UNIVERSITY OF CANTERBURY

Test 1

Prescription Number: **EMTH211-16S2**

Time allowed: 50 minutes.

Attempt ALL 5 questions.

Write your answers in the spaces provided.

There is a *total* of 50 points.

Use black or blue ink. Do not use pencil except for diagrams.

Only UC approved calculators are allowed.

Show all working. Write neatly. Marks will be lost for poorly presented answers.

Family name:	
Given names:	
Student ID:	

MARKS Office Use Only	
Q1	
Q2	
Q3	
Q4	
Q5	
Total	

Question 1 [8 points]

Consider the system of linear equations

$$\begin{aligned} x + ky &= 1 \\ kx + y &= 1 \ . \end{aligned}$$

How many solutions does this system have depending on k?
Note that it was not specifically required to compute these solutions.

Question 2 [12 points]

Consider the matrix

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 4 & 0 & 3 \\ 8 & 3 & 5 \end{bmatrix} .$$

1. Compute a LU-decomposition of A.
2. Use the above decomposition to solve

$$A\mathbf{x} = \begin{bmatrix} -4 \\ 4 \\ 16 \end{bmatrix}$$

Question 3 [10 points]

Remember that a subspace of $W \subset \mathbb{R}^n$ is a set that is closed under vector addition and scalar multiplication. (That is if $u, v \in W$ and $k \in \mathbb{R}$ then $u + v \in W$ and $ku \in W$).
Let A be a $m \times n$ matrix.

1. Show that $W = \{\mathbf{x} \in \mathbb{R}^n \mid A\mathbf{x} = \mathbf{0}\}$ is a subspace
2. Why is, for $V_b = \mathbf{b} \in \mathbb{R}^n$ with $\mathbf{b} \neq \mathbf{0}$, the set $\{\mathbf{x} \in \mathbb{R}^n \mid A\mathbf{x} = \mathbf{b}\}$ not a subspace?

Question 4 [8 points]

Find a basis for the column space, the row space, and the null space for the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

Question 5 [12 points]

1. Give an example of two vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^2$ such that $\|\mathbf{a}\|_1 < \|\mathbf{b}\|_1$, but $\|\mathbf{a}\|_\infty > \|\mathbf{b}\|_\infty$.
2. Are the matrix norms $\|\cdot\|_1$ and $\|\cdot\|_\infty$ independent of row swaps?

Page for rough working ...

END OF PAPER