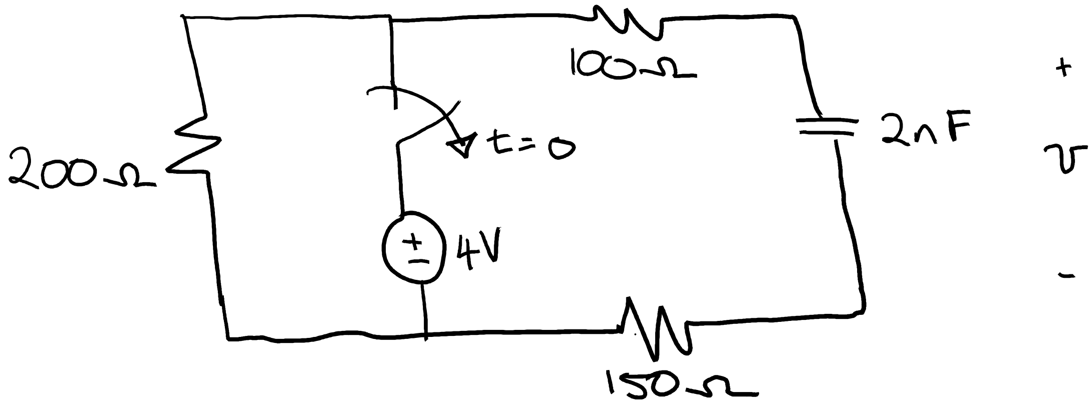
Name: Student ID:

Pre-tutorial 5 Questions (to be attempted before class on May 17th, 2019)

Chapter 8, Ex 20: Source-free RC circuit

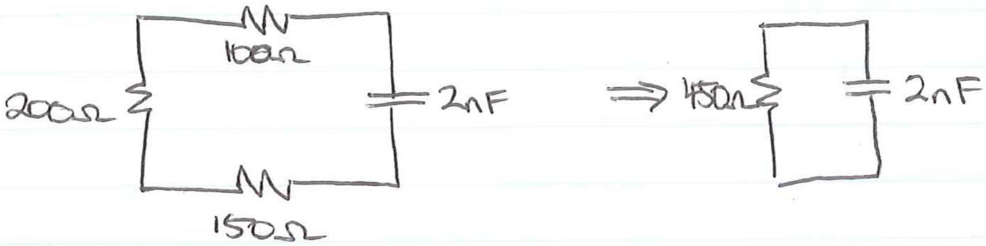
The switch drawn in the circuit below has been closed for such a long time that any transients which might have arisen from first connecting the voltage source have disappeared.



a) Determine the circuit time constant

$\tau = RC$

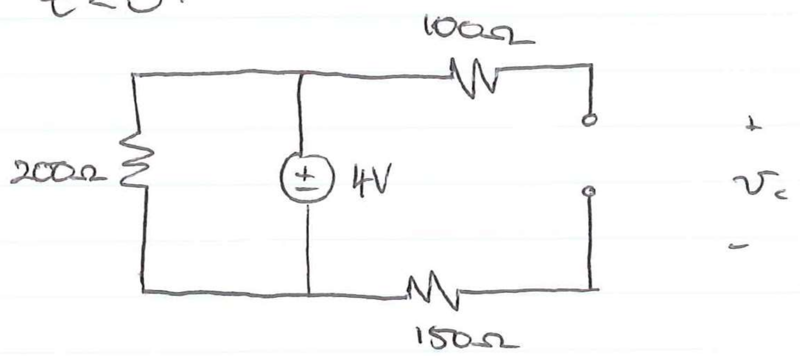
For $t > 0$ circuit is:



$RC = 450 \times 2 \times 10^{-9}$
$= 900ns$

b) Calculate the voltage $v(t)$ at $t = \tau, 2\tau$, and 5τ

$t<0$:



$v_c(0^-) = 4V$

$v_c(0^-) = v_c(0^+) = 4V$

For $t>0$: $v(t) = Ae^{-t/\tau}$

$= Ae^{-t/900\times10^{9}}$

$= Ae^{-1.11\times10^{6}t}$

$v(0) = A\cancel{e^{-1.11\times10^{6}\times0}}^{\,1} = 4$

$A = 4$

$\therefore\ v(t) = 4e^{-1.11\times10^{6}t}$ V $\quad t>0$

$v(\tau) = 4e^{-1}$

$= 1.47$ V

$v(2\tau) = 4e^{-2}$

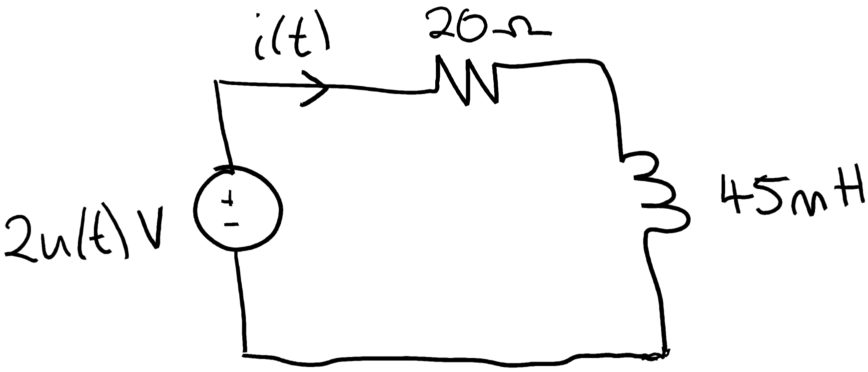
$= 0.54V$

$v(5\tau) = 4e^{-5}$

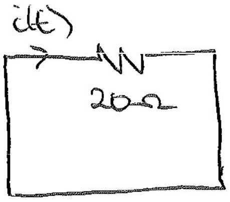
$= 0.027$ V

Chapter 8, Ex 49: Driven RL circuit

The circuit shown below is powered by a source which is inactive for t < 0. Obtain an expression for i(t) valid for all t.

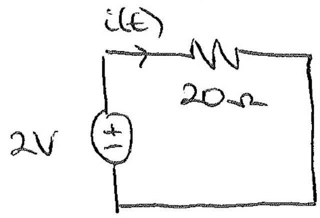


For $t<0$:



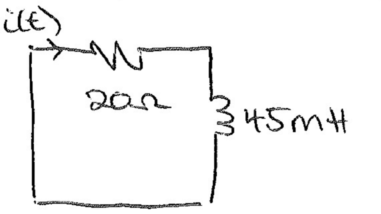
$i(t) = 0\ A$

For $t>0$, forcing function circuit:



$$i_F(t) = \frac{2}{20} = 0.1\ A$$

For $t>0$, natural function circuit:



$$i_n(t) = I_o e^{-t/\tau}$$

$$i_n(t) = I_o e^{-444.4t}$$

$$\tau = \frac{L}{R} = \frac{45\times10^{-3}}{20} = 2.25\times10^{-3}\ s$$

$$1/\tau = 444.4\ s^{-1}$$

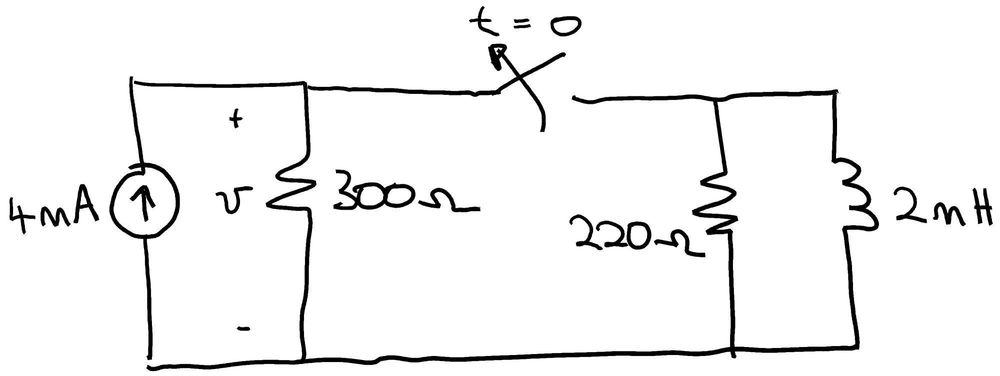
$$i(t) = i_F(t) + i_n(t) \quad t>0$$
$$= 0.1 + I_o e^{-444.4t}$$

$$i(0^-) = i(0^+) = 0 \qquad i(0) = 0.1 + I_o = 0$$
$$I_o = -0.1$$

$$i(t) = 0.1 - 0.1e^{-444.4t}\ A \quad t>0$$
$$= 0.1\left(1 - e^{-444.4t}\right)u(t)\ A$$

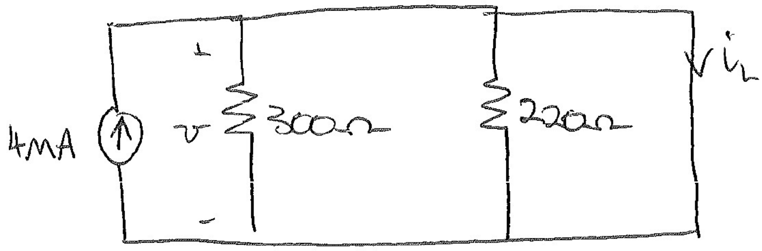
At Tutorial 5 – Marked Question (17th May 2019)

Chapter 8, Ex 8: Source-free RL circuit



The switch in the circuit above has been closed a very long time. Calculate the voltage v as well as the energy stored in the inductor at:

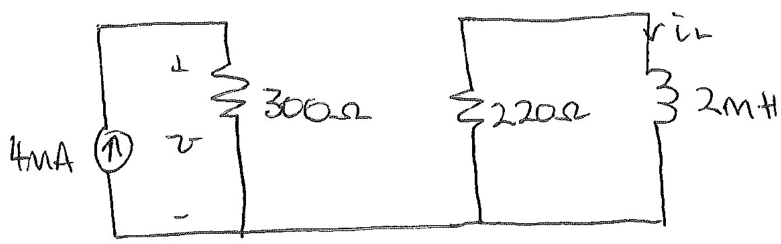
a) The instant just prior to the switch being thrown open



$i_L(0^-) = 4mA$
$v(0^-) = 0V$

$$w(0^-) = \frac{1}{2}Li^2$$
$$= \frac{1}{2} \times 2\times10^{-3} \times (4\times10^{-3})^2$$
$$= 16\times10^{-9}\ J$$

b) The instant just after the switch is opened



$v(0^+) = 4\times10^{-3} \times 300$
$= 1.2V$

$i_L(0^+) = i_L(0^-) = 4mA$

$$w_L(0^+) = \frac{1}{2} \times 2\times10^{-3} \times (4\times10^{-3})^2$$
$$= 16\times10^{-9}\ J$$

c) t = 8 µs

$v(8\mu s) = v(0^+) = 1.2V$

$i = I_o e^{-t/\tau}$

$= 4\times10^{-3} e^{-110\times10^3 t}$

$\frac{1}{\tau} = \frac{R}{L}$

$= \frac{220}{2\times10^{-3}}$

$= 110\times10^3$

$i(8\mu s) = 4\times10^{-3} e^{-110\times10^3 \times 8\times10^{-6}}$

$= 1.66 \text{ mA}$

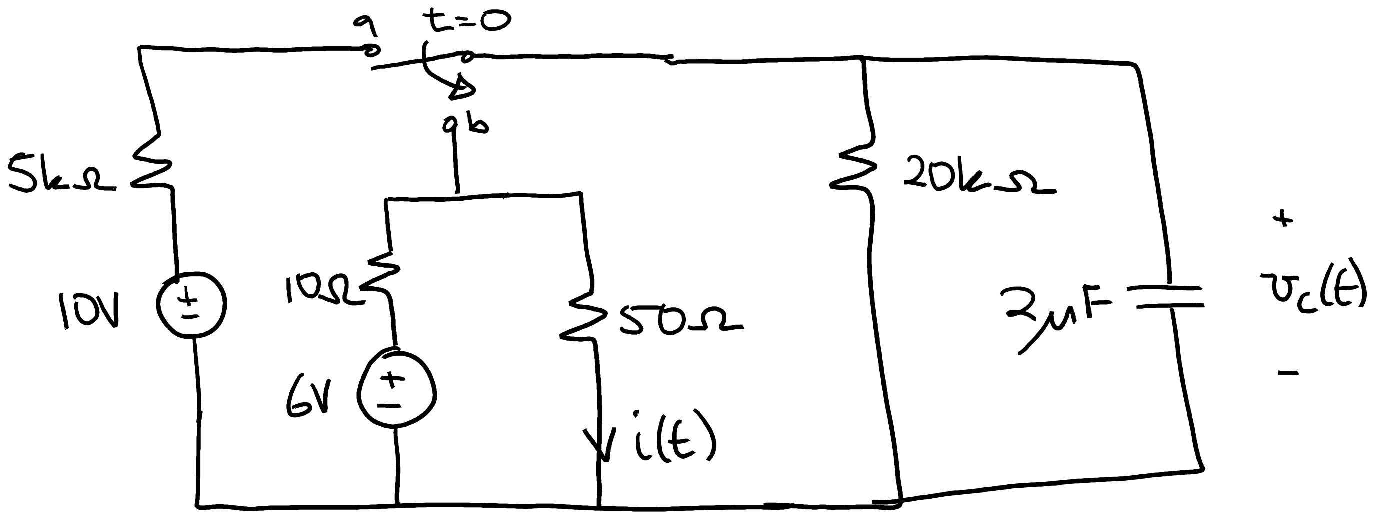
$w(8\mu s) = \frac{1}{2}\times 2\times10^{-3}\times (1.66\times10^{-3})^2$

$= 2.7\times10^{-9} \text{ J}$

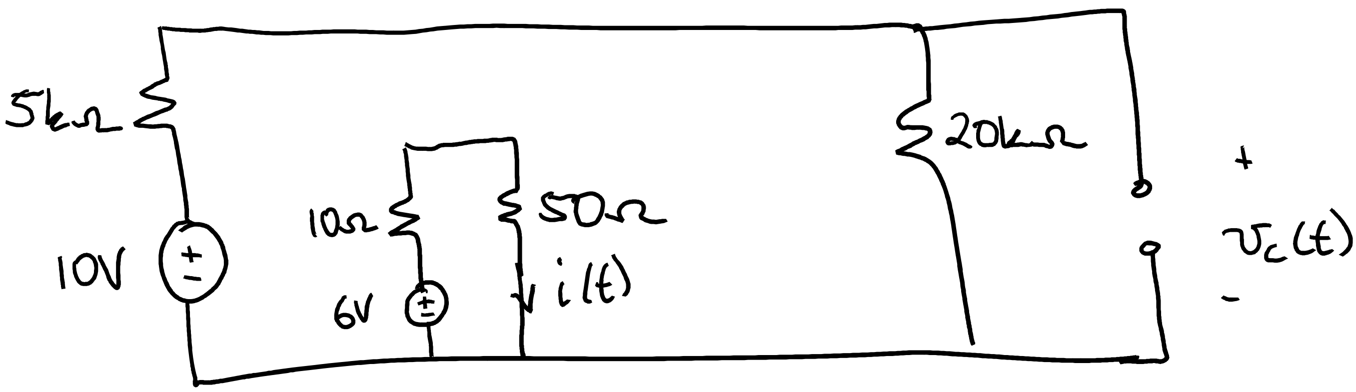
At Tutorial 5 – Unmarked Question (17th May 2019)

Chapter 8, Ex 60: Driven RC circuit

The switch shown in the circuit below has been in position *a* since the original Battlestar Galactica aired on TV. It is moved to position *b* at time t = 0. Obtain expressions for i(t) and $v_c(t)$ valid for all values of t.



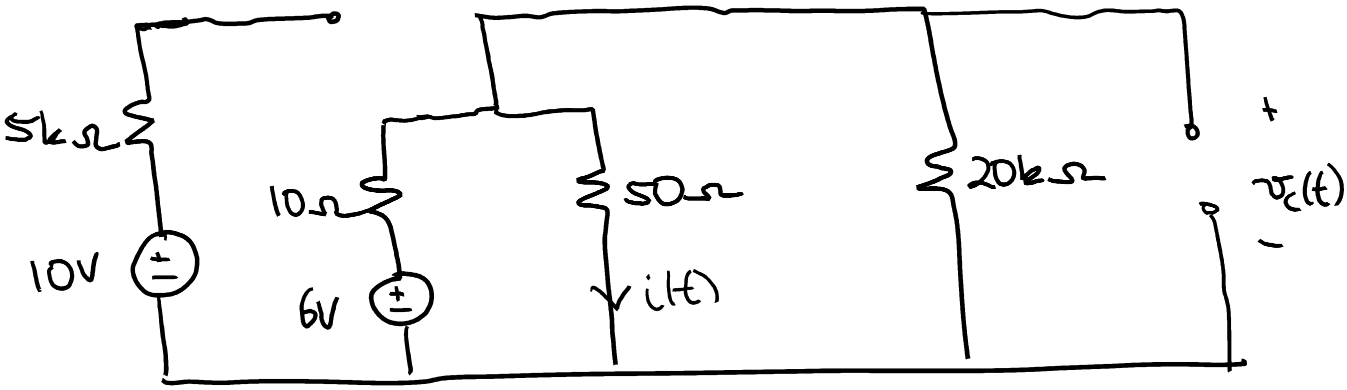
t<0:

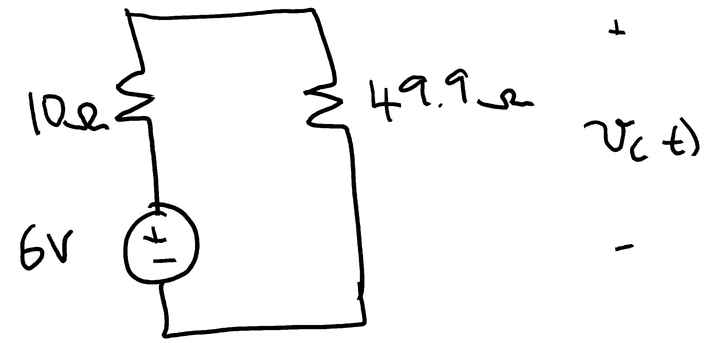


$i(0^-) = 6/60 = 0.1\,A$

$v_c(0^-) = 10 \times \frac{20}{25} = 8V$ (voltage divider)

t > 0 (forced):

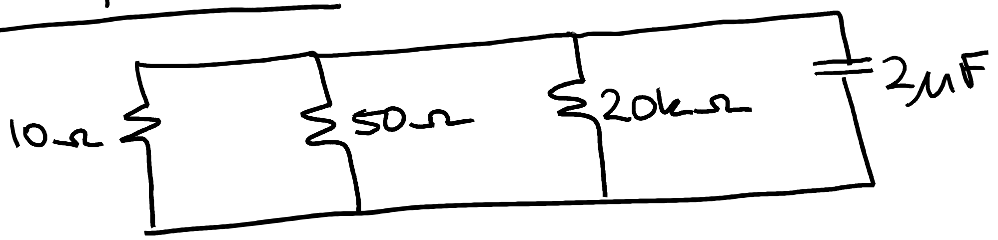


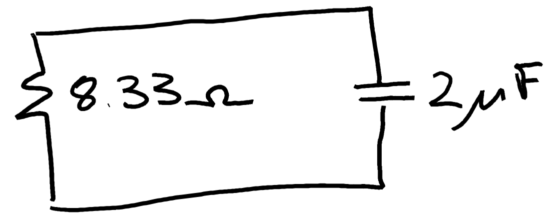
$\Rightarrow$ 

$$R_{eq} = \frac{50 \times 20k}{50 + 20k} = 49.9\,\Omega$$

$$v_{c,F} = 6 \times \frac{49.9}{59.9} \text{ (voltage divider)}$$
$$= 5\text{V}$$
$$i_F = 5/50 = 0.1\text{A}$$

<u>$t > 0$, natural:</u>



$\Rightarrow$ 

$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{50} + \frac{1}{20k}$$
$$R_{eq} = 8.33\,\Omega$$

$$v_{c,n} = V_0 e^{-t/\tau}$$

$$\tau = RC$$
$$\frac{1}{\tau} = \frac{1}{8.33 \times 2\mu}$$
$$= 60 \times 10^3 \text{ s}^{-1}$$
$$v_{c,n} = V_0 e^{-60\times10^3 t}$$

$$v_c = v_{c,F} + v_{c,n} \quad t > 0$$
$$= 5 + V_0 e^{-60\times10^3 t}$$
$$v_c(0^-) = v_c(0^+) = 8$$

$v_c(0) = 5 + V_0 = 8$

$V_0 = 3$

$$v_c = \begin{cases} 8\ V & t \leq 0 \\ 5 + 3e^{-60 \times 10^3 t} & t > 0 \end{cases}$$

$$= 8 - 3\left(1 - e^{-60 \times 10^3 t}\right) u(t) \quad V$$

$$i(t) = v/R$$

$$= \frac{5 + 3e^{-60 \times 10^3 t}}{50} \qquad t > 0$$

$$= 0.1 + 0.06e^{-60 \times 10^3 t}\ A \quad t > 0$$

$$= \begin{cases} 0.1\ A & t \leq 0 \\ 0.1 + 0.06e^{-60 \times 10^{3t}}\ A & t > 0 \end{cases}$$