Name: **Student ID:**

Pre-tutorial 8 Questions (to be attempted before class on August 9th, 2019)

Chapter 14, Ex 21: Laplace Transforms

Using the one-sided Laplace transform equation, calculate (showing intermediate steps) the Laplace transform of the following:

d) $2.1u(t)$

$$F(s) = \int_{0^-}^{\infty} e^{-st}\, 2.1u(t)\,dt$$

$$= 2.1 \int_{0^-}^{\infty} e^{-st}\, dt$$

$$= 2.1 \left[-\frac{1}{s} e^{-st} \right]_{0^-}^{\infty}$$

$$= 2.1 \left[0 - -\frac{1}{s} \right]$$

$$= \frac{2.1}{s}$$

c) $5u(t-2) - 2u(t)$

$$F(s) = \int_{0^-}^{\infty} e^{-st} \left(5u(t-2) - 2u(t) \right) dt$$

$$= 5 \int_{2}^{\infty} e^{-st}\, dt - 2 \int_{0^-}^{\infty} e^{-st}\, dt$$

$$= 5 \left[-\frac{1}{s} e^{-st} \right]_{2}^{\infty} - 2 \left[-\frac{1}{s} \right]_{0^-}^{\infty}$$

$$= 5 \left[0 - -\frac{1}{s} e^{-2s} \right] - 2 \left[0 - -\frac{1}{s} \right]$$

$$= \frac{5}{s} e^{-2s} - \frac{2}{s}$$

Chapter 14, Ex 59: IVT and FVT

Apply the initial- or final-value theorems as appropriate to determine $f(0^+)$ and $f(\infty)$ for the following functions:

a) $\frac{s+2}{s^2+8s+4}$

$$f(0^+) = \lim_{s \to \infty} sF(s)$$

$$= \lim_{s \to \infty} \frac{s^2 + 2s}{s^2 + 8s + 4}$$

$$= \lim_{s\to\infty} \frac{1 + 2/s}{1 + 8/s + 4/s^2}$$

$$= 1$$

For FVT, need to check poles:

poles $\Rightarrow s^2 + 8s + 4 = 0$

$$s = \frac{-8 \pm \sqrt{8^2 - 4\times 4}}{2}$$

$$= -0.54, -7.46 \quad \checkmark \text{ all in LHP}$$

$$f(\infty) = \lim_{s\to 0} \frac{s^2+2s}{s^2+8s+4}$$

$$= 0/4$$

$$= 0$$

c) $\frac{4s^2+1}{(s+1)^2(s+2)^2}$

$$f(0^+) = \lim_{s\to\infty} sF(s)$$

$$= \lim_{s\to\infty} \frac{4s^3+s}{(s+1)^2(s+2)^2}$$

$$= \lim_{s\to\infty} \frac{4s^3+s}{(s^2+2s+1)(s^2+4s+4)}$$

$$= \lim_{s\to\infty} \frac{4s^3+s}{s^4+4s^3+4s^2+2s^3+8s^2+8s+s^2+4s+4}$$

$$= \lim_{s\to\infty} \frac{4s^3+s}{s^4+6s^3+13s^2+12s+4}$$

$$= 0$$

check poles for FVT poles $\Rightarrow (s+1)^2(s+2)^2 = 0$

$s = -1, -2$

(repeated)

all LHP $\therefore$ OK

$$f(\infty) = \lim_{s \to 0} \frac{4s^3 + s}{s^4 + 6s^3 + 13s^2 + 12s + 4}$$

$$= 0/4$$

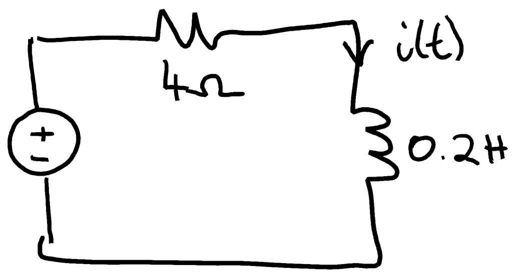
$$= 0$$

At Tutorial 8 – Marked Question (9th August 2019)

Chapter 14 Ex 48a: Laplace Transformations

For the circuit below, write the **s**-domain KVL equation in terms of **I(s)**. Rearrange and simplify the equation to get an **s**-domain expression for I**(s)**.

$$V(s) = \frac{2}{s} - \frac{1}{s+1} \text{ V}$$



$$i(0^-) = 2\text{A}$$

$$-\frac{2}{s} + \frac{1}{s+1} + 4I(s) + 0.2\frac{dI(s)}{dt} = 0$$

$$4I(s) + 0.2\left(sI(s) - i(0^-)\right) = \frac{2}{s} - \frac{1}{s+1}$$

$$4I(s) + 0.2sI(s) - 0.4 = \frac{2s+2-s}{s(s+1)}$$

$$I(s)(4+0.2s) = \frac{s+2}{s(s+1)} + 0.4$$

$$= \frac{s+2+0.4s^2+0.4s}{s(s+1)}$$

$$I(s) = \frac{0.4s^2 + 1.4s + 2}{s(s+1)(0.2s+4)} \text{ A}$$

At Tutorial 8 – Unmarked Questions (9th August 2019)

Chapter 14, Ex 27: Laplace Transformations

Using the Laplace transform tables, determine **F(s)** if $f(t)$ is equal to:

a) $3u(t-2)$

$$F(s) = \frac{3}{s}e^{-2s}$$

b) $3e^{-2}\,u(t) + 5u(t)$

$$F(s) = \frac{3}{s+2} + \frac{5}{s} = \frac{3s+5s+10}{s(s+2)}$$

$$= \frac{8s+10}{s(s+2)}$$

c) $\delta(t) + u(t) - tu(t)$

$$F(s) = 1 + \frac{1}{s} - \frac{1}{s^2} = \frac{s^2+s-1}{s^2}$$

d) $5\delta(t)$

$$F(s) = 5$$

Chapter 14, Ex 35: Laplace Transforms

Determine the inverse transform of **F(s)** equal to:

a) $5 + \frac{5}{s^2} - \frac{5}{(s+1)}$

$$f(t) = 5\delta(t) + 5tu(t) - 5e^{-t}u(t)$$

b) $\frac{1}{s} + \frac{5}{0.1s+4} - 3$

$$F(s) = \frac{1}{s} + \frac{50}{s+40} - 3$$

$$f(t) = u(t) + 50e^{-40t}u(t) - 3\delta(t)$$

c) $-\frac{1}{2s}+\frac{1}{(0.5s)^2}+\frac{4}{(s+5)(s+5)}+2$

$$F(s) = -\frac{1}{2s} + \frac{1}{0.25s^2} + \frac{4}{(s+5)^2} + 2$$

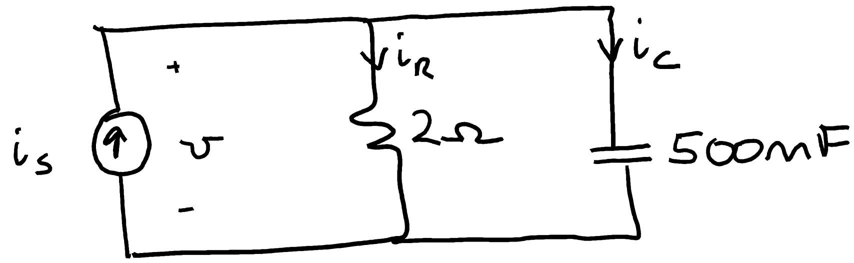
$$f(t) = -\frac{1}{2}u(t) + 4t\,u(t) + 4te^{-5t}u(t) + 2\delta(t)$$

d) $\frac{4}{(s+5)(s+5)}+\frac{2}{s+1}+\frac{1}{s+3}$

$$f(t) = 4te^{-5t}u(t) + 2e^{-t}u(t) + e^{-3t}u(t)$$

Chapter 14, Ex 46: Laplace Transformations

For the circuit below, the initial voltage across the capacitor is $v(0^-) = 1.5$ V and the current source is $i_s = 700u(t)$ mA.



a) Write the differential equation which arises from KCL, in terms of the nodal voltage $v(t)$.

$$i_s = i_R + i_c$$

$$0.7u(t) = \frac{v}{2} + 0.5\,dv/dt$$

b) Take the Laplace transform of the differential equation.

$$\frac{0.7}{s} = 0.5V(s) + 0.5(sV(s) - v(0^-))$$

$$= 0.5V(s) + 0.5sV(s) - 0.5\times1.5$$

$$0.7 = 0.5sV(s) - 0.5s^2V(s) - 0.75s$$

c) Determine the frequency-domain representation of the nodal voltage

$$V(s)(0.5s^2 + 0.5s) = 0.7 + 0.75s$$

$$V(s) = \frac{0.7 + 0.75s}{s(0.5s + 0.5)}$$

Chapter 14, Ex 61: Impedance

The voltage $v(t) = 8e^{-2t}u(t)$ V is applied to a two-terminal device. Your assistant misunderstands you and only records the **s**-domain current which results. Determine what type of element it is and its value if **I(s)** is equal to:

a) $\frac{1}{s+2}$ A

$$V(s) = \frac{8}{s+2}$$

$$Z(s) = \frac{8/(s+2)}{1/(s+2)}$$

$$= \frac{8}{s+2} \times \frac{s+2}{1}$$

$$= 8\Omega$$

$\therefore$ an 8Ω resistor

b) $\frac{4}{s(s+2)}$ A

$$Z(s) = \frac{8/(s+2)}{4/(s(s+2))}$$

$$= \frac{8}{s+2} \times \frac{s(s+2)}{4}$$

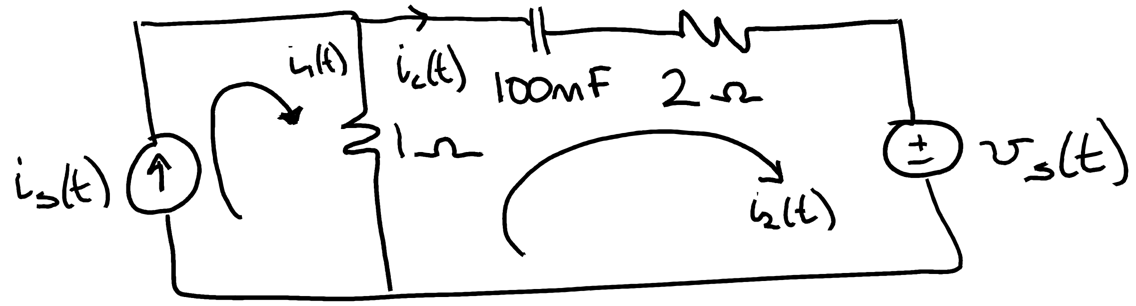
$$= \frac{8s}{4}$$

$$= 2s\Omega$$

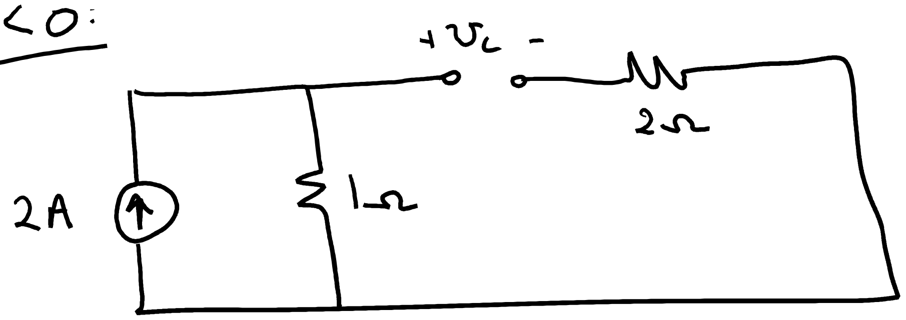
$\therefore$ a 2H inductor

Chapter 14, Ex 64a: Time-Domain Mesh Analysis, LT, Inverse LT

Referring to the circuit below and keeping the circuit in the time-domain, develop an expression for $\mathbf{I_c(s)}$, then determine $i_C(t)$ for t > 0 if $i_s(t) = 2u(t+2)$ A and $v_s(t) = 2u(t)$ V. HINTS: You can work out the initial conditions using techniques from term 2. Use mesh analysis in the time domain for t > 0. Voltage drop across a capacitor is $v_c(t) = \frac{1}{c}\int_{t_0}^{t} i(T)\,dT + v_c(t_0)$.



t<0:



$v_c(0^-) = 2 \times 1$
$= 2V$
$= v_c(0^+)$

t>0:

mesh 2: $1(i_2 - i_1) + \frac{1}{0.1}\int_0^t i_2\,d\tau + 2 + 2i_2 + 2 = 0$

$i_2 = i_c$ & $i_1 = 2$ by inspection

$\therefore\ i_c - 2 + 10\int_0^t i_c\,d\tau + 2i_2 + 4 = 0$

$$I_c(s) + \frac{10}{s}I_c(s) + 2I_c(s) = \frac{-2}{s}$$

$$I_c(s)\left(1 + \frac{10}{s} + 2\right) = \frac{-2}{s}$$

$$I_c(s)\left(\frac{3s+10}{\cancel{s}}\right) = \frac{-2}{\cancel{s}}$$

$$I_c(s) = \frac{-2}{3s+10}$$

$$I_c(s) = \frac{-2}{3}\left(\frac{1}{s + 10/3}\right)$$

$$i_c(t) = \frac{-2}{3} e^{-10/3t} u(t) \text{ A}$$