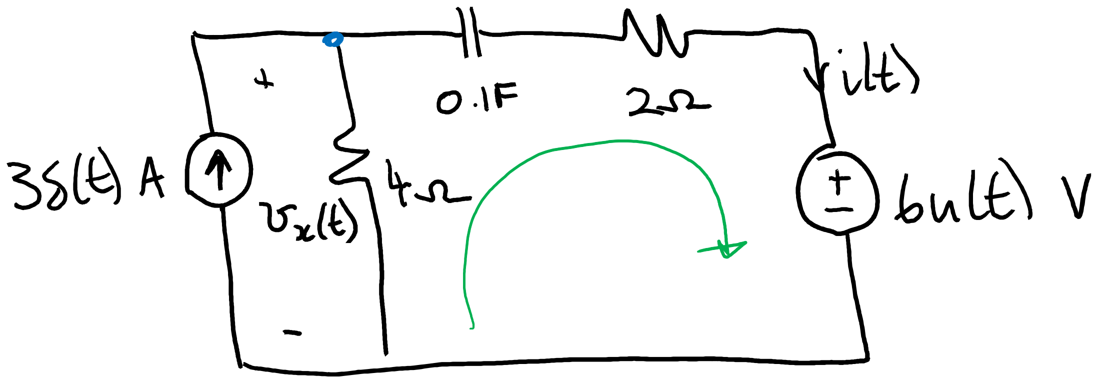
Example:

For the circuit below, write an equation with $i(t)$ as the only unknown. Take the LT, solve for **I(s)**, and then find $i(t)$ by taking the inverse LT. Assume the initial conditions are zero.



$$3\delta(t) = i(t) + \frac{v_x(t)}{4} \qquad (KCL \Rightarrow \Sigma I_{in} = \Sigma I_{out})$$

$$v_x(t) = 12\delta(t) - 4i(t) \qquad (1)$$

$$-v_x + \frac{1}{0.1}\int_{0^-}^{\infty} i(t)\,dt + 2i(t) + 6u(t) = 0 \quad (2) \qquad (KVL)$$

(1) into (2):

$$-12\delta(t) + 4i(t) + 10\int_{0^-}^{\infty} i(t)\,dt + 2i(t) + 6u(t) = 0$$

$$-12 + 4I(s) + \frac{10}{s}I(s) + 2I(s) + \frac{6}{s} = 0$$

$$I(s)\left(6 + \frac{10}{s}\right) = 12 - \frac{6}{s}$$

$$I(s) = \frac{\frac{12s-6}{s}}{\frac{6s+10}{s}}$$

$$= \frac{12s-6}{6s+10}$$

$$= \frac{12s-6}{6(s+10/6)}$$

$$= \frac{A}{6} + \frac{B}{s+10/6}$$

$$= \frac{A(s+10/6) + 6B}{6(s+10/6)}$$

$\therefore \; A(s + 10/6) + 6B = 12s - 6$

$s = -10/6$

$6B = \frac{-120}{6} - 6$

$= -26$

$B = -4.33$

$As = 12s$

$A = 12$

$I(s) = \frac{12}{6} - \frac{4.33}{s + 10/6}$

$= 2 - \frac{4.33}{s + 1.67}$

$i(t) = 2\delta(t) - 4.33e^{-1.67t}\,u(t)$ A

Z(s) and Y(s)

Readings: Section 14.7

Admittance and Impedance

Remember from the start of the term:

Impedance => voltage-current ratio

Admittance => current-voltage ratio

These definitions hold true in the **s**-domain too.

$$\mathbf{Z}(\mathbf{s}) = \frac{\mathbf{V}(\mathbf{s})}{\mathbf{I}(\mathbf{s})} \text{ and } \mathbf{Y}(\mathbf{s}) = \frac{\mathbf{I}(\mathbf{s})}{\mathbf{V}(\mathbf{s})} = \frac{1}{Z(s)}$$

Resistors

Very straightforward:

$$v(t) = i(t)R$$

$$V(s) = I(s)R$$

$$Z(s) = \frac{V(s)}{I(s)} = R \quad (\text{unit } \Omega)$$

$$Y(s) = \frac{I(s)}{V(s)} = \frac{1}{R} \quad (\text{unit } S)$$

Inductors

A little more complicated:

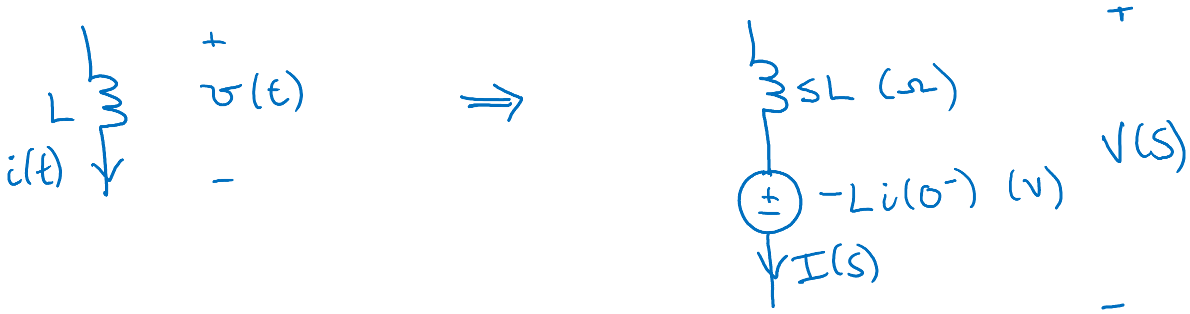
$$v(t) = L\, di/dt$$

$$V(s) = L\left(sI(s) - i(0^-)\right)$$

If $i(0^-) = 0 \Rightarrow V(s) = LsI(s)$

& $Z(s) = sL \quad (\Omega)$ $\quad (i(0^-) = 0A)$

We can now draw an inductor in the **s**-domain:



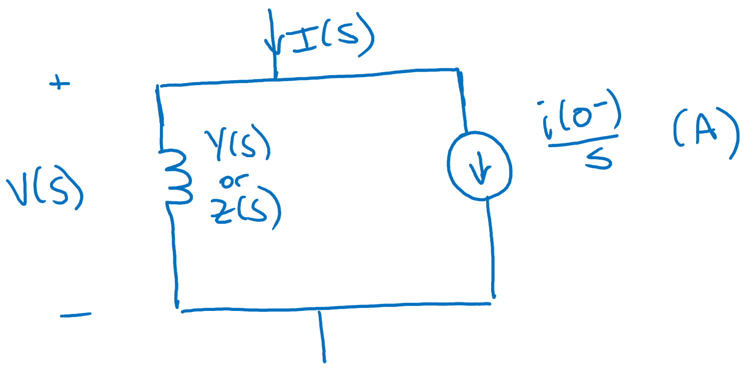
$\Rightarrow Li(0^-)$ will be a constant, which is why it can be modelled as a DC source.

Alternatively, if we want a current-based representation, we can rearrange our formula for **V(s)** to:

$$I(s) = \frac{V(s)}{sL} + \frac{i(0^-)}{s}$$

$$Y(s) = \frac{1}{sL} \ (S)$$

$$Z(s) = sL \ (\Omega)$$



Capacitors

We can do a similar analysis for capacitors:

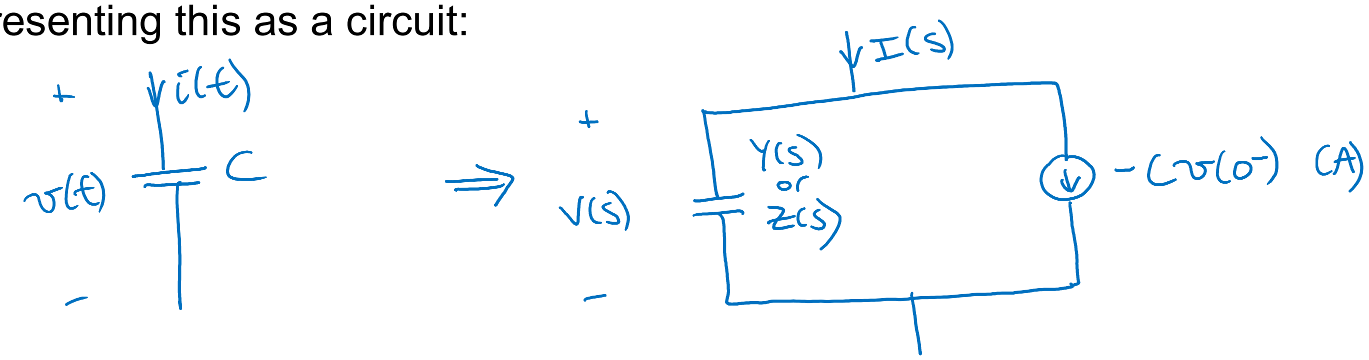
$$i(t) = C \, dv/dt$$

$$I(s) = C(sV(s) - v(0^-))$$

If $v(0^-) = 0 \Rightarrow I(s) = CsV(s)$

$$Z(s) = \frac{1}{sC} \ (\Omega) \qquad (v(0^-) = 0V)$$

Representing this as a circuit:



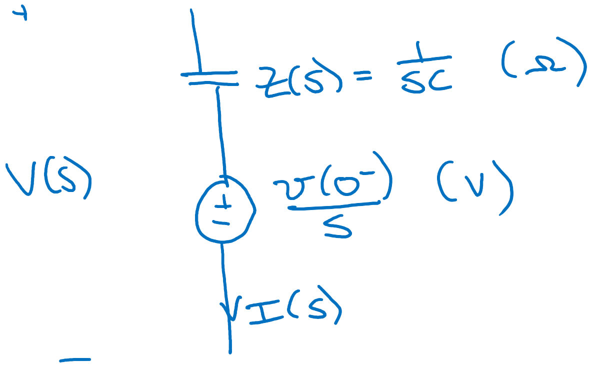
$$Y(s) = sC \ (S), \quad Z(s) = \frac{1}{sC} \ (\Omega)$$

Again, $\mathrm{C}v(0^{-})$ will be a constant, which is why we can represent it as a DC current source.

If we want a series representation, we can rearrange the equation to get:

$$V(s) = \frac{I(s) + Cv(0^-)}{sC}$$

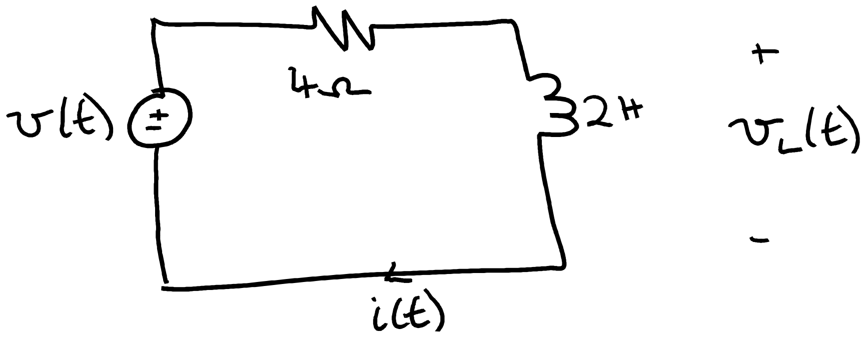
$$= \frac{I(s)}{sC} + \frac{v(0^-)}{s}$$



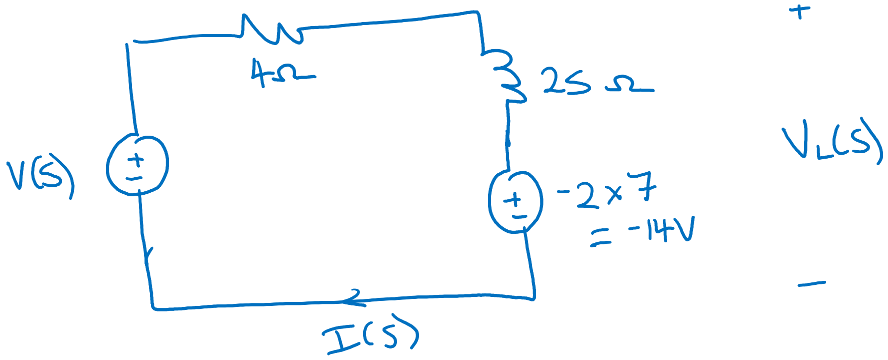
We use these circuit representations to make analysis in the **s**-domain easier. Now we redraw the circuit in the **s**-domain and then analyse it, rather than taking the LT of everything after writing an expression in the time-domain.

Example

For the circuit below, find $v_L(t)$ if $v(t) = 6u(t)$ V, and $i(0^-) = 7$ A.



Redraw cct in s-domain:



$$V(s) = \frac{6}{s} \quad V$$

$$-\frac{6}{s} + 4I(s) + 2s\,I(s) - 14 = 0 \qquad \text{(KVL)}$$

$$I(s)(4+2s) = 14 + \frac{6}{s}$$

$$I(s) = \frac{14s+6}{s(4+2s)}$$

$$V_L(s) = 2s\,I(s) - 14$$

$$= 2s\left(\frac{14s+6}{s(4+2s)}\right) - 14$$

$$= \frac{14s+6}{2+s} - 14$$

$$V_L(s) = \frac{14s + 6 - 28 - 14s}{2+s}$$

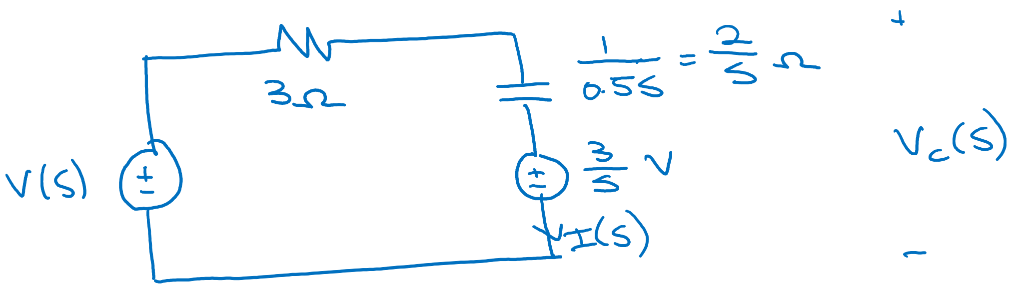
$$= \frac{-22}{s+2}$$

$$v_L(t) = -22e^{-2t}u(t) \text{ V}$$

Example

For the circuit below, what is $i(t)$ if $v(t) = 7e^{-3t}u(t)$ V and $v_c(0^-) = 3$ V?

⇓



$$V(s) = \frac{7}{s+3}$$

$$\frac{-7}{s+3} + 3I(s) + \frac{2}{s}I(s) + \frac{3}{s} = 0 \qquad \text{(KVL)}$$

$$I(s)\left(3 + \frac{2}{s}\right) = \frac{7}{s+3} - \frac{3}{s}$$

$$\left(\frac{3s+2}{s}\right)I(s) = \frac{7s - 3s - 9}{s(s+3)}$$

$$I(s) = \frac{s(4s-9)}{s(s+3)(3s+2)}$$

$$= \frac{4s-9}{(s+3)(3s+2)}$$

$$= \frac{3}{s+3} - \frac{5}{3s+2}$$

(need to do partial fractions)

$$i(t) = \left(3e^{-3t} - \frac{5}{3}e^{-2/3\, t}\right)u(t) \quad A$$