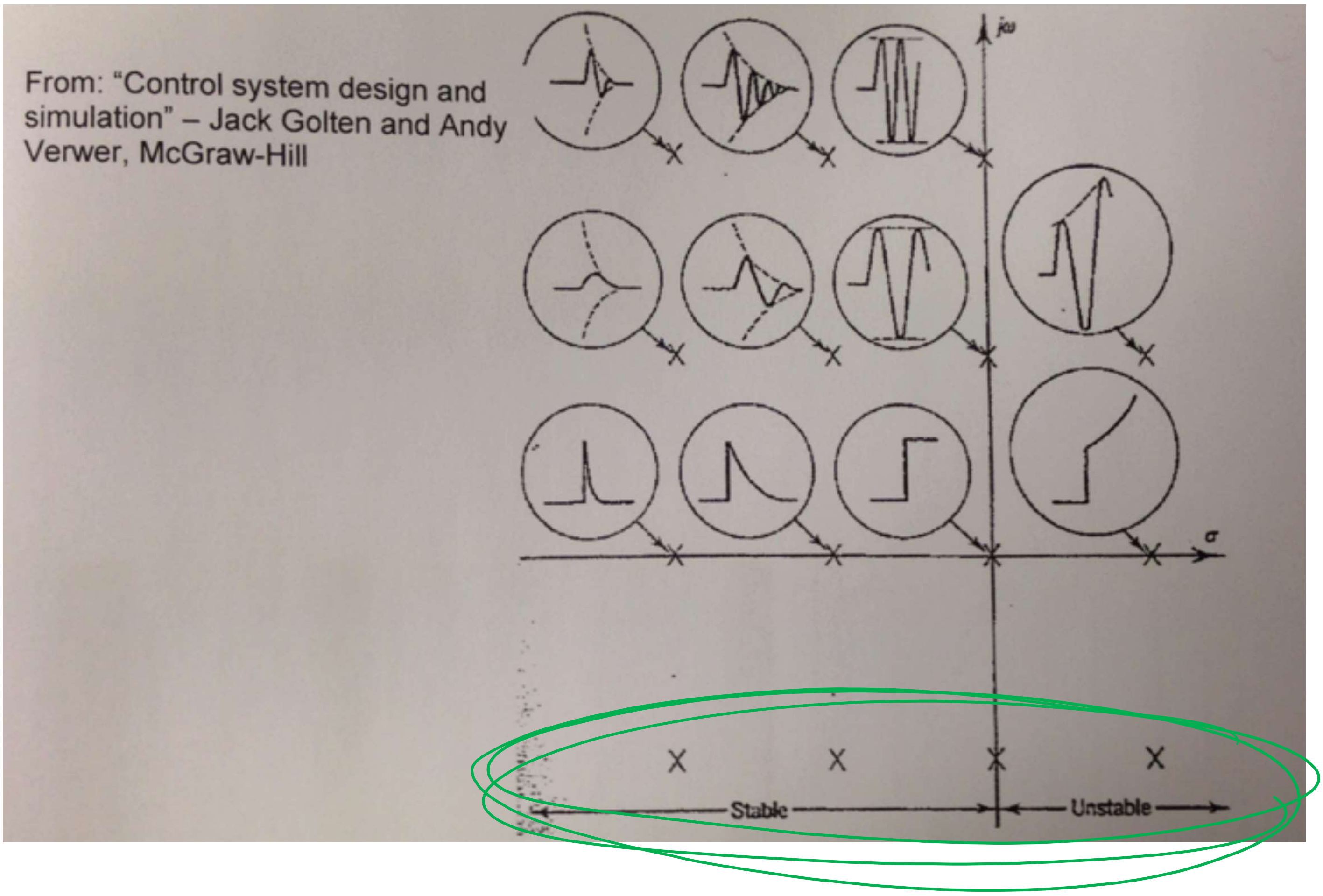


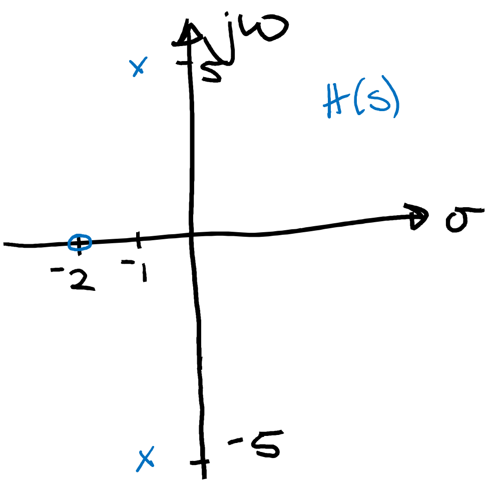
Example

Plot the pole-zero diagram of $\mathbf{H}(\mathbf{s}) = \frac{13(s+2)}{s^2+2s+26}$, and from it determine the shape of the system response.

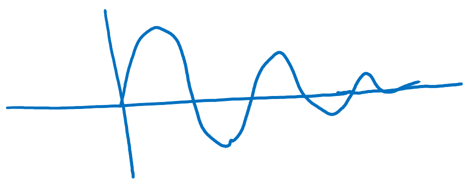
$$H(s) = \frac{13(s+2)}{(s+1+j5)(s+1-j5)}$$

zeros $\Rightarrow$ $s = -2$

poles $\Rightarrow$ $s = -1 \pm j5$

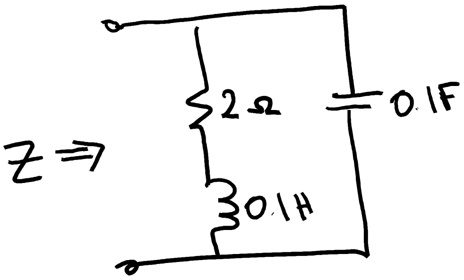


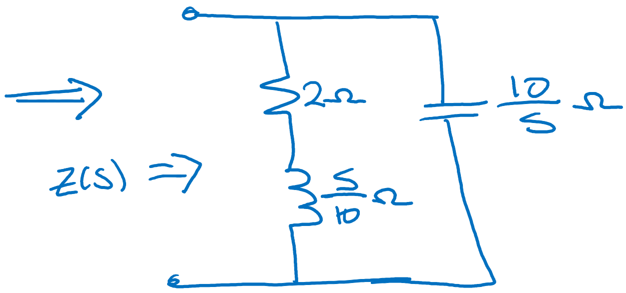
$\Rightarrow$ system response has an exponentially decreasing sinusoidal shape



Example:

For the circuit below, sketch the magnitude of the impedance **Z(s)** and the magnitude of the admittance **Y(s)** as a function of σ and jω.





$$Z(s) = \left(2 + \frac{s}{10}\right) // \frac{10}{s}$$

$$= \frac{\left(2+\frac{s}{10}\right)\frac{10}{s}}{2 + \frac{s}{10} + \frac{10}{s}}$$

$\left(\frac{Z_1 Z_2}{Z_1 + Z_2}\right.$ for two impedances in $\left.//\right)$

$$= \frac{\left(\frac{20}{s} + 1\right)}{\left(\frac{20s + s^2 + 100}{10s}\right)}$$

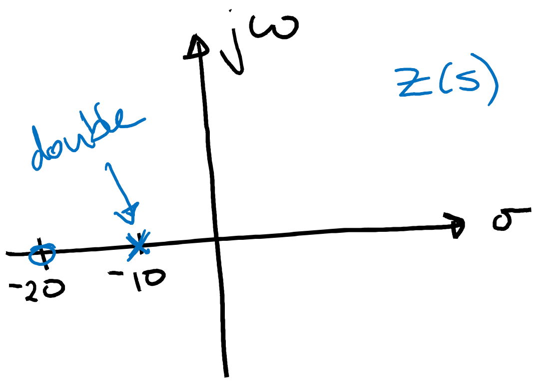
$$= \frac{20 + s}{\cancel{s}} \times \frac{10\cancel{s}}{s^2 + 20s + 100}$$

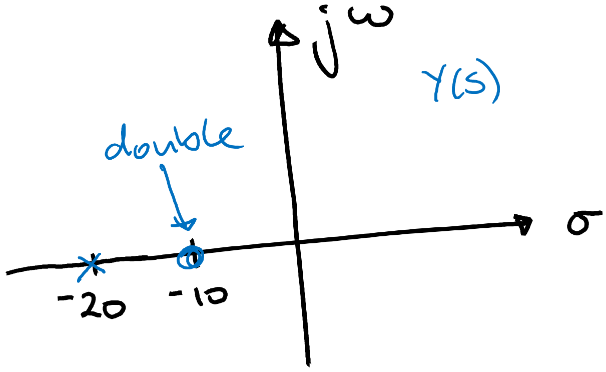
$$= \frac{10(s+20)}{(s+10)^2}$$

zeros ⇒ $s = -20$

poles ⇒ $s = -10$ &

$s = -10$ (double pole)





$$Y(s) = \frac{1}{Z(s)}$$

$$= \frac{(s+10)^2}{10(s+20)}$$

$\Rightarrow$ poles become zeros,
zeros become poles

Convolution

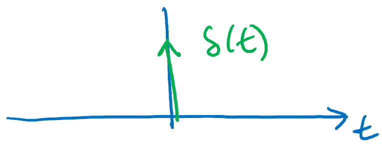
<u>Readings</u>: Section 14.11

The basic idea of convolution is that two waves are interacting, and one is reflected. An example of this is an echo. In the time domain, the maths is not very nice, but it's very useful in the **s**-domain, as it turns into multiplication. The convolution integral uses the impulse response, so we will talk about that first.

Impulse Response

If we're interested in a circuit with initial conditions = 0, and a transfer function **H(s)**, and the input is a unit impulse, $\delta(t)$, then we have:

$v_{in}(t) = \delta(t)$



$V_{in}(s) = 1$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

$$= V_{out}(s)$$

⇒ assuming interested in voltages for input & output

$$\mathcal{L}^{-1}[H(s)] = v_{out}(t)$$

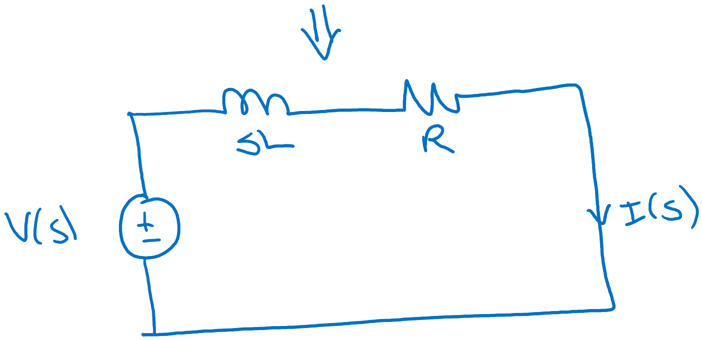
$$= h(t)$$ ← impulse response

⇒ what happens when have impulse as input

Example:

Determine the impulse response for the circuit below.

$\Downarrow$



$$H(s) = \frac{I(s)}{V(s)}$$

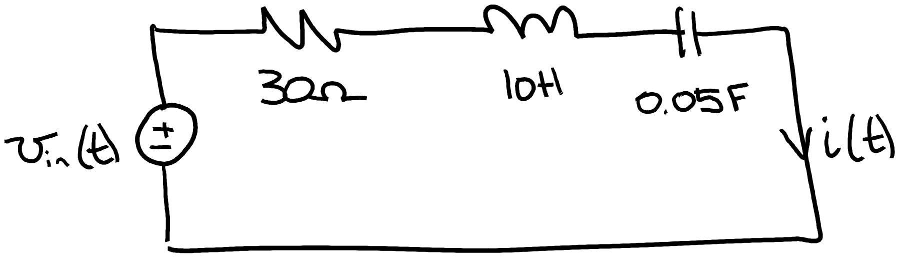
$$= \frac{1}{R + sL}$$

$$= \frac{1}{L}\left(\frac{1}{s + R/L}\right)$$

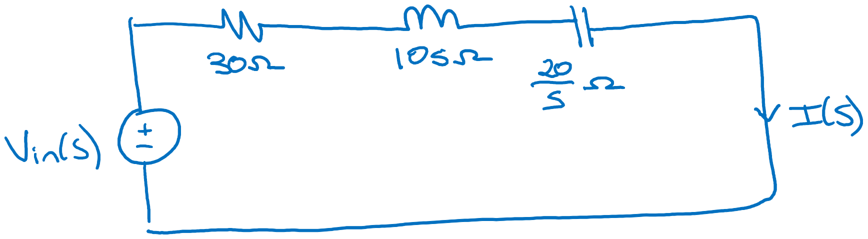
$$h(t) = \frac{1}{L} e^{-R/L\, t}\, u(t)$$

Example

Find the impulse response of the circuit below.



$$\Downarrow$$



$$H(s) = \frac{I(s)}{V_{in}(s)}$$

$$\left(V = IZ \Rightarrow \frac{I}{V} = \frac{1}{Z}\right)$$

$$= \frac{1}{Z(s)}$$

$$= \frac{1}{30 + 10s + \frac{20}{s}}$$

$$= \frac{s}{30s + 10s^2 + 20}$$

$$= \frac{1}{10}\left(\frac{s}{s^2+3s+2}\right)$$

$$= \frac{1}{10}\left(\frac{s}{(s+2)(s+1)}\right)$$

$$= \frac{1}{10}\left(\frac{A}{s+2} + \frac{B}{s+1}\right)$$

$$A(s+1) + B(s+2) = s$$

$s = -1$

$B = -1$

$s = -2$

$-A = -2$

$A = 2$

$$H(s) = \frac{1}{10}\left(\frac{2}{s+2} - \frac{1}{s+1}\right)$$

$$h(t) = \frac{1}{10}\left(2e^{-2t} - e^{-t}\right)u(t)$$

Convolution

The mathematical definition is:

$*$ = symbol for convolution

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(z)h(t-z)\,dz$$

In the **s**-domain however it is:

$$Y(s) = X(s)H(s) \iff y(t) = x(t) * h(t)$$

Remember, **H(s)** is the transfer function, and h(t) is the impulse response.

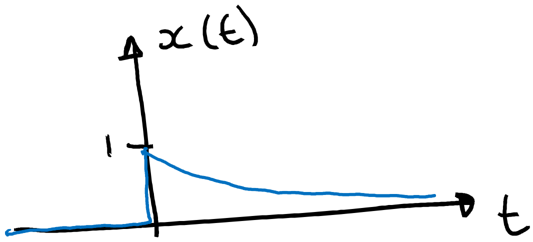
Note that using the transfer function and convolution we can find the output for any given input:

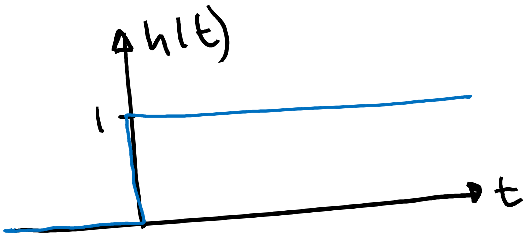
$$V_{in}(s)H(s) = V_{out}(s)$$

(assuming we are interested in both input & output voltages)

Example

If $x(t) = e^{-\alpha t}u(t)$ and $h(t) = u(t)$, what is $y(t)$?





Putting everything into the **s**-domain, we can work it out:

$$X(s) = \frac{1}{s+\alpha} \qquad H(s) = \frac{1}{s}$$

$$Y(s) = X(s)H(s)$$

$$= \frac{1}{s+\alpha} \cdot \frac{1}{s}$$

$$= \frac{A}{s} + \frac{B}{s+\alpha}$$

$$A(s+\alpha) + Bs = 1$$

$\underline{s = -\alpha}$

$-\alpha B = 1$

$B = -1/\alpha$

$\underline{s = 0}$

$\alpha A = 1$

$A = 1/\alpha$

$$Y(s) = \frac{1/\alpha}{s} - \frac{1/\alpha}{s+\alpha}$$

$$y(t) = \frac{1}{\alpha}u(t) - \frac{1}{\alpha}e^{-\alpha t}u(t)$$