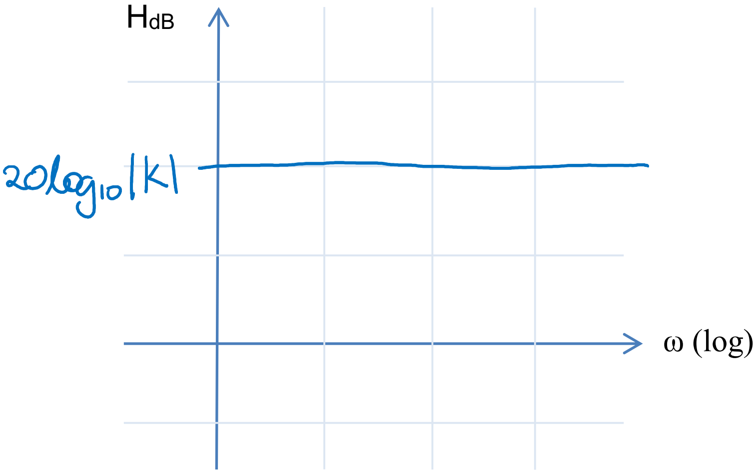
Values

$\lvert H(j\omega)\rvert$	H_{dB}
1	0
2	6.02
10	20

- An increase of $|H(j\omega)|$ by a factor of 10 leads to an increase in H_{dB} by 20 dB.

Multiplying by factor K in H(s)

- Horizontal straight line at $20\log_{10}|K|$ dB above (below if |K| < 1) abscissa.



⇒ if -ve sign it goes on the phase plot ⇒ can't take log of a -ve number.

Asymptotes

- Need to factor H(s) to show poles and zeros.

A simple zero

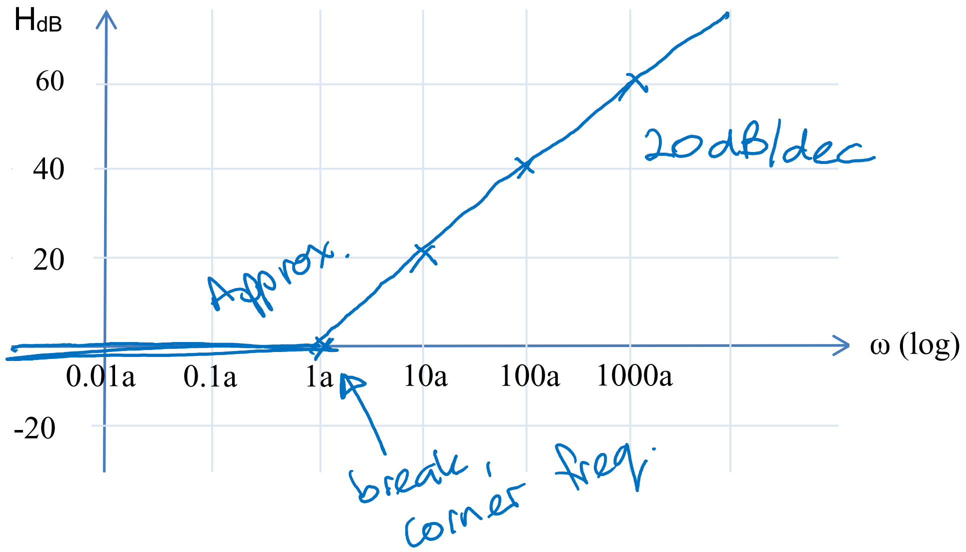
standardised form

- Consider a zero at **s** = -a.
  - $H(s) = 1 + \frac{s}{a}$ $= \cancel{\frac{s+a}{a}}$
  - $|H(j\omega)| = \left|1 + \frac{j\omega}{a}\right| = \sqrt{1 + \frac{\omega^2}{a^2}}$
  - $H_{dB} = 20\log_{10}\sqrt{1 + \frac{\omega^2}{a^2}}$
  - When ω « a, $H_{dB} \approx 20\log_{10} 1 = 0$
  - When ω » a, $H_{dB} \approx 20\log_{10}\left(\frac{\omega}{a}\right)$

Values

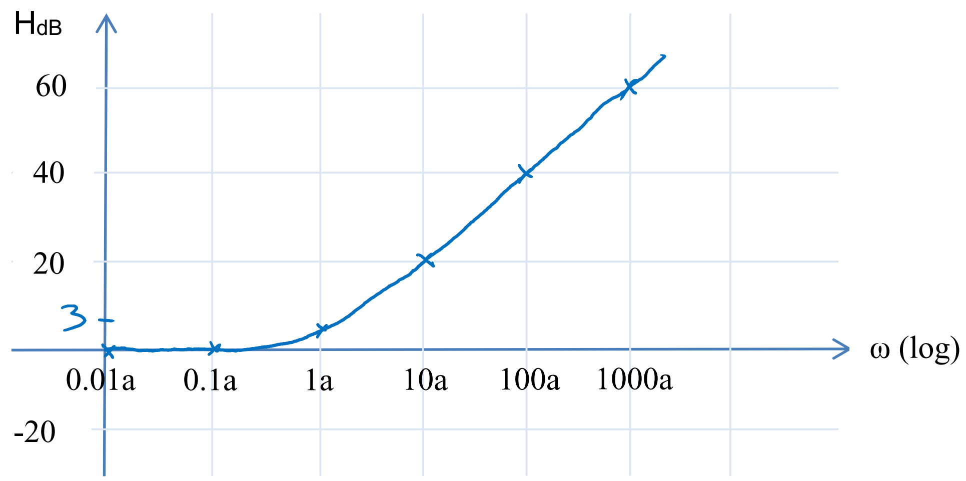
ω = a	$H_{dB} \approx 20\log_{10} 1 = 0$
10a	20
100a	40
1000a	60

- Every factor of 10 you go up 20dB. So, plot dB vs powers of 10 (20 dB/ decade).



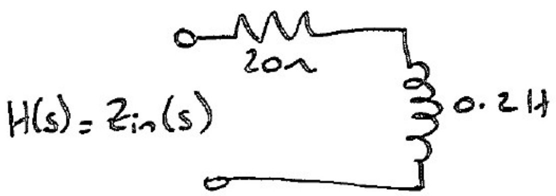
At ω = a, $H_{dB} = 20\log_{10}\sqrt{1+\frac{\omega^2}{a^2}} = 20\log_{10}\sqrt{2} \approx 3dB$.

Therefore, a better approximation is



Example

Draw the bode plot of the input impedance of this network.



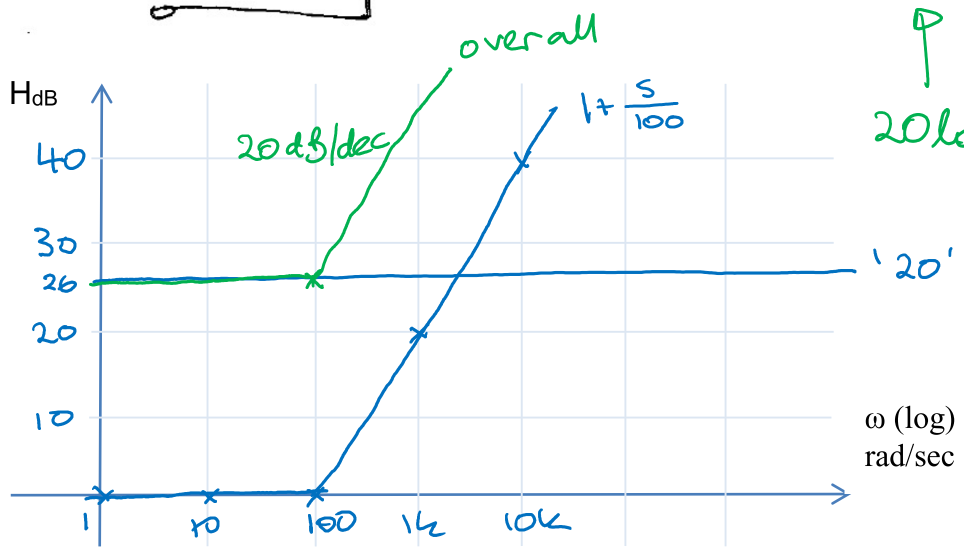
$$Z_{in}(s) = 20 + 0.2s$$

sL

$$= 20\left(1 + \frac{s}{100}\right)$$

break freq. $\omega = 100$

$20\log_{10}|20| = 26\text{dB}$

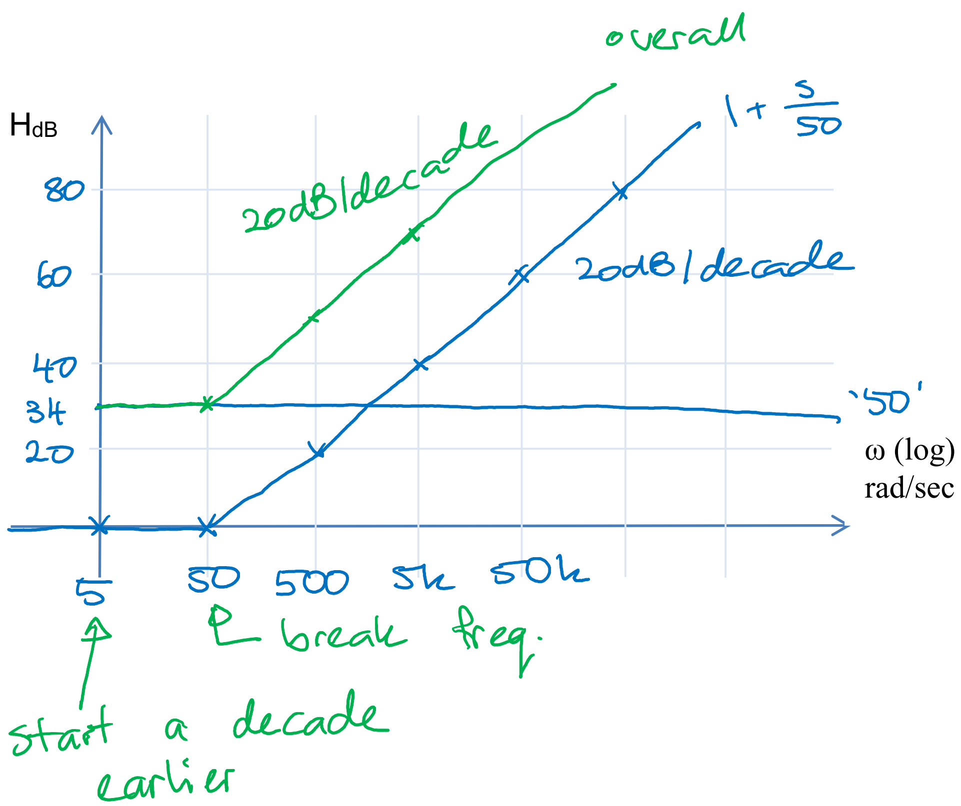


Example

Draw the bode magnitude plot for

$$H(s) = 50 + s = 50\left(1 + \frac{s}{50}\right)$$

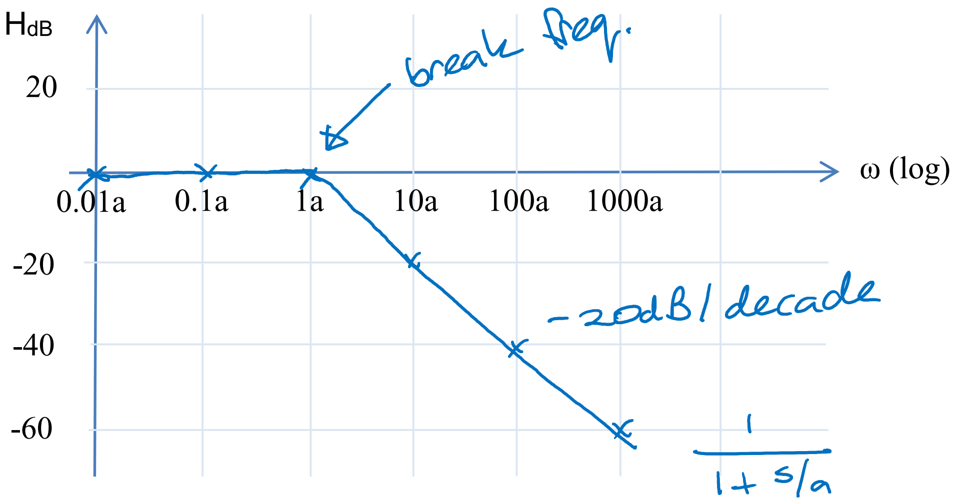
$20\log_{10}(50) = 34\text{dB}$



A simple pole

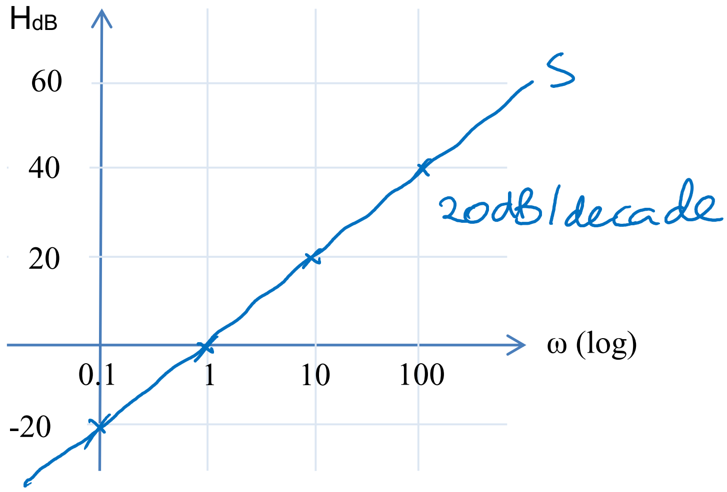
- Consider a pole at **s** = -a.
  - $H(s) = \frac{1}{1+\frac{s}{a}}$ ⟸ standard form
  - $|H(j\omega)| = \frac{1}{\sqrt{1+\frac{\omega^2}{a^2}}}$
  - $H_{dB} = -20\log_{10}\sqrt{1+\frac{\omega^2}{a^2}}$
  - When ω « a, $H_{dB} \approx -20\log_{10} 1 = 0$
  - When ω » a, $H_{dB} \approx -20\log_{10}\left(\frac{\omega}{a}\right)$

- Every factor of 10 you go down 20 dB. So, plot dB vs powers of 10 (-20 dB/decade).



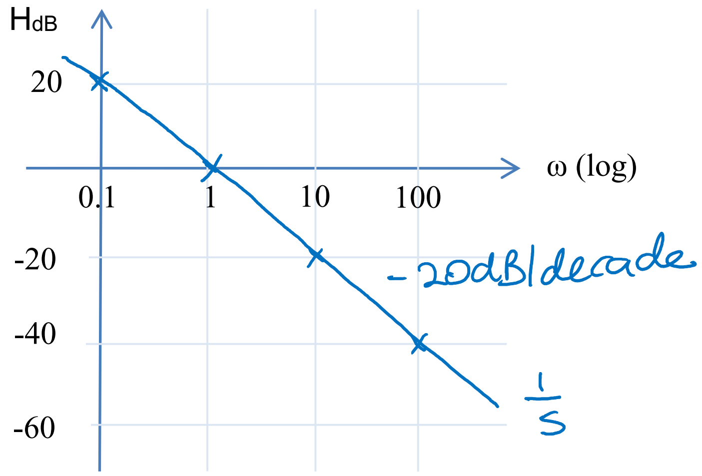
H(s) = s

- $H_{dB} = 20\log_{10}|\omega|$
- Infinite straight line passing up through 0 dB at ω = 1 rad/s.



H(s) = 1/s

- $H_{dB} = 20\log_{10}\left|\frac{1}{\omega}\right| = -20\log_{10}|\omega|$
- Infinite straight line passing down through 0 dB at ω = 1 rad/s.



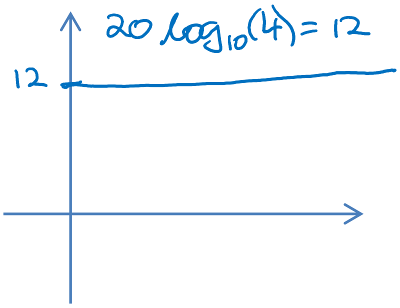
Example: Combinations

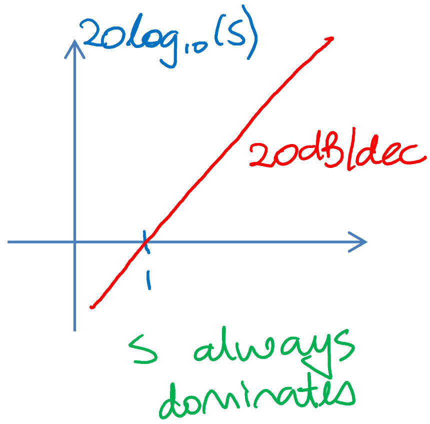
$$H(s) = \frac{4s(1+s)}{\left(1+\frac{s}{2}\right)^2}$$

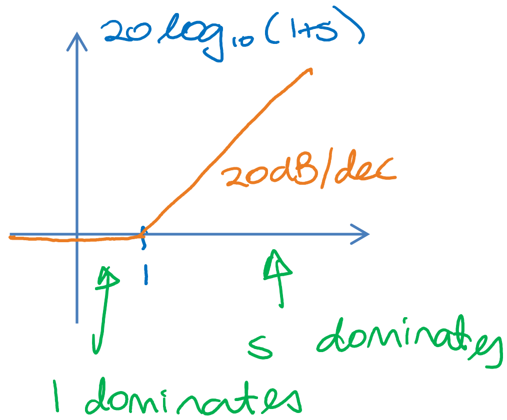
$$20\log_{10}|H(s)| = 20\log_{10}(4) + 20\log_{10}(s) + 20\log_{10}(1+s) - \underbrace{2\times 20\log_{10}\left(1+\frac{s}{2}\right)}_{-40\,\text{dB/dec}}$$

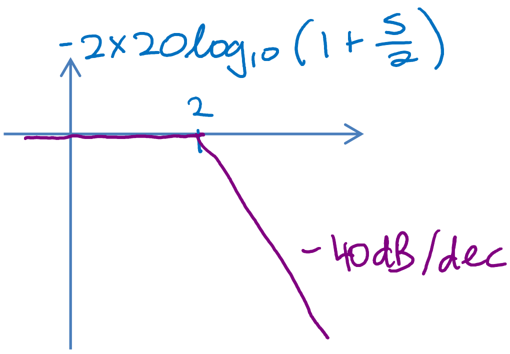
double pole $= \log(a^2) = 2\log a$

Plot each and add.

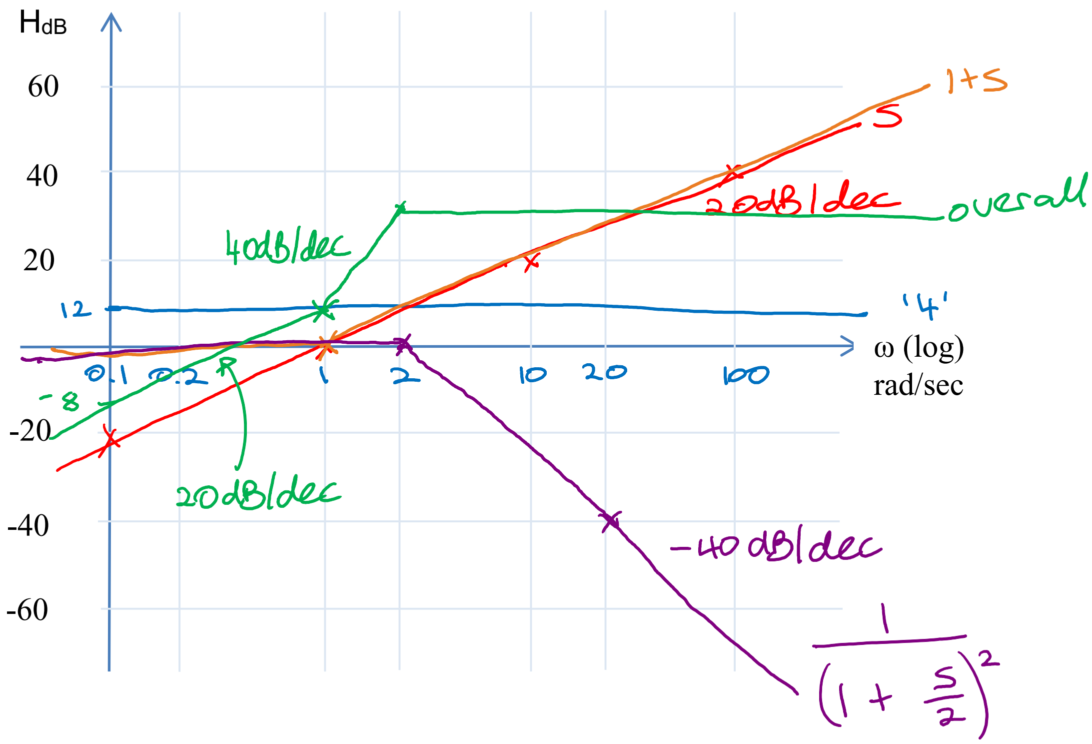




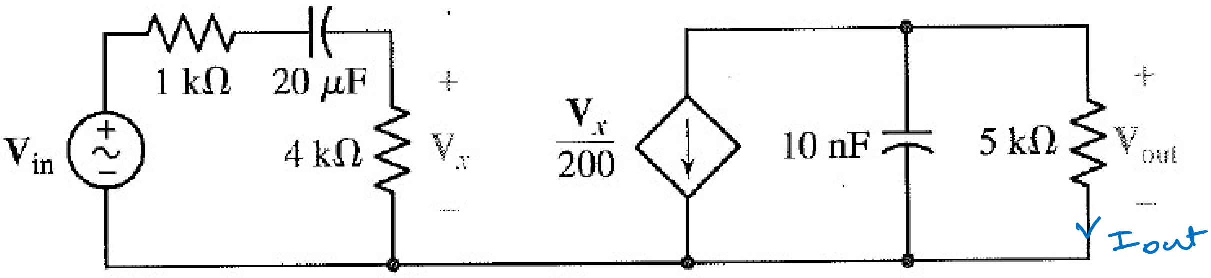




So, break frequencies at $\omega = 1$ & $\omega = 2$ rad/s



Example: Draw Bode plot for gain of circuit



Gain $H(s) = \frac{V_{out}}{V_{in}}$.

Work LHS to RHS of circuit.

$$V_x = \left(\frac{4k}{5k + \frac{10^6}{20s}}\right) V_{in} \quad (1)$$

← V divider

$$\frac{1}{10\times10^{-9}s} = \frac{10^8}{s} \quad \left(\frac{1}{sC}\right)$$

$$V_{out} = I_{out} \times 5k$$

$$= \left(\frac{10^8/s}{5k + \frac{10^8}{s}}\right)\left(\frac{-V_x}{200}\right) \times 5k \quad (2)$$

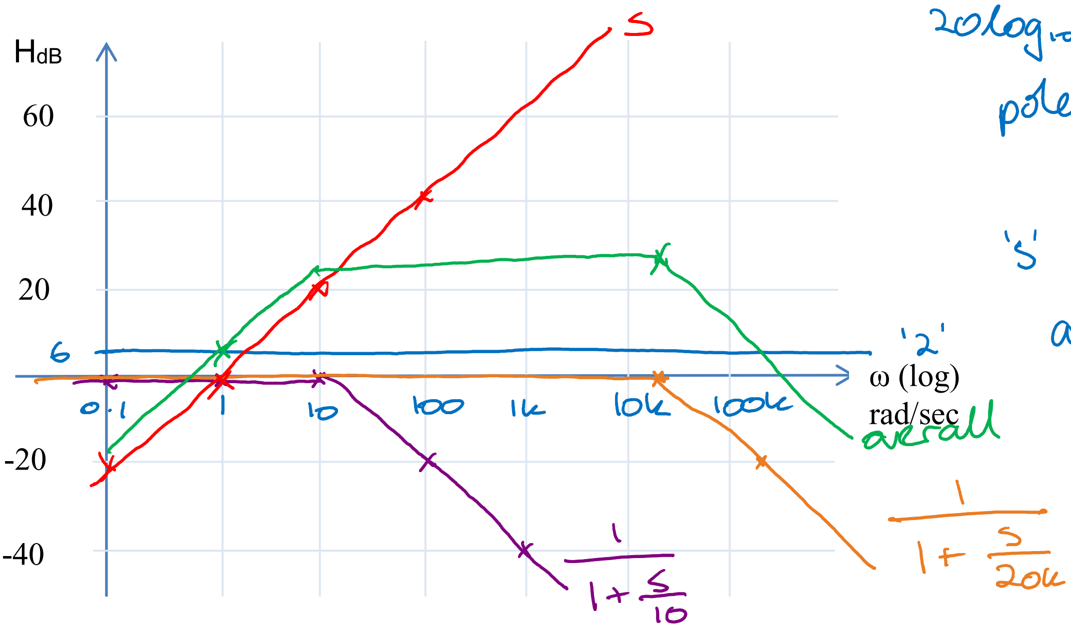
← I divider

Sub. (1) into (2) & rearrange:

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{-1}{200} \times \underbrace{\frac{4k}{5k + \frac{10^6}{20s}}}_{V_x/V_{in}} \times \frac{5k \times 10^8/s}{5k + 10^8/s}$$

$$\frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_x} \cdot \frac{V_x}{V_{in}}$$

$$= \frac{-2s}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{20k}\right)}$$



$20\log_{10}|-2| = 6\text{dB}$

poles ⇒ ω = 10 rad/s

& 20k rad/s

's'

all slopes ± 20dB/dec