Nodal and mesh analysis in the s-domain; Additional circuit analysis techniques

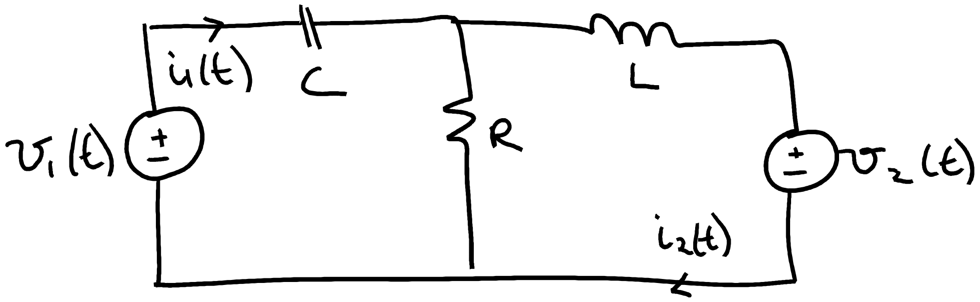
Readings: Sections 14.8, 14.9

Term 1 Techniques in the s-domain

We can do nodal and mesh analysis on **s**-domain circuits, as well as source transformations, superposition, and using Thévenin and Norton's theorems.

Example

Convert the below circuit into the **s**-domain, and apply mesh analysis to it. Assume initial conditions are zero.



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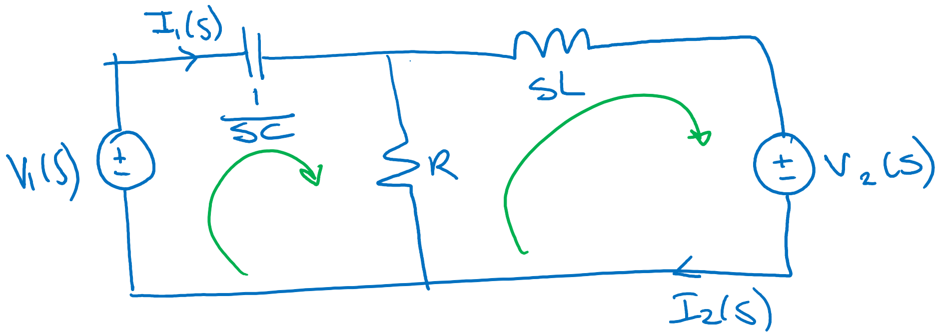


$$-V_1(s) + \frac{I(s)}{sC} + R(I_1(s) - I_2(s)) = 0$$

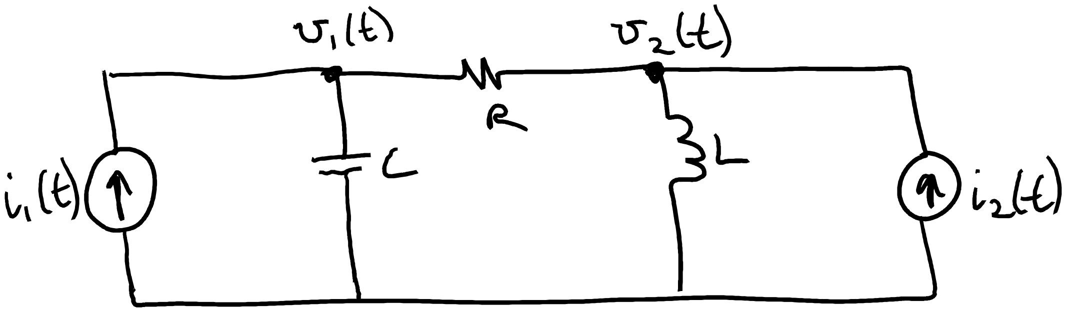
$$R(I_2(s) - I_1(s)) + sLI_2(s) + V_2(s) = 0$$

$\Rightarrow$ 2 eqns & 2 unknowns ($I_1(s)$ & $I_2(s)$)

$\therefore$ can solve

Example

Convert the below circuit into the **s**-domain, and apply nodal analysis to it. Assume the initial conditions are zero.



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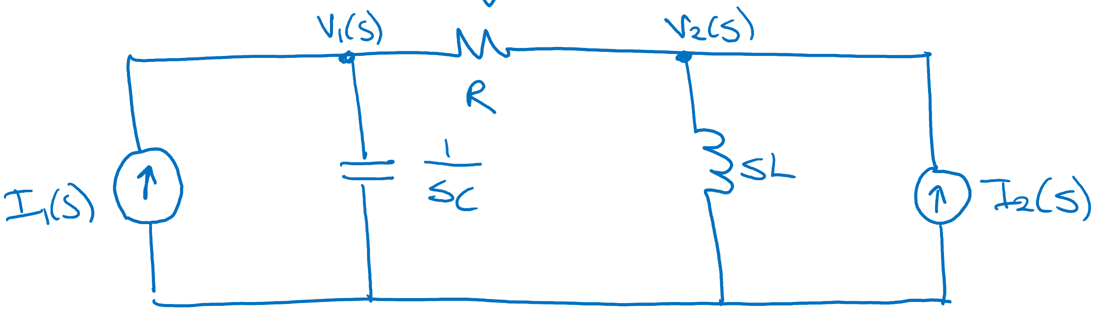


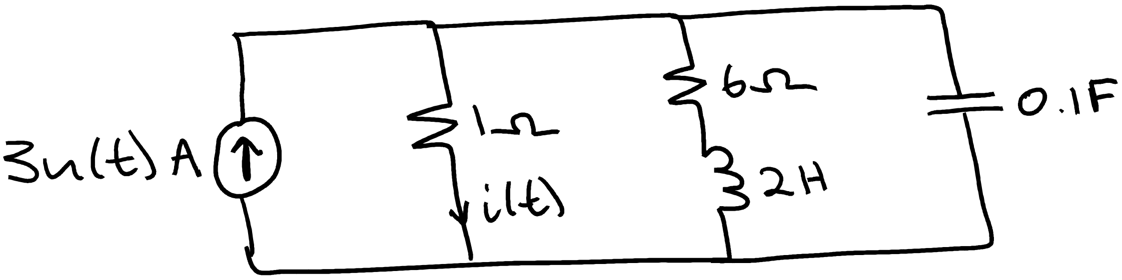
$$I_1(S) = V_1(S) \cdot SC + \frac{V_1(S) - V_2(S)}{R}$$

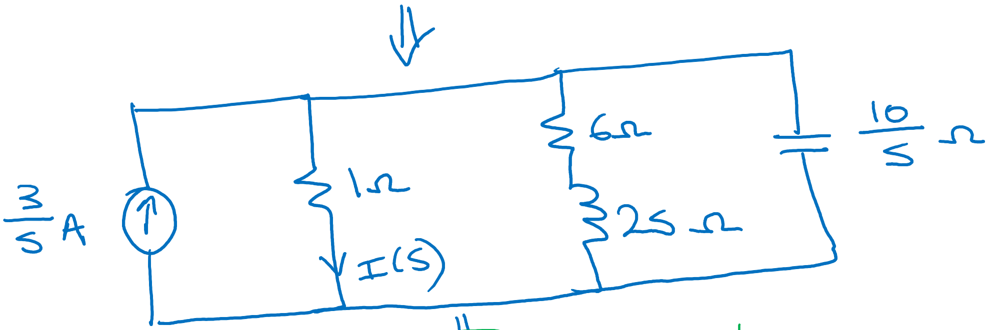
$$I_2(S) = \frac{V_2(S)}{SL} + \frac{V_2(S) - V_1(S)}{R}$$

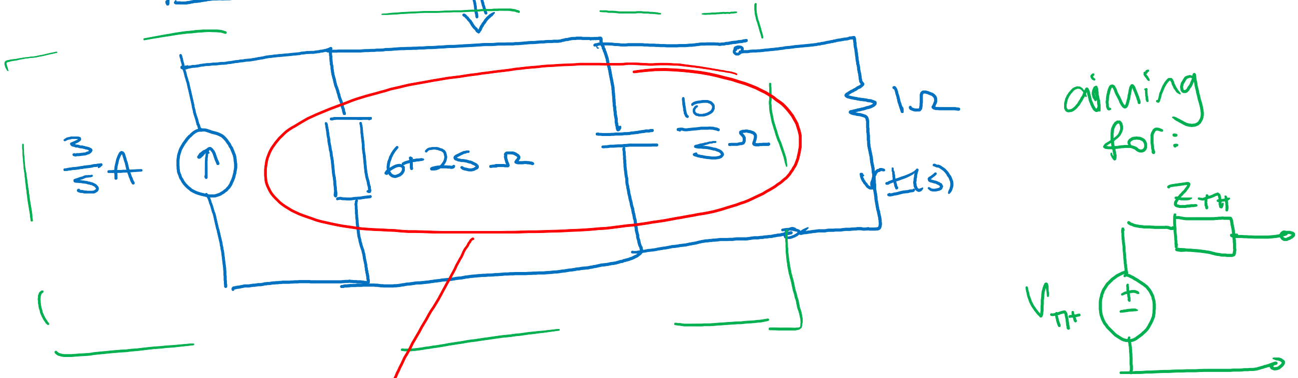
$\Rightarrow$ 2 eqns, 2 unknows, can solve ☺

Example

For the circuit below, find the Thévenin equivalent circuit seen by the 1 Ω resistor. Assume the initial conditions are zero.





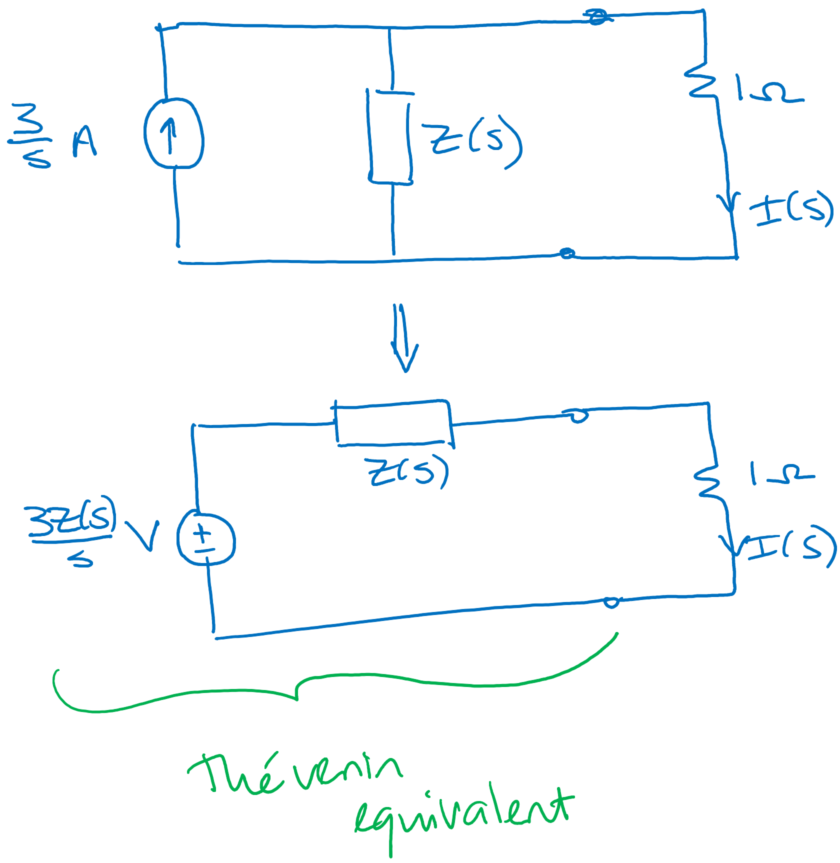


$$Y(s) = \frac{1}{6+2s} + \frac{s}{10}$$

$$= \frac{10+6s+2s^2}{60+20s}$$

$$= \frac{s^2+3s+5}{10s+30}$$

$$Z(s) = \frac{10s+30}{s^2+3s+5}$$


3/s A
Z(s)
1Ω
I(s)
Z(s)
3Z(s)/s V
1Ω
I(s)
Thévenin equivalent

Poles, zeros, and transfer functions

Readings: Section 14.10

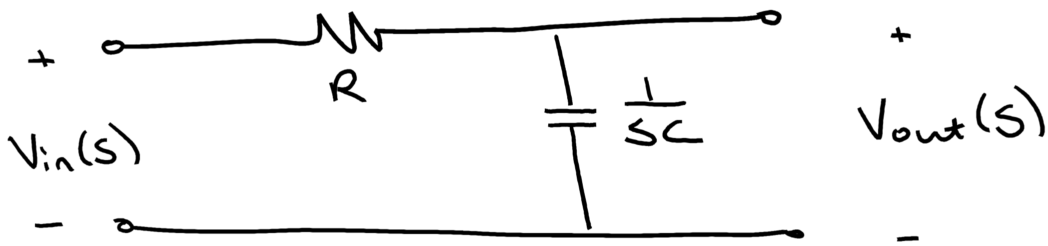
Transfer Function, H(s)

The transfer function **H(s)** is defined as:

$$H(s) = \frac{Y(s)}{X(s)} \quad \begin{matrix} \leftarrow \text{output} \\ \leftarrow \text{input} \end{matrix}$$

Y(s) and **X(s)** can be either voltages or currents, and they don't need to be the same (one could be a voltage and one a current). Always assume the initial conditions are zero when working out the transfer function.

Example



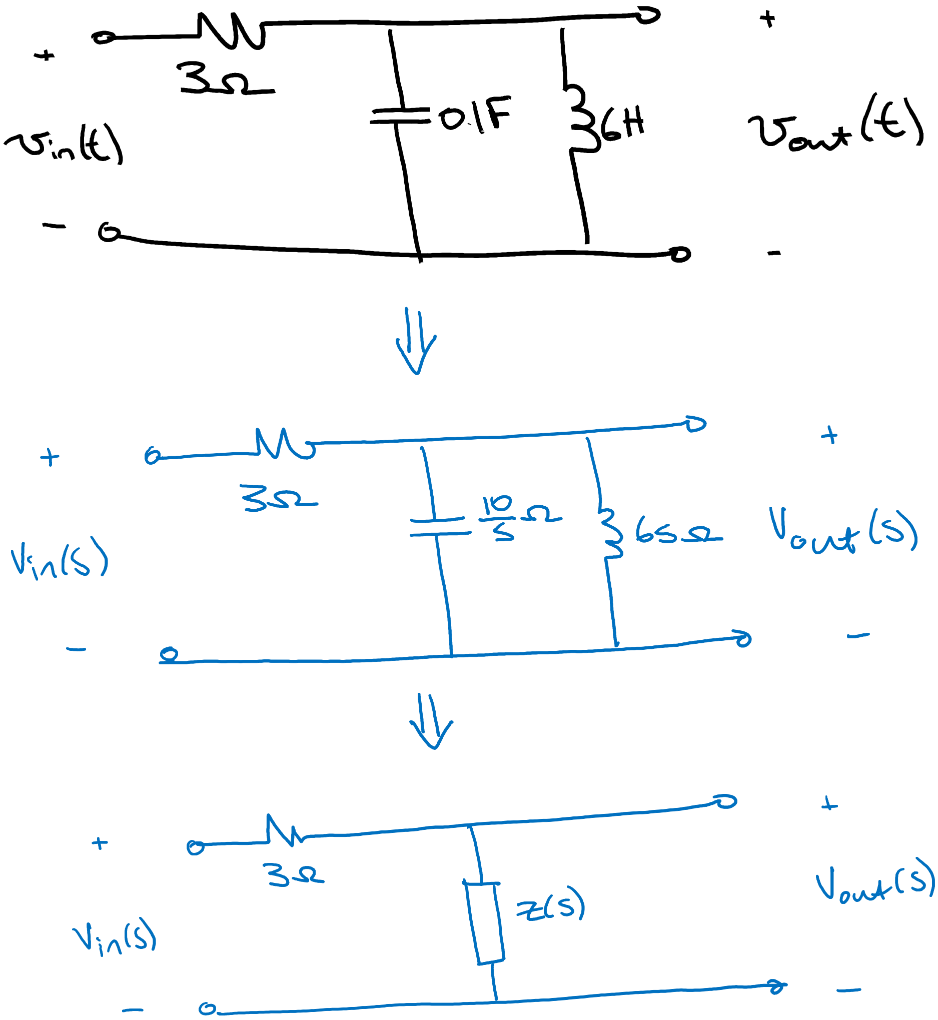
$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$$

H(s) has poles and zeros; these poles and zeros can be used to determine the stability of the system and the shape of the system response.

Remember: zeroes are calculated from the numerator of **H(s)**, while poles are calculated from the denominator.

Example

Find **H(s)** for the circuit below, and determine its poles and zeros.



$$Y(s) = \frac{s}{10} + \frac{1}{6s} = \frac{6s^2+10}{60s}$$

$$Z(s) = \frac{60s}{6s^2+10}$$

$$V_{out} = V_{in}\left(\frac{Z(s)}{3+Z(s)}\right)$$ (voltage divider)

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{\frac{60s}{6s^2+10}}{3 + \frac{60s}{6s^2+10}}$$

$$H(s) = \frac{60s}{18s^2 + 30 + 60s}$$

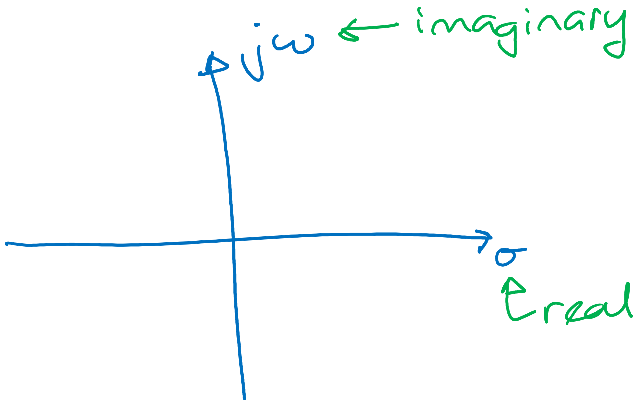
$$= \frac{20s}{6s^2 + 20s + 10}$$

zeros $\Rightarrow$ $20s = 0$
$s = 0$

poles $\Rightarrow$ $6s^2 + 20s + 10 = 0$
$(s + 2.7)(s + 0.6) = 0$
$s = -0.6, -2.7$

Pole-zero Diagrams

Pole-zero diagrams are a graphical representation of a circuit response or transfer function - we plot the poles and zeros on the **s** plane (remember, $s = \sigma + j\omega$, so it has a real part and an imaginary part).



$s = \sigma + j\omega$

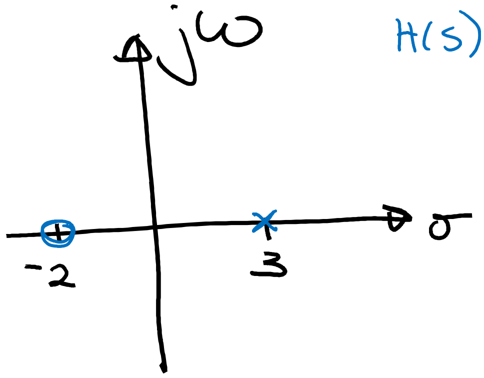
Poles are marked by ‘x’, zeros by ‘o’. It can be useful to think of the **s** plane as a sheet of rubber; the poles are poles holding the sheet up, while the zeros are tacks or pins holding the sheet down (at that point the response is zero, and the height of the sheet must be zero).

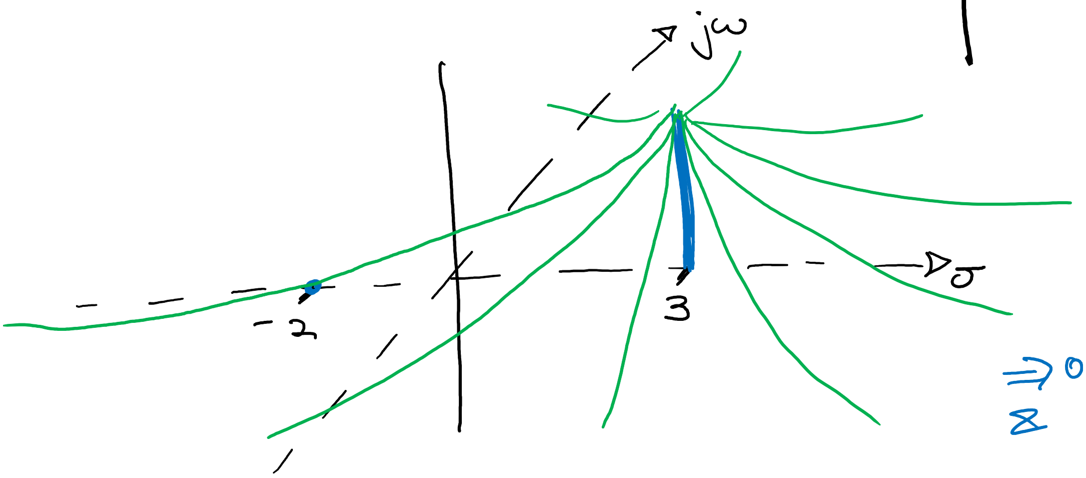
Example:

Plot the pole-zero diagram of $H(s) = \frac{s+2}{s-3}$.

zero at $s+2=0$
$s=-2$

pole at $s-3=0$
$s=3$





⇒ one 'mountain' & one 'crater'

The location of the poles tells you the shape of the system response (very useful for determining if you have a stable system or not).