A Dictionary of terms from Linear Algebra

1. Linear combinations and dependence relations

linear combination — A vector $\mathbf{v}$ is a *linear combination* of the vectors $\mathbf{u}_1,\ \mathbf{u}_2,\ \ldots,\ \mathbf{u}_n$ if there are scalars $s_1, s_2,\ \ldots,\ s_n$ such that

$$\mathbf{v} = s_1\mathbf{u}_1 + s_2\mathbf{u}_2 + \ldots + s_n\mathbf{u}_n.$$

span *(noun)* — The *span* of the vectors $\mathbf{u}_1,\ \mathbf{u}_2,\ \ldots,\ \mathbf{u}_n$ is the set of all linear combinations of $\mathbf{u}_1,\ \mathbf{u}_2,\ \ldots,\ \mathbf{u}_n$.

span *(verb)* — The vectors $\mathbf{u}_1,\ \mathbf{u}_2,\ \ldots,\ \mathbf{u}_n$ *span* the space U if every vector in U is a linear combination of $\mathbf{u}_1,\ \mathbf{u}_2,\ \ldots,\ \mathbf{u}_n$.

spanning set — The vectors $\mathbf{u}_1,\ \mathbf{u}_2,\ \ldots,\ \mathbf{u}_n$ form a *spanning set* for the space U if every vector in U is a linear combination of $\mathbf{u}_1,\ \mathbf{u}_2,\ \ldots,\ \mathbf{u}_n$.

dependence relation — A *dependence relation* among the vectors $\mathbf{u}_1,\ \mathbf{u}_2,\ \ldots,\ \mathbf{u}_n$ is an equation of the form

$$s_1\mathbf{u}_1 + s_2\mathbf{u}_2 + \ldots + s_n\mathbf{u}_n = \mathbf{0}$$

where at least one of the scalars $s_1, s_2,\ \ldots,\ s_n$ is nonzero.

linearly dependent — The vectors $\mathbf{u}_1,\ \mathbf{u}_2,\ \ldots,\ \mathbf{u}_n$ are *linearly dependent* if they satisfy a dependence relation.

linearly independent — The vectors $\mathbf{u}_1,\ \mathbf{u}_2,\ \ldots,\ \mathbf{u}_n$ are *linearly independent* if they are not linearly dependent.

Comments

- These terms are all inter-connected. Notice that the basic terms, needed for all the others, are *linear combination* and *dependence relation*.
- From these definitions it follows that, to show that the vectors $\mathbf{u}_1,\ \mathbf{u}_2,\ \ldots,\ \mathbf{u}_n$ are linearly independent, you need to show that the only way to get

  $$s_1\mathbf{u}_1 + s_2\mathbf{u}_2 + \ldots + s_n\mathbf{u}_n = \mathbf{0}$$

  is by having $s_1 = s_2 = \cdots = s_n = 0$.

2. Subspaces

closed	A set S is *closed* under an operation if performing that operation on elements of S always gives you elements of S.
subspace	A *subspace* of $\mathbb{R}^n$ is a (nonempty) subset S of $\mathbb{R}^n$ such that (i) if $\mathbf{u}$, $\mathbf{v}$ are in S, then so is $\mathbf{u}+\mathbf{v}$, and (ii) if $\mathbf{u}$ is in S, and if c is any real number (or scalar), then $c\mathbf{u}$ is in S too. Equivalently, S is a subspace if it is closed under the operations of addition and scalar multiplication.
null space	The *null space* of an $m \times n$ matrix A is the subspace of $\mathbb{R}^n$ consisting of all solutions $\mathbf{x}$ to the equation $A\mathbf{x} = \mathbf{0}$.
column space	The *column space* of an $m \times n$ matrix A is the subspace of $\mathbb{R}^m$ consisting of all vectors $\mathbf{b}$ such that the equation $A\mathbf{x} = \mathbf{b}$ is solvable. The column space of A is sometimes also called the *range* of A because it consists of all images of the transformation $\mathbf{x} \to A\mathbf{x}$. Equivalently, the column space of A consists of all linear combinations of the columns of A.
row space	The *row space* of an $m \times n$ matrix A is the subspace of $\mathbb{R}^n$ consisting of all linear combinations of the rows of A.
basis	A *basis* for a subspace S is a subset B of S such that (i) the vectors in B span the subspace S, and (ii) the vectors in B are linearly independent.
dimension	The *dimension* of a subspace S is the number of vectors in a basis for S.
rank	The *rank* of an $m \times n$ matrix A is the dimension of the row space of A. The rank of A is also equal to the dimension of the column space of A.
nullity	The *nullity* of an $m \times n$ matrix A is the dimension of the null space of A.