EMTH211 — Exam Questions

1. Consider the matrix
$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 3 \\ -2 & 5 & 5 \end{bmatrix}$$

   (a) Compute the LU decomposition of A.

   (b) Use the LU decomposition from above to solve
$$A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(a) As usual, we use the following sequence of row operations to get A into upper row-echelon form:
$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 3 \\ -2 & 5 & 5 \end{bmatrix}$$
$$\xrightarrow{R_2-2R_1} \begin{bmatrix} 2 & 1 & 3 \\ 0 & -3 & -3 \\ -2 & 5 & 5 \end{bmatrix}$$
$$\xrightarrow{R_3+R_1} \begin{bmatrix} 2 & 1 & 3 \\ 0 & -3 & -3 \\ 0 & 6 & 8 \end{bmatrix}$$
$$\xrightarrow{R_3+2R_2} \begin{bmatrix} 2 & 1 & 3 \\ 0 & -3 & -3 \\ 0 & 0 & 2 \end{bmatrix} = U$$

We can write this as
$$E_1E_2E_3A = U\ ,$$
where
$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

Thus we have $A = LU$ where (beware the order!)
$$L = E_3^{-1}E_2^{-1}E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix}$$

(b) We first solve $L\mathbf{y} = \mathbf{b}$ by forwardsubstitution, and get

$$\mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} .$$

Secondly we solve $U\mathbf{x} = \mathbf{y}$ by backsubstitution and get

$$\mathbf{x} = \begin{bmatrix} -1.5 \\ -2 \\ 2 \end{bmatrix} .$$

2. The set of all 2×2 matrices $M_{2\times 2}$ with real entries is vector space with the usual matrix addition and matrix scalar multiplication. Are the following subsets W and U subspaces? (Give reasons)

- $$W = \left\{ A \in M_{2\times 2} \ : \ A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \right\}$$
- $$U = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2\times 2} \ : \ a + c = 1 \right\}$$

W is a subspace, since it is closed under addition:

$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} + \begin{bmatrix} d & e \\ 0 & f \end{bmatrix} = \begin{bmatrix} a+d & b+e \\ 0 & c+f \end{bmatrix} \in W$$

and under scalar multiplication:

$$k \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} ka & kb \\ 0 & kc \end{bmatrix} \in W .$$

U isn't a subspace, since it is neither closed under addition nor scalar multiplication. As a specific example:

$$\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \in U ,$$

but

$$2 \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & -2 \end{bmatrix}$$

isn't, since $4 - 2 \neq 1$.

3. Consider the following matrix

$$A = \begin{bmatrix} 1 & 3 & 1 & 4 \\ 2 & 3 & 0 & 1 \\ 1 & 0 & 4 & 0 \\ 0 & 0 & 5 & 3 \end{bmatrix}$$

(a) Determine the row rank (that is the dimension of the row space) of the matrix A

(b) Find a basis for the null-space

$$\text{null}(A) = \{\mathbf{x} \in V : A\mathbf{x} = \mathbf{0}\}$$

of A.

(c) How are the row rank and the nullity (the dimension of the null space) of a matrix related in general?

(a) To find the row rank, we get A into row reduced form

$$\begin{bmatrix} 1 & 3 & 1 & 4 \\ 2 & 3 & 0 & 1 \\ 1 & 0 & 4 & 0 \\ 0 & 0 & 5 & 3 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 3 & 1 & 4 \\ 0 & -3 & -2 & -7 \\ 0 & -3 & 3 & -4 \\ 0 & 0 & 5 & 3 \end{bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 3 & 1 & 4 \\ 0 & -3 & -2 & -7 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 5 & 3 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 3 & 1 & 4 \\ 0 & -3 & -2 & -7 \\ 0 & 0 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The first three rows of this reduced matrix form a basis. Thus the (row rank here is 3)

(b) To find a basis for the null space we would like to find a general solution to $A\mathbf{x} = \mathbf{0}$, so we would also try and find the row reduced form of A, but we have already done this above. So we can solve by back-substitution: $x_4 = t$, since x_4 is a free variable. Then $5x_3 = -3t$ or $x_3 = -\frac{3}{5}t$. $-3x_2 - 2x_3 - 7x_4 = 0$, so $-3x_2 = 2x_3 + 7x_4 = -\frac{6}{5}t + 7t = \frac{29}{5}t$ and therefore $x_2 = -\frac{29}{15}t$. Finally $x_1 = -3x_2 - x_3 - 4x_4 = \frac{29}{5}t + \frac{3}{5}t - 4t = \frac{12}{5}t$ So the general solution can be written as

$$\mathbf{x} = \begin{bmatrix} \frac{12}{5} \\ -\frac{29}{15} \\ -\frac{3}{5} \\ 1 \end{bmatrix} t$$

That vector, or any multiple (such as 5 here) form a basis.

(c) If A is a $n \times m$ matrix then $(\text{row})\text{rank}(A) + \text{nullity}(A) = n$

4a) Eigenvector and eigenvalue

$$A\underset{\sim}{v} = \lambda \underset{\sim}{v} \qquad \underset{\sim}{v} \neq 0$$

if A $n \times n$.

Those directions which remain unchanged under a 'tranformation' by A. &c...

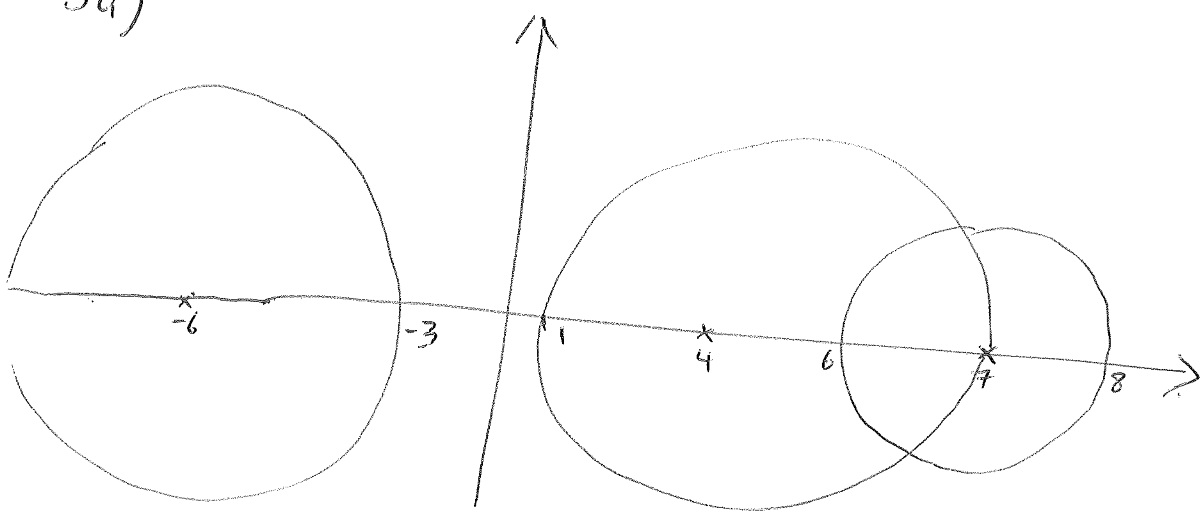
b) $\cancel{AA}$ $\underset{\sim}{x} = \underset{\sim}{v}_1 + 3\underset{\sim}{v}_2$

$$\begin{aligned} A^4 \underset{\sim}{x} &= A^4(\underset{\sim}{v}_1 + 3\underset{\sim}{v}_2) \\ &= A^4 \underset{\sim}{v}_1 + 3A^4 \underset{\sim}{v}_2 \\ &= \lambda_1^4 \underset{\sim}{v}_1 + 3\lambda_2^4 \underset{\sim}{v}_2 \\ &= 1^4 \underset{\sim}{v}_1 + 3 \cdot 4^4 \underset{\sim}{v}_2 \\ &= \underset{\sim}{v}_1 + 768 \underset{\sim}{v}_2 \\ &= \begin{bmatrix} 769 \\ 766 \end{bmatrix} \end{aligned}$$

c) Yes it is possible. The remaining eigenspace could be one-dimensional. The union of the bases of eigenspaces must have 7 vectors in order for the matrix to be diagonalisable.

Hannes + Clem + Blair
24 32 44

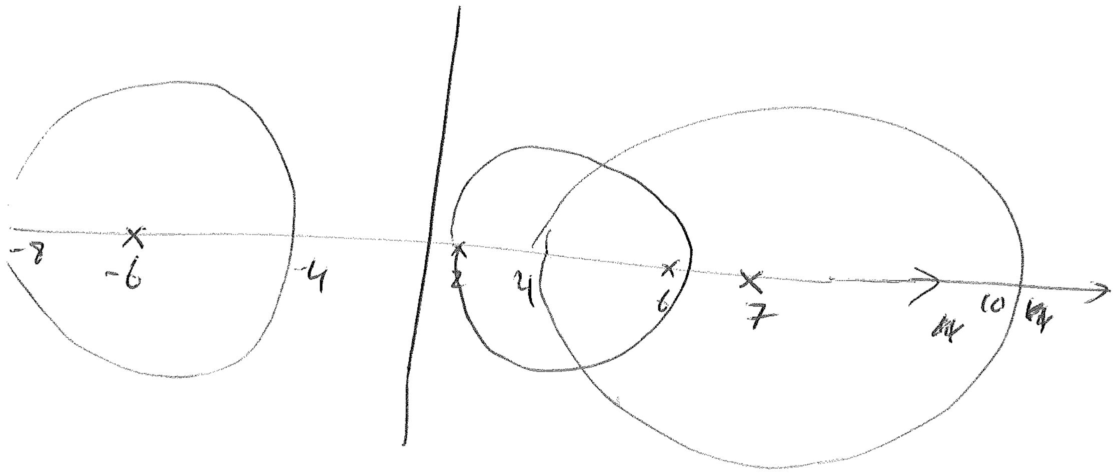
5a) Rows



r=3

r=3 r=1

Columns



r=2

r=2 r=3.

Two circles overlapping
One circle disjoint - if single disk is disjoint from the other disks, then this must contain exactly one eigenvalue and as A is real, this e-value must be real.

other eigenvalues will be located in region

$$(D_2 \cup D_3) \cap (C_2 \cup C_3)$$

b)

$$C = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

$$\underset{\sim}{x}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(1)

$$\lambda_1 = 2$$

$$A - 2I = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$$

$$(A - 2I)\underset{\sim}{x}_0 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

(2)

$$\underset{\sim}{x}_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

scale $\underset{\sim}{y}_1 = \begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$

$$(A - 2I)\underset{\sim}{y} = \begin{bmatrix} 1.5 \\ -3 \end{bmatrix}$$ scale $\begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix}$ (-3)

Produced eigenvector -3

$$\lambda_2 - \lambda_1 = -3$$

$$\lambda_2 - 2 = -3 \Rightarrow \lambda_2 = -1$$

ii) Solve $\underset{\sim}{A}\underset{\sim}{x}_1 = y_0$ where $\underset{\sim}{x}_0 = y_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Row reduce to determine $\underset{\sim}{x}_1$

so $y_1 = \dfrac{\underset{\sim}{x}_1}{\|x_1\|}$

$\underset{\sim}{x}_2$ from $\underset{\sim}{A}x_2 = y_1$

until Smallest eigenvalue of A is the reciprocal of the "Scaler"

"If A is invertible with eigenvalue λ, then A^{-1} has eigenvalue $1/\lambda$."

→ Apply power method to A^{-1}, its dominant e-value will be the reciprocal of the smallest (in magnitude) e-value of A.

&c....

6\. a) Gram- Schmidt process converts a basis for a subspace U into an orthogonal basis for U.

It does this by "subtracting" projections

set $\underset{\sim}{v}_1 = \underset{\sim}{u}_1$

$\underset{\sim}{v}_2 = \underset{\sim}{u}_2 -$ (its proj on $\underset{\sim}{v}_1$)

$\underset{\sim}{v}_3 = u_3 -$ (the projs of $\underset{\sim}{u}_3$ on $\underset{\sim}{v}_1$ & $\underset{\sim}{v}_2$)

b) $R = Q^T A = \begin{bmatrix} & \end{bmatrix} \begin{bmatrix} 5 & -9 \\ 1 & -3 \\ -3 & 7 \\ 1 & -3 \end{bmatrix}$

$= \begin{bmatrix} 6 & -12 \\ 0 & 2 \end{bmatrix}$

c) check $QR = A$ ✓.

d) $QR\underset{\sim}{x} = \begin{bmatrix} 11 \\ 1 \\ -5 \\ 1 \end{bmatrix}$

$R\underset{\sim}{x} = Q^T \begin{bmatrix} 11 \\ 1 \\ -5 \\ 1 \end{bmatrix}$

or $\begin{bmatrix} 6 & -12 \\ 0 & 2 \end{bmatrix} \underset{\sim}{x} = \begin{bmatrix} 12 \\ 2 \end{bmatrix}$

$$x_2 = \cancel{2} \quad 1$$

$$x_1 = 4$$

$$\underset{\sim}{x} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

check!

$$\begin{bmatrix} 5 & -9 \\ 1 & -3 \\ -3 & 7 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 11 \\ 1 \\ -5 \\ 1 \end{bmatrix} \checkmark$$

d) as only RHS has changed, expect new system $A\underset{\sim}{x} = \underset{\sim}{b}$ to be inconsistent.

QR still compute $\underset{\sim}{x}$, but it is the least squares solution to $A\underset{\sim}{x} = \underset{\sim}{b}$, i.e, the value of $\underset{\sim}{x}$ which makes $\| A\underset{\sim}{x} - \underset{\sim}{b} \|_2$ as small as possible.

7(a).

Given

$$\underset{\sim}{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \text{ and } \underset{\sim}{x}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad (\text{from } E_1)$$

$$\underset{\sim}{x}_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad (\text{from } E_2)$$

Want orthogonal basis vectors $\underset{\sim}{v}_1$, $\underset{\sim}{v}_2$, & $\underset{\sim}{v}_3$

$$\underset{\sim}{v}_1 = \underset{\sim}{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\underset{\sim}{v}_2 = \underset{\sim}{x}_2 - \left(\frac{\underset{\sim}{v}_1 \cdot \underset{\sim}{x}_2}{\underset{\sim}{v}_1 \cdot \underset{\sim}{v}_1} \right) \underset{\sim}{v}_1$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \left(\frac{2}{2}\right) \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\underset{\sim}{v}_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

Normalise

$$q_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} \quad q_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad q_3 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$A = \lambda_1 q_1 q_1^T + \lambda_2 q_2 q_2^T + \lambda_3 q_3 q_3^T$$

$$= \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + (-2) \begin{bmatrix} 1/2 & -1/2 & 0 \\ -1/2 & 1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 & 3/2 & 0 \\ 3/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

b)

$$B = \begin{bmatrix} 2 & -1 \\ 2 & 2 \end{bmatrix}$$

b) $B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$$B^T B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

evalues are $(1-\lambda)^2 - 1 = 0$

P

$$1 - 2\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 2\lambda = 0$$

$$\lambda(\lambda - 2) = 0$$

evalues $\lambda = 0, 2$

~~evectors are~~

V

evectors are: $\begin{array}{cc} (2) & (0) \\ \begin{bmatrix} 1 \\ 1 \end{bmatrix} & \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{array}$ &

normalise: $\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$ $\begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$

U

&

$$u_1 = \frac{1}{\sigma_1} = \frac{A v_1}{\sigma_1} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

where $\sigma_1 = \sqrt{2}$ & $\sigma_2 = 0$

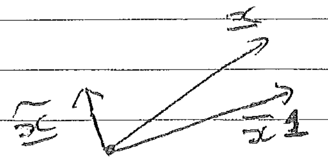
$\sigma_2 = 0$: $u_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (by inspection!)

$$A = U \Sigma V^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

EMTH 211 EXAM SOLUTIONS 2015.

1(a) i) To center a variable $\underline{x}$, the mean $\bar{x}$ is subtracted from each element of $\underline{x}$. The centered vector $\tilde{\underline{x}}$ will have a mean of zero. Centering decomposes $\underline{x}$ into two orthogonal componets.

$$\Rightarrow \quad \underline{x} = \tilde{\underline{x}} + \bar{x}\mathbb{1}$$



ii) $$(\underline{x} - \bar{x}\mathbb{1})\cdot\mathbb{1} = \sum_{i=1}^{n}(x_i - \bar{x})$$
$$= \sum_{i=1}^{n} x_i - n\bar{x}$$
$$= n\bar{x} - n\bar{x}$$
$$= 0.$$

iii) $$s_x = \frac{\|\tilde{\underline{x}}\|}{\sqrt{n-1}} = \frac{\|\underline{x} - \bar{x}\mathbb{1}\|}{\sqrt{n-1}}$$

$$s_{\tilde{x}} = \frac{\|\tilde{\underline{x}} - \bar{\tilde{x}}\mathbb{1}\|}{\sqrt{n-1}}$$
$$= \frac{\|\underline{x} - \bar{x}\mathbb{1} - 0\|}{\sqrt{n-1}}$$
$$= s_x$$

(b) $s_x = \dfrac{\|\tilde{x}\|}{\sqrt{n_x - 1}} = \dfrac{25}{\sqrt{36}} = 4.17$

$s_y = \dfrac{\|\tilde{y}\|}{\sqrt{n_y - 1}} = \dfrac{36}{\sqrt{81}} = 4$

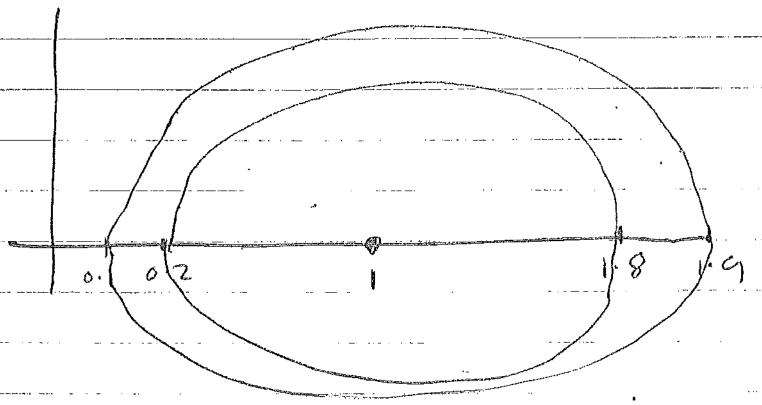
Hence, x has greater spread.

(c) Let

$$C = \begin{pmatrix} r_{xx} & r_{xy} & r_{xz} \\ r_{xy} & r_{yy} & r_{yz} \\ r_{xz} & r_{yz} & r_{zz} \end{pmatrix} = \begin{pmatrix} 1 & 0.7 & 0.2 \\ 0.7 & 1 & -0.1 \\ 0.2 & -0.1 & 1 \end{pmatrix}$$

If C is a correlation matrix, the eigenvalues of C must be positive.

Gerschgorin's Disks:



Hence, the three eigenvalues are positive and so C is a correlation matrix and the given correlations are valid.

(d) i) $s_y = \frac{\|\tilde{y}\|}{\sqrt{n-1}} = 3$

$\Rightarrow \sqrt{n-1} = 15/3 = 5$

$n = 26$

ii) $s_x = \frac{\|\tilde{x}\|}{5} = \frac{20}{5} = 4$

$s_z = \frac{\|\tilde{z}\|}{5} = \frac{5}{5} = 1$

iii) $r_{xy} = \cos\angle(\tilde{x}, \tilde{y}) = \cos(30)$

$= 0.866$

$r_{xz} = \cos\angle(\tilde{x}, \tilde{z}) = \cos(110)$

$= {}^{-}0.342$

$r_{yz} = \cos\angle(\tilde{y}, \tilde{z}) = \cos(80)$

$= 0.1736$

3

2 (a) $\tilde{x} \cdot \hat{e} = 0$

$$\tilde{x} \cdot (\tilde{y} - \hat{b}_1 \tilde{x}) = 0$$

$$\tilde{x} \cdot \tilde{y} - \hat{b}_1 \tilde{x} \cdot \tilde{x} = 0$$

$$\hat{b}_1 = \frac{\tilde{x} \cdot \tilde{y}}{\|\tilde{x}\|^2}$$

(b) $r_{xy} = \cos \angle (\tilde{x}, \tilde{y})$

$$= \frac{\tilde{x} \cdot \tilde{y}}{\|\tilde{x}\| \|\tilde{y}\|}$$

Now.

$$r_{xy} \frac{\|\tilde{y}\|}{\|\tilde{x}\|} = \frac{\tilde{x} \cdot \tilde{y}}{\|\tilde{x}\| \|\tilde{y}\|} \cdot \frac{\|\tilde{y}\|}{\|\tilde{x}\|}$$

$$= \frac{\tilde{x} \cdot \tilde{y}}{\|\tilde{x}\|^2}$$

$$= \hat{b}_1$$

c) ① Independent

② Common Variance

③ Normally distributed with mean 0.

d) $F = \frac{(n-2)R^2}{1-R^2} = \frac{30(0.64)}{0.36}$

$$\approx 53.3$$

The 95% percentile of $F(1,30)$ is

$$F_{0.95}(1,30) = 4.17.$$

Since $F = 53.3 > F_{0.95}(1,30) = 4.17$,

we conclude at level 0.05 that the SLR model is a good fit.

(e) The type 1 error in this case is concluding that the model is a good fit, when it is actually a bad fit (false positive).

(f) The type 2 error in this case is concluding that the model is not a good fit, when it is actually a good fit (false negative).

(g) If $\underline{x}' = -2\underline{x}$, then

$$\underset{\sim}{\tilde{x}}' = -2\underline{x} + 2\bar{x}\mathbb{1}$$

$$= -2\underset{\sim}{\tilde{x}}$$

Hence,

$$r_{x'y} = \frac{-2\,\tilde{\underline{x}}\cdot\tilde{\underline{y}}}{\|-2\tilde{\underline{x}}\|\,\|\tilde{\underline{y}}\|}$$

$$= \frac{-2\,\tilde{\underline{x}}\cdot\tilde{\underline{y}}}{2\,\|\tilde{\underline{x}}\|\,\|\tilde{\underline{y}}\|}$$

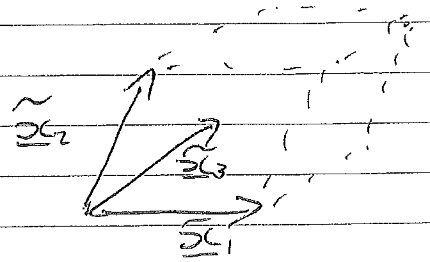
$$= -r_{xy}$$

$$= -10.$$

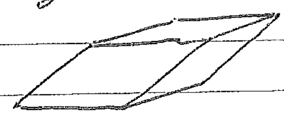
5

3 (a) When the three predictor vectors almost fall into a subspace of dimension less than three.

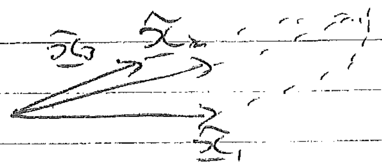
(b) If the standardized vectors are well-conditioned, the volume of the parallelepiped formed by the three vectors will be close to one.



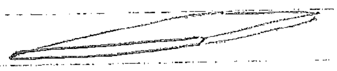
(Large Volume)



If the standardized vectors are ill-conditioned, the volume of the parallelepiped formed by the three vectors will be close to zero.



("Flat" with small volume)



(c) The normalized volume is given by

$$A_{norm} = \sqrt{\det(C)}$$

$$= \sqrt{0.1 \times 1 \times 3}$$

$$= 0.548$$

(d) $H = \left(1 + \frac{2p+5}{6} - n\right) \ln\left(1 - \det(c)\right)$

$= \left(1 + \frac{11}{6} - 34\right) \ln\left(1 - 0.3\right)$

$= 11.1164$

The 95% percentile of $\chi^2(3)$ is

$\chi^2_{0.95}(3) = 7.81$

Since $H = 11.1164 > \chi^2_{0.95}(3) = 7.81$,

we conclude at level 0.05 that the MLR model is stable.

(e) The 95% percentile of $F(3, 30)$ is

$F_{0.95}(3, 30) = 2.92$.

Since $F = 3 > F_{0.95}(3,30) = 2.92$,

we conclude at level 0.05 that the MLR model is a good fit.

(f) The decision in part (e) would not change because

$F_{0.90}(3, 30) < F_{0.95}(3, 30)$.