

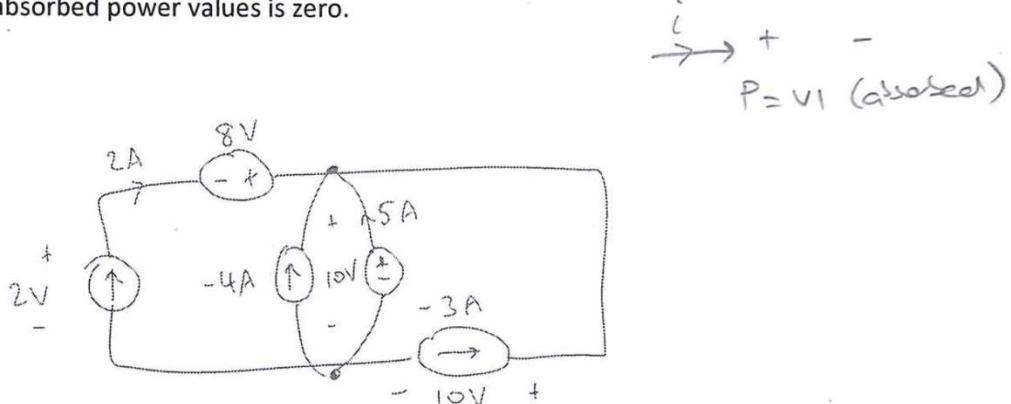
Name:

Student ID:

Pre-tutorial 4 Questions (to be attempted before class on May 3rd, 2019)

Chapter 2, Ex 20: Power absorbed

Determine which of the five sources are being charged (absorbing positive power), and show that the algebraic sum of the five absorbed power values is zero.



2V source $P = V(-I) = 2 \times (-2) = -4W$ absorbed
or 4W generated.

8V source $P = V(-I) = 8 \times (-2) = -16W$ absorbed
or 16W generated.

-4A source $P = V(-I) = 10 \times (-(-4)) = 40W$ absorbed.

10V source $P = V(-I) = 10 \times (-5) = -50W$ absorbed
or 50W generated

-3A source $P = V(-I) = 10 \times (-(-3)) = 30W$ absorbed.

$$\sum P_{\text{abs}} = 0$$

$$-4 - 16 + 40 - 50 + 30 = 0 \quad \checkmark$$

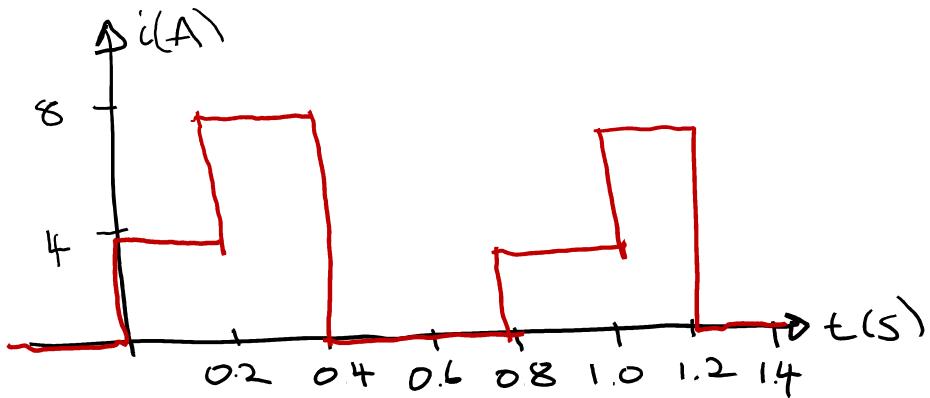
Conservation of energy.

For power absorbed problems you can also try Ex. 22.

Chapter 7, Ex 11a: Capacitors

The current flowing through a 33 mF capacitor is shown graphically below.

- a) Assuming the passive sign convention, sketch the resulting voltage waveform across the device.



Q11a)

$$C = 33 \text{ mF}$$

$$v = \frac{1}{C} \int_{t_0}^t i(\tau) d\tau + v(t_0)$$

⇒ integral of a horizontal straight line (ie. a constant) is a constant slope straight line. If find the end-points can just join them.

$$\begin{aligned} @ t = 0.2 : \quad v &= \frac{1}{33 \times 10^{-3}} \int_0^{0.2} 4 dt + 0 \\ &= \frac{4}{33 \times 10^{-3}} [t]_0^{0.2} \\ &= \frac{4 \times 0.2}{33 \times 10^{-3}} \\ &= 24.2 \text{ V} \end{aligned}$$

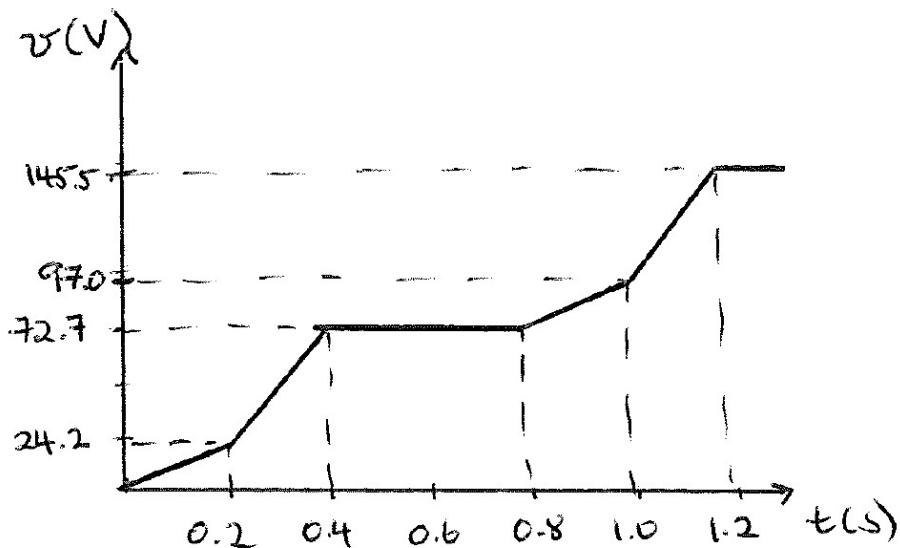
$$\begin{aligned} @ t = 0.4 : \quad v &= \frac{1}{33 \times 10^{-3}} \int_{0.2}^{0.4} 8 dt + 24.2 \\ &= \frac{8}{33 \times 10^{-3}} [t]_{0.2}^{0.4} + 24.2 \\ &= \frac{8 \times (0.4 - 0.2)}{33 \times 10^{-3}} + 24.2 \\ &= 72.7 \text{ V} \end{aligned}$$

$$@ t = 0.8 : \quad V = \frac{1}{33 \times 10^{-3}} \int_{0.4}^{0.8} 0 \, dt + 72.7 \\ = 72.7 \text{ V}$$

(or by inspection)

$$@ t = 1 : \quad V = \frac{1}{33 \times 10^{-3}} \int_{0.8}^1 t \, dt + 72.7 \\ = \frac{4}{33 \times 10^{-3}} [t]_{0.8}^1 + 72.7 \\ = 97.0 \text{ V}$$

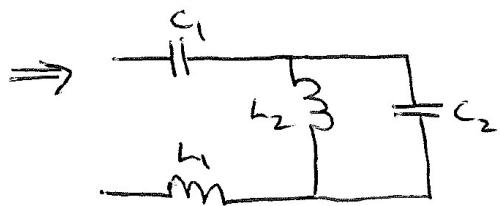
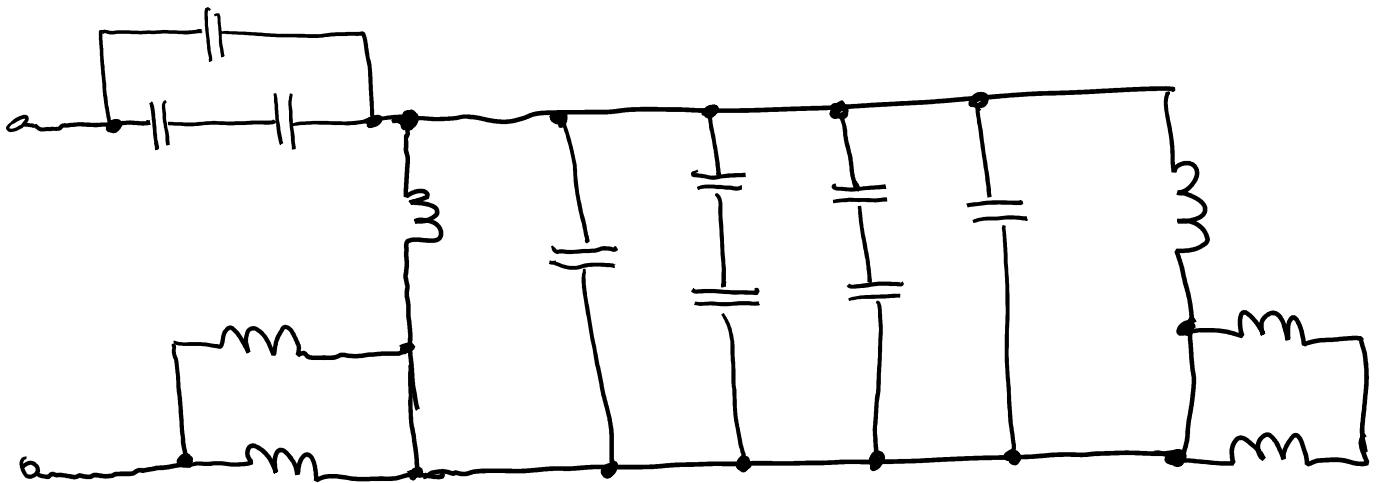
$$@ t = 1.2 : \quad V = \frac{1}{33 \times 10^{-3}} \int_1^{1.2} 8 \, dt + 97 \\ = \frac{8}{33 \times 10^{-3}} [t]_1^{1.2} + 97 \\ = 145.5 \text{ V}$$



At Tutorial 4 – Marked Question (3rd May 2019)

Chapter 7, Ex 41: Equivalent capacitance/inductance

Reduce the network below to the smallest possible number of components if each inductor is 1 nH and each capacitor is 1 mF.



$$C_1 = \frac{(1 \times 10^{-3})^2}{1 \times 10^{-3} + 1 \times 10^{-3}} + 1 \times 10^{-3}$$

$$= 1.5 \times 10^{-3} \text{ F}$$

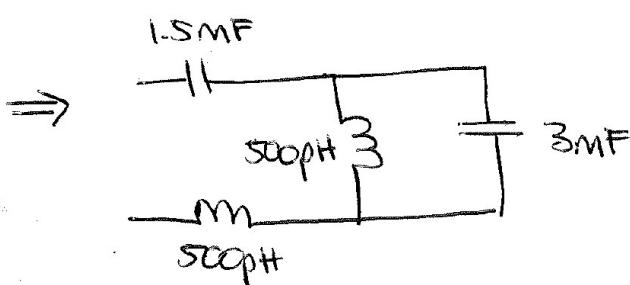
$$C_2 = \left(\frac{(1 \times 10^{-3})^2}{2 \times 10^{-3}} \right) \times 2 + 2 \times 10^{-3}$$

$$= 3 \text{ mF}$$

$$L_1 = \frac{(1 \times 10^{-9})^2}{1 \times 10^{-9} \times 2}$$

$$= 500 \times 10^{-12} \text{ H}$$

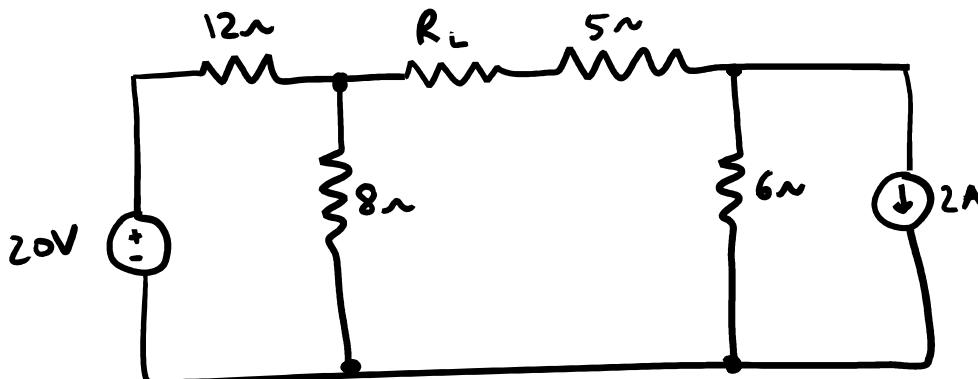
$$L_2 = 500 \times 10^{-12} \text{ H}$$



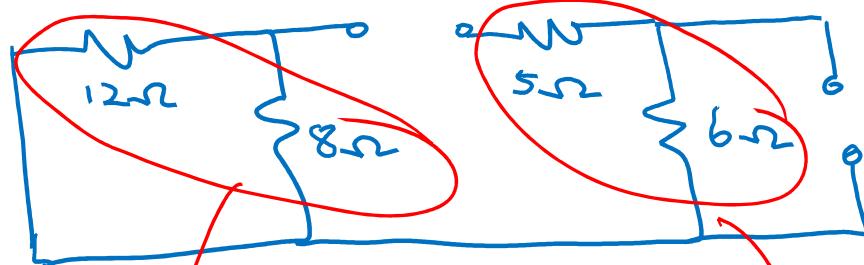
At Tutorial 4 – Unmarked Questions (3rd May 2019)

Ch 5 ex 61: Maximum power transfer

Given you can select any value of R_L , what is the maximum power that could be delivered to R_L ?

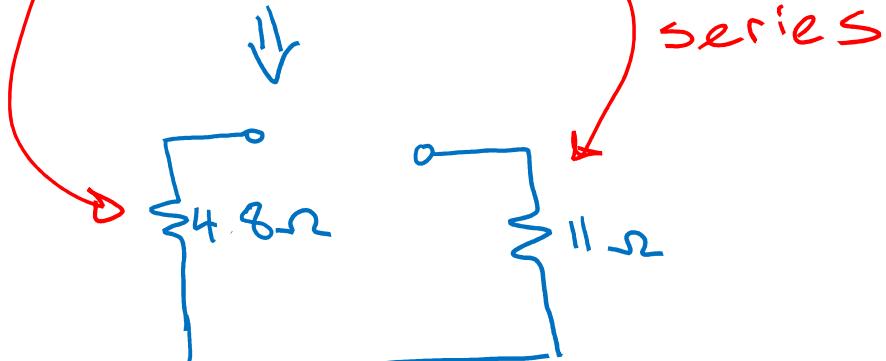


i) Find Thévenin equivalent resistance:



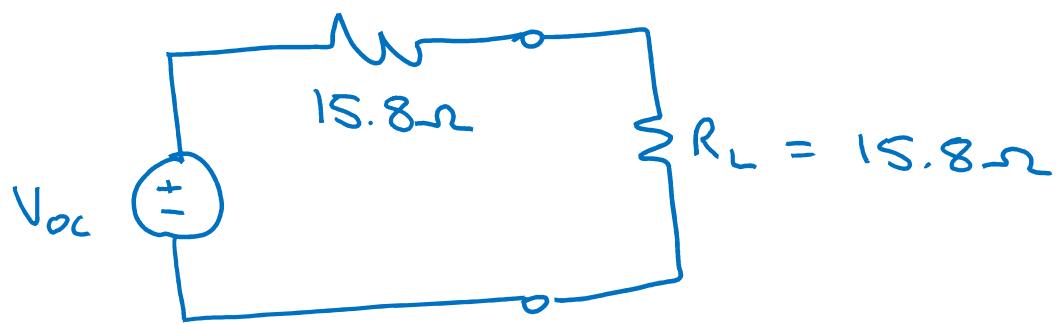
parallel

$$12//8 = \frac{12 \times 8}{12 + 8} \\ = 4.8\Omega$$



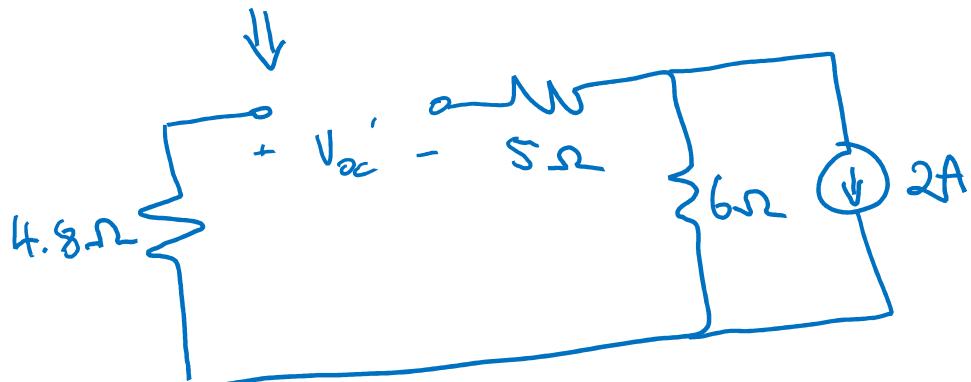
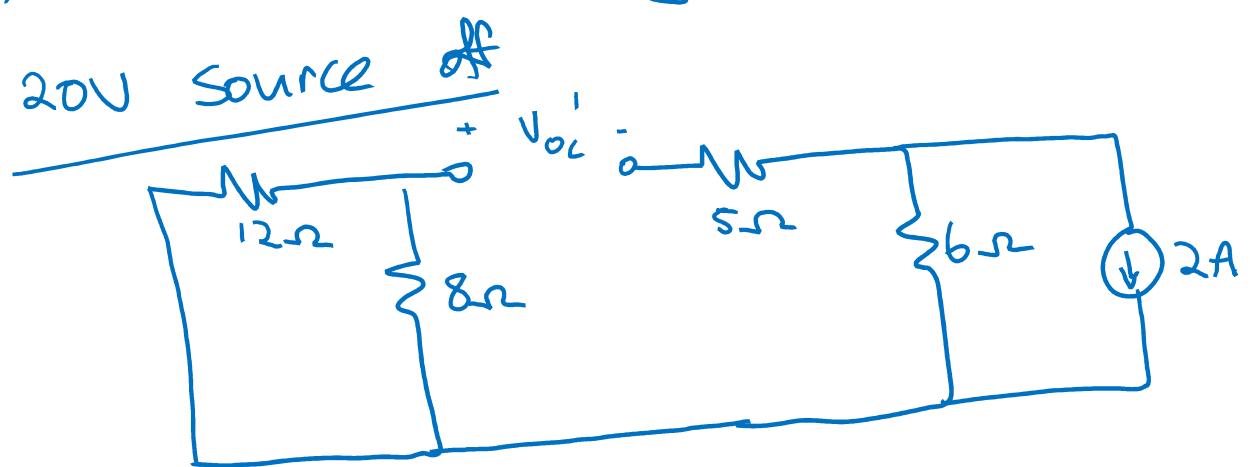
$$R_{Th} = 11 + 4.8$$

$$= 15.8\Omega$$

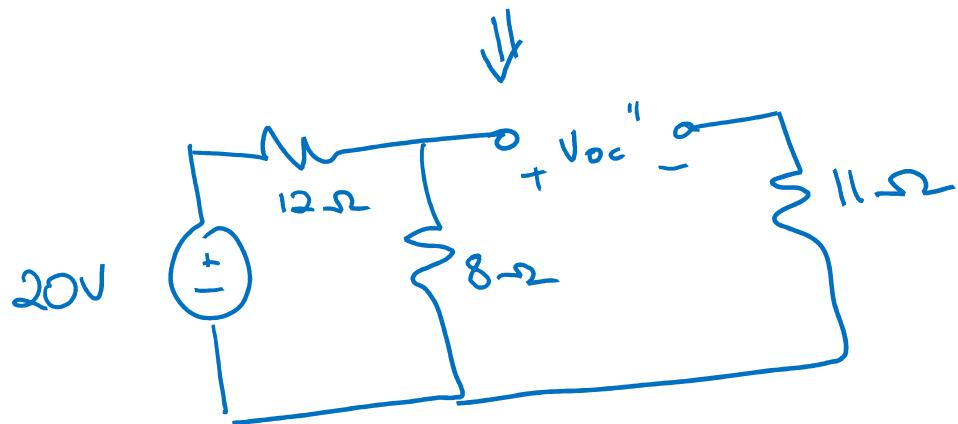
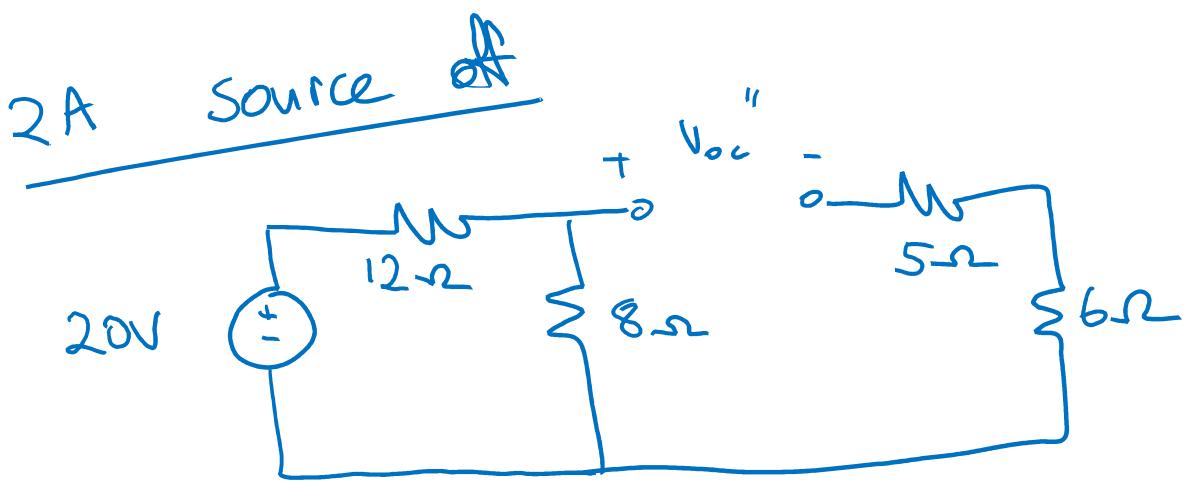


For Max pwr transfer, $R_L = R_{TH}$

2) Find V_{oc} using superposition:

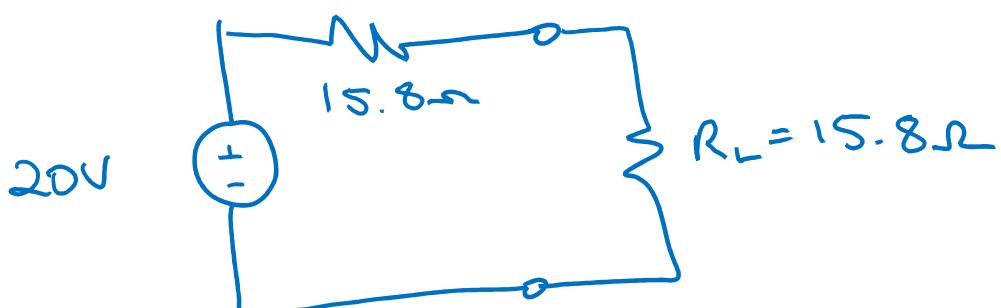


All current goes through $6\ \Omega$ resistor,
 $\therefore V_{oc}' = 6 \times 2 = 12\text{V}$



$$V_{8\Omega} = V_{oc}'' = \frac{20 \times 8}{12 + 8} = 8V$$

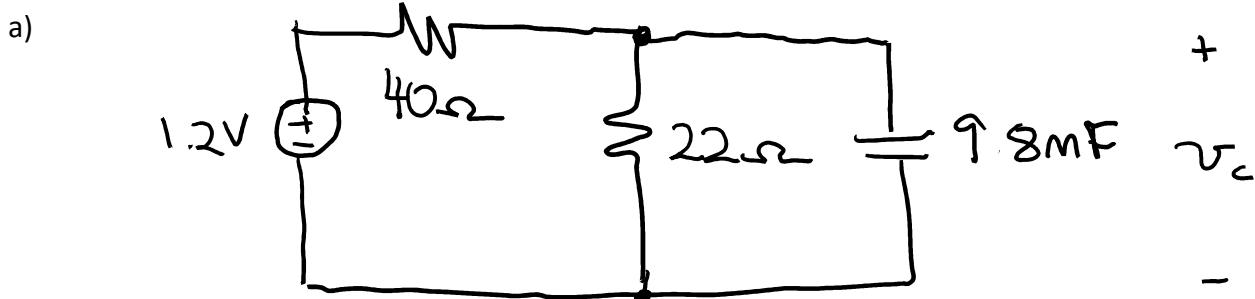
$$V_{oc} = V_{oc}' + V_{oc}'' = 12 + 8 \\ = 20V$$



$$P = \frac{V^2}{R} = \frac{(20/2)^2}{15.8} \\ = 6.33 W$$

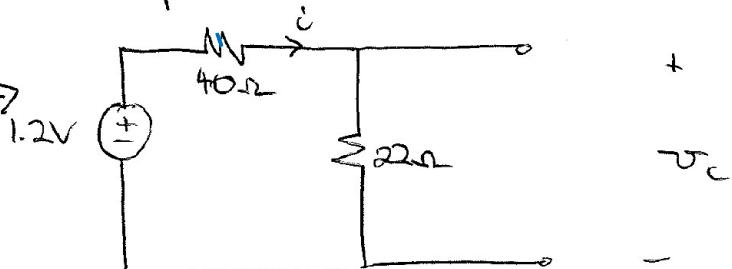
Chapter 7, Ex 14: Power

Assume the circuits below have been connected for a long time. Calculate the power dissipated in the $40\ \Omega$ resistor and the voltage labeled v_c in the circuits below:



capacitor acts as an open circuit to DC

\therefore circuit is \Rightarrow

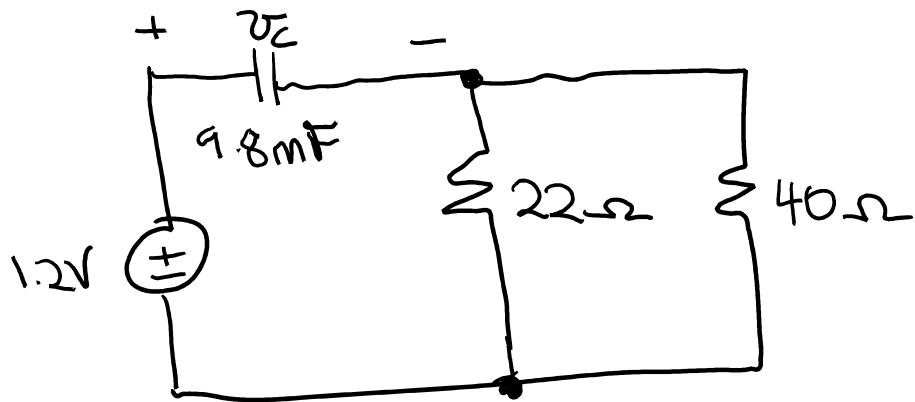


$$i = \frac{1.2}{(40+22)} \\ = 19\text{ mA}$$

$$P_{40\Omega} = i^2 R \\ = (19 \times 10^{-3})^2 \times 40 \\ = 14.98\text{ mW}$$

$$v_c = iR \\ = 19 \times 10^{-3} \times 22 \\ = 0.43\text{ V}$$

b)

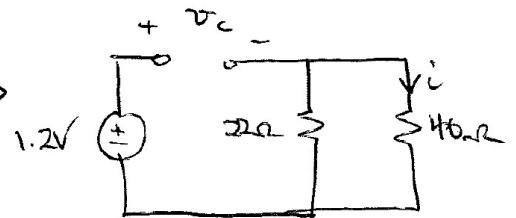


as above. \therefore circuit becomes \Rightarrow

$$P_{40\Omega} = 0W$$

$$(i = 0A)$$

$$v_C = 1.2V$$



Chapter 7, Ex 27: Inductors

Determine the amount of energy stored in a 33 mH inductor at $t = 1 \text{ ms}$ as a result of a current i_L given by:

- a) 7 A

$$i = 7 \text{ A}$$

$$\omega_L = \frac{1}{2} L i^2$$

$$= \frac{1}{2} \times 33 \times 10^{-3} \times 7^2$$

$$= 868.5 \text{ mJ}$$

b) $3 - 9e^{-10^3 t}$ mA

$$i = 3 - 9e^{-10^3 t} \text{ mA}$$

$$\begin{aligned}\omega_r &= \frac{1}{2} L i^2 \\ &= \frac{1}{2} \times 33 \times 10^{-3} \times (3 \times 10^{-3} - 9e^{-10^3 t} \times 10^{-3})^2 \\ &= 16.5 \times 10^{-3} (3 \times 10^{-3} - 3.3 \times 10^{-3})^2 \\ &= 16.5 \times 10^{-3} \times 96.67 \times 10^{-9} \\ &= 1.595 \text{ rad/s}\end{aligned}$$

Chapter 7, Ex 25: Inductors

The voltage across a 2 H inductor is given by $v_L = 4.3t$, $-0.1 \text{ s} \leq t \leq 50 \text{ ms}$. Knowing that $i_L(-0.1) = 100 \mu\text{A}$, calculate the current (assuming it is defined consistent with the passive sign convention) as t equal to:

- a) 0
- b) 1.5 ms
- c) 45ms

$$i(t) = \frac{1}{L} \int_{t_0}^t v(\tau) d\tau + i(t_0)$$

$$= \frac{1}{2} \left[\int_{-0.1}^t 4.3\tau d\tau \right] + 100 \times 10^{-6}$$

$$= \frac{1}{2} \times 4.3 \int_{-0.1}^t \tau d\tau + 100 \times 10^{-6}$$

$$= 2.15 \left[\frac{1}{2} \tau^2 \right]_{-0.1}^t + 100 \times 10^{-6}$$

$$= 2.15 \left(\frac{1}{2} t^2 - 5 \times 10^{-3} \right) + 100 \times 10^{-6}$$

a) $i(0) = -2.15 \times 5 \times 10^{-3} + 100 \times 10^{-6} = -10.65 \text{ mA}$

b) $i(1.5 \text{ ms}) = 2.15 \left(\frac{1}{2} \times (1.5 \times 10^{-3})^2 - 5 \times 10^{-3} \right) + 100 \times 10^{-6}$
 $= -10.65 \text{ mA}$

c) $i(45 \text{ ms}) = 2.15 \left(\frac{1}{2} \times (45 \times 10^{-3})^2 - 5 \times 10^{-3} \right) + 100 \times 10^{-6}$
 $= -8.47 \text{ mA}$

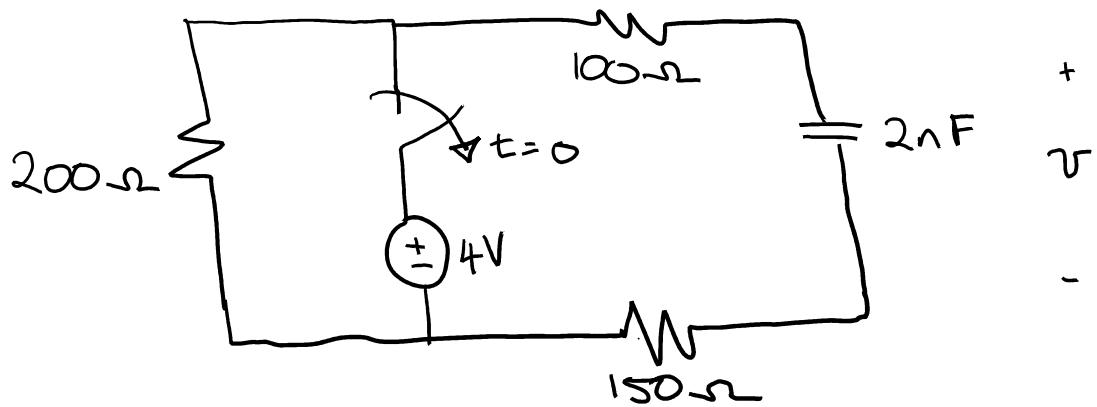
Name:

Student ID:

Pre-tutorial 5 Questions (to be attempted before class on May 17th, 2019)

Chapter 8, Ex 20: Source-free RC circuit

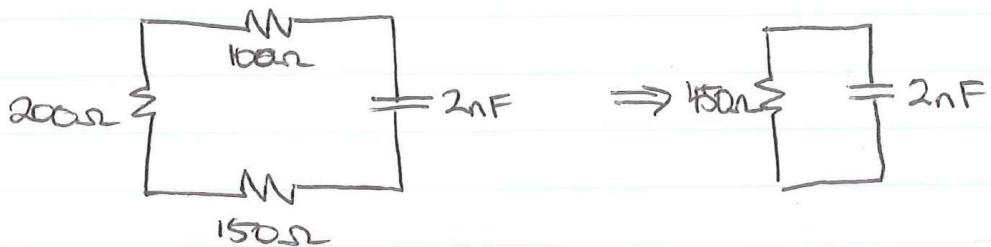
The switch drawn in the circuit below has been closed for such a long time that any transients which might have arisen from first connecting the voltage source have disappeared.



- a) Determine the circuit time constant

$$\tau = RC$$

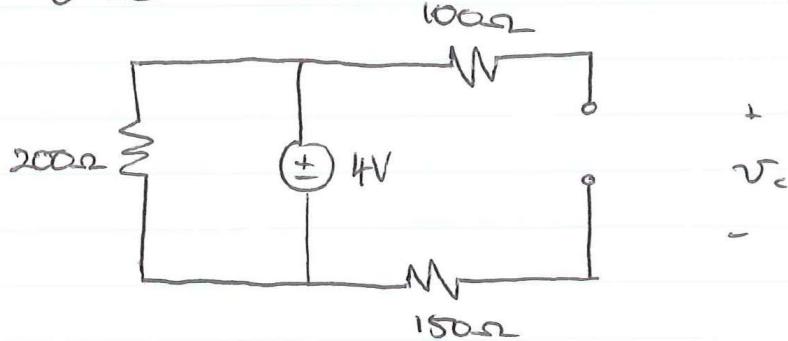
For $t > 0$ circuit is:



$$RC = 150 \times 2 \times 10^{-9}$$
$$= 900 \text{ ns}$$

b) Calculate the voltage $v(t)$ at $t = \tau$, 2τ , and 5τ

$t < 0$:



$$v_c(0^-) = 4V$$

$$v_c(0^-) = v_c(0^+) = 4V$$

$$\begin{aligned} \text{For } t > 0: \quad v(t) &= Ae^{-t/\tau} \\ &= Ae^{-t/900 \times 10^9} \\ &= Ae^{-1.11 \times 10^6 t} \end{aligned}$$

$$\begin{aligned} v(0) &= Ae^{-1.11 \times 10^6 \times 0} = 4 \\ A &= 4 \end{aligned}$$

$$\therefore v(t) = 4e^{-1.11 \times 10^6 t} V \quad t > 0$$

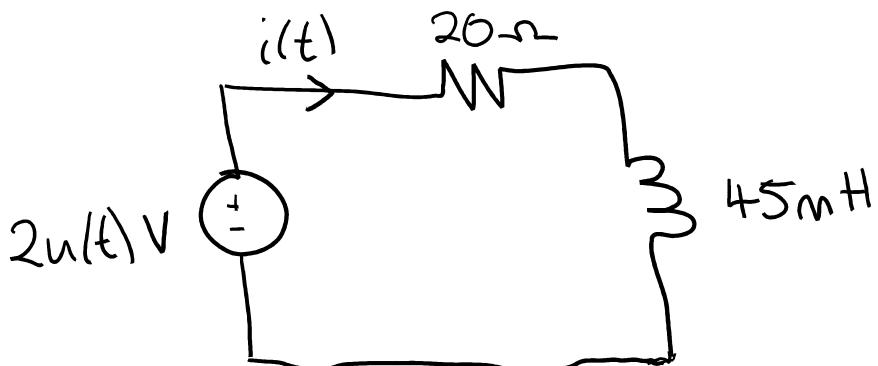
$$\begin{aligned} v(\tau) &= 4e^{-1} \\ &= 1.47 V \end{aligned}$$

$$\begin{aligned} v(2\tau) &= 4e^{-2} \\ &= 0.54 V \end{aligned}$$

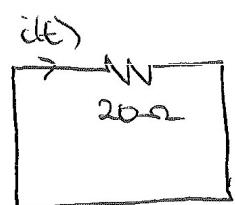
$$\begin{aligned} v(5\tau) &= 4e^{-5} \\ &= 0.027 V \end{aligned}$$

Chapter 8, Ex 49: Driven RL circuit

The circuit shown below is powered by a source which is inactive for $t < 0$. Obtain an expression for $i(t)$ valid for all t .

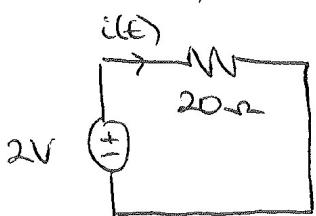


For $t < 0$:



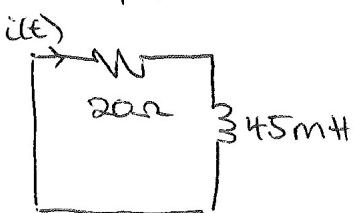
$$i(t) = 0 \text{ A}$$

For $t > 0$, forcing function circuit:



$$\begin{aligned} i_F(t) &= \frac{2}{20} \\ &= 0.1 \text{ A} \end{aligned}$$

For $t > 0$, natural function circuit:



$$i_n(t) = I_0 e^{-t/\tau}$$

$$i_n(t) = I_0 e^{-444.4t}$$

$$\begin{aligned} \tau &= \frac{L}{R} \\ &= \frac{45 \times 10^{-3}}{20} \\ &= 2.25 \times 10^{-3} \text{ s} \end{aligned}$$

$$\tau = 444.4 \text{ s}$$

$$\begin{aligned} i(t) &= i_F(t) + i_n(t) \quad t > 0 \\ &= 0.1 + I_0 e^{-444.4t} \end{aligned}$$

$$i(0^-) = i(0^+) = 0$$

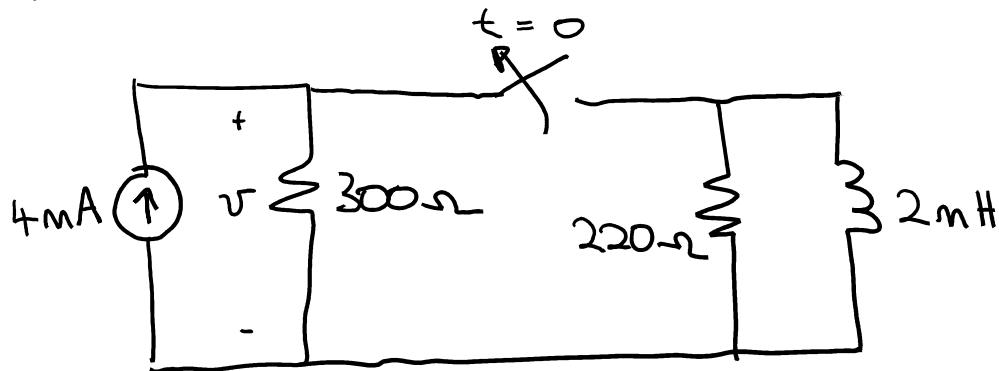
$$i(0) = 0.1 + I_0 = 0$$

$$I_0 = -0.1$$

$$\begin{aligned} i(t) &= 0.1 - 0.1 e^{-444.4t} \text{ A} \quad t > 0 \\ &= 0.1 (1 - e^{-444.4t}) u(t) \text{ A} \end{aligned}$$

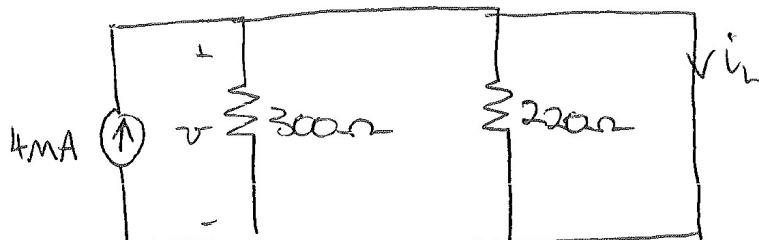
At Tutorial 5 – Marked Question (17th May 2019)

Chapter 8, Ex 8: Source-free RL circuit



The switch in the circuit above has been closed a very long time. Calculate the voltage v as well as the energy stored in the inductor at:

- a) The instant just prior to the switch being thrown open

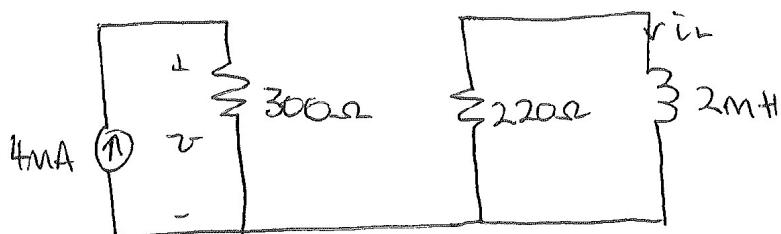


$$i_L(0^-) = 4\text{mA}$$

$$v(0^-) = 0\text{V}$$

$$\begin{aligned}\omega(0^-) &= \frac{1}{2} L i^2 \\ &= \frac{1}{2} \times 2 \times 10^{-3} \times (4 \times 10^{-3})^2 \\ &= 16 \times 10^{-9} \text{ J}\end{aligned}$$

- b) The instant just after the switch is opened



$$\begin{aligned}v(0^+) &= 4 \times 10^{-3} \times 300 \\ &= 1.2\text{V}\end{aligned}$$

$$i_L(0^+) = i_L(0^-) = 4\text{mA}$$

$$\begin{aligned}\omega_L(0^+) &= \frac{1}{2} \times 2 \times 10^{-3} \times (4 \times 10^{-3})^2 \\ &= 16 \times 10^{-9} \text{ J}\end{aligned}$$

c) $t = 8 \mu s$

$$v(8\mu s) = v(0^+) = 1.2V$$

$$i = I_0 e^{-t/r} \\ = 4 \times 10^{-3} e^{-110 \times 10^3 t}$$

$$\frac{1}{T} = \frac{R}{L} \\ = \frac{220}{2 \times 10^{-3}} \\ = 110 \times 10^3$$

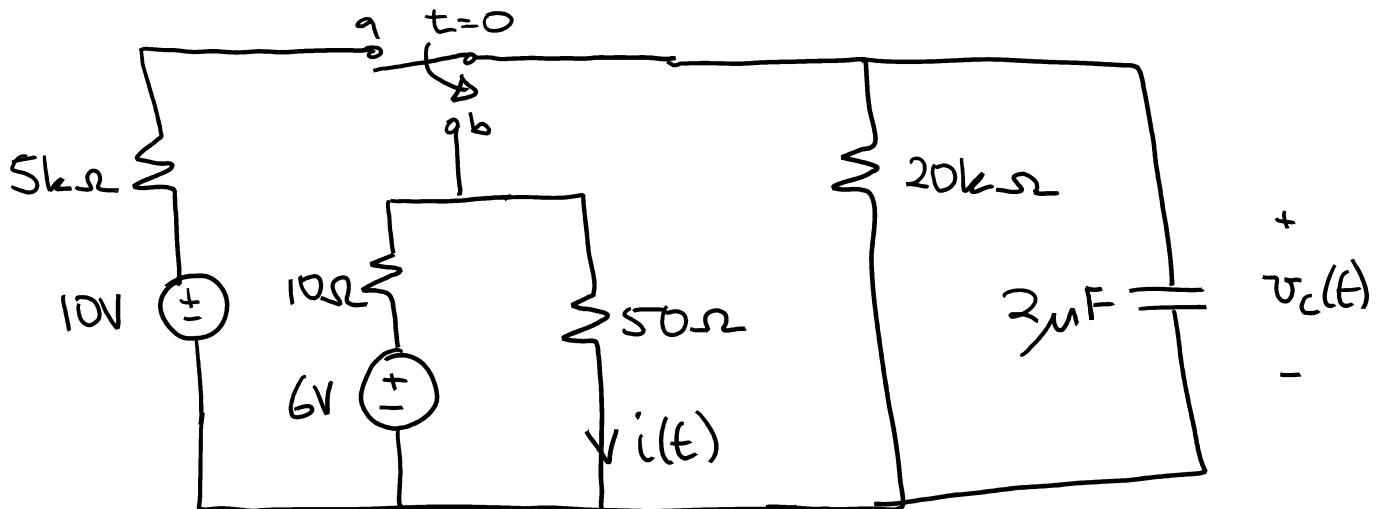
$$i(8\mu s) = 4 \times 10^{-3} e^{-110 \times 10^3 \times 8 \times 10^{-6}} \\ = 1.66 \text{ mA}$$

$$w(8\mu s) = \frac{1}{2} \times 2 \times 10^{-3} \times (1.66 \times 10^{-3})^2 \\ = 2.7 \times 10^{-9} \text{ J}$$

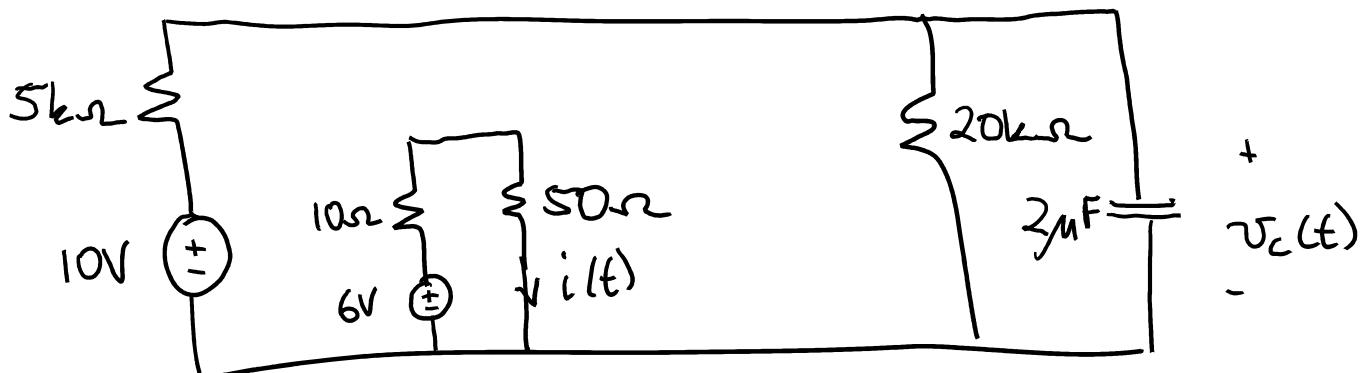
At Tutorial 5 – Unmarked Questions (17th May 2019)

Chapter 8, Ex 60: Driven RC circuit

The switch shown in the circuit below has been in position *a* since the original Battlestar Galactica aired on TV. It is moved to position *b* at time $t = 0$. Obtain expressions for $i(t)$ and $v_c(t)$ valid for all values of t .



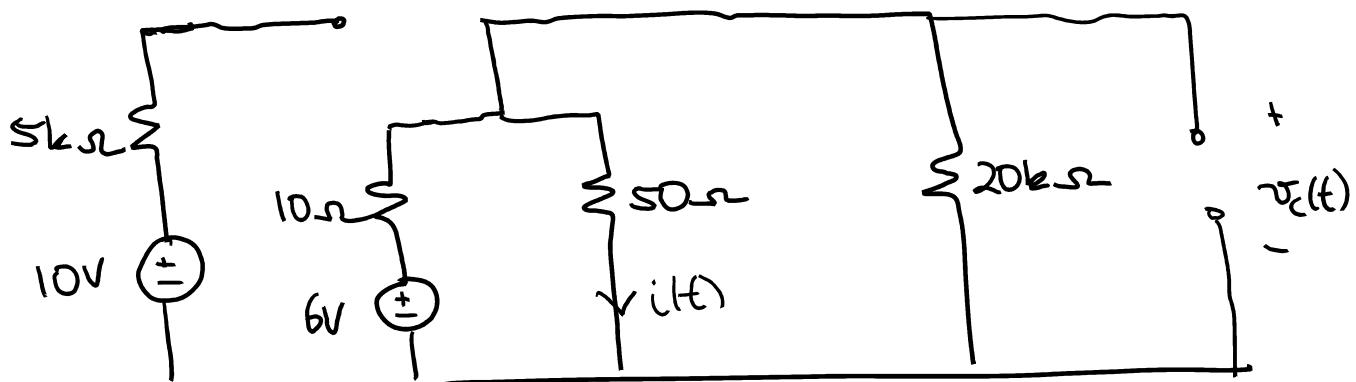
$t < 0$:

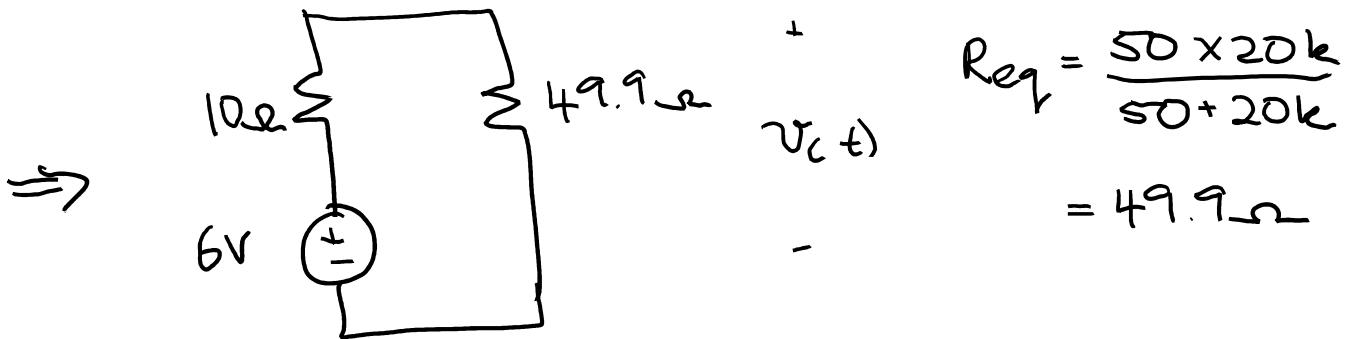


$$i(0^-) = \frac{6}{60} \\ = 0.1 \text{ A}$$

$$v_c(0^-) = 10 \times \frac{20}{25} \text{ (voltage divider)} \\ = 8 \text{ V}$$

$t > 0$ (forced):



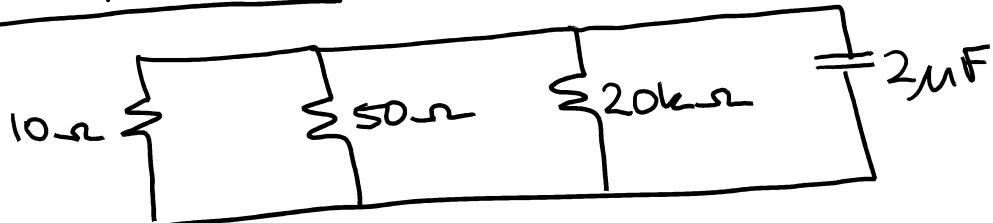


$$v_{c,f} = 6 \times \frac{49.9}{59.9} \text{ (voltage divider)}$$

$$= 5V$$

$$i_F = 5/50 = 0.1A$$

$t > 0$, natural:



$$\Rightarrow \left\{ \begin{array}{l} 8.33\Omega \\ \parallel 2\mu F \end{array} \right. \quad \frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{50} + \frac{1}{20k}$$

$$R_{eq} = 8.33\Omega$$

$$v_{c,n} = V_0 e^{-t/\tau}$$

$$\tau = RC$$

$$\begin{aligned} \frac{1}{\tau} &= \frac{1}{8.33 \times 2\mu F} \\ &= 60 \times 10^3 \text{ s}^{-1} \end{aligned}$$

$$v_{c,n} = V_0 e^{-60 \times 10^3 t}$$

$$\begin{aligned} v_c &= v_{c,f} + v_{c,n} \quad t > 0 \\ &= 5 + V_0 e^{-60 \times 10^3 t} \end{aligned}$$

$$v_c(0^-) = v_c(0^+) = 8$$

$$V_C(0) = 5 + V_0 = 8$$

$$V_0 = 3$$

$$V_C = \begin{cases} 8 & t \leq 0 \\ 5 + 3e^{-60 \times 10^3 t} & t > 0 \end{cases}$$
$$= 8 - 3(1 - e^{-60 \times 10^3 t})u(t) \quad V$$

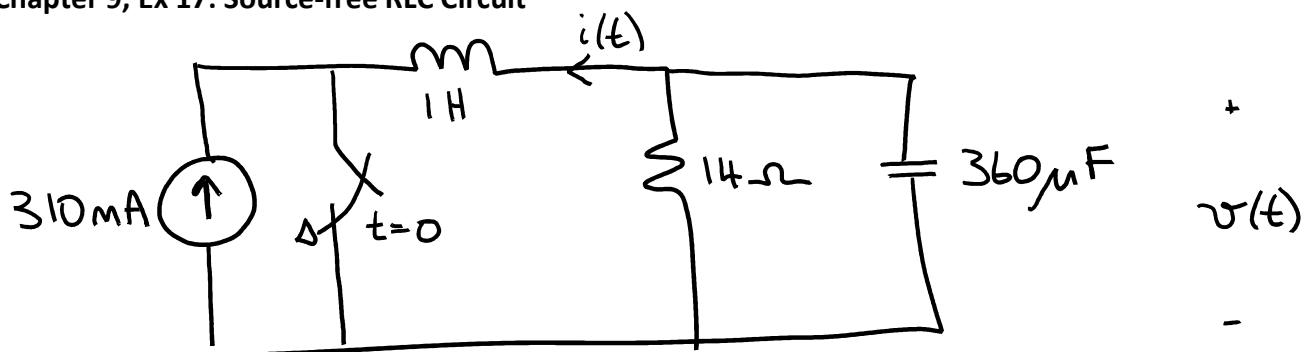
$$i(t) = \frac{v}{R}$$
$$= \frac{5 + 3e^{-60 \times 10^3 t}}{50} \quad t > 0$$
$$= 0.1 + 0.06e^{-60 \times 10^3 t} \text{ A} \quad t > 0$$
$$= \begin{cases} 0.1 \text{ A} & t \leq 0 \\ 0.1 + 0.06e^{-60 \times 10^3 t} \text{ A} & t > 0 \end{cases}$$

Name:

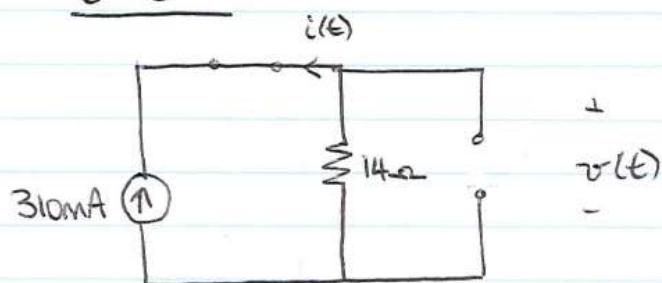
Student ID:

Pre-tutorial 6 Questions (to be attempted before class on May 31st, 2019)

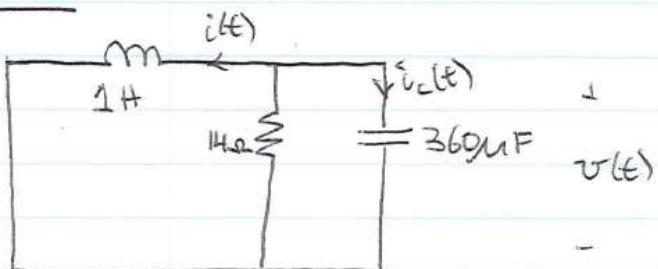
Chapter 9, Ex 17: Source-free RLC Circuit



Obtain expressions for the current $i(t)$ and voltage $v(t)$ as labelled in the circuit above, which are valid for all $t > 0$.

 $t < 0$:

$$\begin{aligned} i(0^-) &= -310\text{mA} \\ v(0^-) &= 310 \times 10^{-3} \times 14 \\ &= 4.34\text{V} \end{aligned}$$

 $t > 0$:

$$\begin{aligned} \omega_0 &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{1 \times 360 \times 10^{-6}}} \\ &= 52.7 \text{ rad/s} \end{aligned}$$

$\alpha > \omega_0 \therefore$ overdamped

$$\begin{aligned} \omega_0 &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{1 \times 360 \times 10^{-6}}} \\ &= 52.7 \text{ rad/s} \end{aligned}$$

$$\Rightarrow v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$\begin{aligned}s_1, s_2 &= -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \\&= -99.2 \pm \sqrt{99.2^2 - 52.7^2} \\&= -15.16, -183.24\end{aligned}$$

$$v(t) = A_1 e^{-15.16t} + A_2 e^{-183.24t}$$

$$\begin{aligned}v(0^+) &= v(0^-) = 4.34 = A_1 + A_2 \\i(0^-) &= i(0^+) = -310 \times 10^{-3}\end{aligned}$$

$$i_c = C \frac{dv}{dt}$$

$$\begin{aligned}&= 360 \times 10^{-6} (-15.16 A_1 e^{-15.16t} - 183.24 A_2 e^{-183.24t}) \\i_c(0^+) &= 360 \times 10^{-6} (-15.16 A_1 - 183.24 A_2) = -5.46 \times 10^{-3} A_1 - 0.066 A_2\end{aligned}$$

$$i(t) + \frac{v(t)}{14} + i_c(t) = 0$$

$$\begin{aligned}\text{@ } t=0: \quad -310 \times 10^{-3} + \frac{4.34}{14} - 5.46 \times 10^{-3} A_1 - 0.066 A_2 &= 0 \\-5.46 \times 10^{-3} A_1 - 0.066 A_2 &= 0 \\A_1 + A_2 &= 4.34 \text{ (from previous page)}\end{aligned}$$

$$\begin{aligned}-5.46 \times 10^{-3} (4.34 - A_2) - 0.066 A_2 &= 0 \\-0.066 A_2 &= 0.0237 \\A_2 &= -0.391\end{aligned}$$

$$\begin{aligned}A_1 &= 4.34 - (-0.391) \\&= 4.73\end{aligned}$$

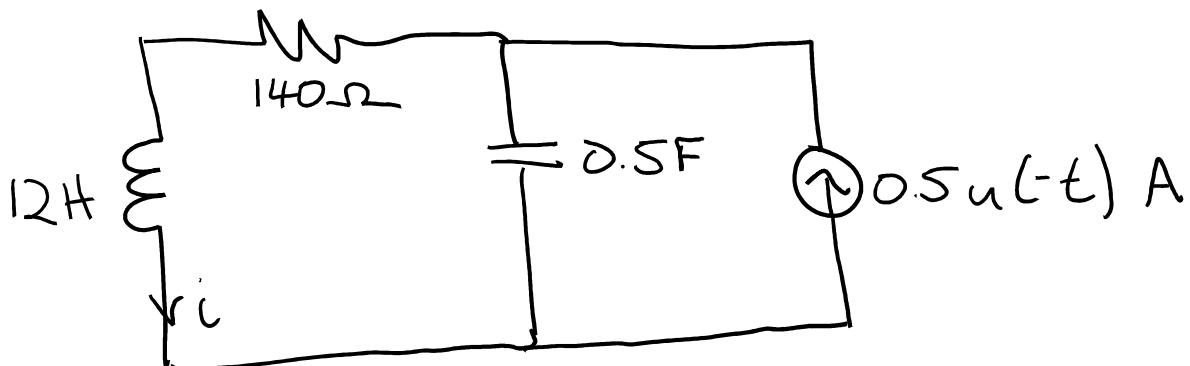
$$v(t) = 4.73 e^{-15.16t} - 0.391 e^{-183.24t} \quad \forall t > 0$$

$$\begin{aligned}i(t) &= -\frac{v(t)}{14} - i_c(t) \\&= -0.34 e^{-15.16t} + 0.028 e^{-183.24t} - \left[-5.46 \times 10^{-3} \times 4.73 e^{-15.16t} \right. \\&\quad \left. - 0.066 \times (-0.391) e^{-183.24t} \right] \\&= -0.31 e^{-15.16t} + 0.002 e^{-183.24t} \quad \forall t > 0\end{aligned}$$

$$\text{OR } i_L = \frac{1}{L} \int_0^t v(\tau) d\tau + i_L(0)$$

$$\begin{aligned} i(t) &= \int_0^t [4.73e^{-15.16\tau} - 0.391e^{-183.24\tau}] d\tau - 310 \times 10^{-3} \\ &= -0.31e^{-15.16t} + 0.002e^{-183.24t} - (-0.312 + 0.002) - \\ &\quad 310 \times 10^{-3} \\ &= -0.31e^{-15.16t} + 0.002e^{-183.24t} \text{ A } t > 0 \end{aligned}$$

Chapter 9, Ex 46: Source-free RLC Circuit

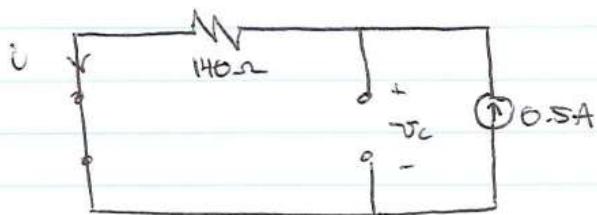


With reference to the circuit above, calculate a) α ; b) ω_0 ; c) $i(0^+)$; d) $\frac{di}{dt}\Big|_{0^+}$

$$\begin{aligned} \text{a)} \quad \alpha &= \frac{R}{2L} \\ &= \frac{140}{2 \times 12} \\ &= 5.83 \text{ s}^{-1} \end{aligned}$$

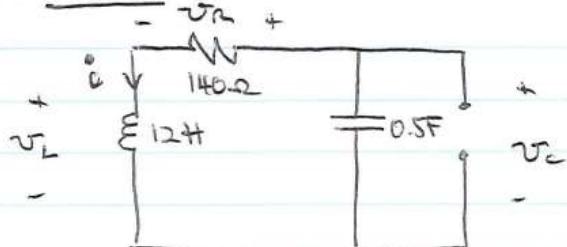
$$\begin{aligned} \text{b)} \quad \omega_0 &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{12 \times 0.5}} \\ &= 0.41 \text{ rad/s} \end{aligned}$$

c) $t < 0$:



$$i(0^-) = 0.5 \text{ A} = i(0^+)$$

d) $t > 0$:



$$v_L = L \frac{di}{dt} = 12 \frac{di}{dt}$$

$$v_L(0^+) - v_c(0^+) + v_R(0^+) = 0$$

$$v_c(0^+) = v_c(0^-) = v_R(0^-) = 0.5 \times 140 \\ = 70V$$

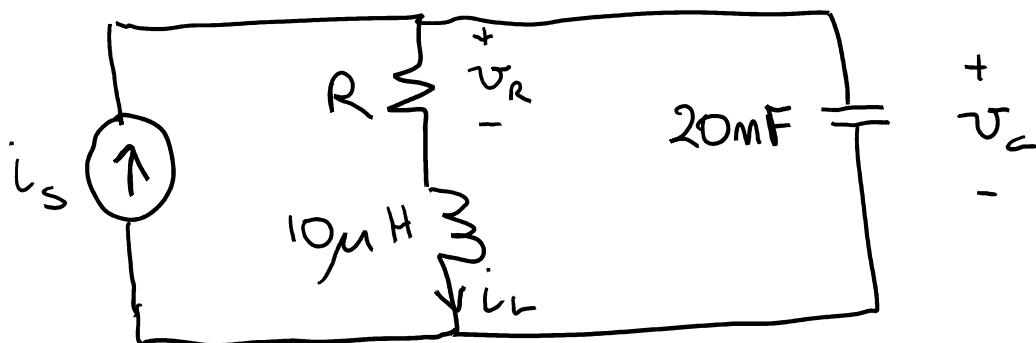
$$v_R(0^+) = iR \\ = 0.5 \times 140 \\ = 70V$$

$$12 \frac{di}{dt} \Big|_{t=0} - 70 + 70 = 0$$

$$\frac{di}{dt} \Big|_{t=0} = 0$$

At Tutorial 6 – Marked Question (31st May 2019)

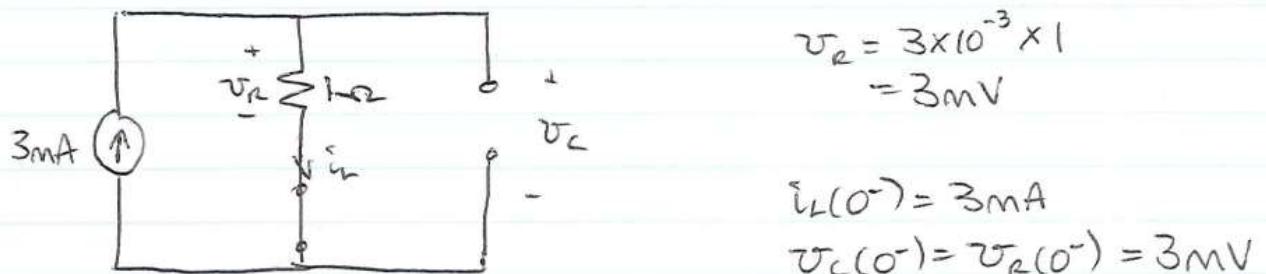
Chapter 9, Ex 50: Driven RLC Circuit



In the series circuit above, set $R = 1 \Omega$. a) Compute α and ω_0 . b) If $i_s = 3u(-t) + 2u(t)$ mA, determine $v_R(0^-)$, $i_L(0^-)$, $v_c(0^-)$, $v_R(0^+)$, $i_L(0^+)$, $v_c(0^+)$, $i_L(\infty)$, and $v_c(\infty)$.

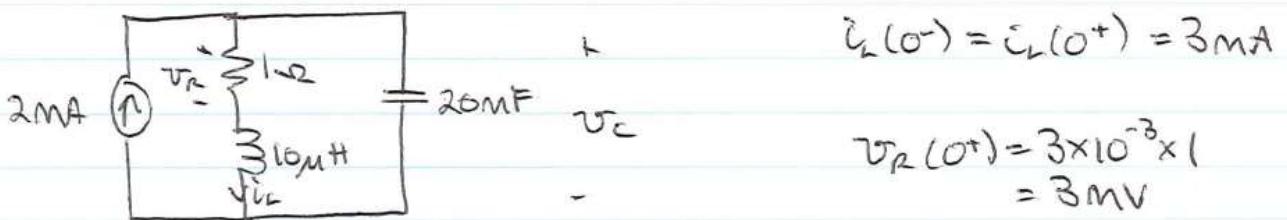
$$\begin{aligned} \text{a) } \alpha &= \frac{R}{2L} & \omega_0 &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{2 \times 10 \times 10^{-6}} & &= \frac{1}{\sqrt{10 \times 10^{-6} \times 20 \times 10^{-3}}} \\ &= 50 \times 10^3 \text{ s}^{-1} & &= 2.236 \times 10^3 \text{ rad/s} \end{aligned}$$

b) $t < 0$:



$$\begin{aligned} v_R &= 3 \times 10^{-3} \times 1 \\ &= 3 \text{ mV} \\ i_L(0^-) &= 3 \text{ mA} \\ v_C(0^-) &= v_R(0^-) = 3 \text{ mV} \end{aligned}$$

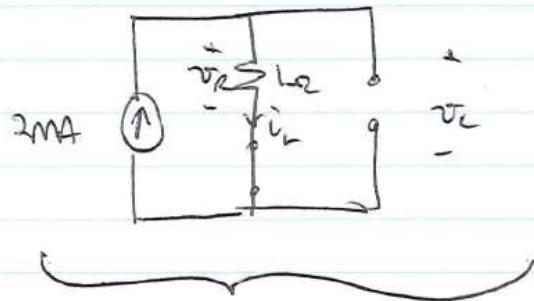
$t > 0$ (complete):



$$\begin{aligned} i_L(0^+) &= i_L(0^-) = 3 \text{ mA} \\ v_R(0^+) &= 3 \times 10^{-3} \times 1 \\ &= 3 \text{ mV} \end{aligned}$$

$$v_C(0^+) = v_C(0^-) = 3 \text{ mV}$$

At $t = \infty$, only the forced response is left, as the natural response $\rightarrow 0$ as $t \rightarrow \infty$:



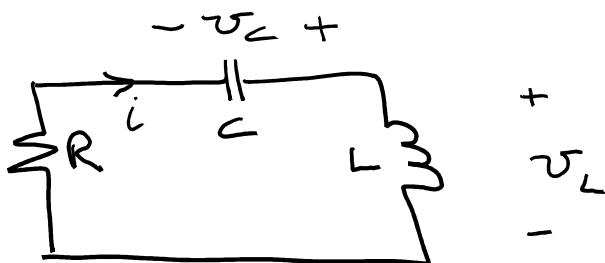
$$i_L(\infty) = 2 \text{ mA}$$

$$v_C(\infty) = 2 \times 10^{-3} \times 1 \\ = 2 \text{ mV}$$

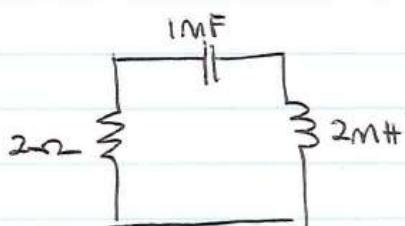
circuit for $t > 0$, forced response

At Tutorial 6 – Unmarked Questions (31st May 2019)

Chapter 9, Ex 42: Source-free RLC Circuit



Component values of $R = 2 \Omega$, $C = 1 \text{ mF}$, and $L = 2 \text{ mH}$ are used to construct the circuit represented above. If $v_c(0^-) = 1 \text{ V}$ and no current initially flows through the inductor, calculate $i(t)$ at $t = 1 \text{ ms}$, 2ms , and 3ms .



$$\begin{aligned}\alpha &= \frac{R}{2L} \\ &= \frac{2}{2 \times 2 \times 10^{-3}} \\ &= 500 \text{ s}^{-1}\end{aligned}$$

$$\begin{aligned}\omega_0 &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{2 \times 10^{-3} \times 1 \times 10^{-3}}} \\ &= 707 \text{ rad/s}\end{aligned}$$

$\alpha < \omega_0$; underdamped

$$\begin{aligned}\Rightarrow i(t) &= e^{-\alpha t} (B_1 \cos \omega_0 t + B_2 \sin \omega_0 t) \\ &= e^{-500t} (B_1 \cos 500t + B_2 \sin 500t)\end{aligned}$$

$$\begin{aligned}\omega_d &= \sqrt{\omega_0^2 - \alpha^2} \\ &= \sqrt{707^2 - 500^2} \\ &= 500 \text{ rad/s}\end{aligned}$$

$$i(0^+) = i(0^-) = 0 = B_1$$

$$\Rightarrow i(t) = B_2 e^{-500t} \sin 500t$$

$$v_c(0^-) = v_c(0^+) = 1 \text{ V}$$

$$v_L = L \frac{di}{dt}$$

$$= 2 \times 10^{-3} (-500 B_2 e^{-500t} \sin 500t + 500 B_2 e^{-500t} \cos 500t)$$

$$\begin{aligned}v_L(0) &= 2 \times 10^{-3} (500 B_2) \\ &= B_2\end{aligned}$$

$$v_L - v_R - v_c = 0$$

$$B_2 - 0 - 1 = 0$$

$$B_2 = 1$$

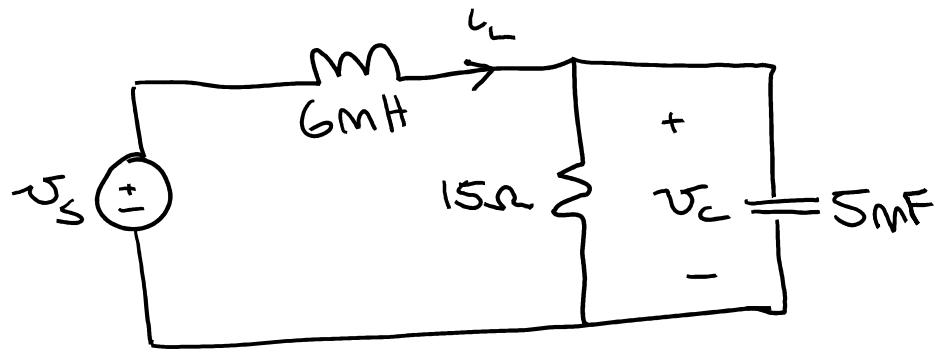
$$i(t) = e^{-500t} \sin 500t \text{ A } t > 0$$

$$i(1\text{ms}) = 0.291 \text{ A}$$

$$i(2\text{ms}) = 0.310 \text{ A}$$

$$i(3\text{ms}) = 0.223 \text{ A}$$

Chapter 9, Ex 52: Forced RLC Circuit



Consider the circuit depicted above. If $v_s(t) = -8 + 2u(t)$ V, determine:

a) $v_c(0^+)$



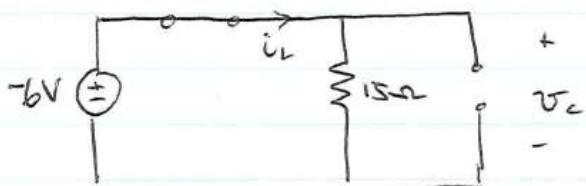
$$v_c(0^+) = v_c(0^-) = -8\text{V}$$

b) $i_L(0^+)$

$$i_L(0^+) = i_L(0^-) = -8/15 \\ = -0.53 \text{ A}$$

c) $v_c(\infty)$

forced response, $t > 0^+$

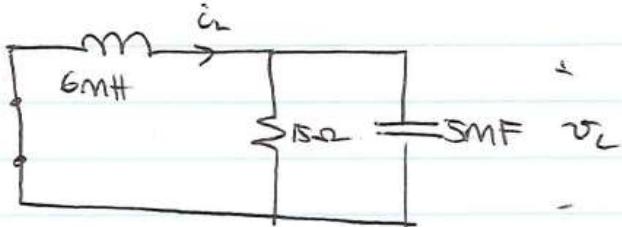


$$v_c(\infty) \Rightarrow \text{forced response only} \\ = -6\text{V}$$

d) $v_C(t = 150\text{ms})$

$v_C(150\text{ms}) \Rightarrow$ need complete response

natural response, $t > 0$:



$$\begin{aligned}\alpha &= \frac{1}{2RC} \\ &= \frac{1}{2 \times 15 \times 5 \times 10^{-3}} \\ &= 6.67 \text{ s}^{-1}\end{aligned}$$

$$\begin{aligned}\omega_0 &= \frac{1}{\sqrt{LC}} \\ &= \frac{1}{\sqrt{6 \times 10^{-3} \times 5 \times 10^{-3}}} \\ &= 182.6 \text{ rad/s}\end{aligned}$$

$$\begin{aligned}\Rightarrow \alpha &< \omega_0 \\ \therefore \text{underdamped} \\ \therefore v_{C,n} &= e^{-\alpha t} (B_1 \cos \omega_0 t + B_2 \sin \omega_0 t)\end{aligned}$$

$$\begin{aligned}\omega_d &= \sqrt{\omega_0^2 - \alpha^2} \\ &= \sqrt{182.6^2 - 6.67^2} \\ &= 182.5 \text{ rad/s}\end{aligned}$$

$$v_{C,n} = e^{-6.67t} (B_1 \cos 182.5t + B_2 \sin 182.5t)$$

$$\begin{aligned}v_C(t) &= v_{C,0} + v_{C,n} \\ &= -6 + e^{-6.67t} (B_1 \cos 182.5t + B_2 \sin 182.5t)\end{aligned}$$

$$\begin{aligned}v_C(0) &= -6 + B_1 = -8 \\ B_1 &= -2\end{aligned}$$

$$\begin{aligned}\frac{dv_C}{dt} &= -6.67e^{-6.67t} (-2 \cos 182.5t + B_2 \sin 182.5t) \\ &\quad + e^{-6.67t} (-365 \sin 182.5t + 182.5 B_2 \cos 182.5t)\end{aligned}$$

$$\begin{aligned}\left. \frac{dv_C}{dt} \right|_{t=0} &= -6.67 \times -2 + 182.5 B_2 \\ &= 182.5 B_2 + 13.33\end{aligned}$$

$$i_C = C \frac{dv}{dt}$$

$$i_C + i_R - i_L = 0$$

at $t=0$:

$$5 \times 10^{-3} (182.5B_2 + 13.33) + \left(-\frac{8}{15}\right) - (-0.53) = 0$$

$$0.9125B_2 + 0.067 - 0.53 + 0.53 = 0$$

$$B_2 = -0.07$$

$$v_C(t) = -6 + e^{-6.67t} (-2 \cos 182.5t - 0.07 \sin 182.5t) \text{ V} \quad t \geq 0$$

$$v_C(150\text{ms}) = -5.56 \text{ V}$$