EMTH211-19S2 LABORATORY 2

JULY 22-26, 2019

2.1 Consider the system

$$\begin{aligned} 100x_1 + 1000000x_2 &= 1000000 \\ -100x_1 + 200x_2 &= 100 \end{aligned} \tag{1}$$

(a) Find the solution (with only partial pivoting) to this system using 3-digit arithmetic.

(b) Rewrite this system as

$$\begin{aligned} 1000000y_1 + 100y_2 &= 1000000 \\ 200y_1 - 100y_2 &= 100 \end{aligned}$$

Solve this system using 3-digit arithmetic (note that this is equivalent to complete pivoting).

(c) Solve (1) using 5-digit arithmetic. What is the minimum number of digits needed to get an accurate answer?

(d) Modify (1) so that 5-digit arithmetic will not give an accurate answer, when using only partial pivoting to solve the system.

(e) Modify (1) so that 16-digit arithmetic will not give an accurate answer, when using only partial pivoting to solve the system (this is roughly the default double-precision in MATLAB).

2.2 In the lectures, it was shown that to solve a system of equations by Gaussian elimination requires

$$\mathcal{O}(\tfrac{2}{3}n^3)$$

flops. This refers to a *generic* or full matrix. Some special forms are much much faster to solve. Compute the flop count for solving a system of equations whose coefficient matrix is

(a) diagonal

(b) upper triangular

(c) lower triangular

2.3 In Lab 1, you solved a *tridiagonal* system. You should have noticed that the speed at which this was achieved depended critically on using the `sparse` function. Compute the flop count to solve a tridiagonal system $A\mathbf{x} = \mathbf{d}$ with

$$A = \begin{bmatrix} a & b & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ b & a & b & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & b & a & b & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & b & a & b \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & b & a \end{bmatrix}$$

efficiently.

OPTIONAL Implement the algorithm you found in MATLAB.