EMTH211–Tutorial 7

Attempt the following problems before the tutorial.

1. (Use Matlab) Use the power method to approximate the dominant eigenvalue and eigenvector of
$$A = \begin{bmatrix} 7 & 2 \\ 2 & 3 \end{bmatrix}$$
using $\mathbf{x}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as the starting vector and carrying out 6 iterations.

In-tutorial problems

2. (Use Matlab) Use the power method to approximate the dominant eigenvalue and eigenvector of
$$A = \begin{bmatrix} 3.5 & 1.5 \\ 1.5 & -0.5 \end{bmatrix}$$
using $\mathbf{x}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as the starting vector and carrying out 6 iterations. Use the shifted power method to find the second eigenvalue of A again using the same starting vector. Calculate a few iterations and see what happens.

3. (Use Matlab) Use the inverse power method to approximate an eigenvector corresponding to the least eigenvalue of
$$A = \begin{bmatrix} 7 & 2 \\ 2 & 3 \end{bmatrix}$$
using $\mathbf{x}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as the starting vector and carrying out 6 iterations. Recall that for the inverse power method, we first find an LU-decomposition for A and then solve the system $A\mathbf{z} = \mathbf{y}^k$.

4. Consider the matrix
$$A = \begin{bmatrix} 6 & 0 & -1 & 0 \\ 1 & 3 & 0 & 0.5 \\ 0 & -2 & 12 & 0 \\ -1 & 0 & 0 & -1 \end{bmatrix}.$$

(a) Draw the Gerschgorin Discs of A. (Column and row based). Mark the area that the eigenvalues of A must lie in.

(b) Show that A is invertible, using the information gained from the previous part of the exercise.

(c) Calculate the eigenvalues of A (use Matlab) and check whether they are indeed contained in the area you indicated in (a).

5. Recall that a matrix is *strictly diagonally dominant* if the absolute value of each diagonal entry is strictly larger than the sum of the absolute values of the remaining entries in that row. Use Gerschgorin's disk theorem to prove that a strictly diagonally dominant matrix must be invertible.