

COSC428 Computer Vision



Filters

What is image filtering?

- Modify the pixels in an image based on some function of a local neighborhood of the pixels.

10	5	3
4	5	1
1	1	7

Local image data

Some function



	7	

Modified image data 9

Linear functions

- Simplest: linear filtering.
 - Replace each pixel by a linear combination of its neighbors.
- The prescription for the linear combination is called the “convolution kernel”.

10	5	3
4	5	1
1	1	7

Local image data

0	0	0
0	0.5	0
0	1	0.5

kernel

	7	

Modified image data

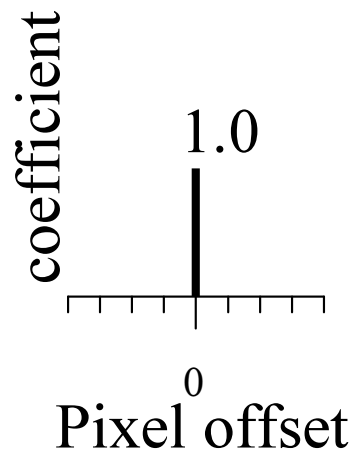
Convolution

$$f[m, n] = I \otimes g = \sum_{k, l} I[m - k, n - l] g[k, l]$$

Linear filtering (warm-up slide)



original

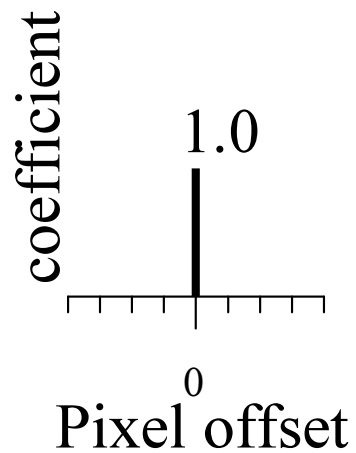


?

Linear filtering (warm-up slide)



original

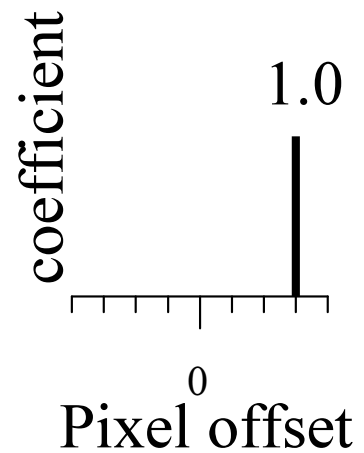


Filtered
(no change)

Linear filtering

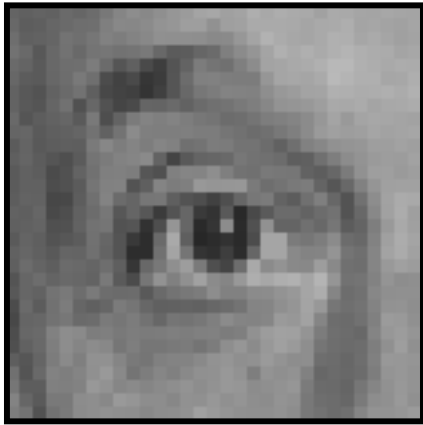


original

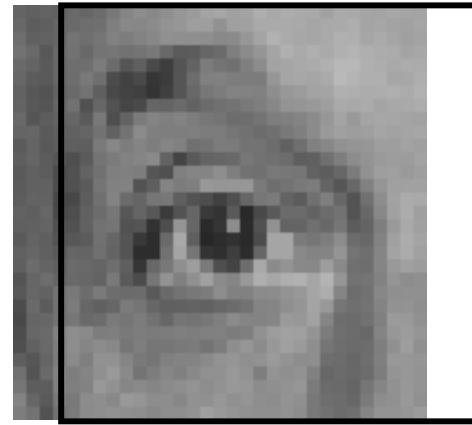
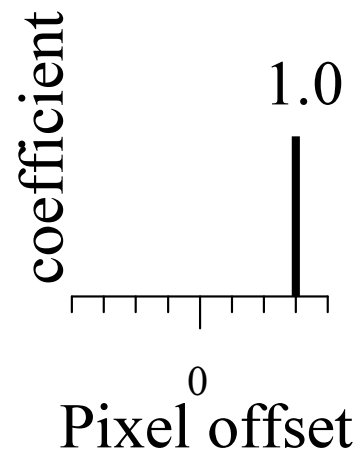


?

shift



original

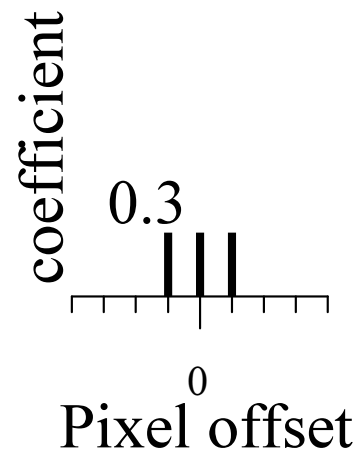


shifted

Linear filtering



original

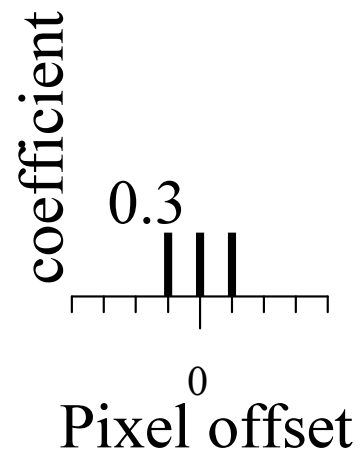


?

Blurring

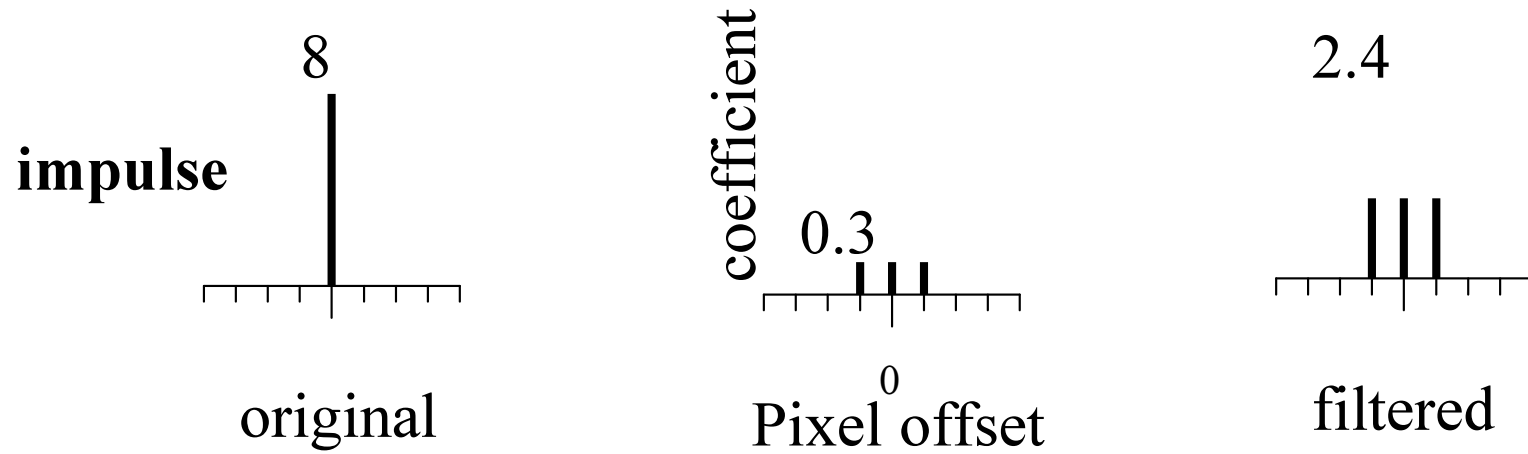


original

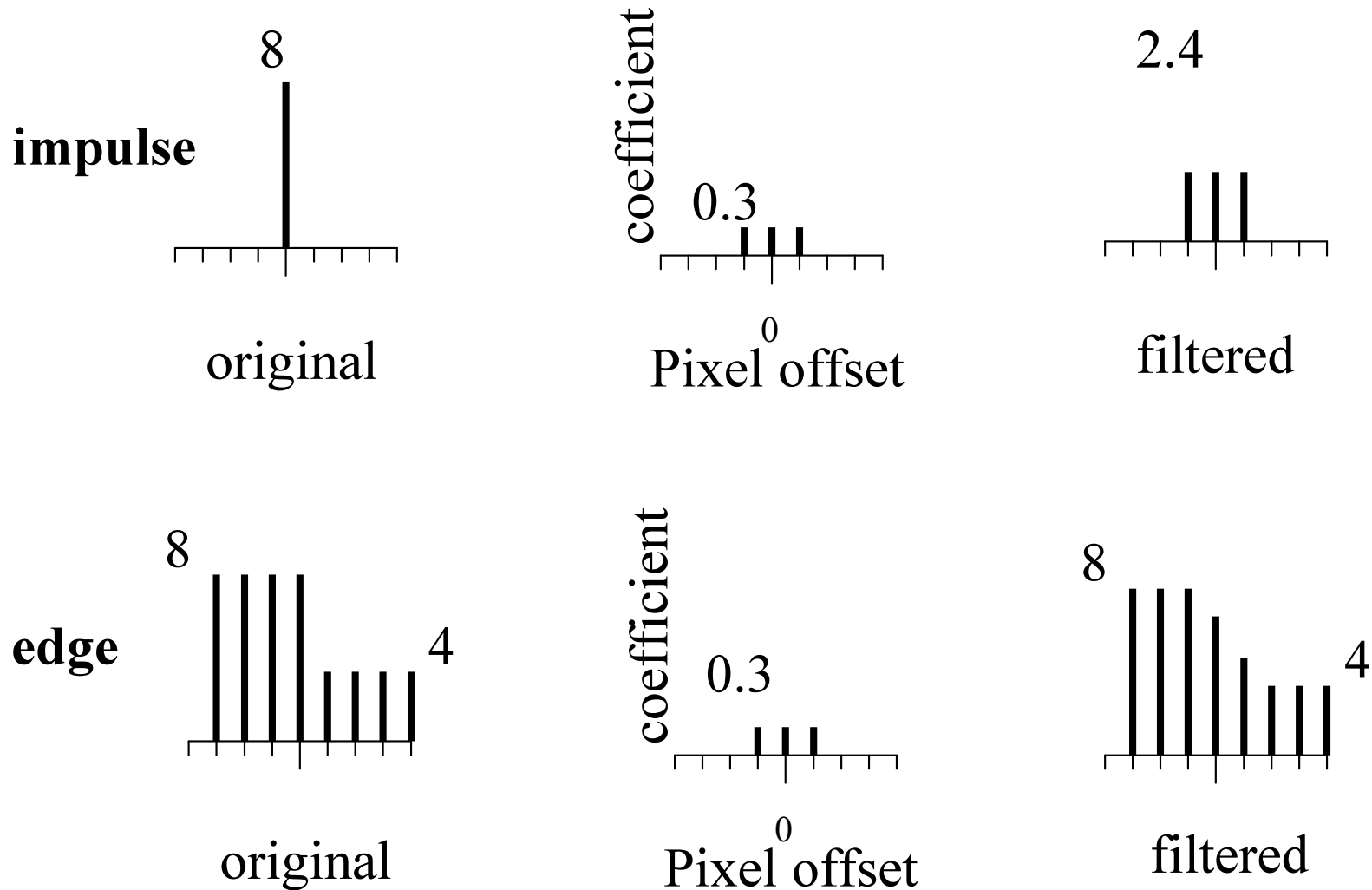


Blurred (filter applied in both dimensions).

Blur examples



Blur examples



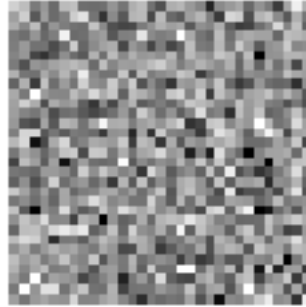
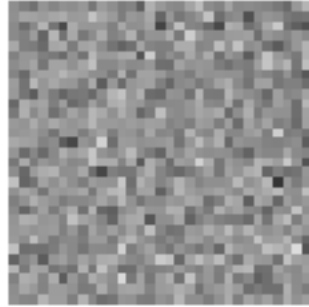
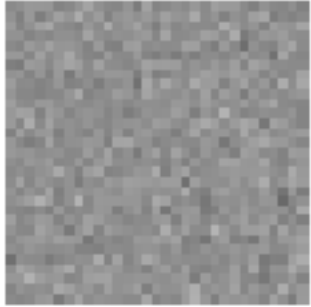
Smoothing reduces noise

- Generally expect pixels to “be like” their neighbours
 - surfaces turn slowly
 - relatively few reflectance changes
- Generally expect noise processes to be independent from pixel to pixel
- Implies that smoothing suppresses noise, for appropriate noise models
- Scale
 - the parameter in the symmetric Gaussian
 - as this parameter goes up, more pixels are involved in the average
 - and the image gets more blurred
 - and noise is more effectively suppressed

$\sigma=0.05$

$\sigma=0.1$

$\sigma=0.2$



no
smoothing



$\sigma=1$ pixel



$\sigma=2$ pixels

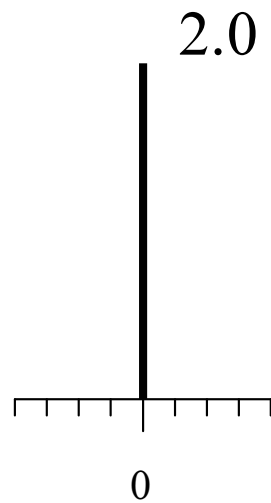
The effects of smoothing

Each row shows smoothing with gaussians of different width; each column shows different realisations of an image of gaussian noise.

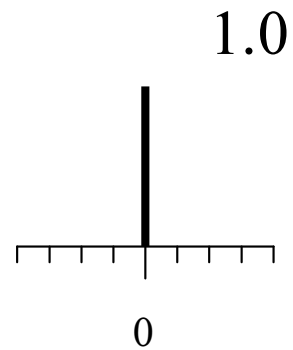
Linear filtering (warm-up slide)



original



—

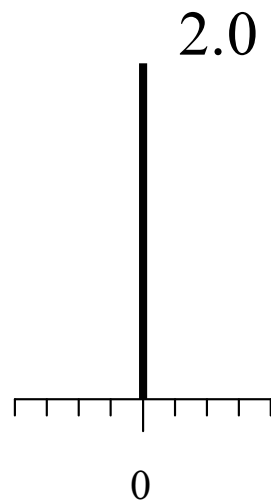


?

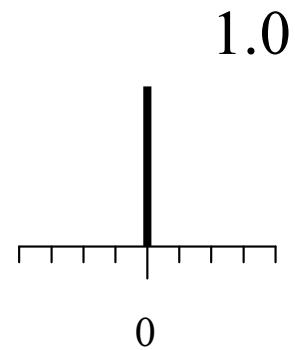
Linear filtering (no change)



original

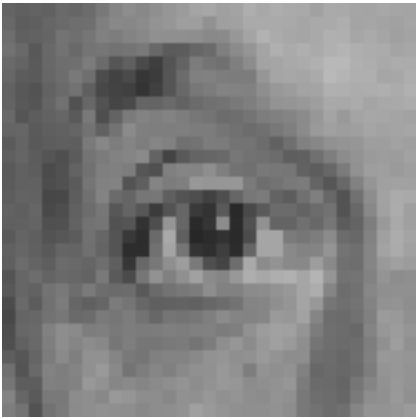


—

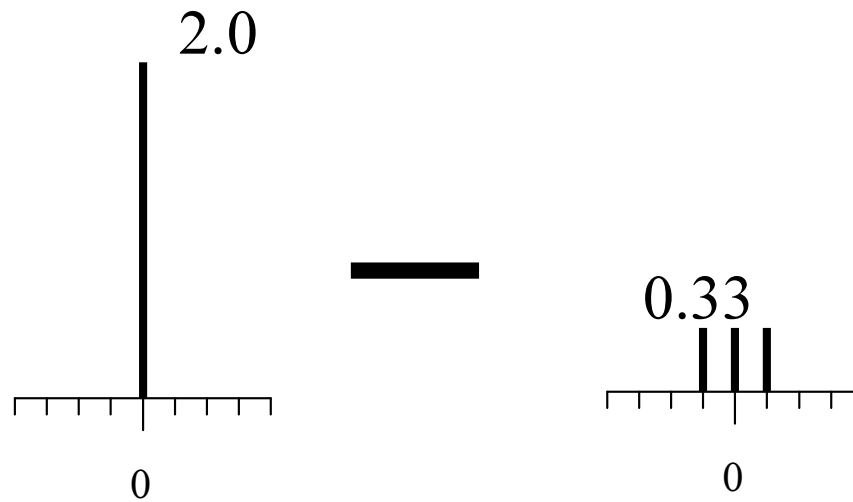


Filtered
(no change)

Linear filtering



original

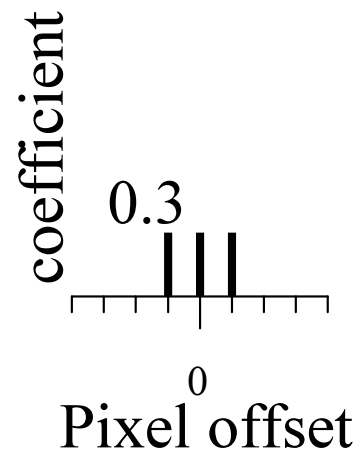


?

(remember blurring)



original

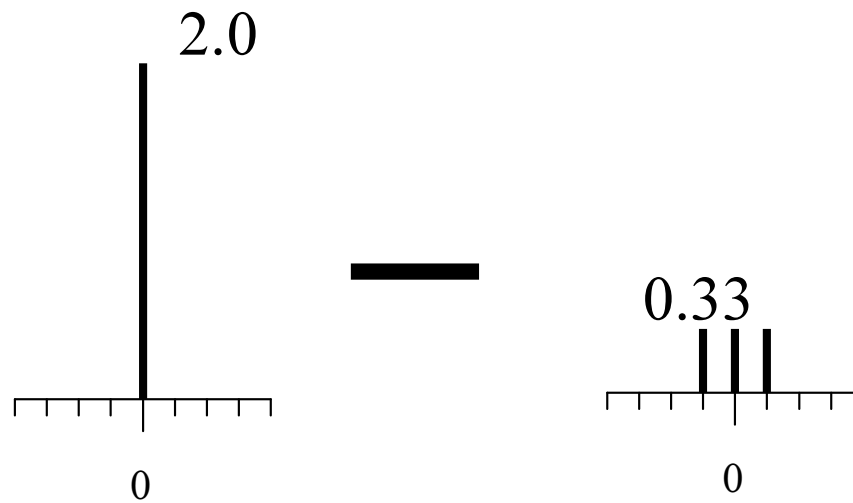


Blurred (filter applied in both dimensions).

Linear filtering

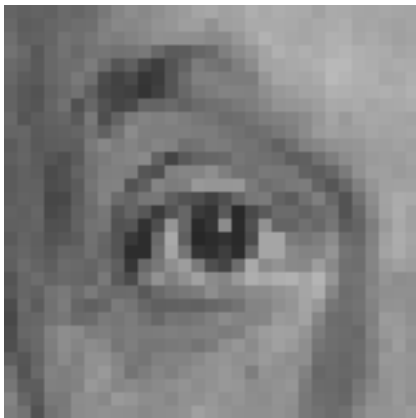


original

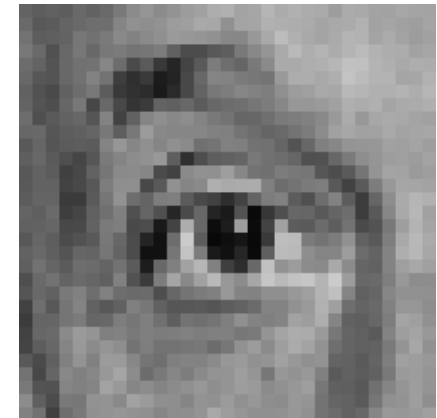
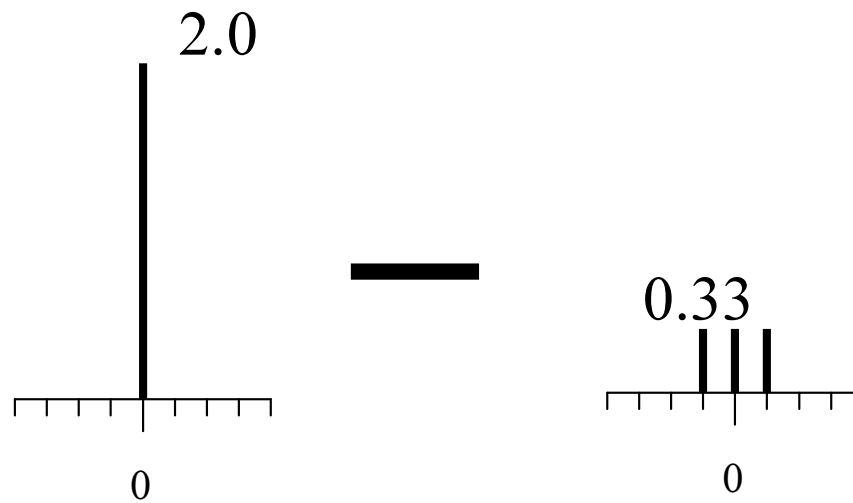


?

Sharpening

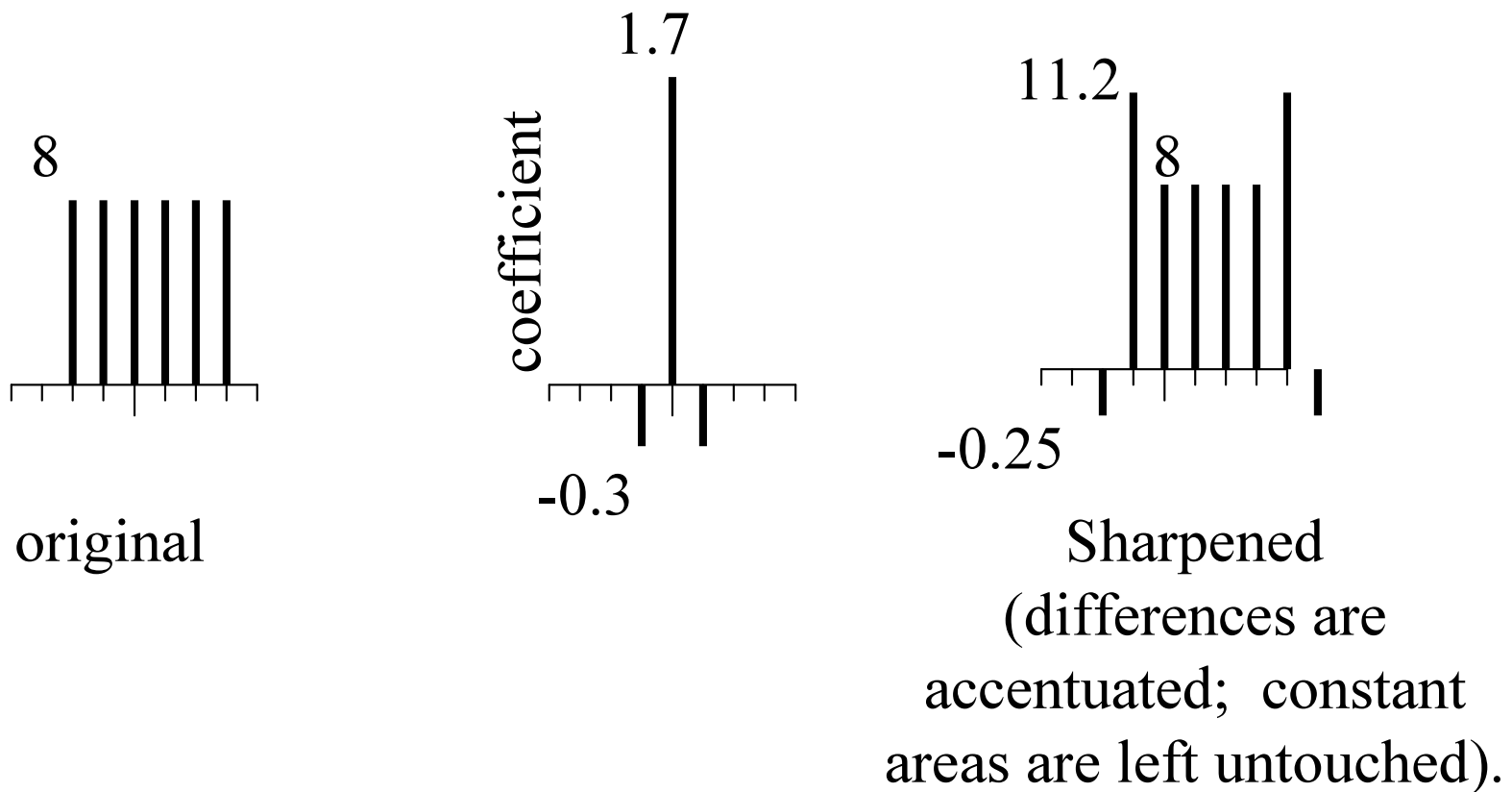


original

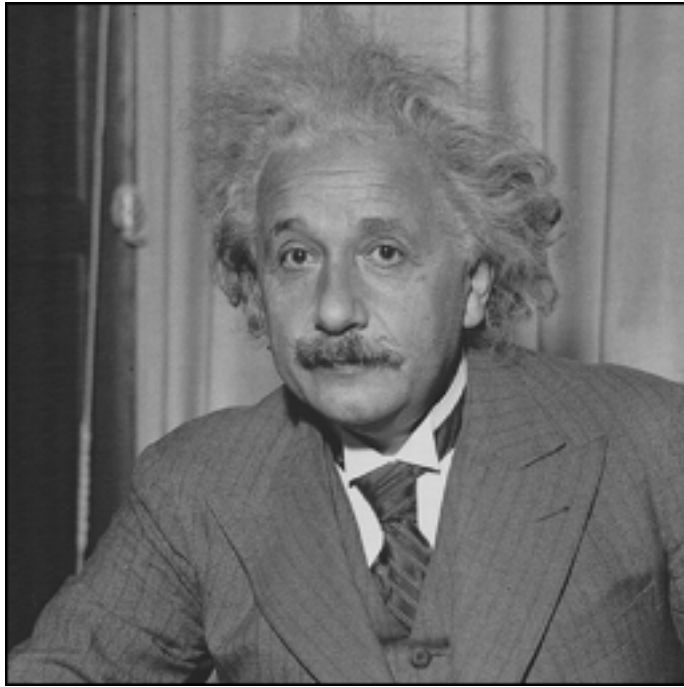


Sharpened
original

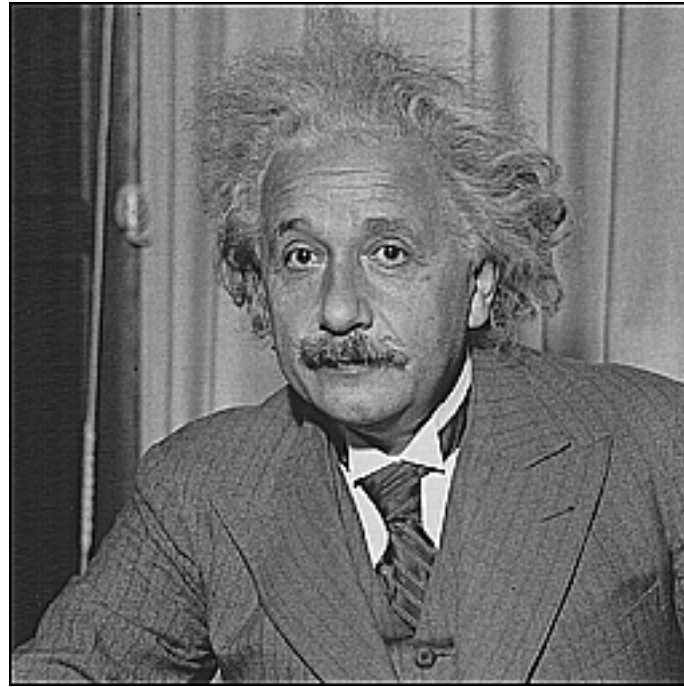
Sharpening example



Sharpening



before



after

Gradients and edges

- Points of sharp change in an image are interesting:
 - change in reflectance
 - change in object
 - change in illumination
 - noise
- Sometimes called **edge points**
- General strategy
 - linear filters to estimate image gradient
 - mark points where gradient magnitude is particularly large wrt neighbours (ideally, curves of such points).

Linear image transformations

- In analyzing images, it's often useful to make a change of basis.


The diagram illustrates the equation $\vec{F} = U\vec{f}$. A blue arrow points from the text "transformed image" to the vector \vec{F} . Another blue arrow points from the text "Vectorized image" to the vector \vec{f} . A third blue arrow points from the text "Fourier transform, or Wavelet transform, or Steerable pyramid transform" to the transformation matrix U .

transformed image $\vec{F} = U\vec{f}$ Vectorized image

Fourier transform, or
Wavelet transform, or
Steerable pyramid transform

Self-inverting transforms

Same basis functions are used for the inverse transform

$$\begin{aligned}\vec{f} &= U^{-1} \vec{F} \\ &= U^+ \vec{F}\end{aligned}$$


U transpose and complex conjugate

An example of such a transform: the Fourier transform

discrete domain

Forward transform

$$F[m, n] = \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} f[k, l] e^{-\pi i \left(\frac{km}{M} + \frac{ln}{N} \right)}$$

Inverse transform

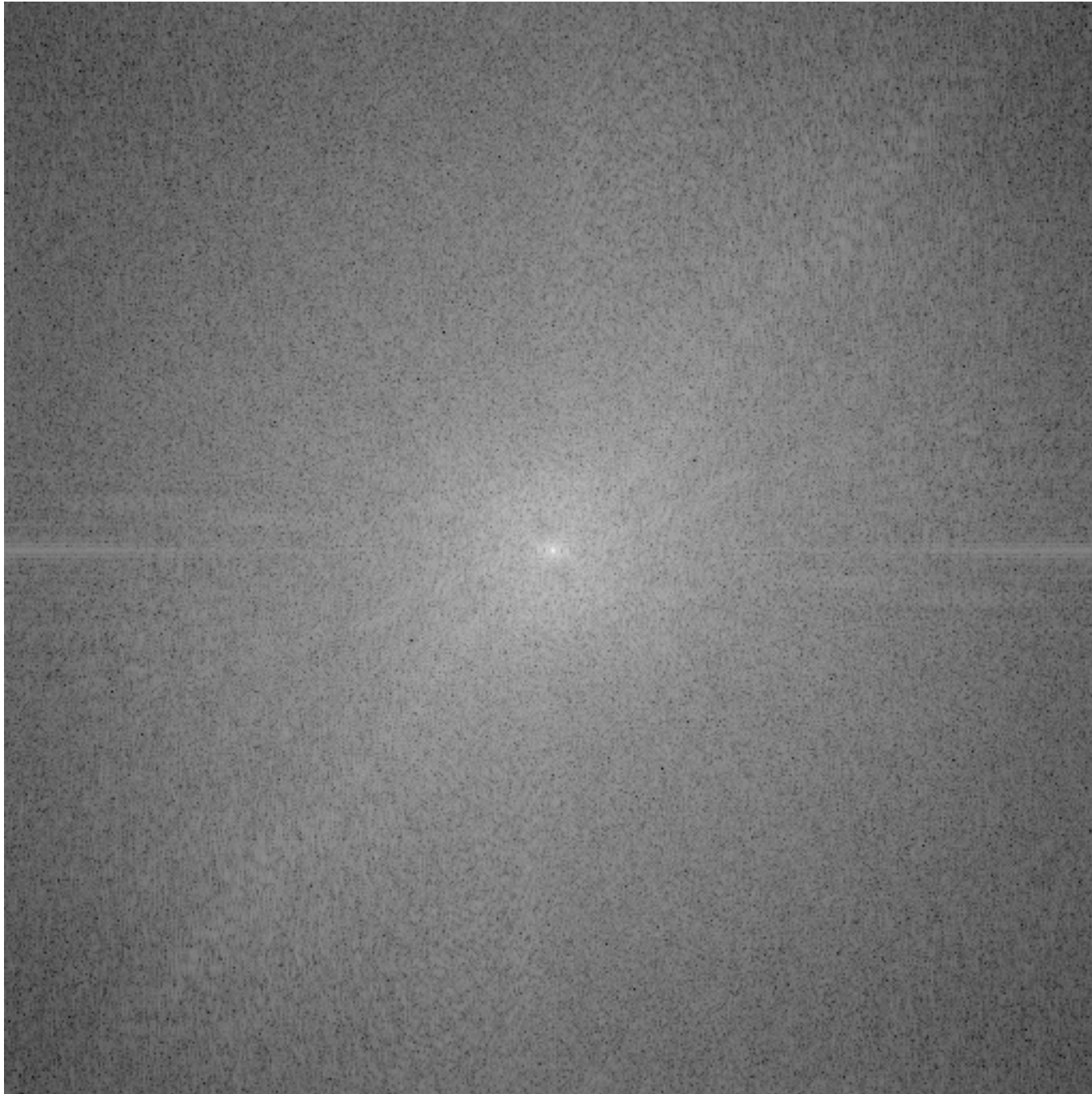
$$f[k, l] = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} F[m, n] e^{+\pi i \left(\frac{km}{M} + \frac{ln}{N} \right)}$$

Phase and Magnitude

- Fourier transform of a real function is complex
 - difficult to plot, visualize
 - instead, we can think of the phase and magnitude of the transform
- Phase is the phase of the complex transform
- Magnitude is the magnitude of the complex transform
- Curious fact
 - all natural images have about the same magnitude transform
 - hence, phase seems to matter, but magnitude largely doesn't
- Demonstration
 - Take two pictures, swap the phase transforms, compute the inverse - what does the result look like?

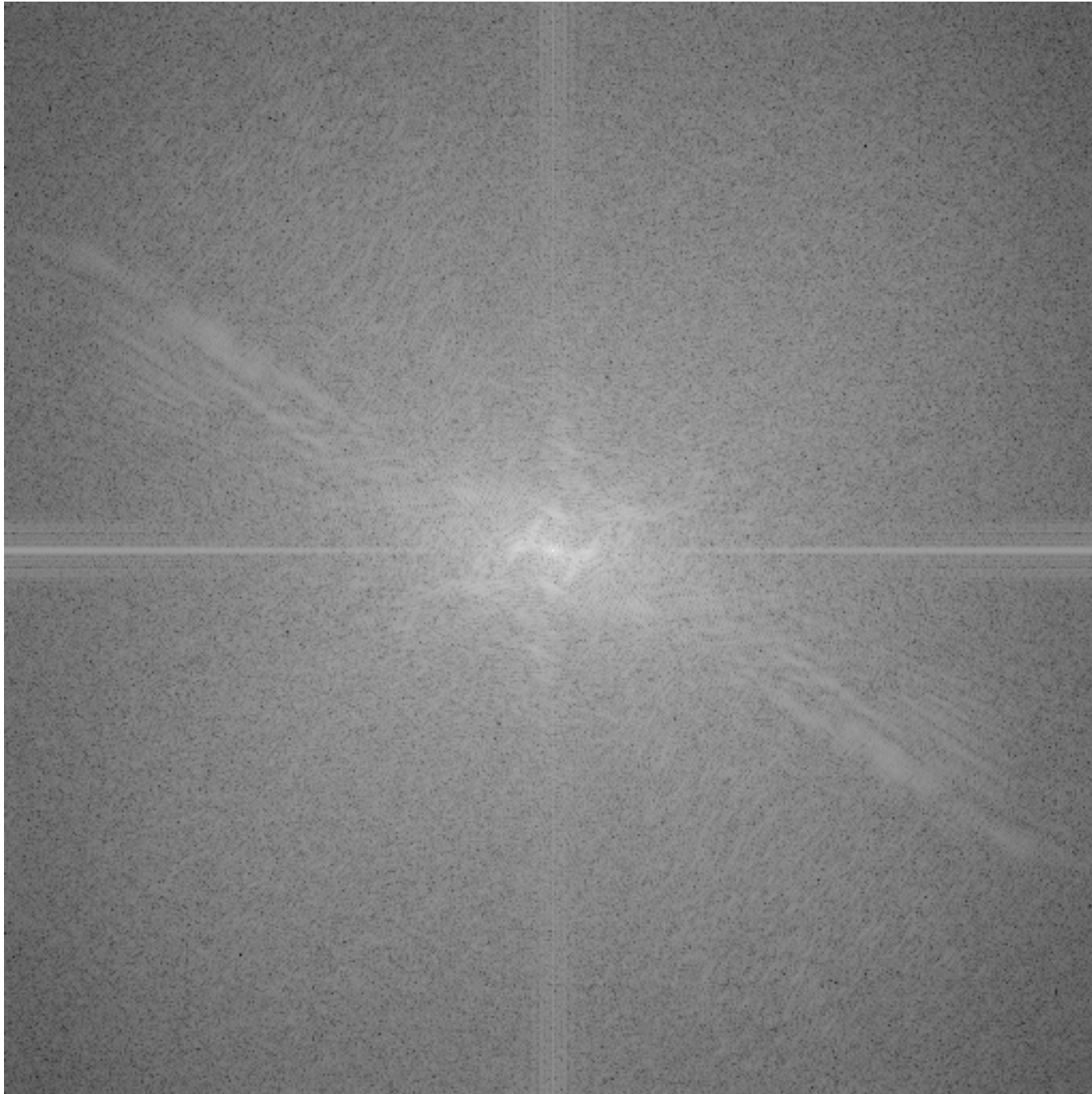


This is the
magnitude
transform
of the
cheetah pic



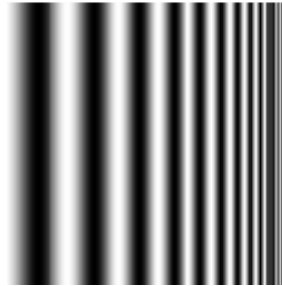


This is the
magnitude
transform
of the zebra
pic



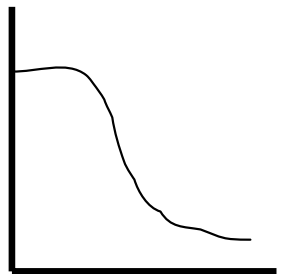
Frequency filtering

sinusoid

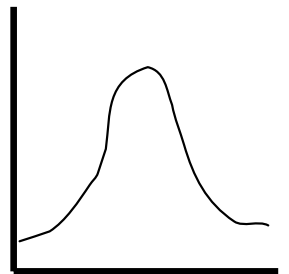
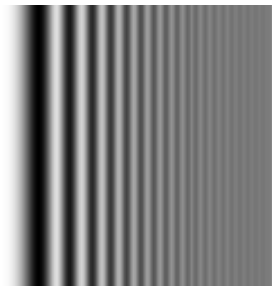


$$\text{in}(x,y) = 1 + \sin(1/x)$$

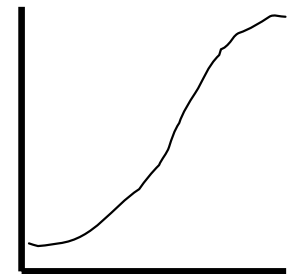
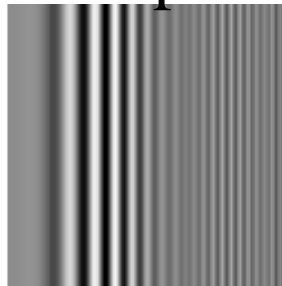
filter:



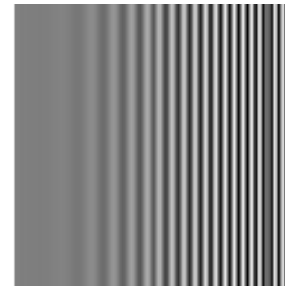
Frequency →
low-pass



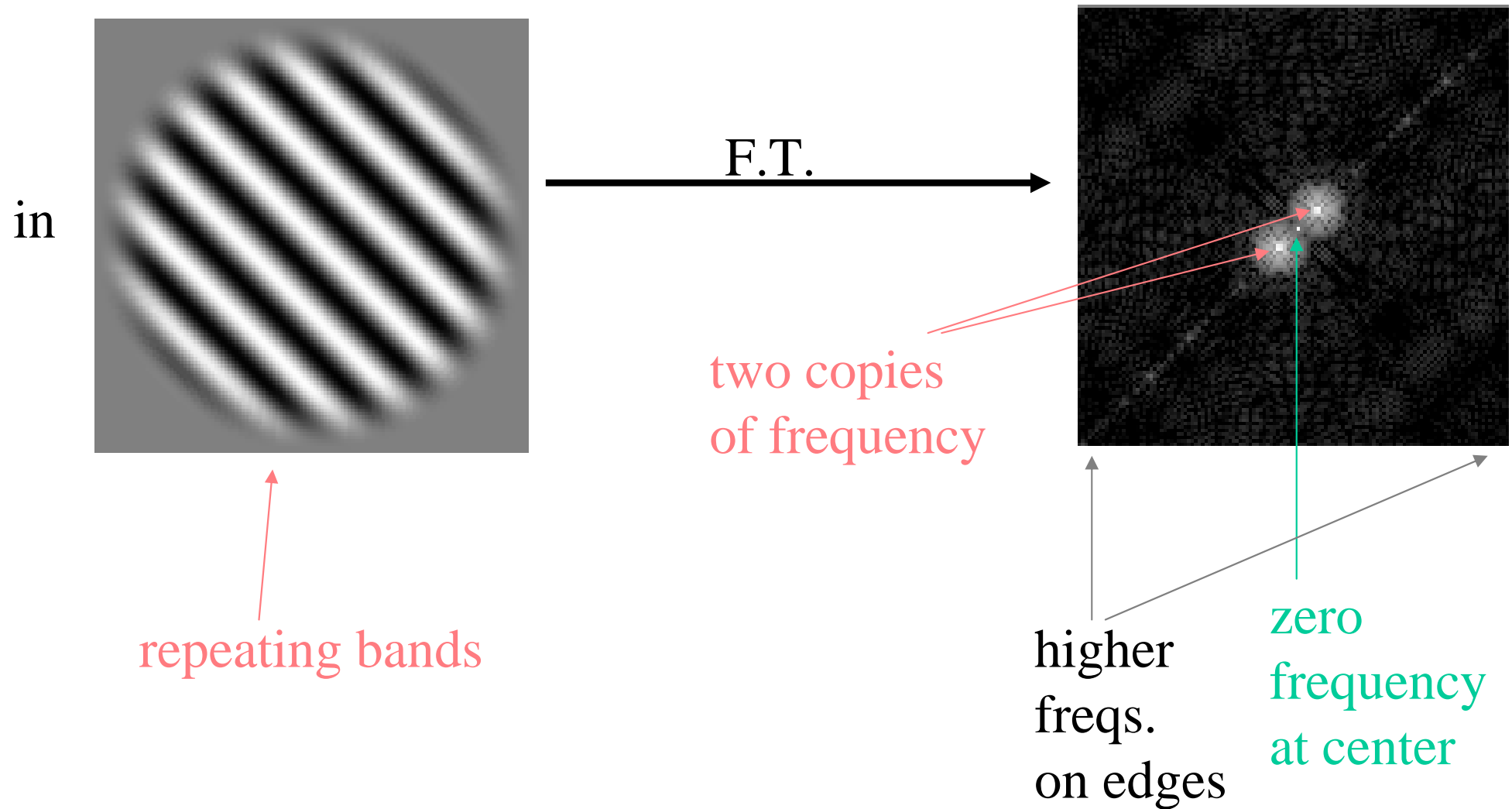
Frequency →
band-pass



Frequency →
high-pass



Fourier Transform

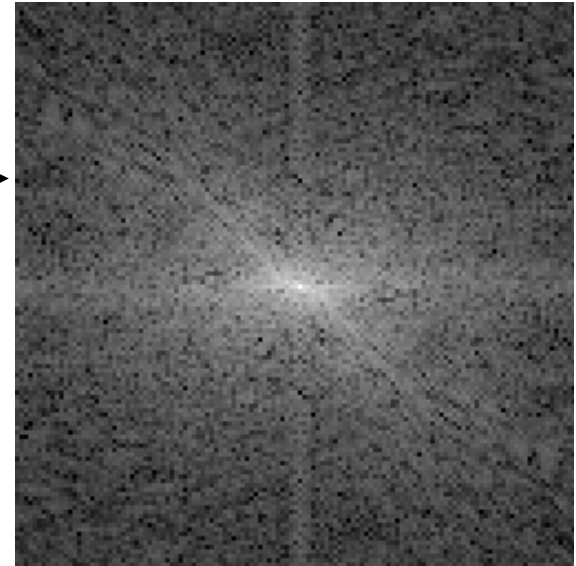


Fourier Low-pass filter

in



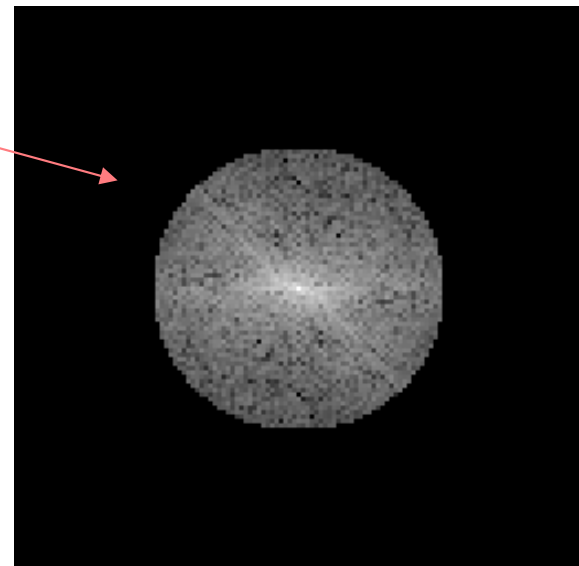
F.T.



high freqs
suppressed



inverse F.T.



blurred,
ringing



out

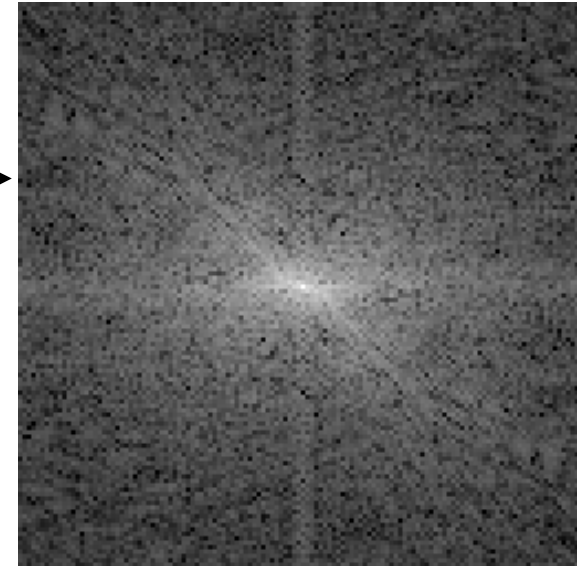


Fourier High-pass filter

in

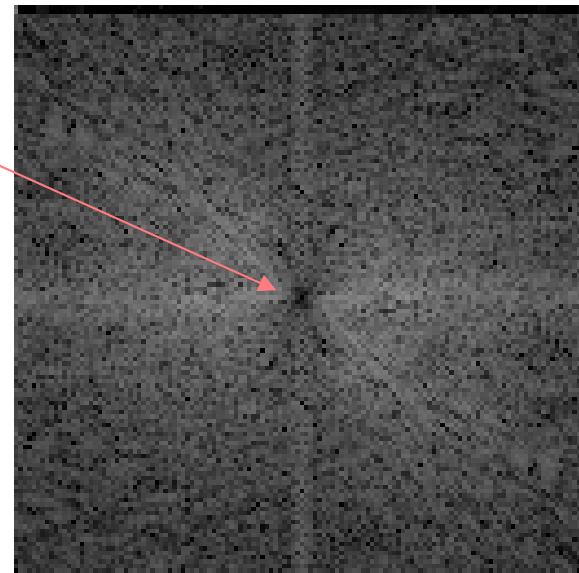


F.T.



low freqs
suppressed

inverse F.T.



edges
detected



out

