Exam Formulas for Term 3 Material

Laplace Transform Pairs

$\mathbf{f(t)} = \mathcal{L}^{-1}\{\mathbf{F(s)}\}$	$\mathbf{F(s)} = \mathcal{L}\{\mathbf{f(t)}\}$	$\mathbf{f(t)} = \mathcal{L}^{-1}\{\mathbf{F(s)}\}$	$\mathbf{F(s)} = \mathcal{L}\{\mathbf{f(t)}\}$
$\delta(t)$	1	$\frac{1}{\beta-\alpha}\left(e^{-\alpha t} - e^{-\beta t}\right)u(t)$	$\frac{1}{(s+\alpha)(s+\beta)}$
$u(t)$	$\frac{1}{s}$	$\sin\omega t\,u(t)$	$\frac{\omega}{s^2+\omega^2}$
$tu(t)$	$\frac{1}{s^2}$	$\cos\omega t\,u(t)$	$\frac{s}{s^2+\omega^2}$
$\frac{t^{n-1}}{(n-1)!}u(t)$, n = 1,2,…	$\frac{1}{s^n}$	$\sin(\omega t+\theta)u(t)$	$\frac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\cos(\omega t+\theta)u(t)$	$\frac{s\cos\theta-\omega\sin\theta}{s^2+\omega^2}$
$te^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^2}$	$e^{-\alpha t}\sin\omega t\,u(t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$, n = 1,2,…	$\frac{1}{(s+\alpha)^n}$	$e^{-\alpha t}\cos\omega t\,u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

Laplace Transform Operations

Operation	$\mathbf{f(t)}$	$\mathbf{F(s)}$
Addition	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
Scalar Multiplication	$kf(t)$	$kF(s)$
Time Differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - sf'(0^-) - f''(0^-)$
Time Integration	$\int_{0^-}^{t} f(t)\,dt$	$\frac{1}{s}F(s)$
	$\int_{-\infty}^{t} f(t)\,dt$	$\frac{1}{s}F(s) + \frac{1}{s}\int_{-\infty}^{0^-} f(t)\,dt$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Time Shift	$f(t-a)u(t-a), a \geq 0$	$e^{-as}F(s)$
Frequency Shift	$f(t)e^{-at}$	$F(s+a)$
Frequency Differentiation	$-tf(t)$	$\frac{dF(s)}{ds}$
Frequency Integration	$\frac{f(t)}{t}$	$\int_{s}^{\infty} F(s)\,ds$
Scaling	$f(at), a \geq 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Initial Value	$f(0^+)$	$\lim_{s\to\infty} sF(s)$
Final Value	$f(\infty)$	$\lim_{s\to 0} sF(s)$ All poles of $sF(s)$ in LHP
Time Periodicity	$f(t) = f(f + nT)$, n = 1,2,…	$\frac{1}{1-e^{-Ts}}F_1(s)$ Where $F_1(s) = \int_{0^-}^{T} f(t)e^{-st}\,dt$