ENEL220 Circuits and Signals Term 3

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Office hours: Mon 1.30 – 4pm, Thurs 11am – 4pm (term time only). I will have a break for lunch on Thurs though ☺. Email me if you want to meet outside of this time.

Note that I am a contract lecturer, and not full time staff, so I am not usually at the university. I may be in my office outside my office hours, so feel free to come and check, but no guarantees!

References for these notes:

- W. H. Hayt, Jr., J. E Kemmerly and S. M. Durbin, "Engineering Circuit Analysis", 7th, 8th, or 9th Editions, McGraw-Hill.

Please reference the textbook directly rather than these notes.

I strongly recommend purchasing a copy of the text book. Either Edition is fine, although the readings in this study guide refer to the 9th edition. Second hand is a good option. Library also has copies.

ENEL220 Term 3 Checklist 2019

Chapter 10

By the end of the Chapter 10 notes you should be able to:

- ☐ Do complex algebra.
- ☐ Analyse a circuit using phasors.

Chapter 14

By the end of the Chapter 14 notes you should be able to:

- ☐ Take the Laplace Transform of a function.
- ☐ Take the Inverse Laplace Transform of a function.
- ☐ Analyse a circuit with a damped sinusoidal input, R, L, and C components using the LT
- ☐ Analyse a circuit in the s-domain using techniques already learnt (e.g. mesh analysis, Norton's theorem etc).
- ☐ Work out the transfer function H(s) of a circuit.
- ☐ Work out the poles and zeroes of a circuit.
- ☐ Explain what convolution is.
- ☐ Work out the output of a circuit using convolution and the impulse response.

Exam Content

Remember you can look up old exams on the UC library website. These are a very good guide to the type of questions you are likely to get! Basic things to remember:

- ☐ Always show <u>all</u> working, even if you're doing something in your head, or if you think it's obvious (for example, write "by inspection"). This makes it easy for me to give you carried error marks if you make a silly mistake.
- ☐ Always put units on your answers!

Exam Formulas for Term 3 Material

Laplace Transform Pairs

$\mathbf{f(t)} = \boldsymbol{\mathcal{L}^{-1}}\{\mathbf{F(s)}\}$	$\mathbf{F(s)} = \boldsymbol{\mathcal{L}}\{\mathbf{f(t)}\}$	$\mathbf{f(t)} = \boldsymbol{\mathcal{L}^{-1}}\{\mathbf{F(s)}\}$	$\mathbf{F(s)} = \boldsymbol{\mathcal{L}}\{\mathbf{f(t)}\}$
$\delta(t)$	1	$\frac{1}{\beta-\alpha}\left(e^{-\alpha t} - e^{-\beta t}\right)u(t)$	$\frac{1}{(s+\alpha)(s+\beta)}$
$u(t)$	$\frac{1}{s}$	$\sin \omega t\, u(t)$	$\frac{\omega}{s^2+\omega^2}$
$tu(t)$	$\frac{1}{s^2}$	$\cos \omega t\, u(t)$	$\frac{s}{s^2+\omega^2}$
$\frac{t^{n-1}}{(n-1)!}u(t)$, n = 1,2,…	$\frac{1}{s^n}$	$\sin(\omega t+\theta)u(t)$	$\frac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\cos(\omega t+\theta)u(t)$	$\frac{s\cos\theta-\omega\sin\theta}{s^2+\omega^2}$
$te^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^2}$	$e^{-\alpha t}\sin \omega t\, u(t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$, n = 1,2,…	$\frac{1}{(s+\alpha)^n}$	$e^{-\alpha t}\cos \omega t\, u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

Laplace Transform Operations

Operation	$\mathbf{f(t)}$	$\mathbf{F(s)}$
Addition	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
Scalar Multiplication	$kf(t)$	$kF(s)$
Time Differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
Time Integration	$\int_{0^-}^{t} f(t)\,dt$	$\frac{1}{s}F(s)$
	$\int_{-\infty}^{t} f(t)\,dt$	$\frac{1}{s}F(s) + \frac{1}{s}\int_{-\infty}^{0^-} f(t)\,dt$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Time Shift	$f(t-a)u(t-a), a \geq 0$	$e^{-as}F(s)$
Frequency Shift	$f(t)e^{-at}$	$F(s+a)$
Frequency Differentiation	$-tf(t)$	$\frac{dF(s)}{ds}$
Frequency Integration	$\frac{f(t)}{t}$	$\int_{s}^{\infty} F(s)\,ds$
Scaling	$f(at), a \geq 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Initial Value	$f(0^+)$	$\lim_{s\to\infty} sF(s)$
Final Value	$f(\infty)$	$\lim_{s\to 0} sF(s)$ All poles of $sF(s)$ in LHP
Time Periodicity	$f(t) = f(f + nT)$, n = 1,2,…	$\frac{1}{1-e^{-Ts}}F_1(s)$ Where $F_1(s) = \int_{0^-}^{T} f(t)e^{-st}\,dt$

Complex Numbers; The Phasor; Impedance and Admittance

Readings: Appendix 5, Sections 10.4, 10.5

Complex Numbers Representations

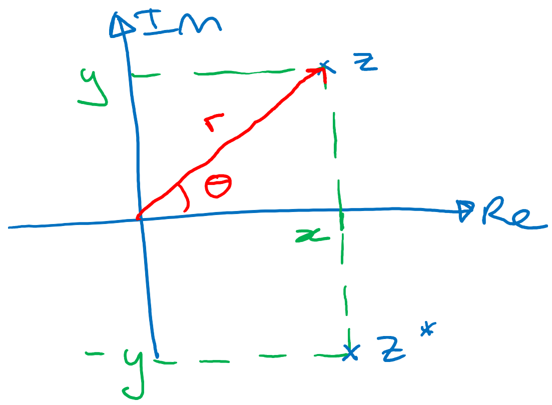
A complex number in Cartesian form is represented as $z = x + jy$.

Where $x = \mathrm{Re}\{z\}$, $y = \mathrm{Im}\{z\}$, and $j = \sqrt{-1}$; x, y are real numbers.

Real part of z   Imaginary part of z

The complex conjugate of z is $z^* = x - jy$. The sign for the imaginary part of the number changes, but the real part stays the same.

Complex numbers can be represented on Real/Imaginary axes - the Complex Plane. The terms r and θ are used if representing a complex number in Polar form ($z = re^{j\theta}$).



Maths with Complex Numbers

Addition and subtraction is most easily done in Cartesian form (simply add/subtract the real components and the imaginary components). For multiplication and division, it depends if you want to end up with a Polar or Cartesian answer.

Multiplication: $z_1 = r_1 e^{j\theta_1}$ & $z_2 = r_2 e^{j\theta_2}$

$$z_1 z_2 = r_1 r_2 e^{j(\theta_1+\theta_2)}$$

or $$z_1 z_2 = (x_1 + jy_1)(x_2 + jy_2) = x_1 x_2 + jx_2 y_1 + jx_1 y_2 - y_1 y_2$$
$$= (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1)$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{j(\theta_1-\theta_2)}$$

or $$\frac{z_1}{z_2} = \frac{x_1 + jy_1}{x_2 + jy_2} \times \frac{x_2 - jy_2}{x_2 - jy_2}$$
$$= \frac{x_1 x_2 + j(x_2 y_1 - x_1 y_2) - j^2 y_1 y_2}{x_2^2 + j(x_2 y_2 - x_2 y_2) - j^2 y_2^2}$$
$$= \frac{(x_1 x_2 + y_1 y_2) - j(x_1 y_2 - x_2 y_1)}{x_2^2 + y_2^2}$$

It is best to swap between forms using the appropriate functions on your calculator! You can do it using trigonometry, but you will need to be careful to get the angle correct.

Example:

Multiply $z_1 = 3 + j2$ and $z_2 = 8 - j7$ in both Cartesian and Polar forms.

$$z_1 z_2 = (3 + j2)(8 - j7)$$
$$= 24 - j21 + j16 - j^2 14$$
$$= 24 + 14 - j5$$
$$= 38 - j5$$

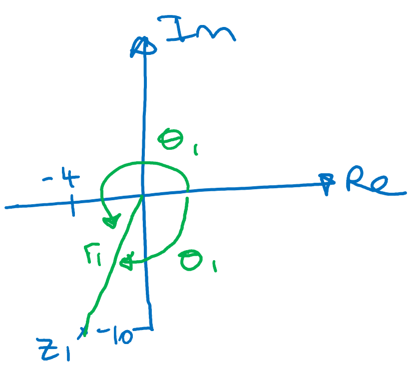
$z_1 = 3.6e^{j0.6}$, $z_2 = 10.6e^{-j0.7}$

$$z_1 z_2 = (3.6 \times 10.6)e^{j(0.6-0.7)}$$
$$= 38e^{-j0.1}$$
$$= 38 - j5$$

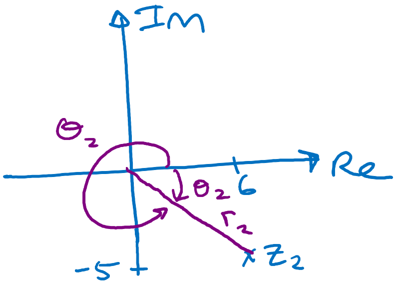
Example:

Work out $\frac{z_1}{z_2}$ if $z_1 = -4 - j10$ and $z_2 = 6 - j5$. First use the Cartesian form and the complex conjugate. Then use the Polar form, and do the conversion to Polar form using trigonometry.

$$\frac{z_1}{z_2} = \frac{-4-j10}{6-j5} \times \frac{6+j5}{6+j5}$$
$$= \frac{-24-j20-j60-j^2 50}{36-j^2 25}$$
$$= \frac{26-j80}{61}$$
$$= 0.43 - j1.31$$



$$r_1 = \sqrt{4^2 + 10^2}$$
$$= 10.77$$
$$\theta_1 = \tan^{-1}(10/4) + \pi$$
$$= 4.33$$
$$z_1 = 10.77e^{j4.33}$$



$$r_2 = \sqrt{5^2 + 6^2}$$
$$= 7.81$$
$$\theta_2 = \tan^{-1}\left(-5/6\right)$$
$$= -0.69$$
$$z_2 = 7.81 e^{-j0.69}$$

$$\frac{z_1}{z_2} = \frac{10.77}{7.81} e^{j(4.33 - -0.69)}$$
$$= 1.38 e^{j5.02}$$
$$= 0.43 - j1.31$$ (using func. on calculator)

Notation

1) We use j for imaginary numbers in electrical engineering so we don't get confused with the current (i).

2) The Polar form can also be written as:

$$z = r(\cos\theta + j\sin\theta)$$

(Remember Euler's formula: $e^{j\theta} = \cos\theta + j\sin\theta$)

3) Sometimes you will see cis (cos i sine) notation, although not in this course.

$$z = r \operatorname{cis} \theta$$

4) We will use Angular or Phasor notation a lot:

$$z = r\angle\theta$$

Angular notation is quick and easy to write, but is <u>not</u> mathematically equal to the other Polar forms (see following section). It can be useful to change between different Polar forms – you will possibly come across this in other courses.

Phasor Representation

In Term 2 we mainly looked at circuits with DC sources. Phasors are useful for solving circuits with AC sources in the time domain.

If we have a source $v(t) = 5\cos(3t + 10°)$ then simply using the techniques from last term will quickly get fairly complicated. Using phasors simplifies things.

Using Euler's identity:

$$\cos(\omega t) = Re\{e^{j\omega t}\}$$

$$V_m \cos(\omega t + \phi) = Re\{V_m e^{j(\omega t + \phi)}\}$$

$$\therefore v(t) = 5\cos(3t + 10°) = Re\{5e^{j(3t+10°)}\}$$

We simplify further by representing the signal as a complex quantity. We do this by adding an imaginary component to the signal – as this doesn't affect the real component (which is what we care about), then this isn't a problem.

$$v(t) = 5e^{j(3t+10°)}$$

Finally, we supress the $e^{j\omega t}$ factor to write in Phasor form ($\mathbf{V}$) (a frequency-domain representation):

$$\bar{V} = 5e^{j10°}$$

$$= 5\angle 10°$$

To convert back to a time-domain representation, $v(t)$, we need to know ω. We can supress it while doing the calculations, as capacitors, inductors, and resistors have no effect on it.

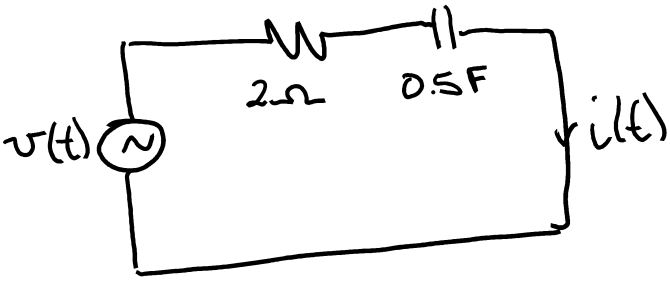
Integrating and Differentiating a Phasor

To integrate a phasor, you divide by $j\omega$. To differentiate you multiply by $j\omega$. If interested in the proof, look in the textbook.

Example:

ω

If $\mathrm{v(t)} = 5\cos(3\mathrm{t} + 10°)$ V, what is $\mathrm{i(t)}$ in the circuit below? Assume there is only a steady-state (forced) response.



Using KVL: $v(t) = 2i(t) + \frac{1}{0.5}\int i(t)\,dt$

Converting to phasors:

$\bar{V} = 5\angle 10°$ $\quad \bar{I} = I_m \angle \phi$

$5\angle 10° = 2\bar{I} + 2\int \bar{I}\,dt$

$= 2\bar{I} + \frac{2}{3j}\bar{I}$

integrating a phasor = divide by $j\omega$

$\bar{I} = \frac{5\angle 10°}{2 + \frac{2}{3j}}$

$= \frac{5\angle 10° \times 3j}{6j + 2}$

$= \frac{5\angle 10° \times 3\angle 90°}{6.32\angle 71.6°}$

$= 2.37\angle 28.4°$

$i(t) = 2.37\cos(3t + 28.4°)$ A