Impedance and Admittance

Impedance, **Z**, is defined as the voltage-current ratio:

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} \quad (\Omega)$$

For resistors, the impedance is the same as the resistance: $\boldsymbol{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \mathrm{R}$.

For capacitors:

$$i(t) = C\frac{dv(t)}{dt}$$

$$\bar{I} = j\omega C\bar{V}$$

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{1}{j\omega C}$$

Similarly for inductors, $\boldsymbol{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \underline{\mathrm{j}\omega\mathrm{L}}$.

You can add impedances in the same way as resistors. Because all components are now in Ohms, you can add capacitors to inductors to resistors.

Admittance, **Y,** is the inverse of impedance, and can also be useful.

$$\bar{Y} = \frac{\bar{I}}{\bar{V}} \quad (S)$$

You can add admittances in the same way that you add capacitors.

Nodal and Mesh Analysis; Superposition, Source Transformations, and Thévenin's Theorem

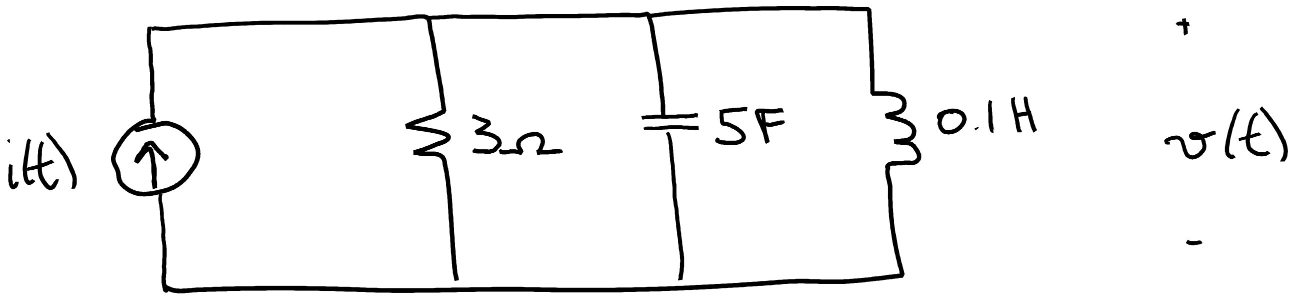
<u>Readings</u>: Sections 10.6, 10.7

The circuits we are looking at are still linear, so we can still apply the techniques from last semester.

Example:

$\omega = 7$

If $\mathrm{i}(\mathrm{t}) = 30\cos(7t + 90°)$, what is $\mathrm{v}(\mathrm{t})$ for the circuit below? Assume there is no natural response.



$\Sigma I_{in} = \Sigma I_{out}$ (nodal analysis)

$$i(t) = \frac{v(t)}{3} + \frac{1}{0.1}\int v(t)\,dt + 5\,\frac{dv(t)}{dt}$$

$$\bar{I} = \frac{\bar{V}}{3} + \frac{10}{7j}\bar{V} + 5 \times 7j\,\bar{V}$$

$$30\angle 90° = \bar{V}\left(\frac{1}{3} + \frac{10}{7j} + 35j\right)$$

$$\bar{V} = \frac{30\angle 90°}{\left(\frac{1}{3} + \frac{10}{7j} + 35j\right)}$$

$$= \frac{21j \times 30\angle 90°}{-705 + 7j} = 0.89\angle 0.6°$$

$$v(t) = 0.89\cos(7t + 0.6°)\ \mathrm{V}$$

As an alternative to using the differential/integral equations for capacitors and inductors, you can use admittances (which are easier when adding in parallel):

$$\bar{Y} = \bar{I}/\bar{V}$$

$$Y = \frac{1}{Z}$$

$$\therefore Y_R = \frac{1}{R}$$

$$Y_C = j\omega C$$

$$Y_L = \frac{1}{j\omega L}$$

$$\bar{V} = \bar{I}/\bar{Y}$$

$$= \frac{30\angle 90^\circ}{\bar{Y}_R + \bar{Y}_C + \bar{Y}_L}$$

$$= \frac{30\angle 90^\circ}{1/3 + j(7\times 5) + \frac{1}{j(7\times 0.1)}}$$

$$= \frac{30\angle 90^\circ}{\frac{1}{3} + 35j - \frac{j}{0.7}}$$

$$= \frac{30\angle 90^\circ}{\frac{1}{3} + 33.6j}$$

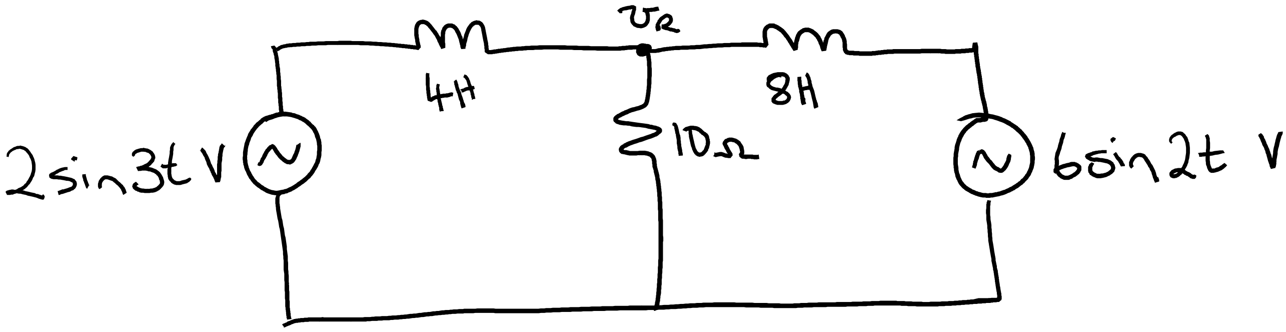
$$= \frac{30\angle 90^\circ}{33.6\angle 89.4^\circ}$$

$$= 0.89\angle 0.6^\circ$$

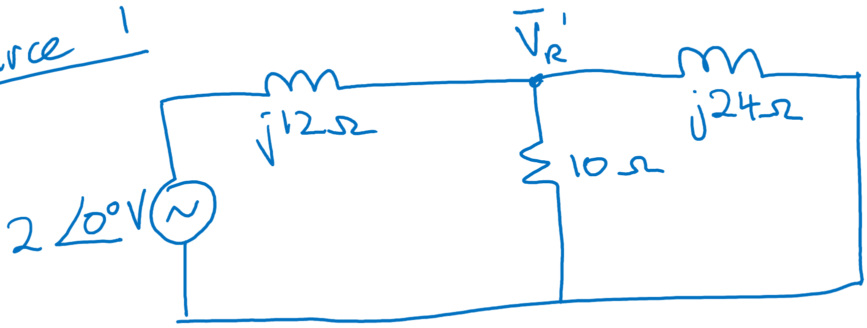
$v(t) = \ldots$ (as before)

If there are two sinusoidal sources with different frequencies, then we can use superposition to solve this, doing the final addition in the time domain. (Note: superposition can also be used if there are multiple sources with the same frequency.)

Example:
For the circuit below, determine the voltage across the resistor. Assume there is no natural response.

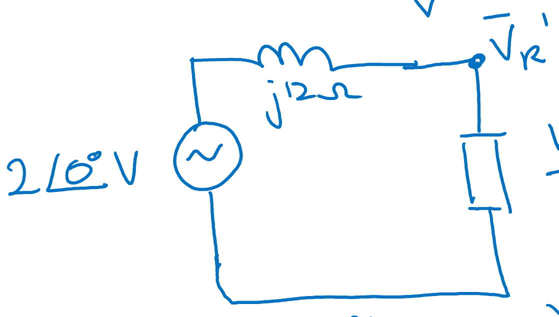


Source 1



($Z_L = j\omega L$)

$\Downarrow$



$$\frac{10 \times j24}{10+j24} = \frac{j240}{10+j24}$$ (add in //)

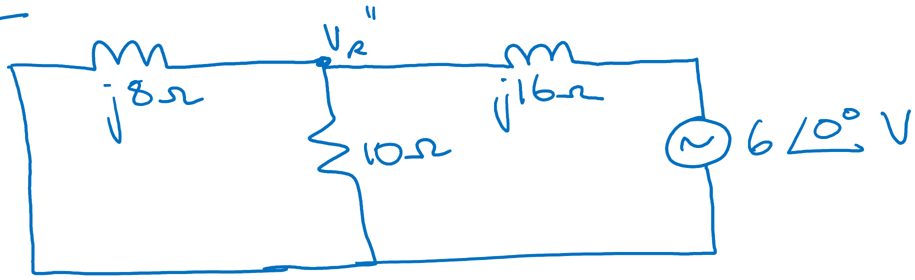
$$\bar{V}_R' = 2\angle 0^\circ \left(\frac{\frac{j240}{10+j24}}{\frac{j240}{10+j24} + j12} \right)$$

$$= \frac{j480}{j240 + j120 - 288}$$

$$= \frac{j480}{-288 + j360}$$

$$= \frac{480\angle 90^\circ}{461\angle 128.7^\circ} = 1.04\angle -38.7^\circ$$

Source 2



$\Downarrow$

$$V_R'' = \frac{\frac{j80}{10+j8}}{\frac{j80}{10+j8} + j16}\, 6\angle 0^\circ = \frac{\frac{j480}{\cancel{10+j8}}}{\frac{j80 + j160 + j^2 128}{\cancel{10+j8}}}$$

$480\angle 90^\circ$

-128

$$= \frac{480\angle 90^\circ}{j80 + j160 - 128}$$

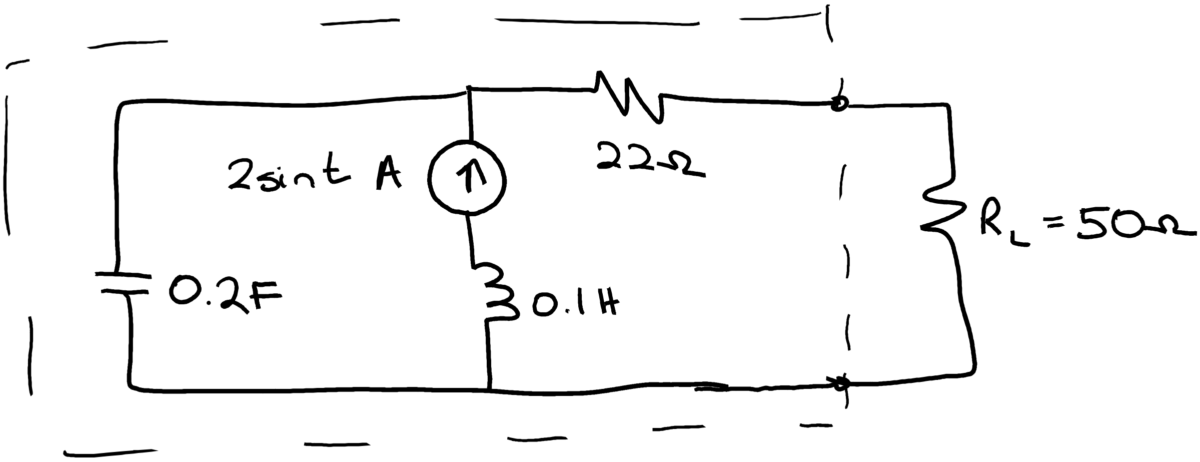
$$= \frac{480\angle 90^\circ}{-128 + j240} = \frac{480\angle 90^\circ}{272\angle 118.1^\circ}$$

$$= 1.76\angle -28.1^\circ \text{ V}$$

$$v_R = v_R' + v_R''$$

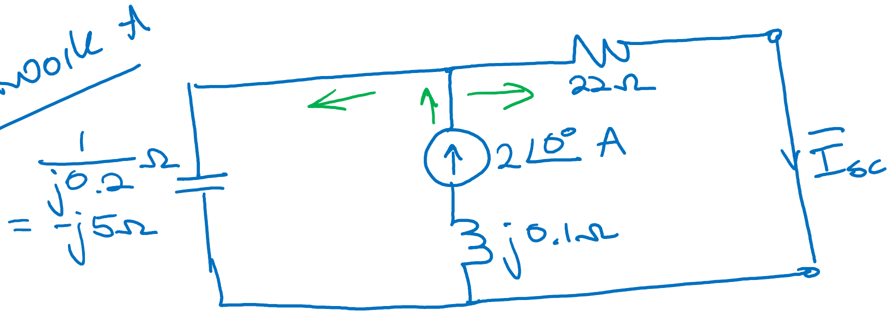
$$= 1.04\sin(3t - 38.7^\circ) + 1.76\sin(2t - 28.1^\circ) \text{ V}$$

Example:
Find the Norton equivalent of circuit A below, then do a source transformation, and find the voltage across the load resistor.



A

network A

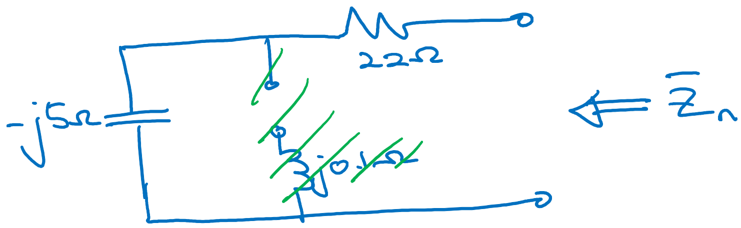


$$I_{sc} = \frac{-j5}{22-j5} \times 2\angle 0^\circ \quad \text{(current divider)}$$

$$= \frac{-5\angle 90^\circ \times 2\angle 0^\circ}{22.56\angle -12.8^\circ}$$

$$= -0.44\angle 102.8^\circ \text{ A}$$

$Z_n = Z_{TH}$



$$\therefore Z_n = 22 - j5\,\Omega$$

Norton Equiv.