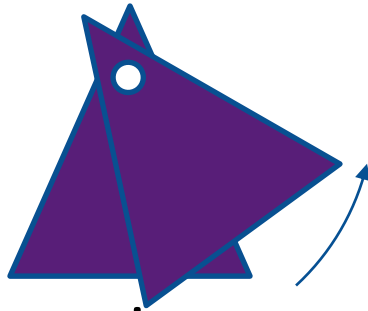


```
glRotatef(30., 0., 0., 1.);  
glTranslatef(2., 0., 0.);  
glutSolidTeapot(1);
```

Order of  
Transformations

# Rotations about a pivot point

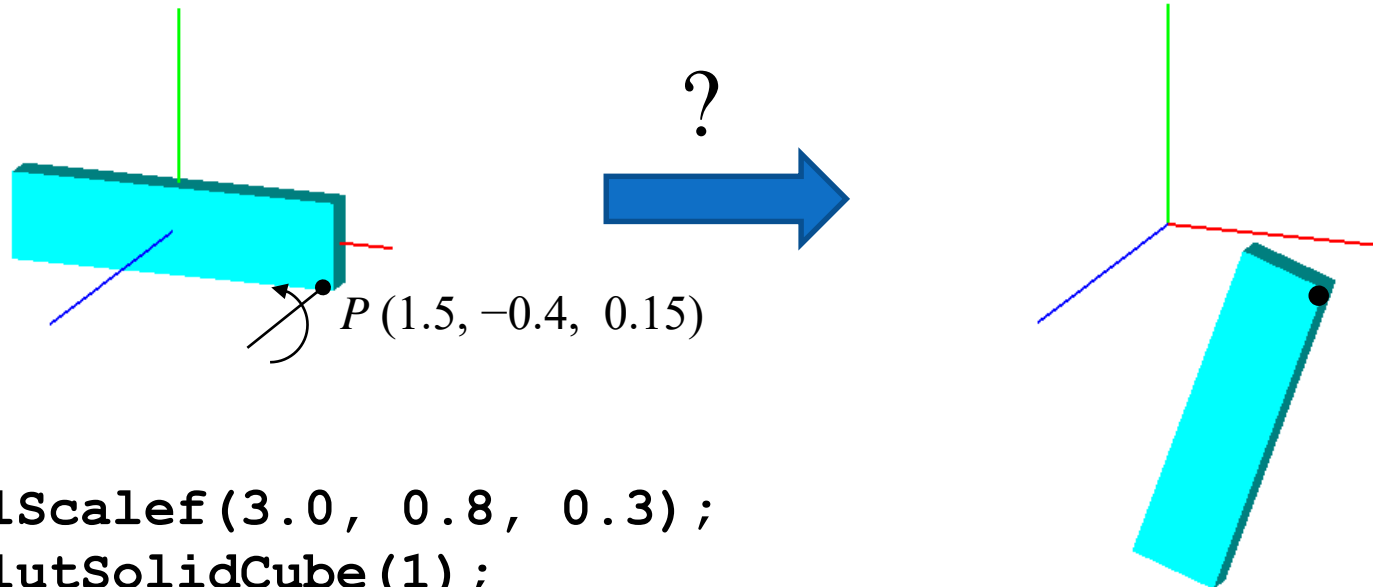
- Rotations of an object about an axis passing through a pivot point are often required.
- A pivot point remains fixed during the rotational transformation.



- OpenGL rotations are always performed with the origin as the pivot point
- How can we perform rotations about an arbitrary pivot point? (Eg. Rotation of an arm about the shoulder joint, rotation of a wheel about its centre)

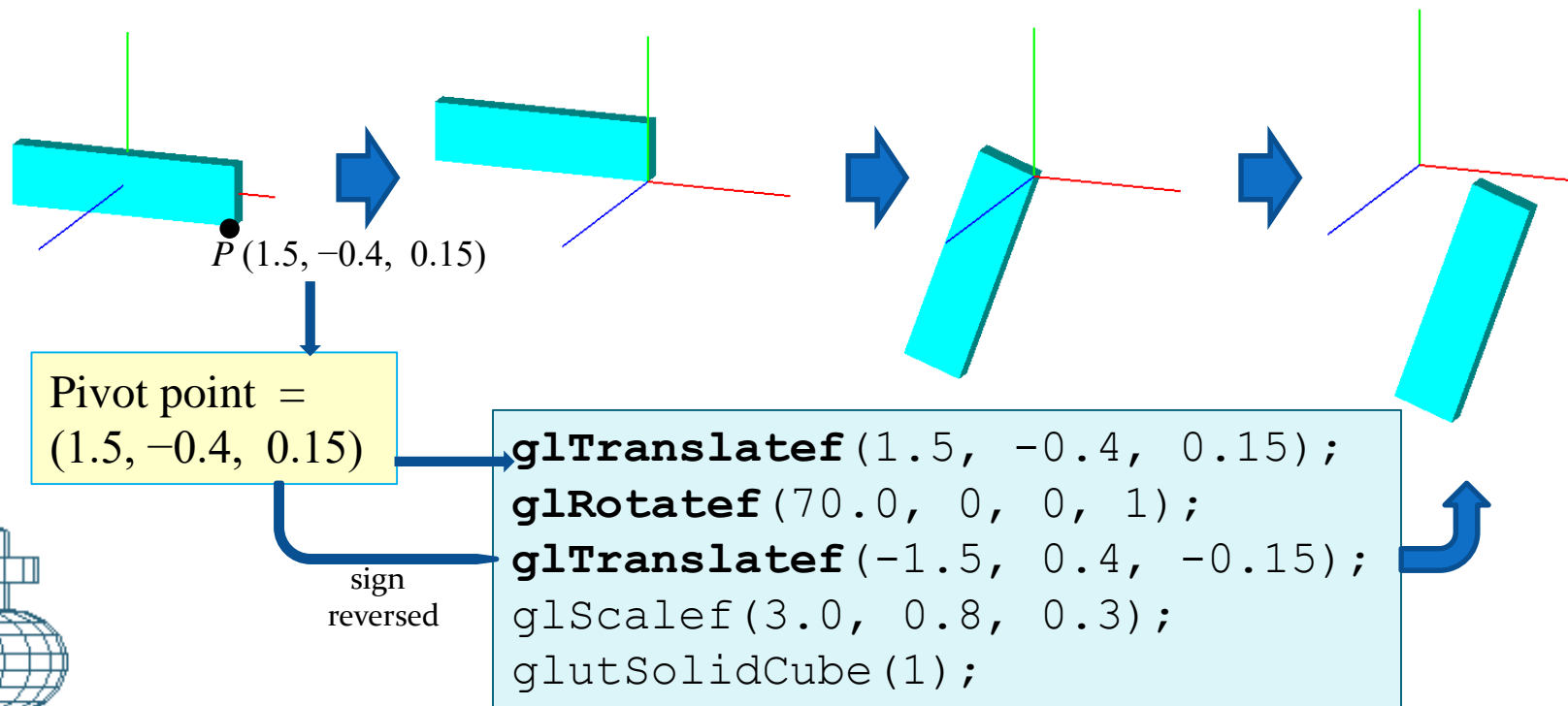
# Rotations about a pivot point

Example: We require a 70 deg rotation of the parallelopiped in the following figure about the pivot point  $P$  and axis of rotation parallel to  $z$ .

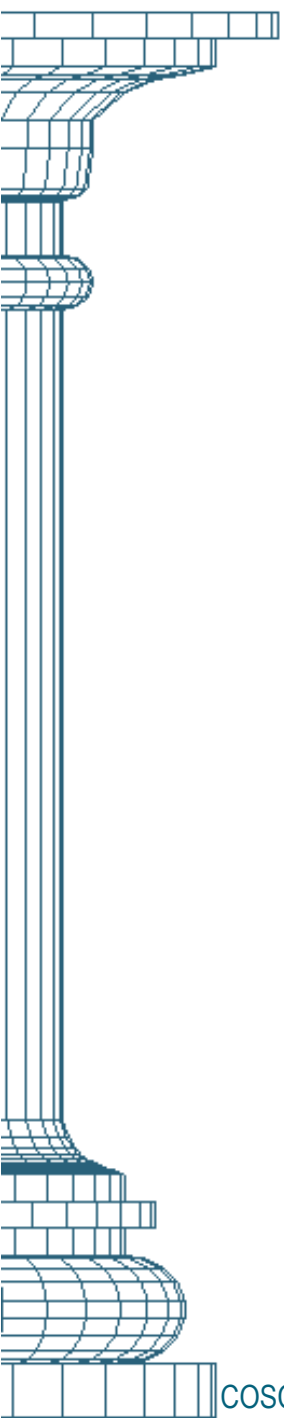


# Rotations about a pivot point

- First, translate the object such that pivot point  $(p_x, p_y, p_z)$  goes to the origin: `glTranslatef( $-p_x$ ,  $-p_y$ ,  $-p_z$ )`
- Perform the required rotation: `glRotatef(theta,  $l$ ,  $m$ ,  $n$ )`
- Translate the object so that the pivot point goes back to its original position: `glTranslatef( $p_x$ ,  $p_y$ ,  $p_z$ )`



# Trivia Quiz

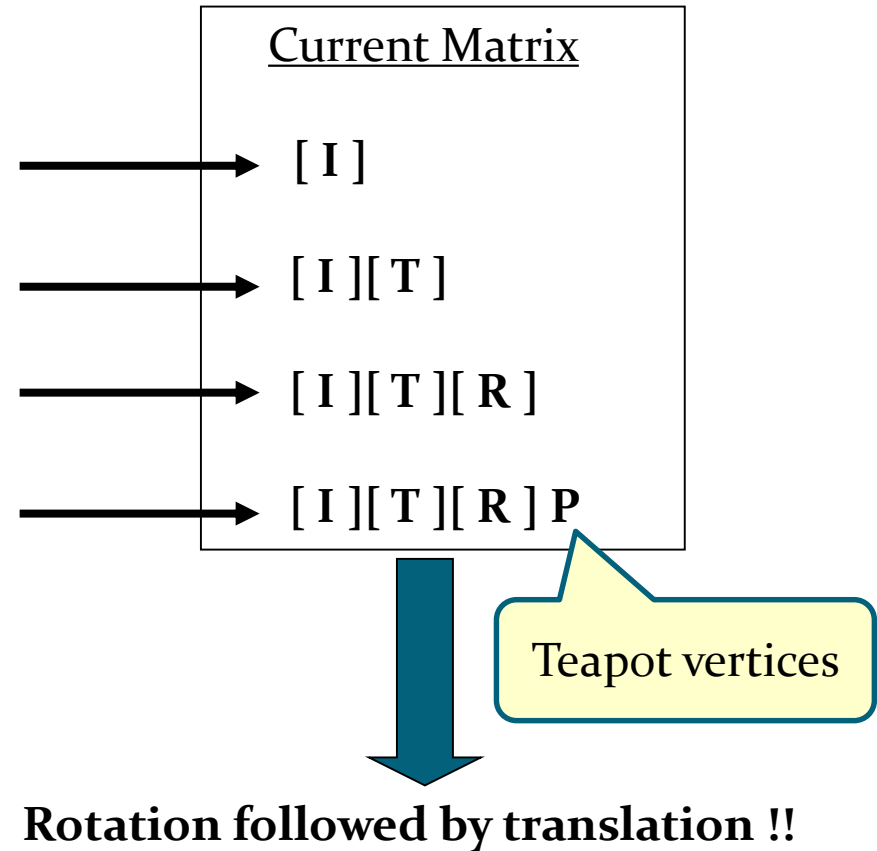




# OpenGL Transformations

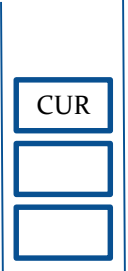
OpenGL **post**-multiplies the current transformation matrix with the new transformation matrix

```
glMatrixMode(GL_MODELVIEW);  
glLoadIdentity();  
glTranslatef(tx, ty, tz);  
glRotatef(theta, 0, 0, 1.0);  
glutSolidTeapot(1);
```

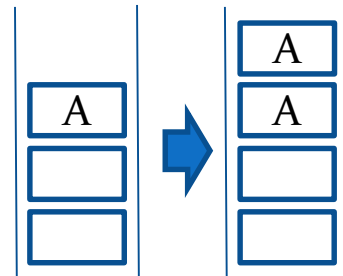


# Matrix Stack

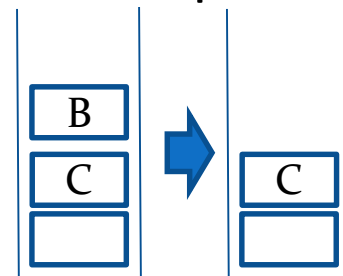
- A transformation matrix can be pushed into the matrix stack, saving it for later use.
- The top of stack represents the current transformation matrix
- The matrix stack is useful for applying different and independent sets of transformations to different objects.
- OpenGL:



- **glPushMatrix()**: Create a copy of the current transformation matrix and push into the stack



- **glPopMatrix()**: Remove the matrix at the top of stack



# Independent Transformations

```
glMatrixMode(GL_MODELVIEW);  
glLoadIdentity();  
gluLookAt(...);
```

View

```
glPushMatrix();
```

View

View

```
    glTranslatef(-2.0, 0.3, 0.0);  
    glScalef(0.5, 0.5, 0.5);  
    glutSolidTeapot(1);
```

VT

View

Applied to the  
teapot

```
glPopMatrix();
```

View

```
glPushMatrix();
```

View

View

```
    glTranslatef(4, 0.5, 3);  
    glutSolidCube(1);
```

VT

View

Applied to the  
cube

```
glPopMatrix();
```

# Independent Transformations

A modified example:

```
glMatrixMode(GL_MODELVIEW);  
glLoadIdentity();  
gluLookAt(...);  
  
glRotatef(30, 0, 1, 0);  
  
glPushMatrix();  
    glTranslatef(-2.0, 0.3, 0.0);  
    glScalef(0.5, 0.5, 0.5);  
    glutSolidTeapot(1);  
glPopMatrix();  
  
glPushMatrix();  
    glTranslatef(4, 0.5, 3);  
    glutSolidCube(1);  
glPopMatrix();
```

→ This transformation will be applied to both the teapot and the cube.

# Transformation of Light Sources

```
void display()
{
    float lgt_pos[4]={0., 10., 10., 1.};
    glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT);
    glMatrixMode(GL_MODELVIEW);
    glLoadIdentity();

    gluLookAt(5., 3., 2., 0., 0., 0., 0., 1., 0.);

    glLightfv(GL_LIGHT0, GL_POSITION, lgt_pos);

    glTranslatef(0.0, 1.2, 0.0);
    glRotatef(angle, 0.0, 1.0, 0.0);

    glutSolidTeapot(1.0);
    glFlush();
}
```

Light source moves with the object  
Light source's position fixed in the scene  
Light source fixed relative to the camera.

A point light source is transformed like any other point