<u>Question 1: Phasors</u> [9 Marks]

Consider the circuit shown in Figure Q1 and then answer the questions below.

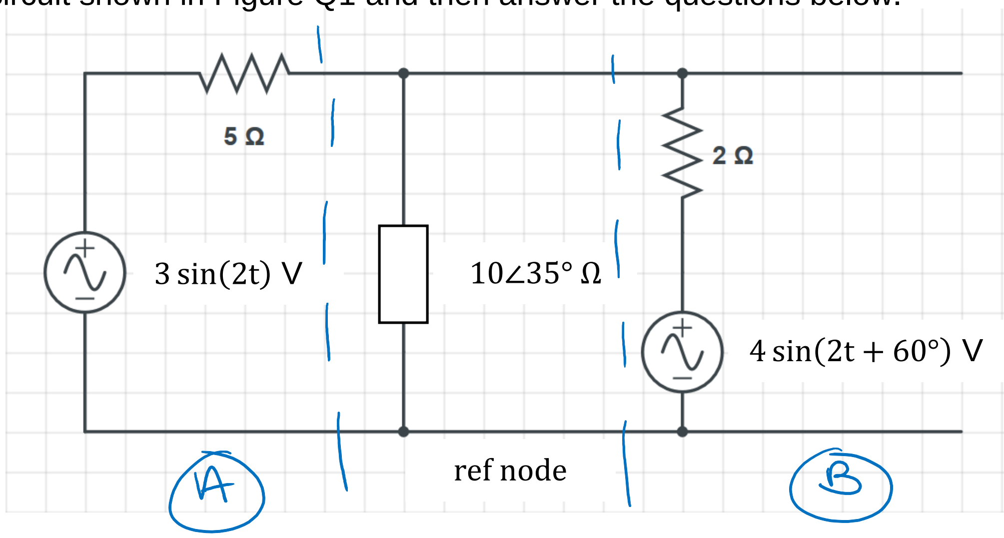
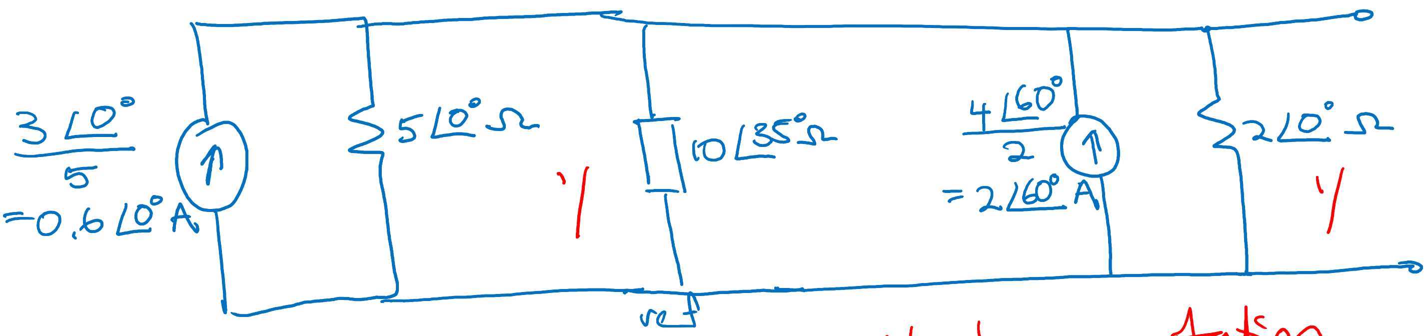


Figure Q1: Circuit diagram for Q1.

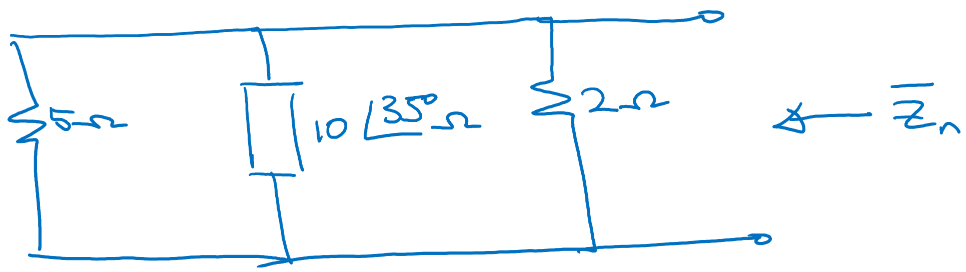
a) Perform a source transformation on both voltage sources, and draw the resulting circuit. Put all values into phasor notation (i.e. r∠θ).



✓ phasor notation

b) Determine the Norton equivalent current, $\bar{I}_N$, and impedance, $\bar{Z}_N$. Draw and label all circuits used for the calculations, as well as the Norton equivalent circuit. Utilise rectangular and polar co-ordinates as required, showing all working. Put the final answers in phasor format (i.e. r∠θ).

$\bar{Z}_n$



$$\overline{Y}_n = \frac{1}{5} + \frac{1}{10\angle 35^\circ} + \frac{1}{2}$$

$$= \frac{7}{10} + \frac{1}{10}\angle -35^\circ$$

$$= 0.7 + 0.082 - j0.057$$

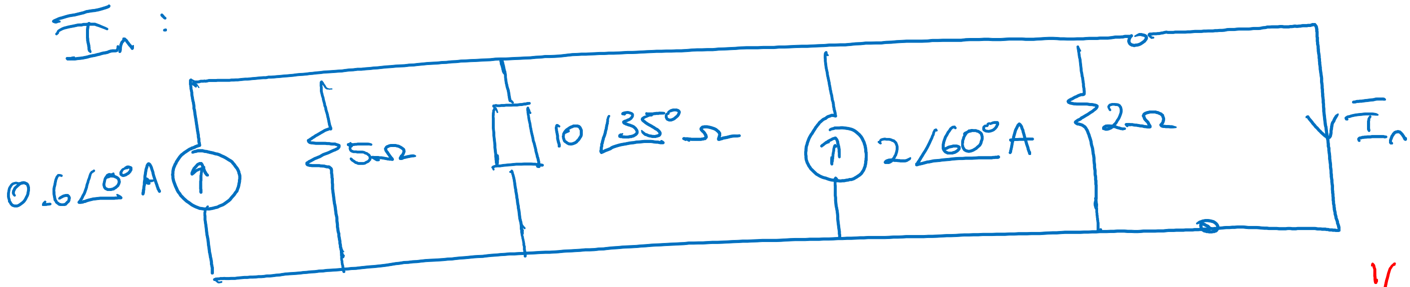
$$= 0.782 - j0.057$$

$$= 0.78\angle -4.2^\circ$$

$$\overline{Z}_n = \frac{1}{\overline{Y}_n} = \frac{1}{0.78\angle -4.2^\circ}$$

$$= 1.28\angle 4.2^\circ\ \Omega$$

$\overline{I}_n$:



All current goes through s.c.

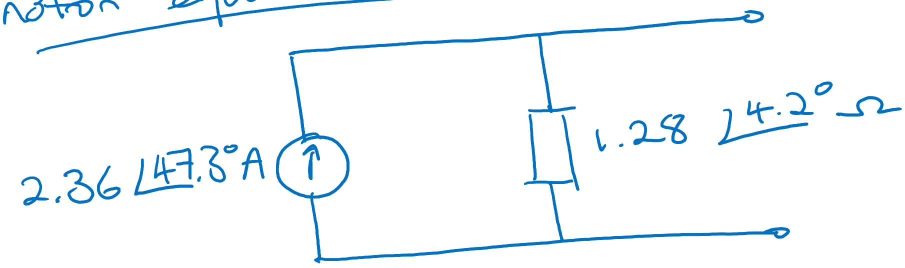
$$\therefore \overline{I}_n = 0.6\angle 0^\circ + 2\angle 60^\circ$$

$$= 0.6 + 1 + j\sqrt{3}$$

$$= 1.6 + j\sqrt{3}$$

$$= 2.36\angle 47.3^\circ\ A$$

notron equiv. circuit



Question 2: Laplace Transforms [10 Marks]

Consider the circuit shown in Figure Q2 and then answer the questions below.

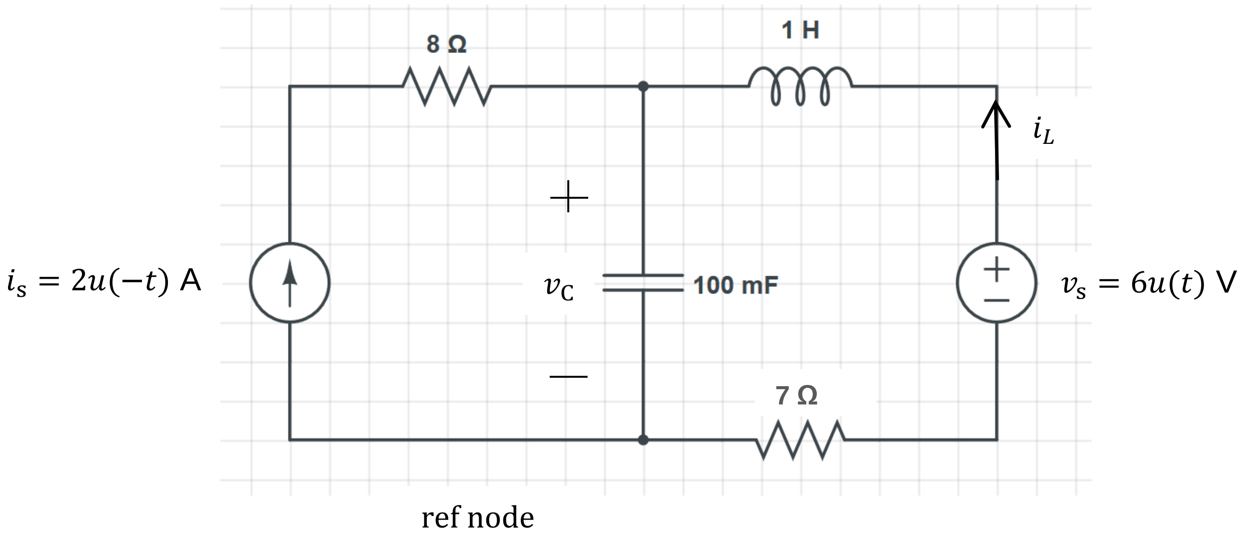
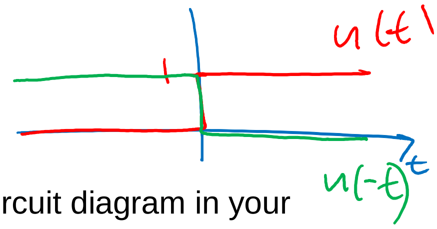
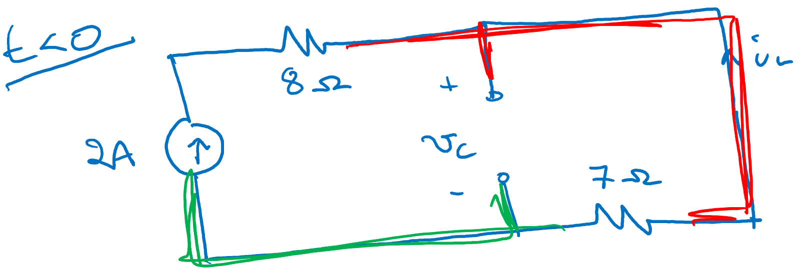


Figure Q2: Circuit diagram for Q2.



a) Find the initial conditions $v_c(0^-)$ and $i_L(0^-)$. Include a labelled circuit diagram in your answer.



1/

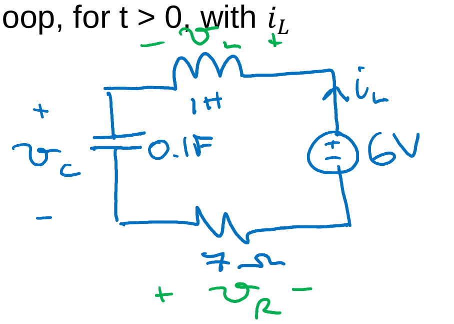
$i_L(0^-) = -2A$ by inspection

$v_c(0^-) = 2 \times 7$

$= 14V$

1/

b) Write the time-domain mesh analysis equation for the right hand loop, for t > 0, with i_L as the only unknown. The following equations may be useful:



$$v_c(\mathrm{t}) = \frac{1}{C}\int_{\mathrm{t_0}}^{\mathrm{t}} i(\mathrm{T})\,\mathrm{dT} + v_\mathrm{c}(\mathrm{t_0})$$

$$v_L(\mathrm{t}) = \mathrm{L}\frac{\mathrm{d}i}{\mathrm{dt}}$$

going anticlockwise:

$$7i_L - 6 + 1\frac{di_L}{dt} + \frac{1}{0.1}\int_0^t i_L(\tau)\,d\tau + 14 = 0 \quad \text{(use PSC)}$$

$$7i_L + \frac{di_L}{dt} + 10\int_0^t i_L(\tau)\,d\tau + 8 = 0$$

2/

c) From your answer for part (b), develop an expression for $\mathbf{I_L}(\mathbf{s})$.

$$\underset{1/2}{7I_L(s)} + \underset{1/2}{\underbrace{sI_L(s) - i_L(0^-)}} + \underset{1/2}{\frac{10}{s}I_L(s)} + \underset{1/2}{\frac{8}{s}} = 0$$

$$I_L(s)\left(7 + s + \frac{10}{s}\right) = i_L(0^-) - \frac{8}{s}$$

$$I_L(s)\left(\frac{7s + s^2 + 10}{\cancel{s}}\right) = -2 - \frac{8}{s}$$

$$= \frac{-2s - 8}{\cancel{s}}$$

$$I_L(s) = \frac{-2s - 8}{s^2 + 7s + 10} \qquad 1$$

d) Derive an expression for $i_L(\mathrm{t})$ using your answer from part (c), partial fractions, and the inverse Laplace Transform.

$$I_L(s) = \frac{-2s - 8}{(s+5)(s+2)}$$

$$= \frac{A}{s+5} + \frac{B}{s+2} \qquad 1$$

$$A(s+2) + B(s+5) = -2s - 8$$

$s = -2$

$3B = 4 - 8$

$B = -4/3$

$s = -5$

$-3A = 10 - 8$

$A = -2/3 \qquad 1$

$$I_L(s) = \frac{-2/3}{s+5} - \frac{4/3}{s+2}$$

$$i_L(t) = \left(-\frac{2}{3}e^{-5t} - \frac{4}{3}e^{-2t}\right)u(t)\ \text{A} \qquad 1$$