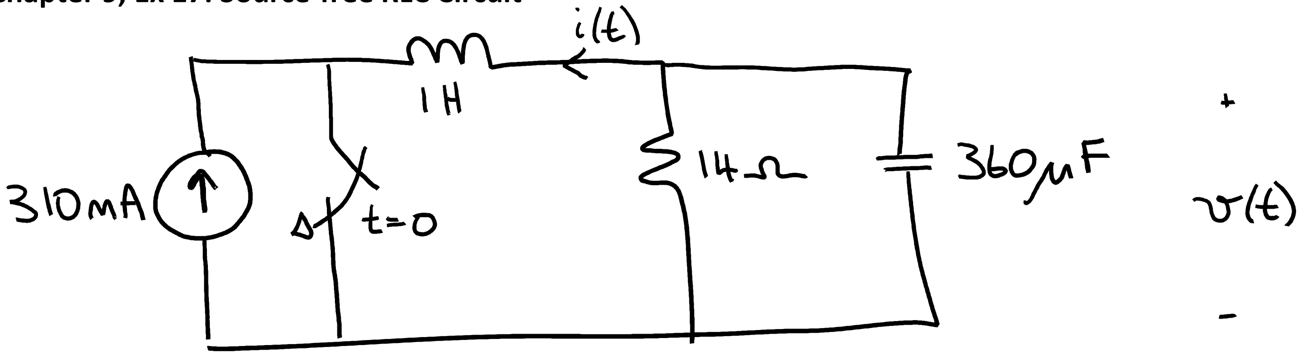
Name: **Student ID:**

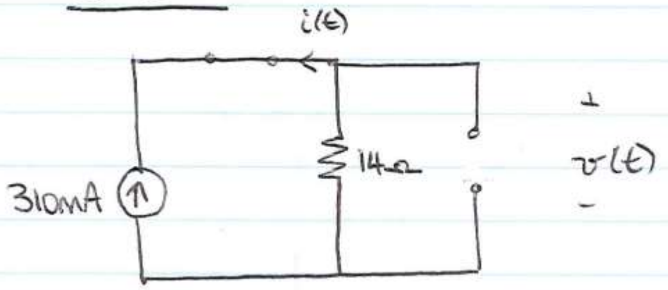
Pre-tutorial 6 Questions (to be attempted before class on May 31st, 2019)

Chapter 9, Ex 17: Source-free RLC Circuit



Obtain expressions for the current i(t) and voltage v(t) as labelled in the circuit above, which are valid for all t > 0.

$t<0$:

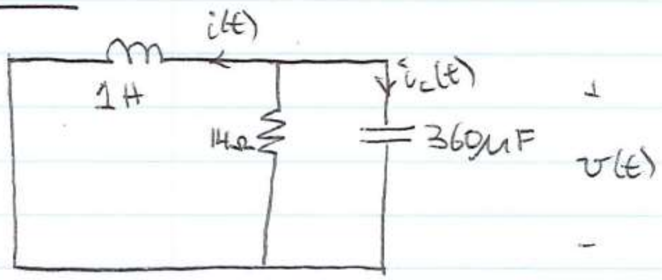


$i(0^-) = -310\text{mA}$

$v(0^-) = 310\times10^{-3}\times14$

$= 4.34\text{V}$

$t>0$:



$\alpha = \dfrac{1}{2RC}$

$= \dfrac{1}{2\times14\times360\times10^{-6}}$

$= 99.2 \ s^{-1}$

$\omega_0 = \dfrac{1}{\sqrt{LC}}$

$= \dfrac{1}{\sqrt{1\times360\times10^{-6}}}$

$= 52.7 \text{ rad/s}$

$\alpha > \omega_0 \quad \therefore$ overdamped

$\Rightarrow v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$
$$= -99.2 \pm \sqrt{99.2^2 - 52.7^2}$$
$$= -15.16, -183.24$$

$$v(t) = A_1 e^{-15.16t} + A_2 e^{-183.24t}$$

$$v(0^+) = v(0^-) = 4.34 = A_1 + A_2$$
$$i(0^-) = i(0^+) = -310 \times 10^{-3}$$

$$i_c = C \frac{dv}{dt}$$
$$= 360 \times 10^{-6} (-15.16 A_1 e^{-15.16t} - 183.24 A_2 e^{-183.24t})$$
$$i_c(0^+) = 360 \times 10^{-6} (-15.16 A_1 - 183.24 A_2) = -5.46 \times 10^{-3} A_1 - 0.066 A_2$$

$$i(t) + \frac{v(t)}{14} + i_c(t) = 0$$

$$@\ t = 0: \quad -310 \times 10^{-3} + \frac{4.34}{14} - 5.46 \times 10^{-3} A_1 - 0.066 A_2 = 0$$
$$-5.46 \times 10^{-3} A_1 - 0.066 A_2 = 0$$
$A_1 + A_2 = 4.34$ (from previous page)

$$-5.46 \times 10^{-3}(4.34 - A_2) - 0.066 A_2 = 0$$
$$-0.06 A_2 = 0.0237$$
$$A_2 = -0.391$$

$$A_1 = 4.34 - (-0.391)$$
$$= 4.73$$

$$v(t) = 4.73 e^{-15.16t} - 0.391 e^{-183.24t} \text{ V} \quad t > 0$$

$$i(t) = \frac{-v(t)}{14} - i_c(t)$$
$$= -0.34 e^{-15.16t} + 0.028 e^{-183.24t} - \Big[-5.46 \times 10^{-3} \times 4.73 e^{-15.16t}$$
$$- 0.066 \times (-0.391) e^{-183.24t} \Big]$$
$$= -0.31 e^{-15.16t} + 0.002 e^{-183.24t} \text{ A} \quad t > 0$$

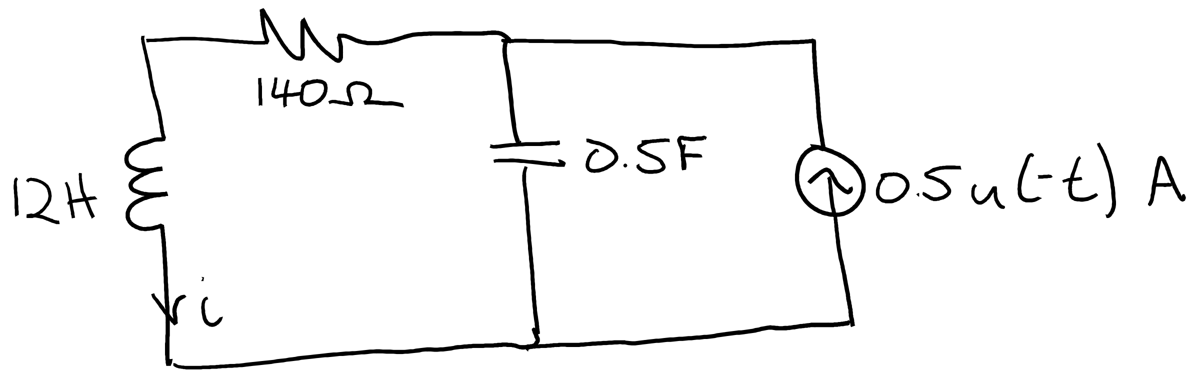
<u>OR</u> $i_L = \frac{1}{L}\int_0^t v(\tau)\,d\tau + i_L(0)$

$$i(t) = \int_0^t \left[4.73e^{-15.16\tau} - 0.391e^{-183.24\tau}\right] d\tau - 310\times10^{-3}$$

$$= -0.31e^{-15.16t} + 0.002e^{-183.24t} - (-0.312+0.002) - 310\times10^{-3}$$

$$= -0.31e^{-15.16t} + 0.002e^{-183.24t} \text{ A} \quad t>0$$

Chapter 9, Ex 46: Source-free RLC Circuit



With reference to the circuit above, calculate a) α; b) ω_0; c) $i(0^+)$; d) $\left.\frac{di}{dt}\right|_{0^+}$

a) $\alpha = \frac{R}{2L}$

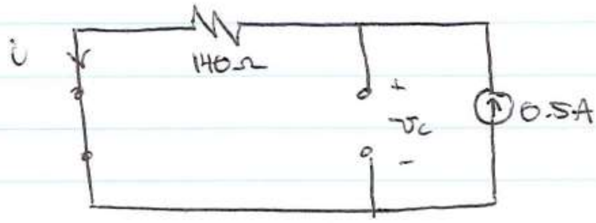
$= \frac{140}{2 \times 12}$

$= 5.83\ s^{-1}$

b) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{12 \times 0.5}}$

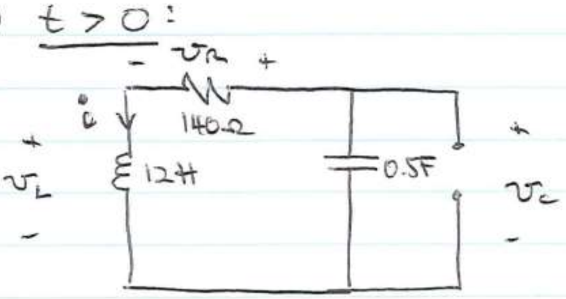
$= 0.41$ rad/s

c) $\underline{t<0:}$



$i(0^-) = 0.5A = i(0^+)$

d) $\underline{t>0:}$



$v_L = L\frac{di}{dt} = 12\frac{di}{dt}$

$$v_L(0^+) - v_c(0^+) + v_R(0^+) = 0$$

$$v_c(0^+) = v_c(0^-) = v_R(0^-) = 0.5 \times 140$$
$$= 70V$$

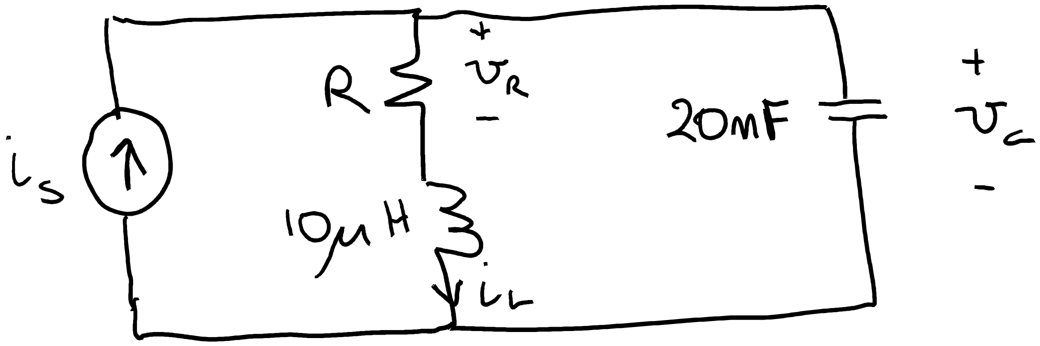
$$v_R(0^+) = iR$$
$$= 0.5 \times 140$$
$$= 70V$$

$$12 \left.\frac{di}{dt}\right|_{t=0} - 70 + 70 = 0$$

$$\left.\frac{di}{dt}\right|_{t=0} = 0$$

At Tutorial 6 – Marked Question (31st May 2019)

Chapter 9, Ex 50: Driven RLC Circuit



In the series circuit above, set R = 1 Ω. a) Compute α and ω_0. b) If $i_s = 3u(-t) + 2u(t)$ mA, determine $v_R(0^-)$, $i_L(0^-)$, $v_c(0^-)$, $v_R(0^+)$, $i_L(0^+)$, $v_C(0^+)$, $i_L(\infty)$, and $v_c(\infty)$.

a) $\alpha = \frac{R}{2L}$

$= \frac{1}{2 \times 10 \times 10^{-6}}$

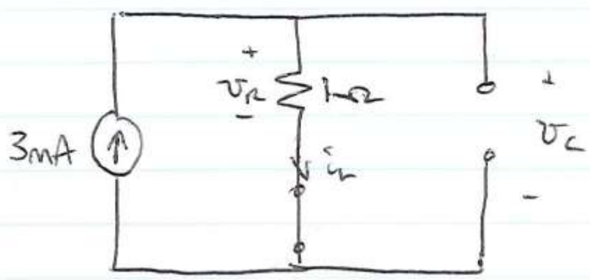
$= 50 \times 10^{3}\ s^{-1}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$= \frac{1}{\sqrt{10 \times 10^{-6} \times 20 \times 10^{-3}}}$

$= 2.236 \times 10^{3}$ rad/s

b) <u>t<0:</u>

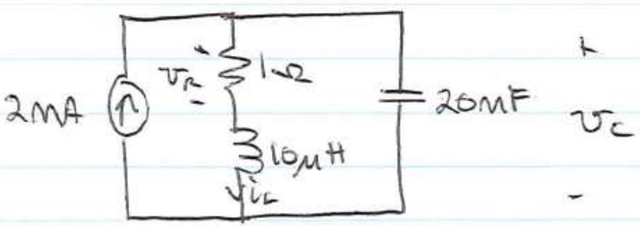


$v_R = 3 \times 10^{-3} \times 1$
$= 3$mV

$i_L(0^-) = 3$mA

$v_c(0^-) = v_R(0^-) = 3$mV

<u>t>0</u> (complete):

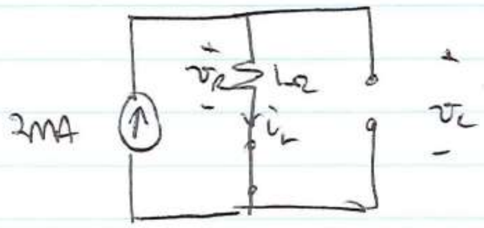


$i_L(0^-) = i_L(0^+) = 3$mA

$v_R(0^+) = 3 \times 10^{-3} \times 1$
$= 3$mV

$v_c(0^+) = v_c(0^-) = 3$mV

At $t=\infty$, only the forced response is left, as the natural response $\to 0$ as $t \to \infty$:



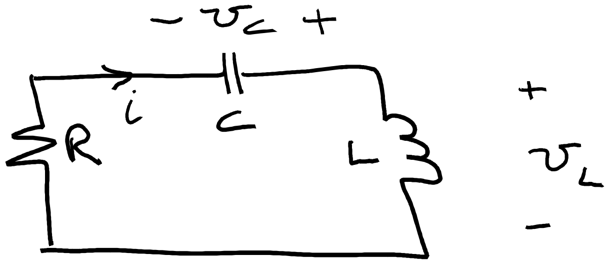
$i_L(\infty) = 2mA$

$v_C(\infty) = 2\times10^{-3}\times1$
$= 2mV$

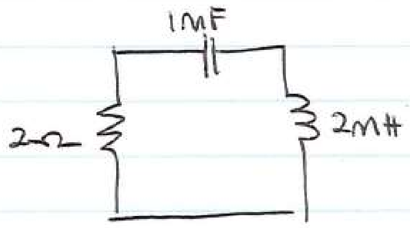
circuit for $t>0$, forced response

At Tutorial 6 – Unmarked Questions (31st May 2019)

Chapter 9, Ex 42: Source-free RLC Circuit



Component values of R = 2 Ω, C = 1 mF, and L = 2 mH are used to construct the circuit represented above. If $v_c(0^-)$ = 1 V and no current initially flows through the inductor, calculate $i(t)$ at t = 1 ms, 2ms, and 3ms.



$$\alpha = \frac{R}{2L} = \frac{2}{2\times2\times10^{-3}} = 500\ s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times10^{-3}\times1\times10^{-3}}} = 707\ \text{rad/s}$$

$\alpha < \omega_0$ $\therefore$ underdamped

$$\Rightarrow i(t) = e^{-\alpha t}(B_1\cos\omega_d t + B_2\sin\omega_d t)$$
$$= e^{-500t}(B_1\cos 500t + B_2\sin 500t)$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{707^2 - 500^2} = 500\ \text{rad/s}$$

$$i(0^+) = i(0^-) = 0 = B_1$$

$$\Rightarrow i(t) = B_2 e^{-500t}\sin 500t$$

$$v_c(0^-) = v_c(0^+) = 1\text{V}$$

$$v_L = L\frac{di}{dt}$$
$$= 2\times10^{-3}(-500B_2e^{-500t}\sin 500t + 500B_2e^{-500t}\cos 500t)$$

$$v_L(0) = 2\times10^{-3}(500B_2)$$
$$= B_2$$

$v_L - v_R - v_C = 0$

$B_2 - 0 - 1 = 0$

$B_2 = 1$

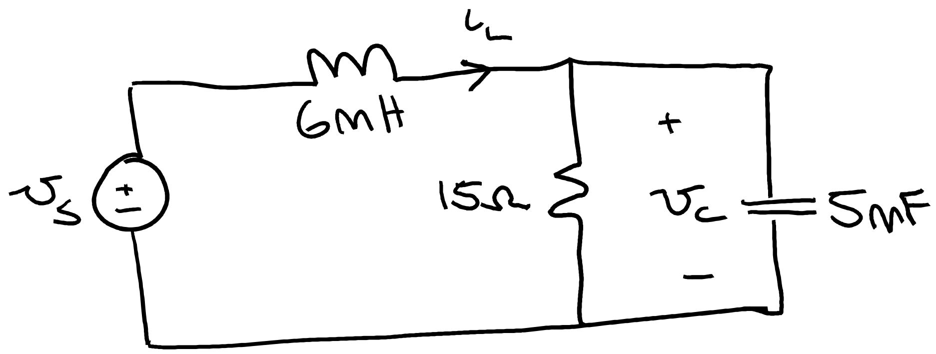
$i(t) = e^{-500t} \sin 500t \text{ A} \quad t > 0$

$i(1ms) = 0.291 \text{ A}$

$i(2ms) = 0.310 \text{ A}$

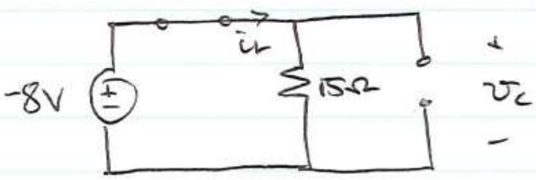
$i(3ms) = 0.223 \text{ A}$

Chapter 9, Ex 52: Forced RLC Circuit



Consider the circuit depicted above. If $v_S(t) = -8 + 2u(t)$ V, determine:

a) $v_C(0^+)$



$v_C(0^+) = v_C(0^-) = -8V$

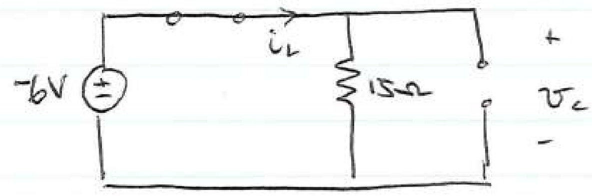
b) $i_L(0^+)$

$i_L(0^+) = i_L(0^-) = -8/15$

$= -0.53$ A

c) $v_c(\infty)$

<u>forced response, t>0:</u>



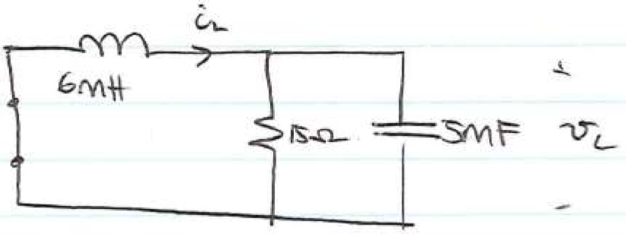
$v_c(\infty) \Rightarrow$ forced response only

$= -6V$

d) $v_C(t = 150\text{ms})$

$v_c(150\text{ms}) \Rightarrow$ need complete response

natural response, $t>0$:



$$\alpha = \frac{1}{2RC} = \frac{1}{2\times15\times5\times10^{-3}} = 6.67\ s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{6\times10^{-3}\times5\times10^{-3}}} = 182.6 \text{ rad/s}$$

$\Rightarrow \alpha < \omega_0$

$\therefore$ underdamped

$\therefore v_{c,n} = e^{-\alpha t}(B_1\cos\omega_d t + B_2\sin\omega_d t)$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{182.6^2 - 6.67^2} = 182.5 \text{ rad/s}$$

$$v_{c,n} = e^{-6.67t}(B_1\cos 182.5t + B_2\sin 182.5t)$$

$$v_c(t) = v_{c,p} + v_{c,n} = -6 + e^{-6.67}(B_1\cos 182.5t + B_2\sin 182.5t)$$

$$v_c(0) = -6 + B_1 = -8$$
$$B_1 = -2$$

$$\frac{dv_c}{dt} = -6.67e^{-6.67t}(-2\cos 182.5t + B_2\sin 182.5t) + e^{-6.67}(-365\sin 182.5t + 182.5B_2\cos 182.5t)$$

$$\left.\frac{dv_c}{dt}\right|_{t=0} = -6.67\times-2 + 182.5B_2 = 182.5B_2 + 13.33$$

$$i_c = C\frac{dv}{dt}$$

$$i_c + i_R - i_L = 0$$

at $t=0$:

$$5\times10^{-3}(182.5B_2 + 13.33) + \left(\frac{-8}{15}\right) - (-0.53) = 0$$

$$0.9125B_2 + 0.067 - 0.53 + 0.53 = 0$$

$$B_2 = -0.07$$

$$v_c(t) = -6 + e^{-6.67t}(-2\cos 182.5t - 0.07\sin 182.5t)\ \text{V} \quad t>0$$

$$v_c(150\text{ms}) = -5.56\text{V}$$