EMTH211–Tutorial 4

Attempt the following problems before the tutorial

1. Determine whether the following sets are a basis for the given vector space:

   (a) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ for $\mathbb{R}^2$

   (b) $\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$ for $\mathbb{R}^2$

   (c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ for $\mathbb{R}^3$

   (d) $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ for $\mathbb{R}^3$

   (e) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ for $\mathbb{R}^4$

2. Let $x = [1, 3, 4, 5]$, and $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$.

   (a) Find $\|x\|_1, \|x\|_2, \|x\|_\infty$ by hand and check your solution with MatLab.

In-tutorial problems

3. Find a basis for the following subspaces.

   (a) $U_1 = \left\{ \begin{bmatrix} r \\ r \\ s \end{bmatrix} \middle| r, s \in \mathbb{R} \right\}$;

   (b) $U_2 = \left\{ \begin{bmatrix} r+s \\ r-s \\ r \end{bmatrix} \middle| r, s \in \mathbb{R} \right\}$;

   (c) $U_3 = \left\{ \begin{bmatrix} r \\ r \\ s+t \end{bmatrix} \middle| r, s, t \in \mathbb{R} \right\}$;

   (d) $U_4 = \left\{ \begin{bmatrix} r+s \\ r-t \\ s+t \end{bmatrix} \middle| r, s, t \in \mathbb{R} \right\}$.

4. Determine the rank of the following matrices over $\mathbb{R}$.

$$(a) \begin{bmatrix} 1 & 2 & 1 & 1 \\ 2 & -1 & 1 & 2 \\ 4 & 3 & 3 & 4 \\ 3 & 1 & 2 & 3 \end{bmatrix} \qquad (b) \begin{bmatrix} 2 & 1 & 4 & 0 & 0 \\ 1 & 0 & -2 & 0 & 1 \\ -2 & 0 & 3 & 4 & 1 \end{bmatrix}$$

5. For which values of $a \in \mathbb{R}$ are the following 3 vectors in $\mathbb{R}^3$ linearly dependent:

$$\begin{bmatrix} 1 \\ 2 \\ a \end{bmatrix}, \quad \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} ?$$

6. Give a basis for the row space, the column space and the null space of the matrix $\begin{bmatrix} 2 & -4 & 0 & 2 & 1 \\ -1 & 2 & 1 & 2 & 3 \\ 1 & -2 & 1 & 4 & 4 \end{bmatrix}$.

7. Show that for $x \in \mathbb{R}^n$

$$\|x\|_\infty \leqslant \|x\|_1 \leqslant n\|x\|_\infty .$$

Hint: for some k between 1 and n we have $|x_k| = \max\{|x_1|, \ldots, |x_n|\}$.