EMTH211-19S2 LABORATORY 10

SEPTEMBER 30-OCTOBER 4, 2019

These exercises deal with

- SVD

Reading guide (Poole, Linear Algebra)

Section 7.4.

10.1 Let

$$\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Find a SVD for both $\mathbf{v}$ and $\mathbf{v}^T$.

10.2 Find a SVD for

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}.$$

10.3 Compute the pseudoinverse of

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

10.4 Every complex number z can be written in the form

$$z = r\, e^{i\theta}$$

where r is a real non-negative number. With this representation, z has been decomposed into a stretching factor r and a rotation θ. There is an analogous decomposition for *any* square matrix A. Show that a square matrix A may be factored

$$A = R\, Q$$

where R is a symmetric matrix with non-negative eigenvalues (and NOT a upper triangular matrix) and Q is an orthogonal matrix. (*Hint:* Write a SVD of $A = U\,\Sigma\,V^T = \left(U\,\Sigma\,U^T\right)\left(U\,V^T\right)$.) This decomposition is called the **polar decomposition** of A. R represents a scaling and Q represents a rotation.

10.5 Compute the polar decomposition for the matrices in questions 2 and 3.