Example: Series resonant circuit with R = 50Ω, L = 4mH and C = 0.1μF.

Calculate the following parameters:

a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\times10^{-3}\times 0.1\times10^{-6}}} = 50k\ rad/s$

b) $f_{0s} = \frac{\omega_{0s}}{2\pi} = \frac{50\times10^3}{2\pi} = 7.957\ kHz$

resonant frequency

c) $Q_{0s} = \frac{\omega_{0s}L_s}{R_s} = \frac{50\times10^3\times4\times10^{-3}}{50} = 4$ ⇒ quality factor (how steep)

d) $B_s = \frac{\omega_{0s}}{Q_{0s}} = \frac{50\times10^3}{4} = 12.5k\ rad/s = \omega_2 - \omega_1$ ⇒ bandwidth (half-power)

e) $\omega_1 = \omega_0\left[\sqrt{1+\left(\frac{1}{2Q_0}\right)^2} - \frac{1}{2Q_0}\right] = 50\times10^3\left[\sqrt{1+\left(\frac{1}{8}\right)^2} - \frac{1}{8}\right]$

$= 44.14\ krad/s$

f) $\omega_2 = \omega_0\left[\sqrt{1+\left(\frac{1}{2Q_0}\right)^2} + \frac{1}{2Q_0}\right] = 50\times10^3\left[\sqrt{1+\left(\frac{1}{8}\right)^2} + \frac{1}{8}\right]$

$= 56.64\ krad/s$

g) Z_{in} at 45 krad/sec = ?

$Z_{in} = R + sL + \frac{1}{sC} = 50 + s\times4\times10^{-3} + \frac{1}{s\times0.1\times10^{-6}}$

$s = j\omega$ $\qquad \frac{1}{j} = -j$

$Z_{in}(j\omega) = Z_{in}(j45\times10^3)$

$= 50 + j45\times10^3\times4\times10^{-3} - \frac{j}{45\times10^3\times0.1\times10^{-6}}$

$= 50 - j42.22$

$= 65.44\angle{-0.701}\ \Omega$

Scaling

Once a network has been designed for a specific Q and ω_0 it can be scaled.

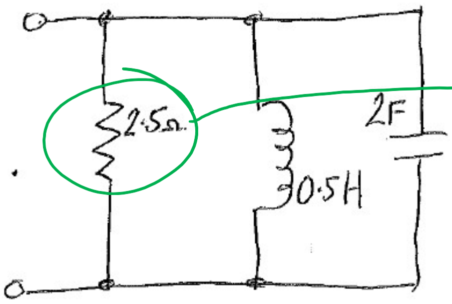
Magnitude scaling by a factor K_m means all impedances are increased by K_m (a real positive value), thus

$$R \to K_m R, \qquad L \to K_m L, \qquad C \to C/K_m$$

Frequency scaling by a factor K_f means that the frequency at which a particular impedance occurs is increased by K_f (a real positive value), thus

$$R \to R, \qquad L \to L/K_f, \qquad C \to C/K_f$$

Example: Scale a prototype circuit



The "prototype" circuit shown has $\omega_0 = 1$ rad/sec, $Q_0 = 5$ and B = 0.2 rad/sec. We wish to scale it so that the maximum impedance is 800 Ω at 2×10^6 rad/sec. Apply both magnitude and frequency scaling.

$R = 2.5\Omega$ & $k_m R = 800\Omega$

$2.5 k_m = 800$

$k_m = 320$

$k_f = \dfrac{2\times10^6}{1} = 2\times10^6$

ω₀ (new)

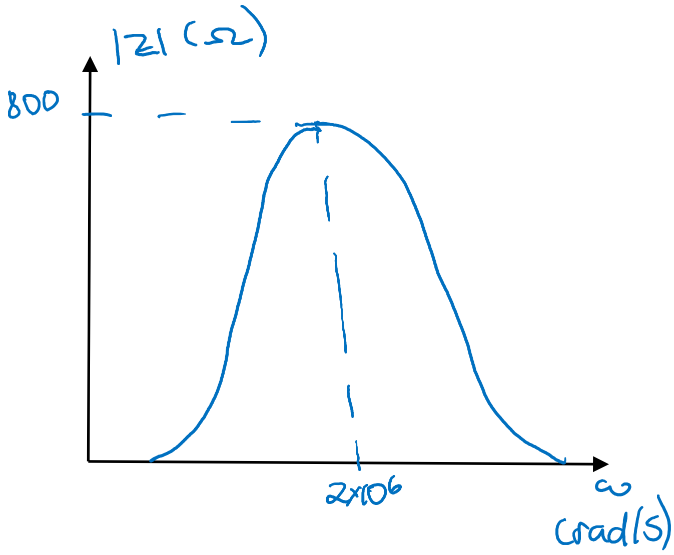
ω₀ (original)

$L' = \dfrac{k_m L}{k_f} = \dfrac{320\times0.5}{2\times10^6}$

$= 80\mu H$

$C' = \dfrac{C}{k_m k_f} = \dfrac{2}{320\times2\times10^6}$

$= 3.125 nF$



Bode Diagrams

<u>Readings</u>: Section 15.2

Cartesian and Polar Coordinates

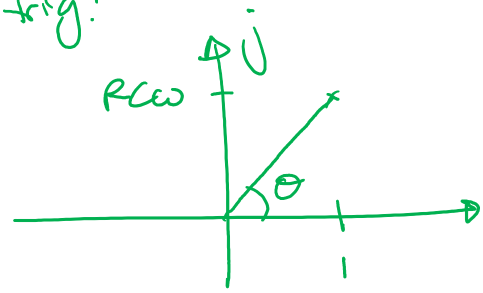
To draw bode plots you must be able to calculate magnitude $|G(j\omega)|$ and phase $\angle G(j\omega)$ from *G(s)* and be able to find the poles and zeros.

Example: Find magnitude $|G(j\omega)|$ and phase $\angle G(j\omega)$ for $G(s) = \frac{1}{RCs+1}$

$$G(j\omega) = \frac{1}{jRC\omega + 1}$$

no numbers, so have to use trig!

$$|G(j\omega)| = \frac{1}{\sqrt{1^2 + (RC\omega)^2}}$$



$$\angle G(j\omega) = -\tan^{-1}\left(\frac{RC\omega}{1}\right)$$

negative because looking at denominator

If we had $\dfrac{a+jb}{(c+jd)(e+jf)}$

then $$|\,| = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}\,\sqrt{e^2+f^2}}$$

$$\angle = \tan^{-1}\left(\frac{b}{a}\right) - \tan^{-1}\left(\frac{d}{c}\right) - \tan^{-1}\left(\frac{f}{e}\right)$$

Example 2: Find $|G(j\omega)|$ and $\angle G(j\omega)$ for $G(s) = \frac{s+4}{(s+1)(s+2)}$.

$$G(j\omega) = \frac{j\omega + 4}{(j\omega+1)(j\omega+2)}$$

$$|G(j\omega)| = \frac{\sqrt{4^2+\omega^2}}{\sqrt{1^2+\omega^2}\sqrt{2^2+\omega^2}} = \frac{\sqrt{16+\omega^2}}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$$

$$\angle G(j\omega) = \tan^{-1}\left(\frac{\omega}{4}\right) - \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

Polar co-ordinates $\Rightarrow |G(j\omega)| \angle G(j\omega)$

Frequency Response

- Assuming a steady-state sinusoidal response.
- Assuming a well-behaved transfer function (LHP poles).
- We can calculate $|G(j\omega)|$ and $\angle G(j\omega)$ and find:

Magnitude of input — Frequency of input

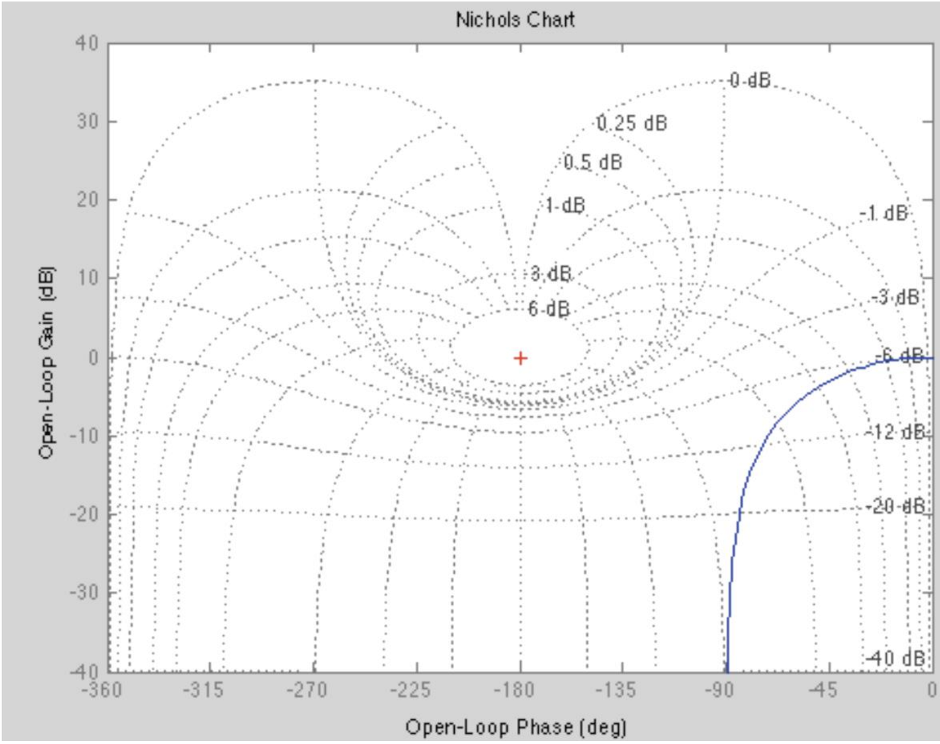
$$y_{ss}(t) = r_0 |G(j\omega)| \sin(\omega t + \angle G(j\omega))$$

Magnitude of transfer function — Phase of transfer function

- Frequency response is the visualisation of $|G(j\omega)|$ and $\angle G(j\omega)$.
- It's generally done in one of 3 ways…

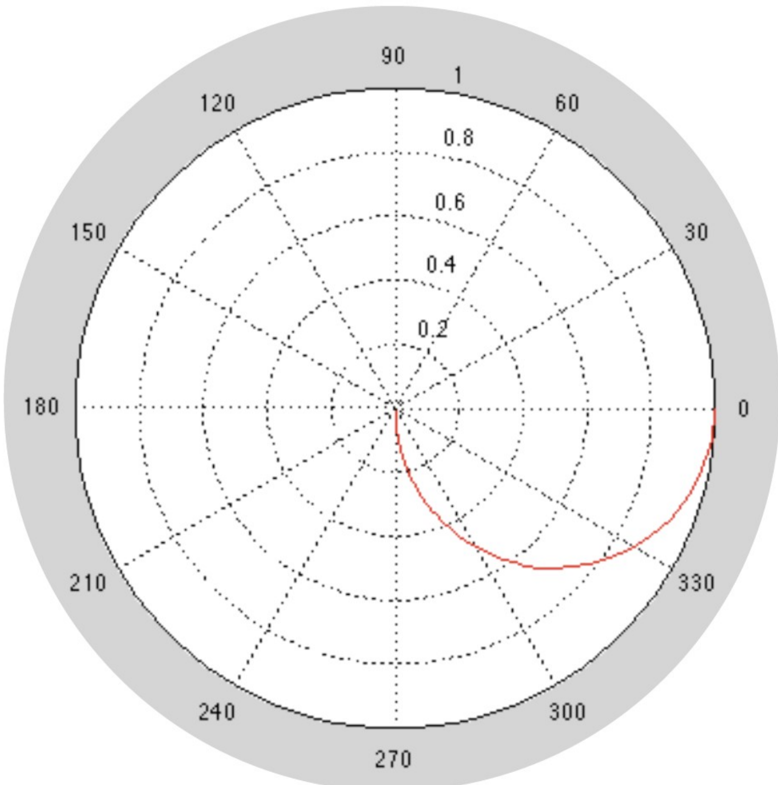
1. Nichols plot: $|G(j\omega)|$ and $\angle G(j\omega)$ are shown in rectangular coordinates as ω varies.

$$G(s)=\frac{1}{s+1}$$



2. Nyquist plot: $|G(j\omega)|$ and $\angle G(j\omega)$ are shown in polar coordinates as ω varies.

$$G(s)=\frac{1}{s+1}$$



3. <u>Bode plot</u>: $|G(j\omega)|$ and $\angle G(j\omega)$ are shown in separate parametric plots as ω varies.

decibels

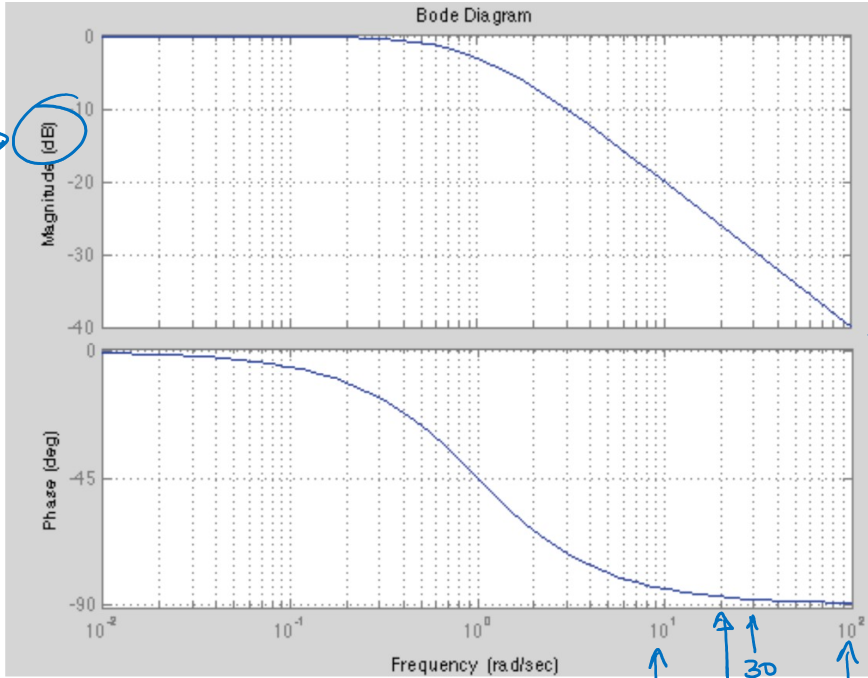
$G(s)=\dfrac{1}{s+1} \Rightarrow \dfrac{1}{j\omega+1}$

$\omega \to 0, \ |G(j\omega)| \to 1$

$\angle G(j\omega) \to 0$

$\omega \to \infty, \ |G(j\omega)| \to 0$

$\angle G(j\omega) \to -j = -90^\circ$



log freq. scale

$10^1 = 10$ 20 30 $10^2 = 100$

Bode diagrams

- A quick way to get approximate picture of amplitude and phase variation of a transfer function as a function of ω.
- Plot using logarithmic frequency scale.
- Magnitude plotted in decibels (dB).
  - Straightens commonly observed curves, easy sketching.
  - Enables superposition of multiple curves, since log(AB) = log(A) + log(B). $\Rightarrow \log(A/B) = \log A - \log B$
- For transfer function $H(j\omega)$
  - Magnitude $|H(j\omega)|$.
  - Magnitude in dB is $H_{dB} = 20\log_{10}|H(j\omega)|$.
  - $|H(j\omega)| = 10^{(H_{dB}/20)}$.

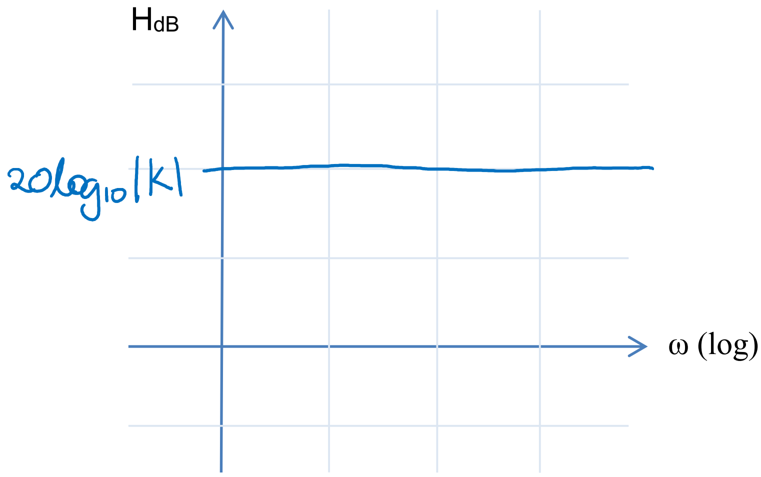
Values

$\lvert H(j\omega)\rvert$	H_{dB}
1	0
2	6.02
10	20

- An increase of $|H(j\omega)|$ by a factor of 10 leads to an increase in H_{dB} by 20 dB.

Multiplying by factor K in H(s)

- Horizontal straight line at $20\log_{10}|K|$ dB above (below if |K| < 1) abscissa.



Asymptotes

- Need to factor H(s) to show poles and zeros.

A simple zero

- Consider a zero at **s** = -a.
  - $H(s) = 1 + \frac{s}{a}$
  - $|H(j\omega)| = \left|1 + \frac{j\omega}{a}\right| = \sqrt{1 + \frac{\omega^2}{a^2}}$
  - $H_{dB} = 20\log_{10}\sqrt{1 + \frac{\omega^2}{a^2}}$
  - When ω « a, $H_{dB} \approx 20\log_{10} 1 = 0$
  - When ω » a, $H_{dB} \approx 20\log_{10}\left(\frac{\omega}{a}\right)$