EMTH211-19S2 LABORATORY 3

JULY 29 - AUGUST 2, 2019

These problems should be done by hand (you can check your answers using MATLAB).

3.1 Suppose that the product $A\,B$ is defined. Show that

$$(A\,B)^T = B^T\,A^T.$$

3.2

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = E_{21}(\alpha) \qquad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \beta & 0 & 1 \end{bmatrix} = E_{42}(\beta).$$

Compute $A\,B$ and $B\,A$.

3.3 Since we can row reduced any non-singular matrix A to the identity matrix (this is Gauss-Jordan)

$$[A \mid I] \to [I \mid A^{-1}]$$

we can write A and A^{-1} as a product of elementary matrices. Let

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}.$$

Find a sequence of elementary matrices L_1, L_2, ..., L_k such that

$$L_k \cdots L_2\,L_1\,A = I.$$

Hence write both A and A^{-1} as products of these elementary matrices.

3.4 Find the LU decomposition of

$$A = \begin{bmatrix} 2 & 2 & 2 & 1 \\ -2 & 4 & -1 & 2 \\ 4 & 4 & 7 & 3 \\ 6 & 9 & 5 & 8 \end{bmatrix}.$$

Note you do not have to use partial pivoting. Hence solve $A\mathbf{x} = \mathbf{b}$ with

$$\mathbf{b} = \begin{bmatrix} 0 \\ 9 \\ 9 \\ 0 \end{bmatrix}.$$

3.5 Use the LU decomposition with partial pivoting to solve the system $A^2\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 6 & 3 & -8 \\ 3 & -1 & 5 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}.$$