Possibly useful equations for Q1-3

Laplace Transform Pairs

$\mathbf{f(t)} = \mathcal{L}^{-1}\{\mathbf{F(s)}\}$	$\mathbf{F(s)} = \mathcal{L}\{\mathbf{f(t)}\}$	$\mathbf{f(t)} = \mathcal{L}^{-1}\{\mathbf{F(s)}\}$	$\mathbf{F(s)} = \mathcal{L}\{\mathbf{f(t)}\}$
$\delta(t)$	1	$\frac{1}{\beta-\alpha}\left(e^{-\alpha t}-e^{-\beta t}\right)u(t)$	$\frac{1}{(s+\alpha)(s+\beta)}$
$u(t)$	$\frac{1}{s}$	$\sin\omega t\, u(t)$	$\frac{\omega}{s^2+\omega^2}$
$tu(t)$	$\frac{1}{s^2}$	$\cos\omega t\, u(t)$	$\frac{s}{s^2+\omega^2}$
$\frac{t^{n-1}}{(n-1)!}u(t), n = 1,2,\ldots$	$\frac{1}{s^n}$	$\sin(\omega t+\theta)u(t)$	$\frac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\cos(\omega t+\theta)u(t)$	$\frac{s\cos\theta-\omega\sin\theta}{s^2+\omega^2}$
$te^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^2}$	$e^{-\alpha t}\sin\omega t\, u(t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t), n = 1,2,\ldots$	$\frac{1}{(s+\alpha)^n}$	$e^{-\alpha t}\cos\omega t\, u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

Laplace Transform Operations

Operation	$\mathbf{f(t)}$	$\mathbf{F(s)}$
Addition	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
Scalar Multiplication	$kf(t)$	$kF(s)$
Time Differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
Time Integration	$\int_{0^-}^{t} f(t)\,dt$	$\frac{1}{s}F(s)$
	$\int_{-\infty}^{t} f(t)\,dt$	$\frac{1}{s}F(s) + \frac{1}{s}\int_{-\infty}^{0^-} f(t)\,dt$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Time Shift	$f(t-a)u(t-a), a \geq 0$	$e^{-as}F(s)$
Frequency Shift	$f(t)e^{-at}$	$F(s+a)$
Frequency Differentiation	$-tf(t)$	$\frac{dF(s)}{ds}$
Frequency Integration	$\frac{f(t)}{t}$	$\int_{s}^{\infty} F(s)\,ds$
Scaling	$f(at), a \geq 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Initial Value	$f(0^+)$	$\lim_{s\to\infty} sF(s)$
Final Value	$f(\infty)$	$\lim_{s\to 0} sF(s)$ All poles of $sF(s)$ in LHP
Time Periodicity	$f(t) = f(f+nT), n = 1,2,\ldots$	$\frac{1}{1-e^{-Ts}}F_1(s)$ Where $F_1(s) = \int_{0^-}^{T} f(t)e^{-st}\,dt$

TURN OVER

Possibly useful equations for Q4-6

Resonance

For parallel RLC circuits: $\omega_0 = \frac{1}{\sqrt{LC}}$, $\alpha = \frac{1}{2RC}$, $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$, $Q_0 = R\sqrt{\frac{C}{L}}$

For series RLC circuits: $\omega_0 = \frac{1}{\sqrt{LC}}$, $\alpha = \frac{R}{2L}$, $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$, $Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$

Bode plots

$$\mathbf{H(s)} = 1 + \frac{\mathrm{s}}{a} \rightarrow H_{dB} = 20\log_{10}\sqrt{1 + (\frac{\omega}{a})^2}, \quad \angle(H(j\omega) = \tan^{-1}\frac{\omega}{a}$$

$$\mathbf{H(s)} = \frac{1}{1 + \frac{\mathrm{s}}{a}} \rightarrow H_{dB} = -20\log_{10}\sqrt{1 + (\frac{\omega}{a})^2}, \quad \angle(H(j\omega) = -\tan^{-1}\frac{\omega}{a}$$

$$\mathbf{H(s)} = 1 + 2\xi(\frac{\mathrm{s}}{\omega_0}) + (\frac{\mathrm{s}}{\omega_0})^2 \rightarrow H_{dB} = 20\log_{10}|1 + j2\xi(\frac{\omega}{\omega_0}) - (\frac{\omega}{\omega_0})^2|,$$

$$\angle(H(j\omega) = \tan^{-1}\frac{2\xi(\frac{\omega}{\omega_0})}{1 - (\frac{\omega}{\omega_0})^2}$$

Fourier

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)\right)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt, \quad a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)dt, \quad b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)dt$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}, \qquad c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_0 t}dt$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega t}F(j\omega)d\omega, \qquad F(j\omega) = \int_{-\infty}^{\infty} e^{-j\omega t}f(t)dt$$

f(t)	**F(jω)**	**f(t)**	**F(jω)**
$\delta(t - t_0)$	$e^{-j\omega t_0}$	$sgn(t)$	$\frac{2}{j\omega}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\cos(\omega_0 t)$	$\pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$	$e^{-\alpha t}u(t)$	$\frac{1}{\alpha + j\omega}$
1	$2\pi\delta(\omega)$	$[e^{-\alpha t}\cos(\omega_d t)]u(t)$	$\frac{\alpha + j\omega}{(\alpha + j\omega)^2 + \omega_d^2}$

End of Examination