Term 3 Additional Tutorial Questions

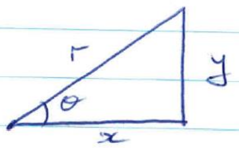
These are tutorial questions that I wrote when I first lectured Term 3. I have since converted to questions from the textbook to be consistent with the other terms. However, the below could be useful for your exam study ☺

1. Complex Numbers

   Let $z_1 = 10 + j15$ and $z_2 = -4 + j3$

   a) Convert z_1 and z_2 to polar notation $z = re^{j\theta}$ using Pythagoras and trigonometry (i.e. not just the calculator function to convert between forms). θ is to be in radians, not degrees.

a)



$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}(y/x)$

$z = x + jy$
$= re^{j\theta}$

$z_1 \Rightarrow r_1 = \sqrt{10^2 + 15^2}$
$= \sqrt{325}$
$= 18.03$

$\theta_1 = \tan^{-1}(15/10)$
$= \tan^{-1}(1.5)$
$= 0.98$ rad

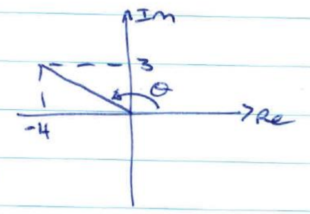
angle OK? Y

$\underline{z_1 = 10 + j15 = 18.03e^{j0.98}}$

$z_2 \Rightarrow r_2 = \sqrt{(-4)^2 + 3^2}$
$= \sqrt{25}$
$= 5$

$\theta_2 = \tan^{-1}(3/-4)$
$= \tan^{-1}(-0.75)$
$= -0.64$ rad

angle OK? N



$\theta_2 = -0.64 + \pi$
$= 2.50$ rad

$\underline{z_2 = -4 + j3 = 5e^{j2.5}}$

b) Work out z_1/z_2 and $z_1 - z_2$

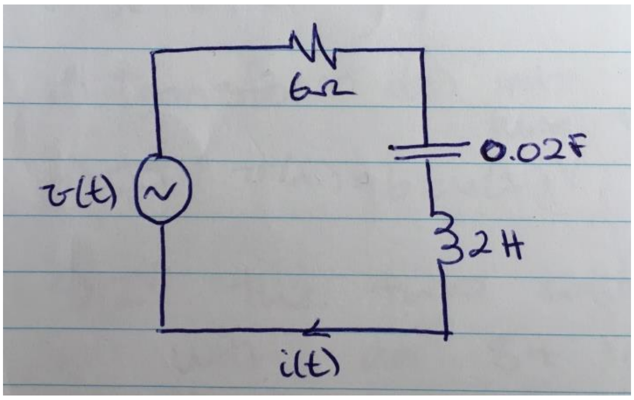
$$z_1/z_2 = \frac{18.03e^{j0.98}}{5e^{j2.5}} = \underline{3.606e^{-j1.52}}$$

$$z_1 - z_2 = 10 + j15 - (-4 + j3) = \underline{14 + j12}$$

c) Let $z_3 = r_3 e^{j\theta_3}$ and $z_4 = r_4 e^{j\theta_4}$. Show that $z_3 z_4 = r_3 r_4 e^{j(\theta_3+\theta_4)}$

$$\begin{aligned} z_3 z_4 &= (r_3 e^{j\theta_3})(r_4 e^{j\theta_4}) \\ &= r_3 r_4 e^{j\theta_3} e^{j\theta_4} \\ &= r_3 r_4 e^{j\theta_3 + j\theta_4} \\ &= \underline{r_3 r_4 e^{j(\theta_3+\theta_4)}} \end{aligned}$$

2. <u>Phasors</u>



For the circuit above, if $v(t) = 10\cos(7t + 30°)\,V$, what is $i(t)$? Use phasors to calculate the answer. Ignore initial conditions.

$\bar{V} = 10\,\angle 30°$

$$v(t) = 6\,i(t) + \frac{1}{0.02}\int i(t)\,dt + 2\frac{di(t)}{dt}$$

$$\bar{V} = 6\bar{I} + 50\int \bar{I}\,dt + 2\,d\bar{I}/dt$$

$$= 6\bar{I} + \frac{50}{7j}\bar{I} + 2\times 7j\,\bar{I}$$

$$\bar{I} = \frac{\bar{V}}{6 + 50/7j + 14j}$$

$$= \frac{10\,\angle 30° \times 7j}{42j + 50 + 98j^2}$$

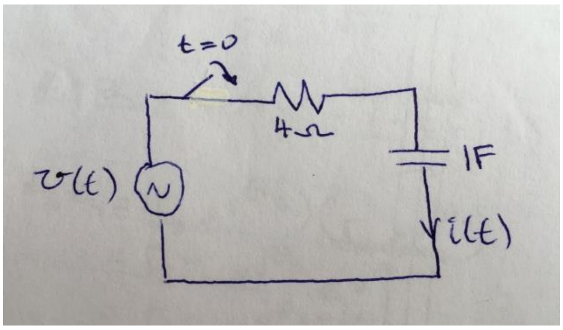
$$= \frac{10\,\angle 30° \times 7\,\angle 90°}{-48 + 42j}$$

$$= \frac{70\,\angle 120°}{63.78\,\angle 138.8°}$$

$$= 1.1\,\angle -18.8°$$

$$i(t) = 1.1\cos(7t - 18.8°)\,A$$

3. <u>Phasors</u>



For the circuit to the left, $v(0^-) = 0V$ and $v(t) = 20\cos(2t + \pi)\,V$. What is $i(t)$?

(See hints on the following page.)

Hints:

- Write an equation for $v(t)$ using KVL, and write equations for **V** and **I**
- Replace the time-domain functions in the first equation with phasors
- Rearrange so have **I =**
- Simplify
- Convert phasors back into time-domain variables

$$v(t) = 4i(t) + 1\int_{t_0}^{t} i(t')\,dt + v(t_0)$$

$$\bar{V} = 20\angle\pi \qquad \bar{I} = I_m\angle\phi$$

$$\bar{V} = 4\bar{I} + \int \bar{I}\,dt$$

$$20\angle\pi = 4\bar{I} + \frac{1}{2j}\bar{I}$$

$$\bar{I} = \frac{20\angle\pi}{4 + \frac{1}{2j}}$$

$$= \frac{20\angle\pi + 2j}{8j + 1}$$

$$= \frac{20\angle\pi + 2\angle\pi/2}{8.06\angle 1.44}$$

$$= 4.96\angle 3.26$$

$$i(t) = 4.96\cos(2t + 3.26)\ A$$

4. Laplace Transform

a) Using the one-sided LT formula, find the LT of $f(t) = 2u(t-6) + e^{-4t}$

$$F(s) = \int_{0^-}^{\infty} e^{-st} f(t)\,dt$$

$$= \int_{0^-}^{\infty} e^{-st} (2u(t-6) + e^{-4t})\,dt$$

$$= \int_{0^-}^{\infty} e^{-st} (2u(t-6))\,dt + \int_{0^-}^{\infty} e^{-st} e^{-4t}\,dt$$

$$= \int_{6}^{\infty} 2e^{-st}\,dt + \int_{0^-}^{\infty} e^{-(s+4)t}\,dt$$

$$= 2\left[-\frac{1}{s}e^{-st}\right]_6^{\infty} + \left[-\frac{1}{s+4}e^{-(s+4)t}\right]_{0^-}^{\infty}$$

$$= 2\left[0-\left(-\frac{1}{s}e^{-6s}\right)\right] + \left[0-\left(\frac{-1}{s+4}\right)\right]$$

$$= \frac{2}{s}e^{-6s} + \frac{1}{s+4}$$

b) Using the tables, find the LT of $f(t) = 7\delta(t) + 9tu(t)$

$$F(s) = 7 + 9/s^2$$

c) For the circuit in Q2, write an equation for I(s) if $v(t) = tu(t)$ and $i(0) = 0A$. Write it in the form $I(s) = \frac{x+ys+zs^2}{a+bs+cs^2}$. Note some of the coefficients could equal zero.

$V(s) = 1/s^2$

$$v(t) = 6i(t) + 50\int i(t)\,dt + 2\,di(t)/dt$$

$$V(s) = 6I(s) + 50\left(\frac{1}{s}I(s)\right) + 2(sI(s) - i(0^-))$$

$$I(s) = \frac{1/s^2}{6 + 50/s + 2s}$$

$$= \frac{1}{2s^3+6s^2+50s} \quad \text{OR} \quad \frac{1}{s(2s^2+6s+50)}$$

5. Laplace Transform

Show that $\mathcal{L}[\sin(\omega t)\, u(t)] = \frac{\omega}{s^2+\omega^2}$ by using $\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$. Do not use the tables.

$$\mathcal{L}[\sin(\omega t)u(t)] = \int_{0^-}^{\infty} e^{-st} \frac{e^{j\omega t} - e^{-j\omega t}}{2j}\, dt$$

$$= \frac{1}{2j} \int_{0^-}^{\infty} e^{-(s-j\omega)t} - e^{-(s+j\omega)t}\, dt$$

$$= \frac{1}{2j} \left[\frac{1}{-(s-j\omega)} e^{-(s-j\omega)t} - \frac{1}{-(s+j\omega)} e^{-(s+j\omega)t} \right]_{0^-}^{\infty}$$

$$= \frac{1}{2j} \left[0 - \left(\frac{-1}{s-j\omega} + \frac{1}{s+j\omega} \right) \right]$$

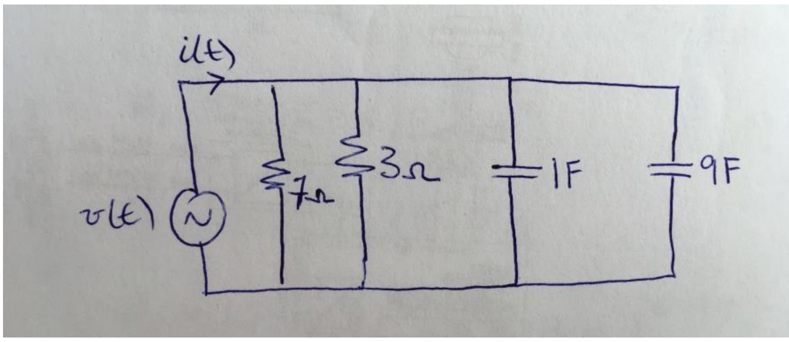
$$= \frac{1}{2j} \left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega} \right]$$

$$= \frac{1}{2j} \left[\frac{(s+j\omega) - (s-j\omega)}{s^2 - j^2\omega^2} \right]$$

$$= \frac{1}{\cancel{2j}} \left(\frac{\cancel{2j}\omega}{s^2+\omega^2} \right)$$

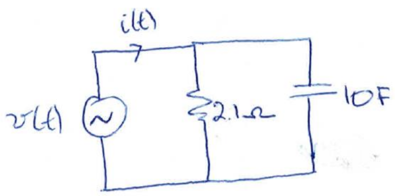
$$= \frac{\omega}{s^2+\omega^2}$$

6. Laplace Transform and Inverse Laplace Transform



For the circuit above, all initial conditions = 0.

a) Simplify the circuit so there is only one capacitor and one resistor.



$R_{eq} = \left(\frac{1}{7} + \frac{1}{3}\right)^{-1} = 2.1\Omega$ OR $R_{eq} = \frac{3 \times 7}{3+7} = 2.1\Omega$

$C_{eq} = 1 + 9 = 10F$

b) Write an equation for $i(t)$ using KCL.

$$i(t) = \frac{v(t)}{2.1} + 10\frac{dv(t)}{dt}$$

c) Transform $i(t)$ into the s-domain (i.e. find I(s)). Use the tables for this.

$$I(s) = \frac{1}{2.1}V(s) + 10(sV(s) - v(0^-))$$

d) If $v(t) = 6tu(t)V$ find $i(t)$ from I(s) for t > 0.

$$V(s) = \frac{6}{s^2}$$

$$I(s) = \frac{1}{2.1}\left(\frac{6}{s^2}\right) + 10s\left(\frac{6}{s^2}\right) - 0$$

$$= \frac{6}{2.1s^2} + \frac{60}{s}$$

$$i(t) = 2.86t\,u(t) + 60u(t)\text{ A}$$

e) If the two capacitors were replaced with an 8H inductor, what would $i(t)$ be? You will need to repeat the above process. Remember for an inductor $i(t) = \frac{1}{L}\int_{t_0}^{t} v(t')dt' + i(t_0)$

$$i(t) = \frac{v(t)}{2.1} + \frac{1}{8}\int_{t_0}^{t} v(t')\,dt' + i(t_0)$$

$$I(s) = \frac{V(s)}{2.1} + \frac{1}{8}\left(\frac{1}{s}V(s)\right) \quad \text{(we know } i(t_0) = 0 \text{ as } v(t) = 6t\,u(t))$$

$$= \frac{6}{2.1s^2} + \frac{6}{8s} \times \frac{1}{s^2}$$

$$i(t) = 2.86t\,u(t) + 0.75\frac{t^2}{2}u(t)$$

$$= 2.86t\,u(t) + 0.375t^2u(t)\text{ A}$$

7. Inverse Laplace Transform

a) Using partial fractions, find the inverse LT of $I(s) = \frac{7}{s^2+s-12}$

$$I(s) = \frac{7}{s^2+s-12}$$

$$= \frac{7}{(s+4)(s-3)}$$

$$= \frac{A}{s+4} + \frac{B}{s-3}$$

$$= \frac{-1}{s+4} + \frac{1}{s-3}$$

$$i(t) = (-e^{-4t} + e^{3t})u(t)$$

$$As - 3A + Bs + 4B = 7$$

$$s(A+B) + (4B-3A) = 7$$

$$A+B=0 \qquad 4B-3A=7$$

$$A=-B \qquad 4B+3B=7$$

$$A=-1 \qquad B=1$$

b) Find the simplified equation for $V(s) = \frac{s+2}{(s^2+4)(s+1)}$ using partial fractions

$$\text{b)}\quad V(s) = \frac{s+2}{(s^2+4)(s+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+4} = \frac{1/5}{s+1} + \frac{(-1/5)s + 6/5}{s^2+4}$$

$$As^2 + 4A + Bs^2 + Cs + Bs + C = s+2$$
$$(A+B)s^2 + s(B+C) + (4A+C) = s+2$$

$$A+B=0 \qquad B+C=1$$
$$A=-B \qquad C=1-B$$

$$4A+C=2$$
$$-4B+C=2$$
$$-4B+1-B=2$$
$$-5B=1$$
$$B=-1/5$$

$$A=\frac{1}{5} \qquad C=\frac{6}{5}$$

8. <u>Laplace Transform and Inverse Laplace Transform</u>

a) If $v(t) = \delta(t-3)u(t-3) + 8e^{-(t-4)}u(t-4)$ V, find $V(s)$. Hint: Use the tables and time delay

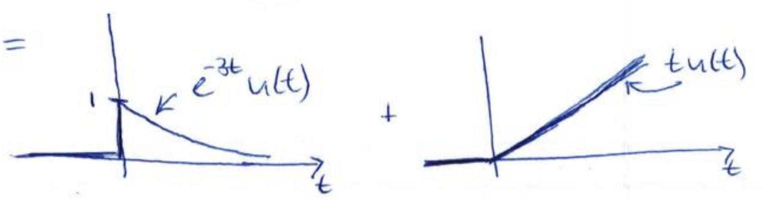
$$V(s) = e^{-3s} + \frac{8e^{-4s}}{s+1}$$

b) Using the IVT, find $v(0^+)$ of $v(t) = (e^{-3t} + t)u(t)$ V

$$v(t) = (e^{-3t} + t)\,u(t)$$
$$V(s) = \frac{1}{s+3} + \frac{1}{s^2}$$
$$v(0^+) = \lim_{s\to\infty}\left[s\left(\frac{1}{s+3} + \frac{1}{s^2}\right)\right]$$
$$= \lim_{s\to\infty}\left[\frac{s}{s+3} + \frac{1}{s}\right]$$
$$= \lim_{s\to\infty}\left[\frac{1}{1} + \frac{0}{1}\right] \quad \text{(l'hôpital's rule)}$$
$$= 1$$

Does this make sense? (Y)

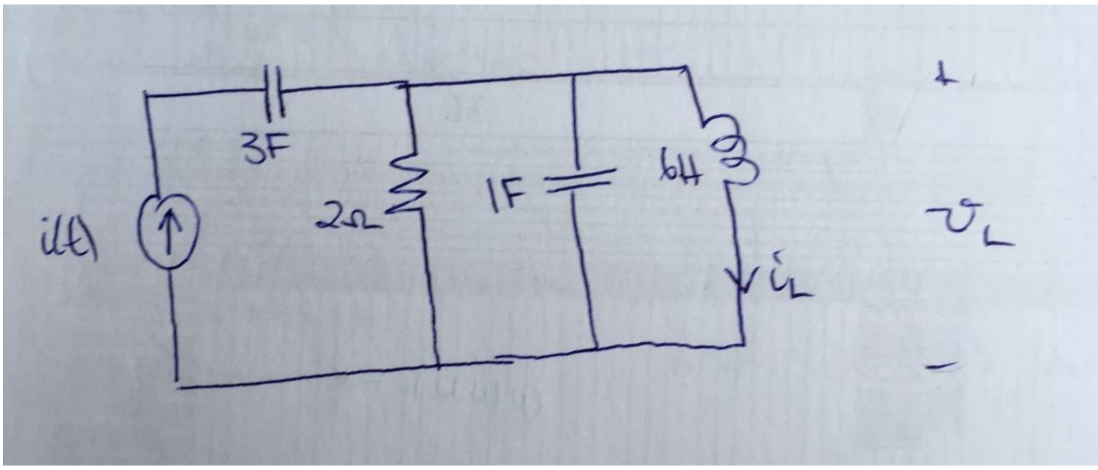
$v(t) =$



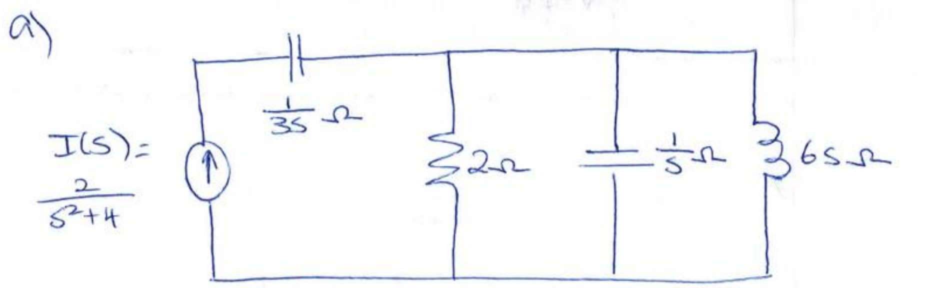
So at $t=0$, expect $v(t)=1$

☺

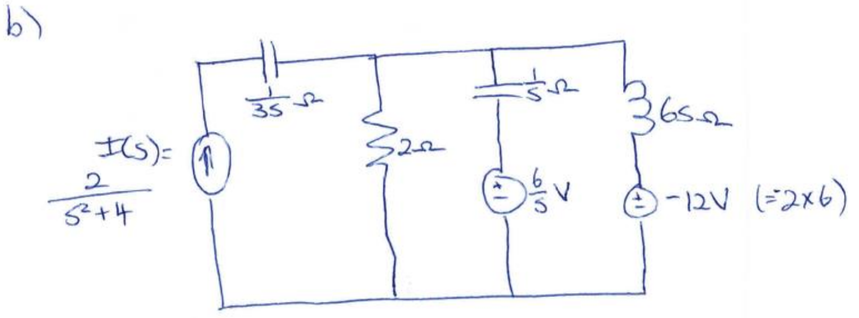
9. <u>s-domain Analysis</u>



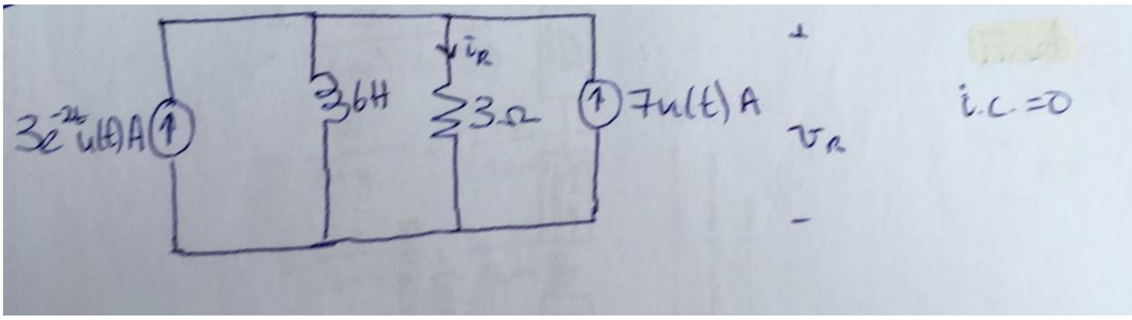
a) Redraw the circuit above in the s-domain if $i(t) = \sin(2t)\,u(t)$ A, and the initial conditions are all 0.



b) Redraw the circuit above in the s-domain if $i_L(0^-) = 2A$ and $v_L(0^-) = 6V$. Use a voltage source model where required, not a current source model.



10. s-Domain Analysis



a) For the circuit above, write an equation for the currents in the circuit using KCL. Carry out a LT on this equation. Rearrange to the form $I_R(s) = \frac{N(s)}{D(s)}$.

Hint: $i_L = \frac{1}{L}\int_0^t v_L\, dT + i_L(0^-)$.

$$3e^{-2t}u(t) - \frac{1}{6}\int_0^t v_R\,dt' + \cancel{i_L(0^-)}^{\,0} - i_R + 7u(t) = 0$$

$$\frac{3}{s+2} - \frac{1}{6s}V_R(s) - I_R(s) + \frac{7}{s} = 0$$

$$v_R = 3i_R$$

$$V_R(s) = 3I_R(s)$$

$$\frac{3}{s+2} + \frac{7}{s} = I_R(s) + \frac{1}{6s}\left(3I_R(s)\right)$$

$$\frac{3s+7s+14}{s(s+2)} = I_R(s)\left(1+\frac{1}{2s}\right)$$

$$I_R(s) = \frac{10s+14}{\cancel{s}(s+2)} \times \frac{2\cancel{s}}{2s+1}$$

$$= \frac{20s+28}{(s+2)(2s+1)}$$

c) Using partial fractions and the LT tables, find the inverse LT of $I_R(s)$, thus finding $i_R(t)$.

$$I_R(s) = \frac{A}{s+2} + \frac{B}{2s+1} = \frac{A(2s+1)+B(s+2)}{(s+2)(2s+1)}$$

$$= \frac{20s+28}{(s+2)(2s+1)}$$

$$\therefore A(2s+1) + B(s+2) = 20s+28$$

$\underline{s=-2:}$

$$A(2\times -2+1) = 20\times -2+28$$
$$-3A = -12$$
$$A = 4$$

$\underline{s=-1/2:}$

$$A(2\times \tfrac{-1}{2}+1)+B(\tfrac{-1}{2}+2) = -20\times \tfrac{-1}{2}+28$$
$$\tfrac{3}{2}B = 18$$
$$B = 12$$

$$I_R(s) = \frac{4}{s+2} + \frac{12}{2s+1}$$
$$= \frac{4}{s+2} + \frac{6}{s+1/2}$$
$$i_R(t) = 4e^{-2t}u(t) + 6e^{-1/2t}u(t) \text{ A}$$

11. Initial value Theorem and Final Value Theorem

$V(s) = \frac{s(s+3)}{s^2+4s+4}$. Use the IVT and the FVT to find $v(0^+)$ and $v(\infty)$.

$$v(0^+) = \lim_{s\to\infty}\left[s\times \frac{s(s+3)}{s^2+4s+4}\right]$$
$$= \lim_{s\to\infty}\left[\frac{s^3+3s^2}{s^2+4s+4}\right]$$
$$= \infty$$

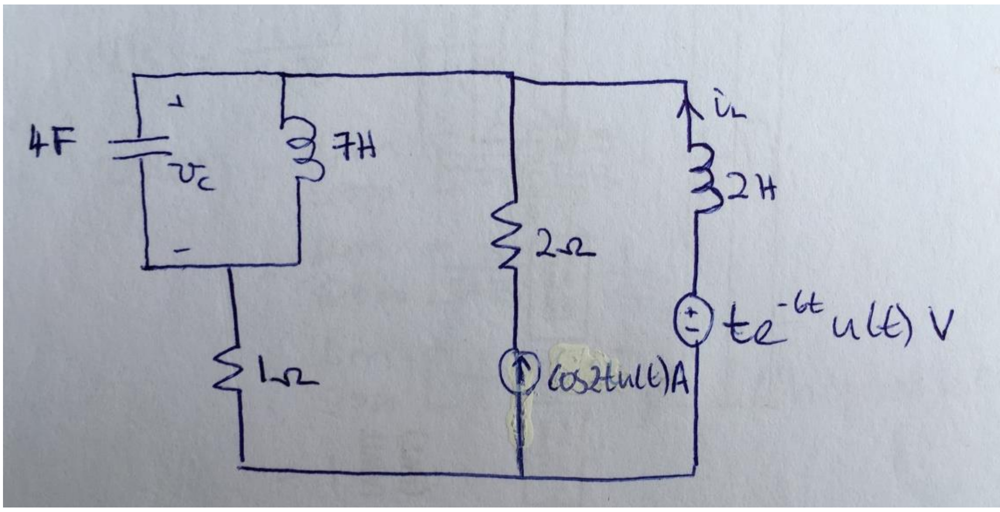
Check poles before use FVT!

$$s^2+4s+4 = 0$$
$$(s+2)^2 = 0$$
$$s = -2$$

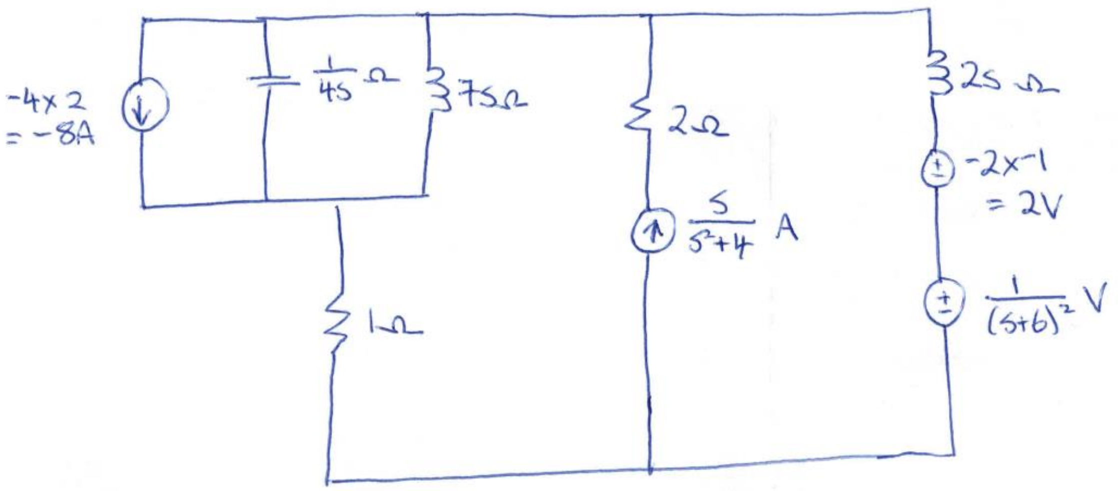
poles at $s=-2<0$ $\therefore$ ok to use FVT

$$v(\infty) = \lim_{s\to 0}\left[\frac{s^3+3s^2}{s^2+4s+4}\right]$$
$$= 0$$

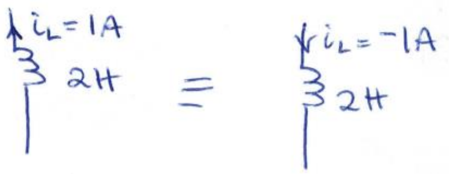
12. <u>s-Domain Circuits</u>



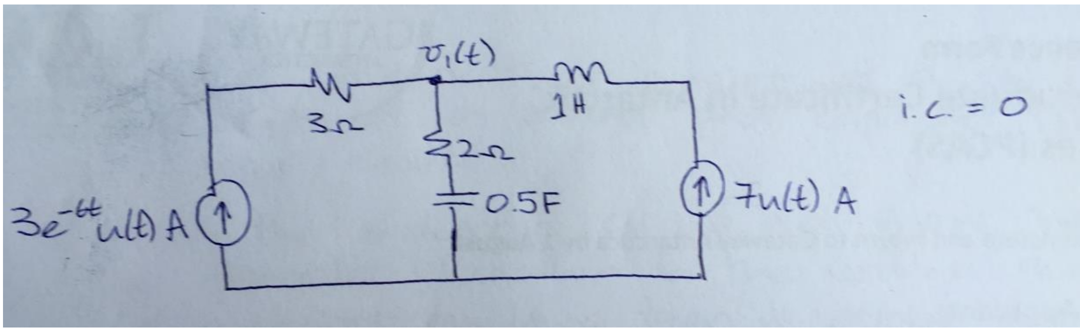
Redraw the circuit above in the s-domain, using a current source for the capacitor and a voltage source for the inductor. $v_c(0^-) = 2V$, $i_L(0^-) = 1A$, no i.c. across 7H inductor.



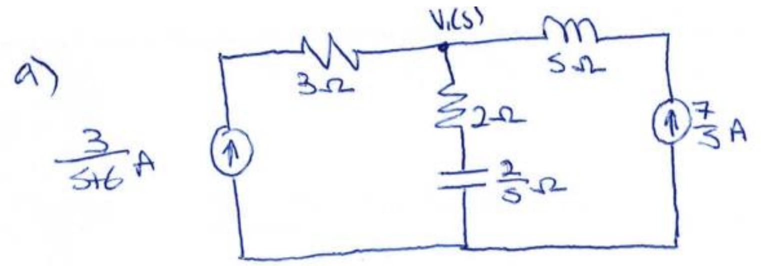
Note:



13. s-Domain Circuit Analysis



a) Draw the above circuit in the s-domain.



b) Write an equation for the node $V_1(s)$ and simplify it.

b) $$\frac{3}{s+6} + \frac{7}{s} - \frac{V_1}{(2+2/s)} = 0$$

$$\frac{3s+7(s+6)}{s(s+6)} - \frac{V_1 s}{2s+2} = 0$$

$$\frac{V_1 s}{2s+2} = \frac{3s+7s+42}{s(s+6)}$$

$$V_1 = \frac{(2s+2)(10s+42)}{s^2(s+6)}$$

$$= \frac{(20s^2+104s+84)}{s^2(s+6)}$$

$$= \frac{20s^2+104s+84}{s^2(s+6)}$$

c) Use partial fractions to prepare for the inverse LT.

c) $$V_1 = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+6}$$

$$As(s+6) + B(s+6) + Cs^2 = 20s^2+104s+84$$
$$As^2+6As+Bs+6B+Cs^2 = 20s^2+104s+84$$
$$s^2(A+C)+s(6A+B)+6B = 20s^2+104s+84$$

$A + C = 20$ $\quad$ $6A + B = 104$ $\quad$ $6B = 84$

$6A + 14 = 104$ $\quad$ $B = 14$

$15 + C = 20$ $\quad$ $A = 5$

$C = 5$

$$V_1 = \frac{15}{s} + \frac{14}{s^2} + \frac{5}{s+6}$$

d) Take the inverse LT to get $v_1(t)$.

$$v_1(t) = (15 + 14t + 5e^{-6t})u(t)$$

14. <u>Impulse Response, Transfer Function, and Convolution</u>

a) If the input to a system is $x(t) = 6tu(t) - 2u(t-4)$, and the impulse response $h(t) = 7u(t)$, what is the output $y(t)$? Hint: Work in the s-domain, and find the convolution of $x(t)$ and $h(t)$.

$$y(t) = x(t) * h(t)$$

$$Y(s) = X(s)H(s)$$

$$X(s) = \frac{6}{s^2} - \frac{2}{s}e^{-4s} \qquad H(s) = \frac{7}{s}$$

$$Y(s) = \left(\frac{7}{s}\right)\left(\frac{6}{s^2} - \frac{2}{s}e^{-4s}\right)$$

$$= \frac{42}{s^3} - \frac{14}{s^2}e^{-4s}$$

$$y(t) = \frac{42}{2}t^2u(t) - 14(t-4)u(t-4)$$

$$= 21t^2u(t) - 14(t-4)u(t-4)$$

b) What if $h(t) = 9u(t-2)$?

$$H(s) = \frac{9}{s}e^{-2s}$$

$$Y(s) = X(s)H(s)$$

$$= \left(\frac{6}{s^2} - \frac{2}{s}e^{-4s}\right)\left(\frac{9}{s}e^{-2s}\right)$$

$$= \frac{54}{s^3}e^{-2s} - \frac{18}{s^2}e^{-4s-2s}$$

$$= \frac{54}{s^3}e^{-2s} - \frac{18}{s^2}e^{-6s}$$

$$y(t) = \frac{54(t-2)^2}{2}u(t-2) - 18(t-6)u(t-6)$$

$$= 27(t-2)^2 u(t-2) - 18(t-6)u(t-6)$$

15. Poles, Zeros, and Transfer Functions

a) If $H(s) = \frac{2s^2+12s}{s^2-3s-28}$ what are its poles and zeroes? Sketch the pole-zero diagram.

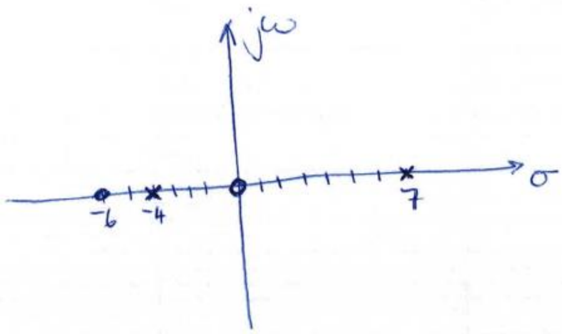
$$H(s) = \frac{2s(s+6)}{(s+4)(s-7)}$$

zeros $\Rightarrow 2s(s+6) = 0$

$s = 0, -6$

poles $\Rightarrow (s+4)(s-7) = 0$

$s = -4, 7$



b) Do the same for $H(s) = \frac{1}{10s^2+6s+1}$. What is the shape of the system response in this case? (e.g. sinusoid, exponentially decreasing sinusoid, exponentially increasing sinusoid, DC exponential)

$$H(s) = \frac{1}{10s^2+6s+1}$$

using quadratic formula $\Rightarrow s = \frac{-6 \pm \sqrt{36 - 4\times10\times1}}{2\times10}$

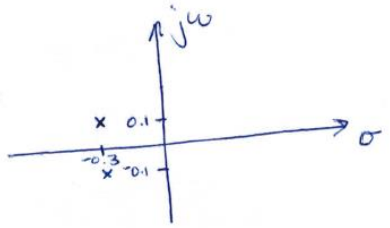
$$= \frac{-6 \pm \sqrt{-4}}{20}$$

$$= \frac{-6}{20} \pm j\frac{2}{20}$$

$$= -0.3 \pm j0.1$$

$$\therefore H(s) = \frac{1}{(s+0.3-j0.1)(s+0.3+j0.1)}$$

poles at $s = -0.3 \pm j0.1$



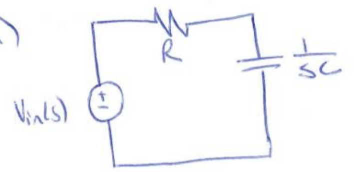
$\Rightarrow$ system response will be an exponentially decreasing sinusoid ($\sigma < 0$, $j\omega \neq 0$)

16. Transfer Functions

Consider a network that consists of a series combination of a resistor, a capacitor, and a voltage source.

a) Draw the s-domain circuit, and show that if a voltage is applied at the voltage source, then the transfer function for the voltage across the capacitor is $H(s) = \frac{1}{RC}\frac{1}{s+(1/_{RC})}$.

a)



$+$

$V_{out}(s)$

$-$

$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$

$$V_{out} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_{in}$$

$$H(s) = \frac{1}{sRC+1}$$

$$= \frac{1}{RC} \cdot \frac{1}{s + 1/RC}$$

b) Find the pole(s) of $H(s)$.

poles $\Rightarrow$ $s + \frac{1}{RC} = 0$; no zeros.

$s = \frac{-1}{RC}$

c) Calculate the output response, $v_{out}(t)$, if $v_{in}(t) = 2u(t)$ V.

c) $V_{in} = \frac{2}{s}$

$$V_{out}(s) = V_{in}(s)H(s)$$

$$= \frac{2}{s} \cdot \frac{1}{RC} \cdot \frac{1}{s + \frac{1}{RC}}$$

$$= \frac{2}{s(1+sRC)}$$

$$= \frac{A}{s} + \frac{B}{1+sRC}$$

$$A(1+sRC) + Bs = 2$$

$\underline{s=0:}$ $A = 2$

$\underline{s = \frac{-1}{RC}:}$ $\frac{-B}{RC} = 2$

$B = -2RC$

$$V_{out}(s) = \frac{2}{s} - \frac{2RC}{1+sRC}$$

$$= \frac{2}{s} - \frac{2}{\frac{1}{RC}+s}$$

$$v_{out}(t) = \left(2 - 2e^{-\frac{1}{RC}t}\right)u(t)\ V$$

17. <u>Poles, Zeros, Impulse Response</u>

If $H(s) = \frac{s^2+4s-6}{s^3+2s^2-15s}$ what are the poles and zeroes of $H(s)$, and what is the impulse response $h(t)$?

$$H(s) = \frac{s^2+4s-6}{s(s+5)(s-3)}$$

zeros $\Rightarrow s^2+4s-6=0$

$s = 1.16, -5.16$

(quadratic formula)

poles $\Rightarrow s(s+5)(s-3)=0$

$s = 0, -5, 3$

$$= \frac{A}{s} + \frac{B}{s+5} + \frac{C}{s-3}$$

$$A(s+5)(s-3) + Bs(s-3) + Cs(s+5) = s^2+4s-6$$

$\underline{s=-3:}$

$C(3)(8) = 9+12-6$

$C = 15/24 = 5/8$

$\underline{s=-5:}$

$B(-5)(-8) = 25-20-6$

$B = -1/40$

$\underline{s=0:}$

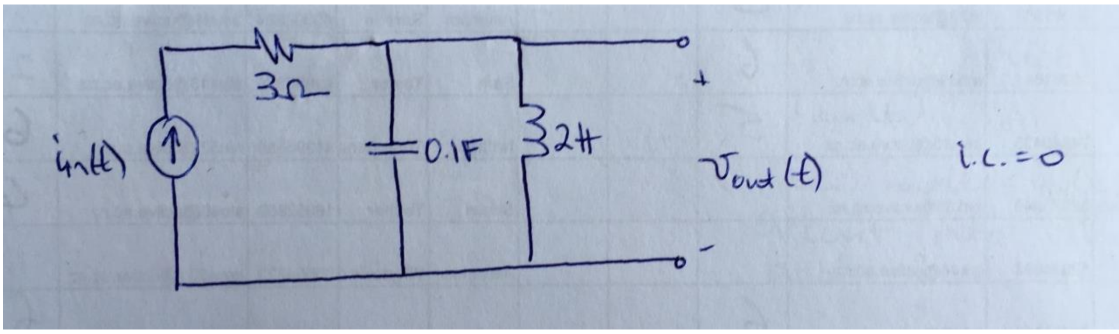
$A(5)(-3) = -6$

$A = 2/5$

$$H(s) = \frac{2/5}{s} - \frac{1/40}{s+5} + \frac{5/8}{s-3}$$

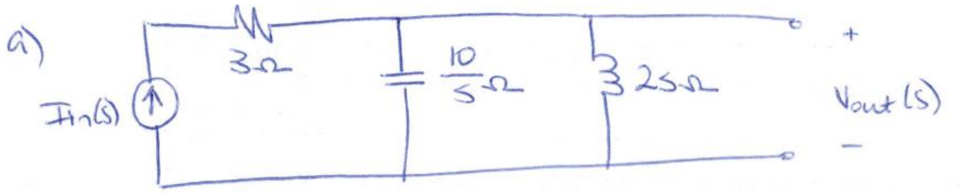
$$h(t) = \left(\frac{2}{5} - \frac{1}{40}e^{-5t} + \frac{5}{8}e^{3t}\right)u(t)$$

18. Transfer Function, Poles, Zeros, Convolution

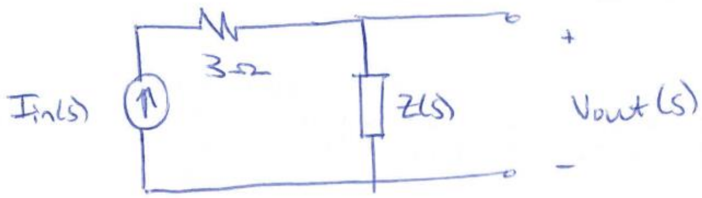


a) Find the transfer function $H(s)$ of the circuit above, the poles and zeroes of $H(s)$, and sketch the pole-zero plot.

a)



⇓



$$Z(s) = \frac{2s \cdot 10/s}{2s + 10/s}$$

$$= \frac{20s}{2s^2 + 10}$$

$$V_{out} = Z(s) I_{in}(s)$$

$$= \left(\frac{20s}{2s^2 + 10}\right) I_{in}(s)$$

$$H(s) = \frac{V_{out}(s)}{I_{in}(s)}$$

$$= \frac{20s}{2s^2 + 10}$$

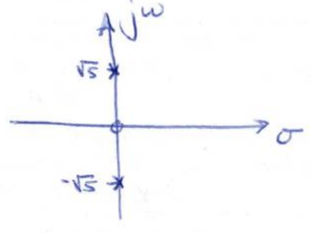
zeros ⇒ $20s = 0$

$s = 0$

poles ⇒ $2s^2 + 10 = 0$

$s^2 = -5$

$s = \pm j\sqrt{5}$



b) If $i_{in}(t) = 6\delta(t)$ A, what is $v_{out}(t)$?

$$I_{in}(s) = 6$$

$$V_{out}(s) = H(s) I_{in}(s)$$

$$= \frac{20s}{2s^2+10} \cdot 6$$

$$= \frac{120s}{2s^2+10}$$

$$= \frac{60s}{s^2+5}$$

$$= 60\left(\frac{s}{s^2+(\sqrt{5})^2}\right)$$

$$v_{out}(t) = 60 \cos \sqrt{5}t \, u(t) \text{ V}$$