Term 3 Additional Tutorial Questions

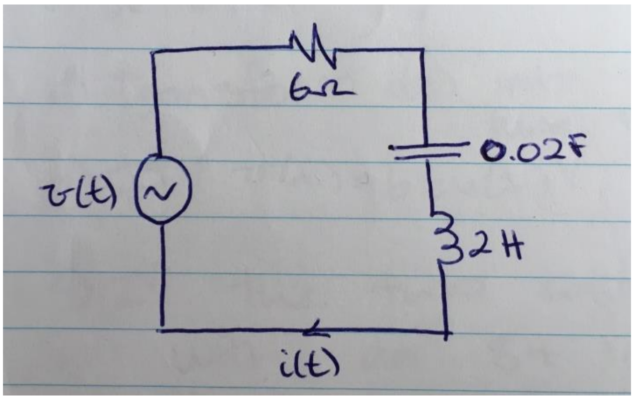
These are tutorial questions that I wrote when I first lectured Term 3. I have since converted to questions from the textbook to be consistent with the other terms. However, the below could be useful for your exam study ☺

1. Complex Numbers

   Let $z_1 = 10 + j15$ and $z_2 = -4 + j3$

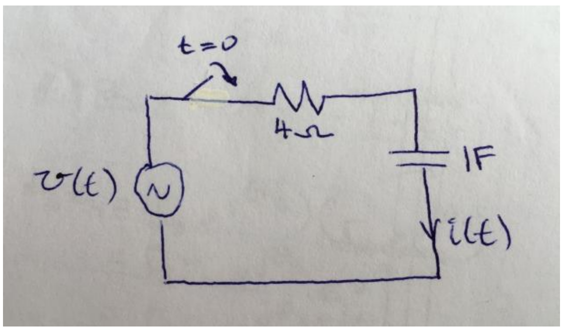
   a) Convert z_1 and z_2 to polar notation $z = re^{j\theta}$ using Pythagoras and trigonometry (i.e. not just the calculator function to convert between forms). θ is to be in radians, not degrees.
   b) Work out ${}^{z_1}/_{z_2}$ and $z_1 - z_2$
   c) Let $z_3 = r_3 e^{j\theta_3}$ and $z_4 = r_4 e^{j\theta_4}$. Show that $z_3 z_4 = r_3 r_4 e^{j(\theta_3 + \theta_4)}$

2. Phasors



   For the circuit above, if $v(t) = 10\cos(7t + 30°)\,V$, what is $i(t)$? Use phasors to calculate the answer. Ignore initial conditions.

3. Phasors



   For the circuit to the left, $v(0^-) = 0V$ and $v(t) = 20\cos(2t + \pi)\,V$. What is $i(t)$?

   (See hints on the following page.)

Hints:

- Write an equation for $v(t)$ using KVL, and write equations for **V** and **I**
- Replace the time-domain functions in the first equation with phasors
- Rearrange so have **I =**
- Simplify
- Convert phasors back into time-domain variables

4. Laplace Transform

a) Using the one-sided LT formula, find the LT of $f(t) = 2u(t-6) + e^{-4t}$

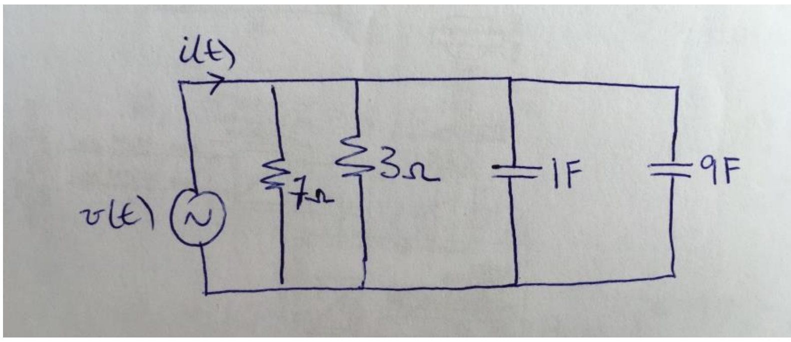
b) Using the tables, find the LT of $f(t) = 7\delta(t) + 9tu(t)$

c) For the circuit in Q2, write an equation for I(s) if $v(t) = tu(t)$ and $i(0) = 0A$. Write it in the form $I(s) = \frac{x+ys+zs^2}{a+bs+cs^2}$. Note some of the coefficients could equal zero.

5. Laplace Transform

Show that $\mathcal{L}[\sin(\omega t)\, u(t)] = \frac{\omega}{s^2+\omega^2}$ by using $\sin(\omega t) = \frac{e^{j\omega t}-e^{-j\omega t}}{2j}$. Do not use the tables.

6. Laplace Transform and Inverse Laplace Transform



For the circuit above, all initial conditions = 0.

a) Simplify the circuit so there is only one capacitor and one resistor.

b) Write an equation for $i(t)$ using KCL.

c) Transform $i(t)$ into the s-domain (i.e. find I(s)). Use the tables for this.

d) If $v(t) = 6tu(t)V$ find $i(t)$ from I(s) for t > 0.

e) If the two capacitors were replaced with an 8H inductor, what would $i(t)$ be? You will need to repeat the above process. Remember for an inductor $i(t) = \frac{1}{L}\int_{t_0}^{t} v(t')dt' + i(t_0)$

7. Inverse Laplace Transform

a) Using partial fractions, find the inverse LT of $I(s) = \frac{7}{s^2+s-12}$

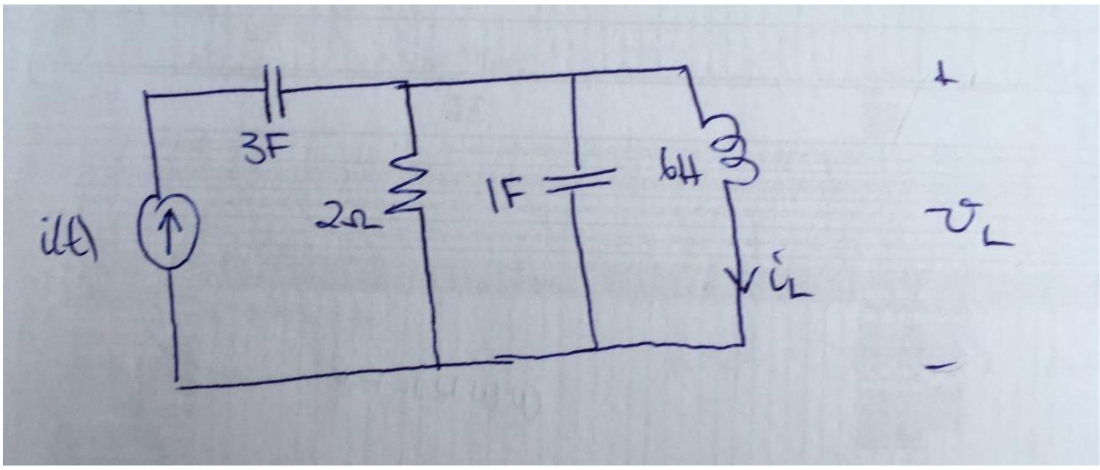
b) Find the simplified equation for $V(s) = \frac{s+2}{(s^2+4)(s+1)}$ using partial fractions

8. Laplace Transform and Inverse Laplace Transform

a) If $v(t) = \delta(t-3)u(t-3) + 8e^{-(t-4)}u(t-4)$ V, find $V(s)$. Hint: Use the tables and time delay

b) Using the IVT, find $v(0^+)$ of $v(t) = (e^{-3t} + t)u(t)$ V

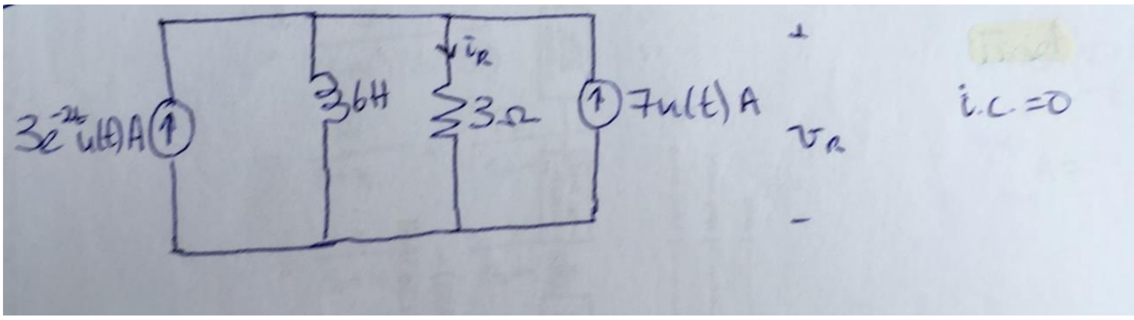
9. s-domain Analysis



a) Redraw the circuit above in the s-domain if $i(t) = \sin(2t)\,u(t)$ A, and the initial conditions are all 0.

b) Redraw the circuit above in the s-domain if $i_L(0^-) = 2A$ and $v_L(0^-) = 6V$. Use a voltage source model where required, not a current source model.

10. s-Domain Analysis



a) For the circuit above, write an equation for the currents in the circuit using KCL. Carry out a LT on this equation. Rearrange to the form $I_R(s) = \frac{N(s)}{D(s)}$.

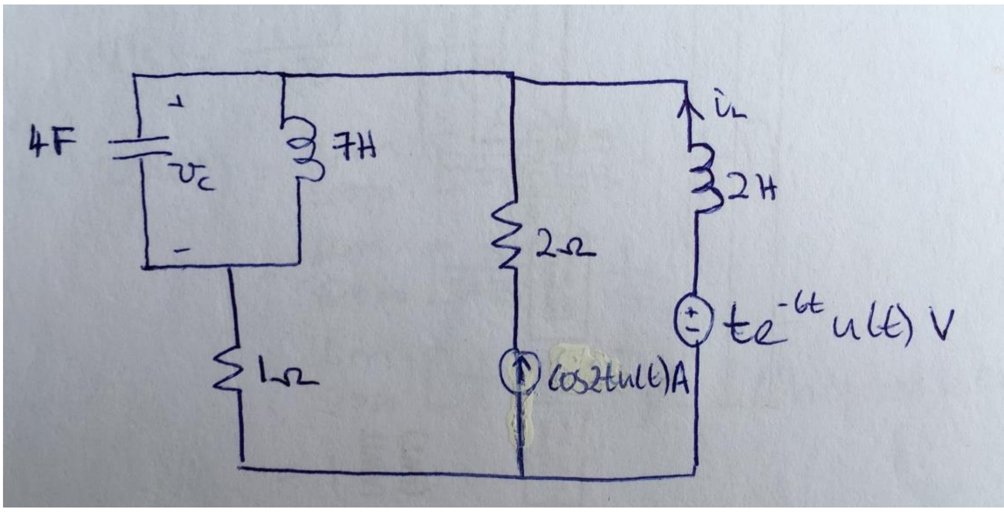
Hint: $i_L = \frac{1}{L}\int_0^t v_L \, dT + i_L(0^-)$.

b) Using partial fractions and the LT tables, find the inverse LT of $I_R(s)$, thus finding $i_R(t)$.

11. Initial value Theorem and Final Value Theorem

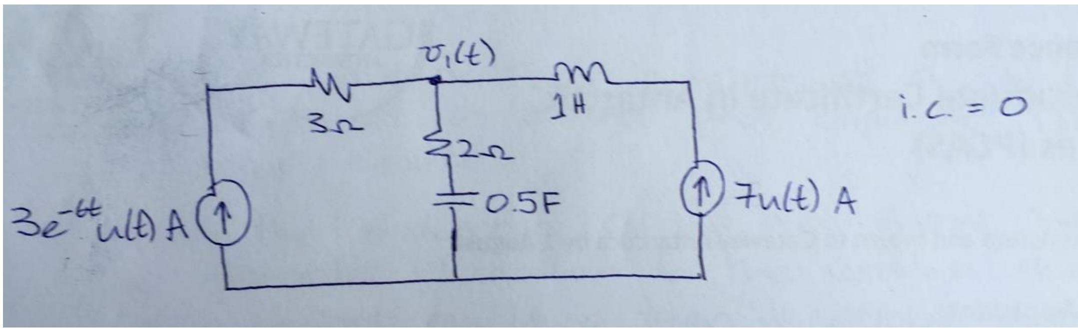
$V(s) = \frac{s(s+3)}{s^2+4s+4}$. Use the IVT and the FVT to find $v(0^+)$ and $v(\infty)$.

12. s-Domain Circuits



Redraw the circuit above in the s-domain, using a current source for the capacitor and a voltage source for the inductor. $v_c(0^-) = 2V$, $i_L(0^-) = 1A$, no i.c. across 7H inductor.

13. s-Domain Circuit Analysis



a) Draw the above circuit in the s-domain.
b) Write an equation for the node $V_1(s)$ and simplify it.
c) Use partial fractions to prepare for the inverse LT.
d) Take the inverse LT to get $v_1(t)$.

14. Impulse Response, Transfer Function, and Convolution

a) If the input to a system is $x(t) = 6tu(t) - 2u(t-4)$, and the impulse response $h(t) = 7u(t)$, what is the output $y(t)$? Hint: Work in the s-domain, and find the convolution of $x(t)$ and $h(t)$
b) What if $h(t) = 9u(t-2)$?

15. Poles, Zeros, and Transfer Functions

a) If $H(s) = \frac{2s^2+12s}{s^2-3s-28}$ what are its poles and zeroes? Sketch the pole-zero diagram.

b) Do the same for $H(s) = \frac{1}{10s^2+6s+1}$. What is the shape of the system response in this case? (e.g. sinusoid, exponentially decreasing sinusoid, exponentially increasing sinusoid, DC exponential)

16. Transfer Functions

Consider a network that consists of a series combination of a resistor, a capacitor, and a voltage source.

a) Draw the s-domain circuit, and show that if a voltage is applied at the voltage source, then the transfer function for the voltage across the capacitor is $H(s) = \frac{1}{RC}\frac{1}{s+(1/RC)}$.

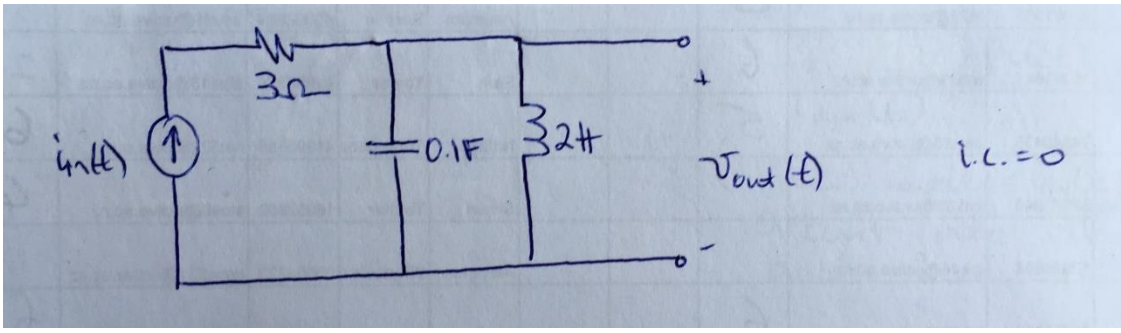
b) Find the pole(s) of $H(s)$.

c) Calculate the output response, $v_{out}(t)$, if $v_{in}(t) = 2u(t)$ V.

17. Poles, Zeros, Impulse Response

If $H(s) = \frac{s^2+4s-6}{s^3+2s^2-15s}$ what are the poles and zeroes of $H(s)$, and what is the impulse response $h(t)$?

18. Transfer Function, Poles, Zeros, Convolution



a) Find the transfer function $H(s)$ of the circuit above, the poles and zeroes of $H(s)$, and sketch the pole-zero plot.

b) If $i_{in}(t) = 6\delta(t)$ A, what is $v_{out}(t)$?