UNIVERSITY OF CANTERBURY

Test

Prescription Number: **EMTH211-17S2**

Time allowed: 60 minutes.

Write your answers in the spaces provided.

There is a *total* of 40 points.

Use black or blue ink. Do not use pencil.

Only UC approved calculators are allowed.

There is no formula sheet for this test.

Show all working. Write neatly. Marks can be lost for poorly presented answers.

Family name:	
Given names:	
Student ID:	

MARKS Office Use Only	
Q1	
Q2	
Q3	
Q4	
Q5	
Q6	
Total	

Question 1 [7 points]

Let A be the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 1 & 2 & -1 & 2 \end{bmatrix}.$$

The reduced row echelon form for A is given by

$$RREF = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Give a basis for the row space of A.

(b) Give a basis for the column space of A

(c) What is the rank of A?

(d) What is the nullity of A?

(e) Give a formula that, for a general $m \times n$-matrix, relates its rank and its nullity.

(f) What is the nullity of A^T? *You should not calculate the null space of A^T in order to solve this question !*

Question 2 [7 points]

The matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 2 & 6 \\ -1 & 2 & 3 \end{bmatrix}$$

can be reduced to the echelon form

$$EF = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

by executing the row operations
$R_2 \to R_2 - 3R_1$
$R_3 \to R_3 + R_1$
$R_3 \to R_3 - R_2$.

(a) Write down the LU-decomposition for A. (*Hint: use the multiplier method)*

(b) Solve the system $A\mathbf{x} = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$ by using the LU decomposition for A.

Question 3 [6 points]

Let A be a 3×3 matrix with eigenvalues $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$. Let $\mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ be an eigenvector of A with associated eigenvalue 1, let $\mathbf{x}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ be an eigenvector of A with associated eigenvalue 2 and let $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ be an eigenvector of A with associated eigenvalue 3.

(a) Calculate $A^{2017}\mathbf{y}$, where $\mathbf{y} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$. *(You can of course leave expressions of the form k^l, where k is a number and l is a large number, in your answer)*

(b) Diagonalise A (i.e. write down matrices P and D such that $A = PDP^{-1}$)

Question 4 [8 points]

Suppose that the characteristic polynomial of a matrix B is given by

$$(2-\lambda)^2(3-\lambda)(1-\lambda).$$

(a) What are the dimensions of B? (i.e. B is a $\_\_\times\_\_$ matrix)

(b) List all eigenvalues of B and their algebraic multiplicities.

(c) What is the determinant of B?

(d) What is the trace of B?

(e) Is B invertible? Explain your answer.

(f) Suppose that B has an eigenspace of dimension 2. Explain why B is diagonalisable.

Question 5 [6 points]

Remember that a subspace of W of a vectorspace V is a set that is closed under taking linear combinations (That is if $u, v \in W$ and $k, l \in \mathbb{R}$ then $ku + lv \in W$).
Let P_2 be the vector space of all polynomials in the variable x of degree at most 2.

(a) Show that the set $\{1 - x, x^2 - 1, 3x\}$ spans P_2.

(b) Let W be the set of all polynomials of the form $a + bx^2$, where $a, b \in \mathbb{R}$. Show that W is a subspace of P_2.

(c) Why is the set $W' = \{a + x^2 | a \in \mathbb{R}\}$ not a subspace of P_2?

Question 6 [6 points]

Let $E = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$. The inverse of E is given by $E^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$.

(a) Calculate $\|E\|_1$, $\|E\|_\infty$ and the condition number $k(E)$ using the ∞-norm.

(b) Describe **briefly** how the condition number of a matrix E may affect the accuracy of a solution to $E\mathbf{x} = \mathbf{b}$. *A formula relating the condition number to the error of a solution might be relevant.*

Page for rough working

Page for rough working

Good luck!

END OF PAPER