Analogue to Digital Conversion I

ENCE361 Embedded Systems 1

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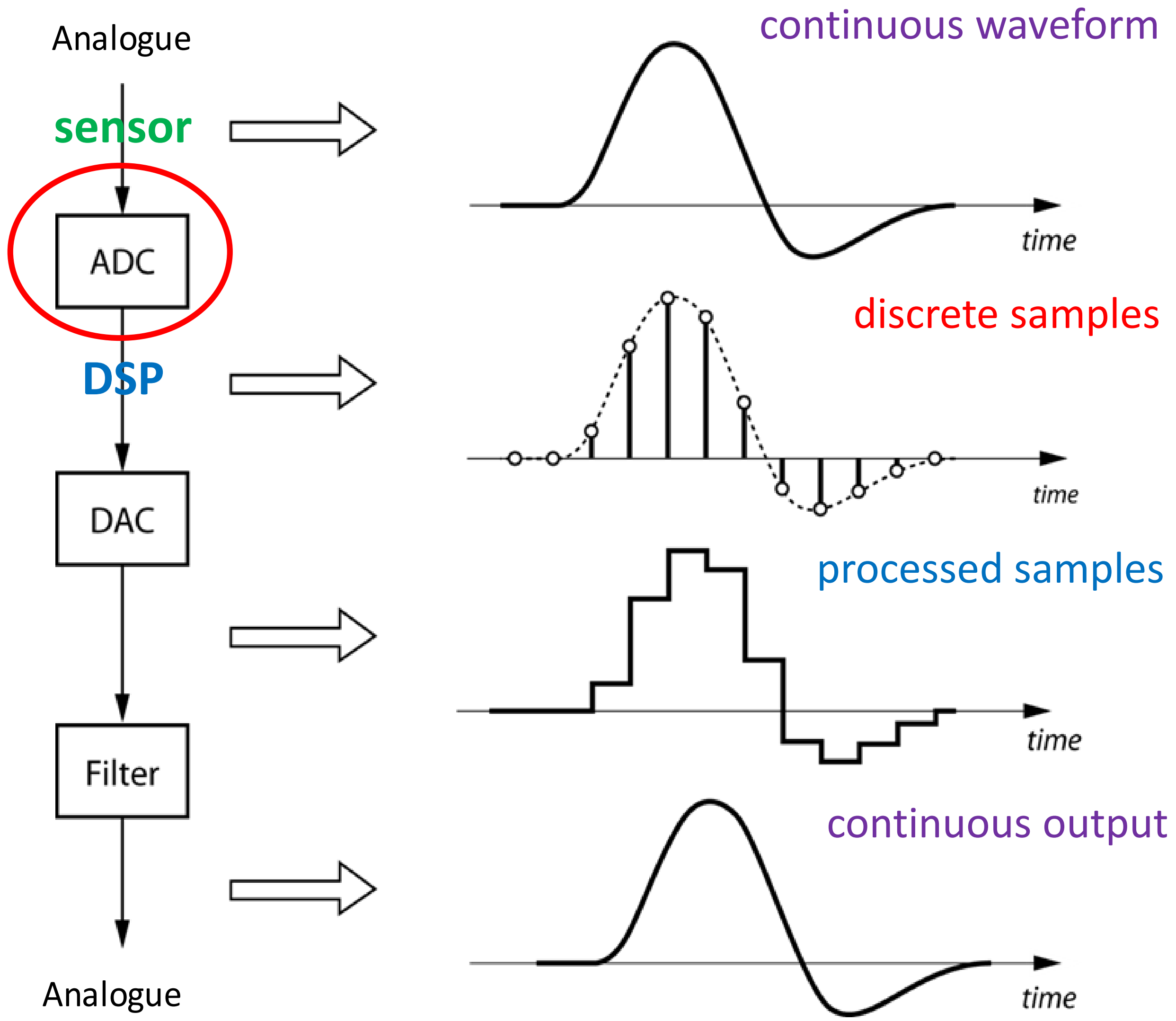
Where we're going today

- **Analogue to digital conversion overview**
- Nyquist-Shannon sampling theorem
- Aliasing and anti-alias filter
- Homework

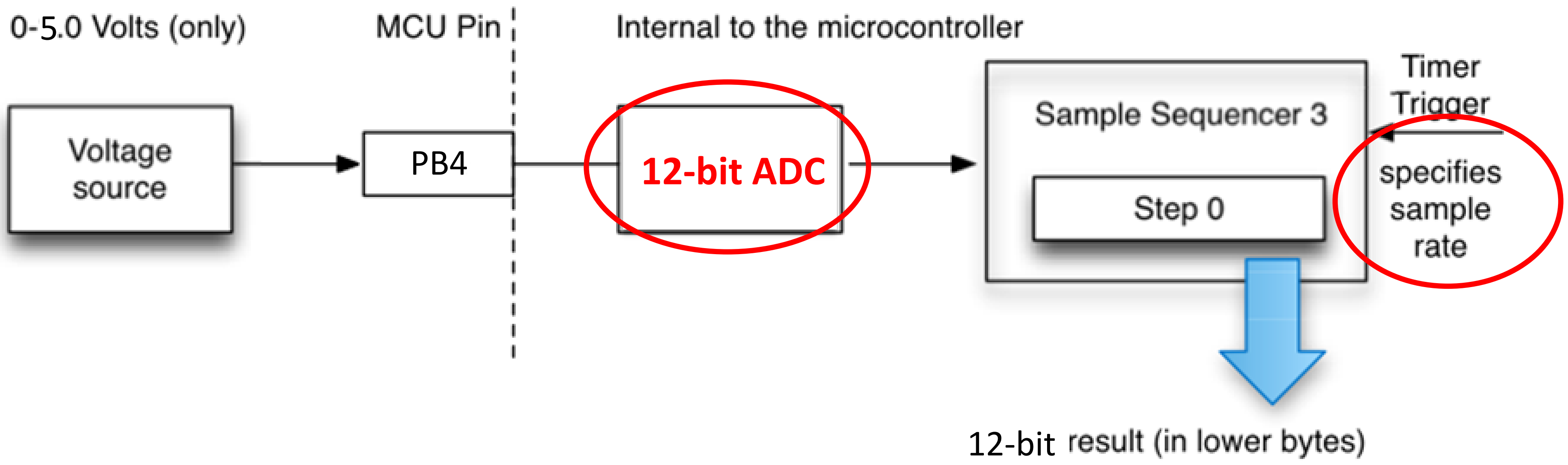
Digital Signal Processing

- Digital processing of analogue signals
  - Analogue to digital converter (ADC)
  - Digital signal processor (DSP)
  - Digital to analogue converter (DAC)

- Advantages:
  - Allow sophisticated algorithms to be realized in software
  - Reproducible and relatively low cost
  - …

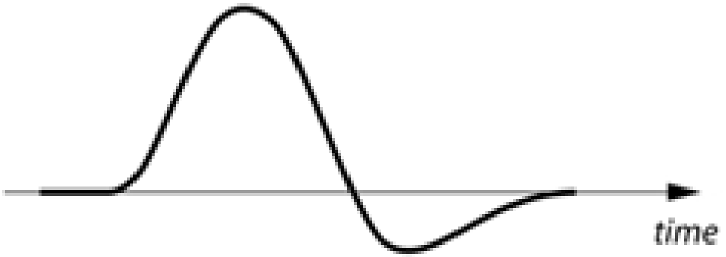


Analogue to Digital Conversion in MCU

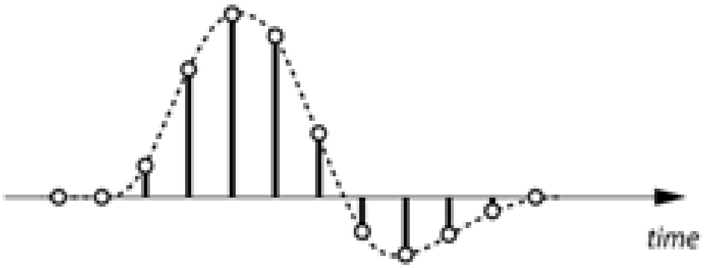


Continuous Signals vs. Digital Signals

- Continuous signals
  - Continuous-time
  - Continuous-amplitude
  - Cannot be directly processed by a DSP



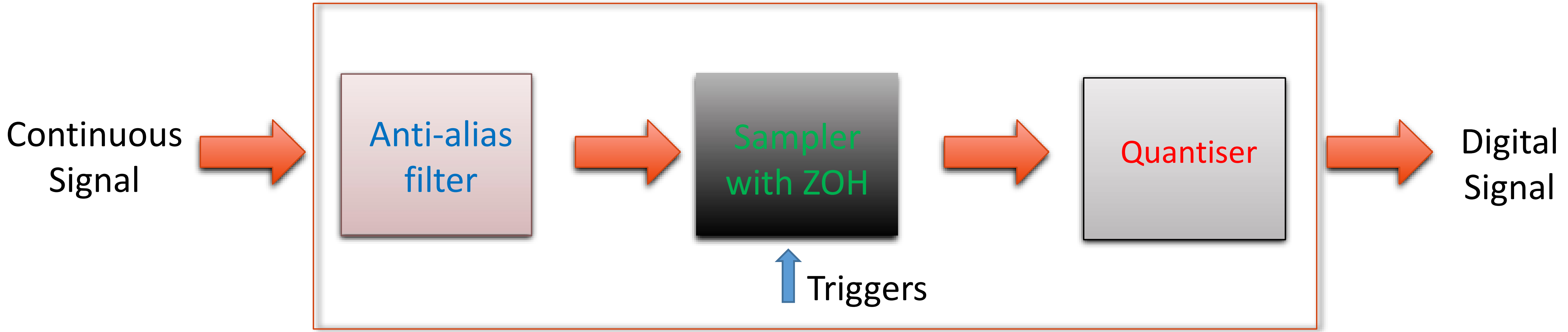
- Digital signals
  - Discrete-time
  - Discrete-amplitude



- Continuous-time to discrete-time: sampling
- Continuous-amplitude to discrete-amplitude: quantization (more in lecture 4)

Analogue to Digital Converter (ADC)

- ADC diagram



- Sampler with zero-order hold (ZOH) holds the input value until next conversion trigger
- Quantiser maps the held input value into one of possible discrete values
- Anti-alias filter removes frequencies of the input that might alias (see next 4 slides)

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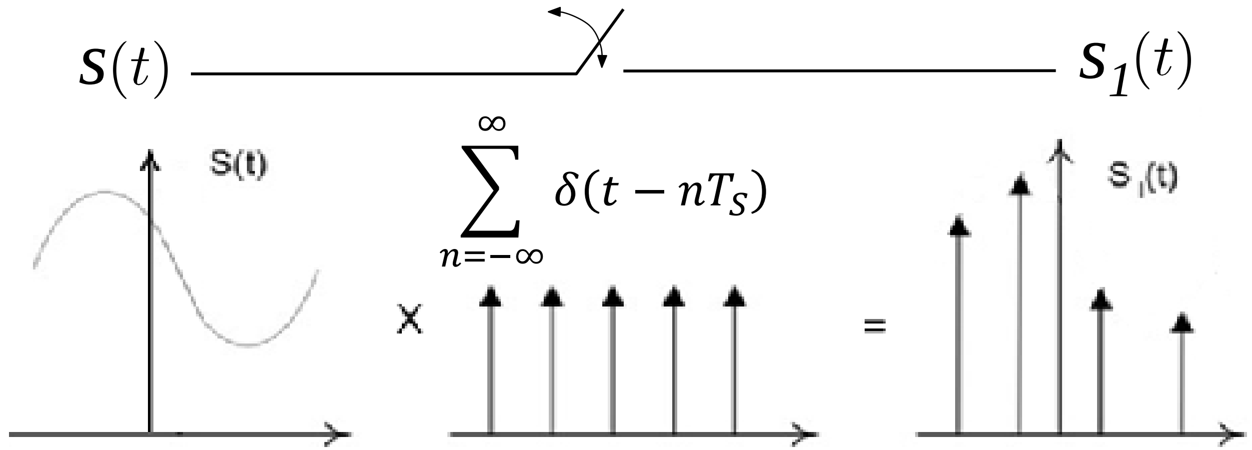
Nyquist-Shannon Sampling Theorem (1)

- Continuous-time to discrete-time: sampling
  - How fast should we sample a continuous signal so that it can be fully recovered from discrete samples?
- Ideal sampling using a uniform unit impulse train (T_s is sampling period)



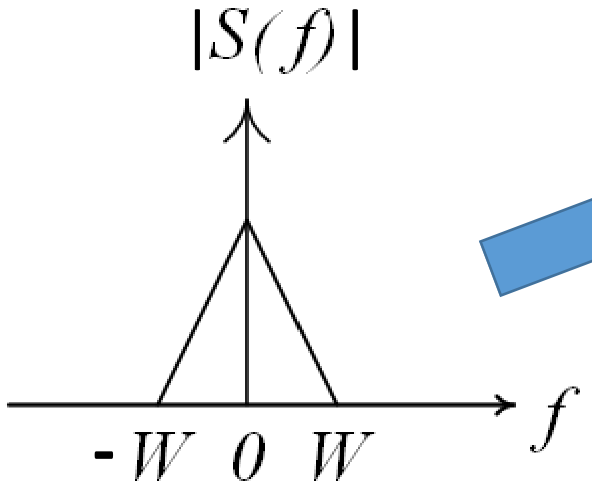
$$S_I(t) = S(t) \times \sum_{n=-\infty}^{\infty} \delta(t - nT_S) = \sum_{n=-\infty}^{\infty} S(nT_S) \cdot \delta(t - nT_S)$$

Nyquist-Shannon Sampling Theorem (2)

Sampling Frequency

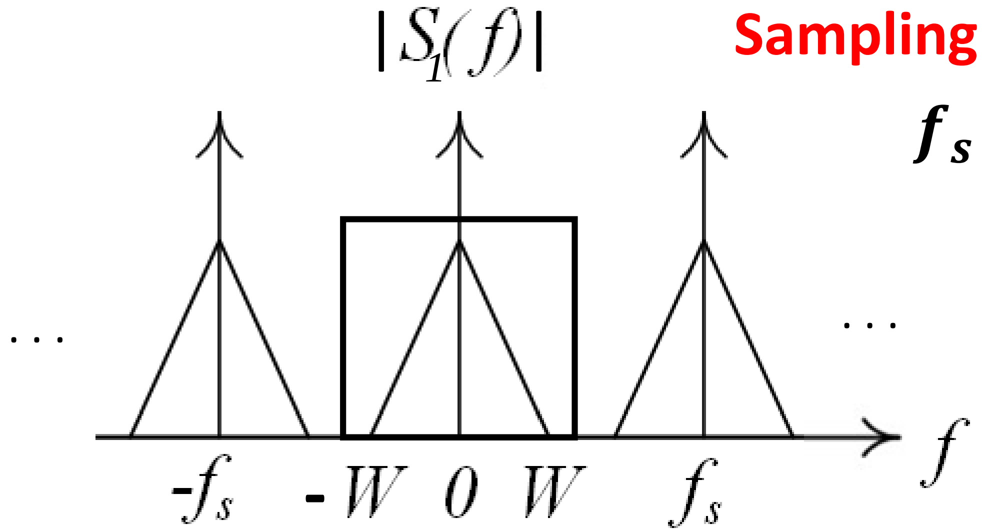
$$f_s = \frac{1}{T_s}$$

Fourier Transform of $S(t)$

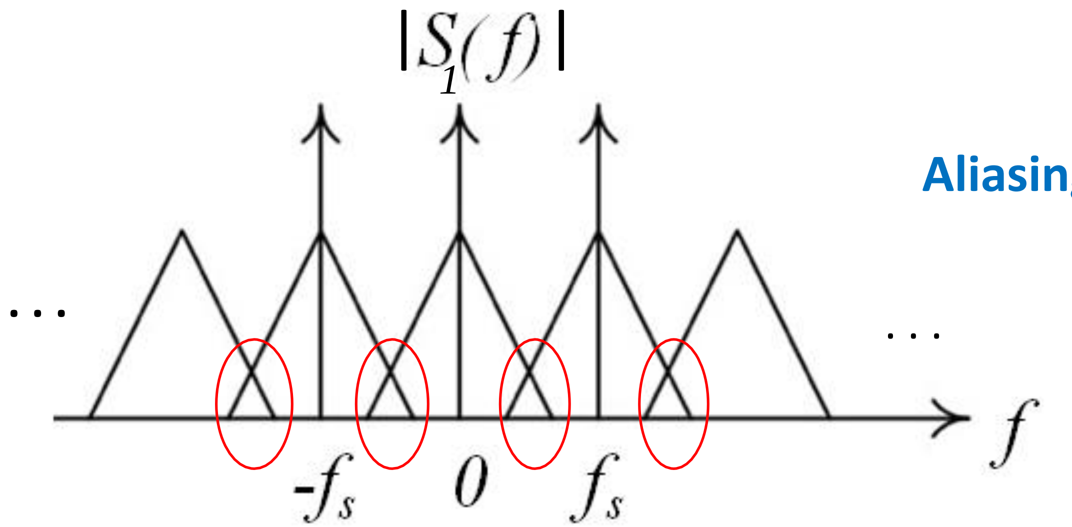


W: bandwidth

$f_s \geq 2W$



$f_s < 2W$



Aliasing occurs

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- **Aliasing and anti-alias filter**
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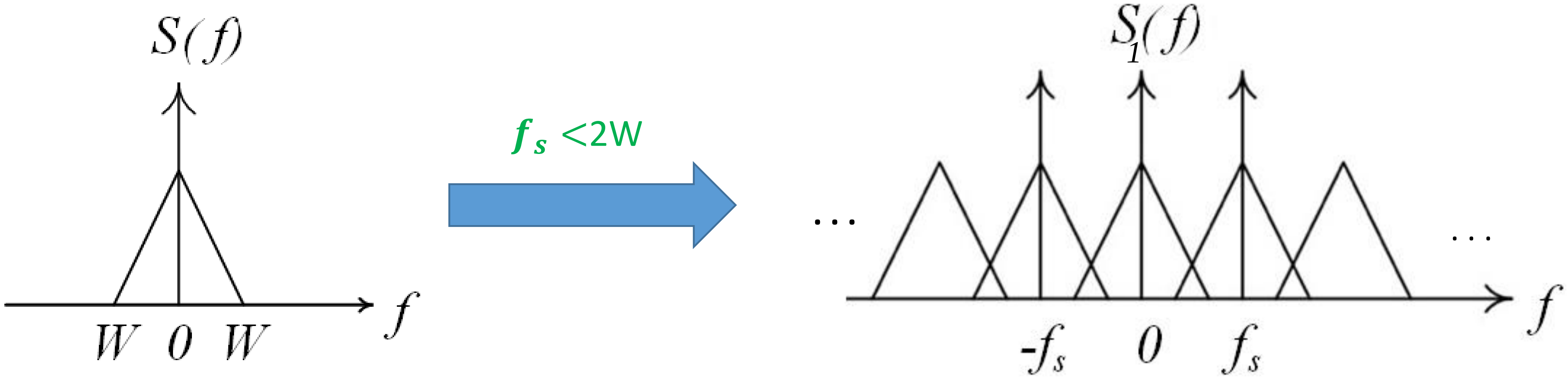
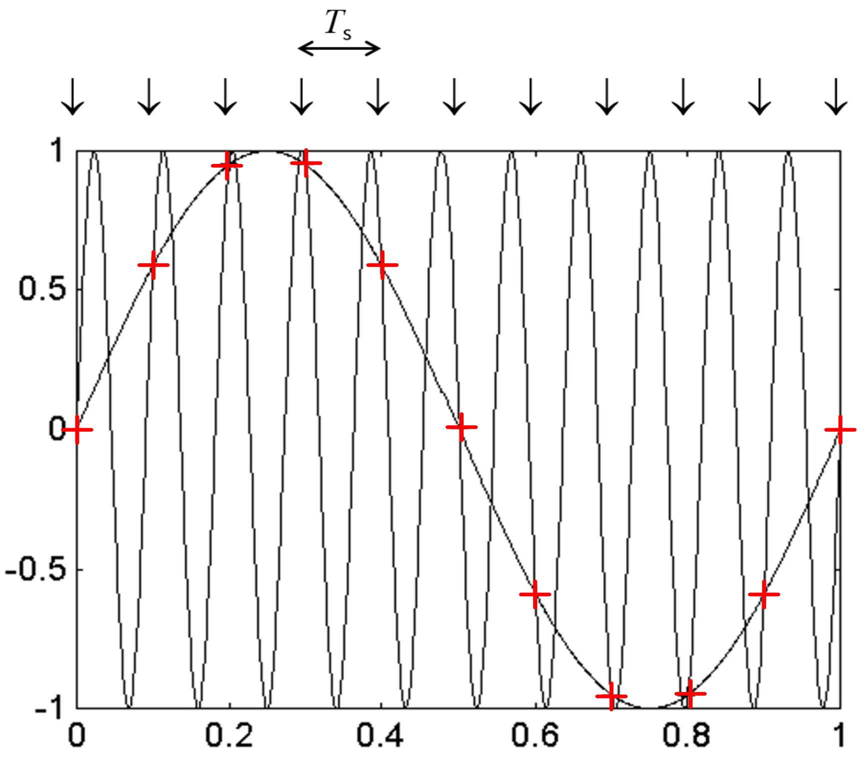
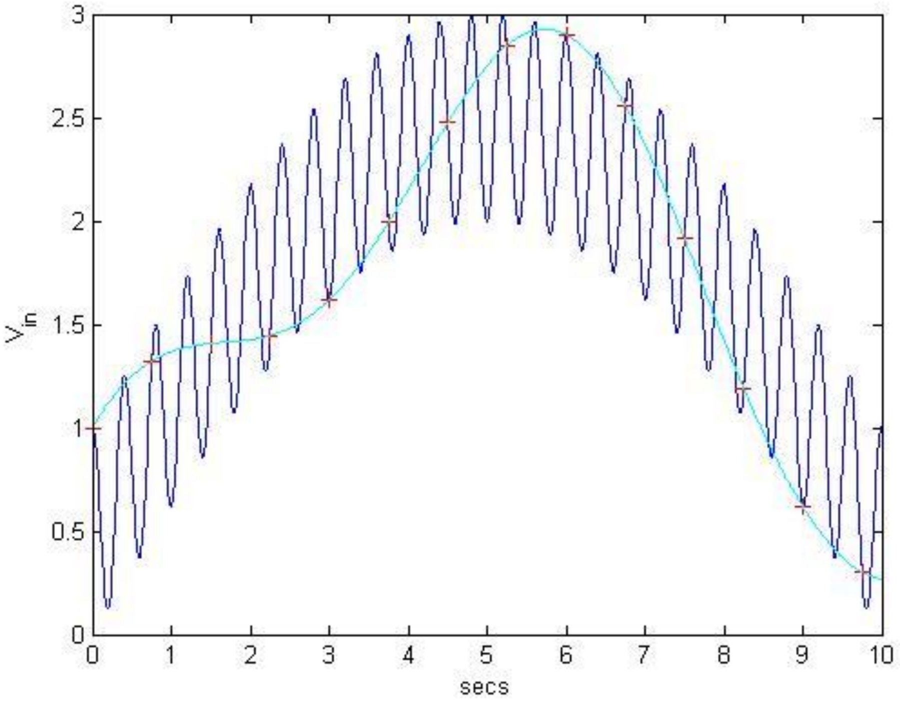
Nyquist Rate



- If sampling frequency f_s <2W, aliasing occurs
  - *W*: highest input signal frequency
- If sampling frequency f_s ≥2W, no aliasing
- 2W: Nyquist rate for fully reconstructing original waveform from samples
- In practice, f_s may need to be **much higher** than Nyquist rate

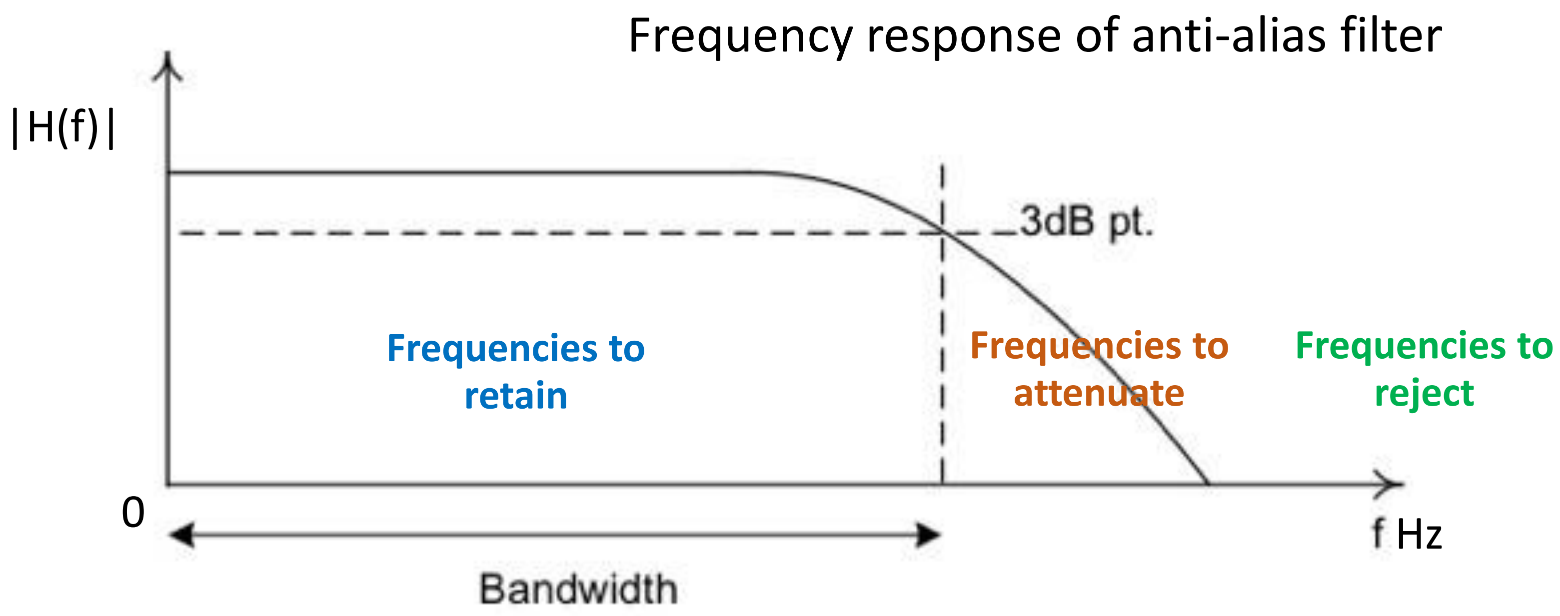
Aliasing Example





- Sample a 11 Hz sinewave at $\boldsymbol{f_s}$ = 10 Hz
  - Cannot tell samples whether the samples are from a 11 Hz sinewave or a 1 Hz sinewave
- Sample analogue altitude signal in helicopter controller at low rate
  - An uncomfortable ride !

Anti-Alias Filter



- Anti-alias filter is a low-pass filter that 'band-limits' the input signal
  - High frequencies are attenuated to reduce aliasing due to sampling
  - Analog signal conditioning
- The characteristics of the anti-alias filter determines how much above the highest frequency of interest the sampling rate needs to be

Digital Signal Conditioning

- Noise is everywhere

$$y(nT_S) = S(nT_S) + e(nT_S)$$

  - $e(nT_S)$: noise (more in lecture 6)
    - Electronic (thermal) noise within analogue circuity
    - Interference from other nearby signals
    - Sampling jitter …

- Digital signal conditioning via signal averaging

$$z(nT_S) = \frac{1}{M}\sum_{m=0}^{M-1} y((n-m)T_S)$$

  - Reduce noise power by a factor of M for independent & identically distributed (i.i.d.) noise samples ☺
  - 'Decrease' the sampling rate by a factor of $\frac{1}{M}$ ☹

Homework

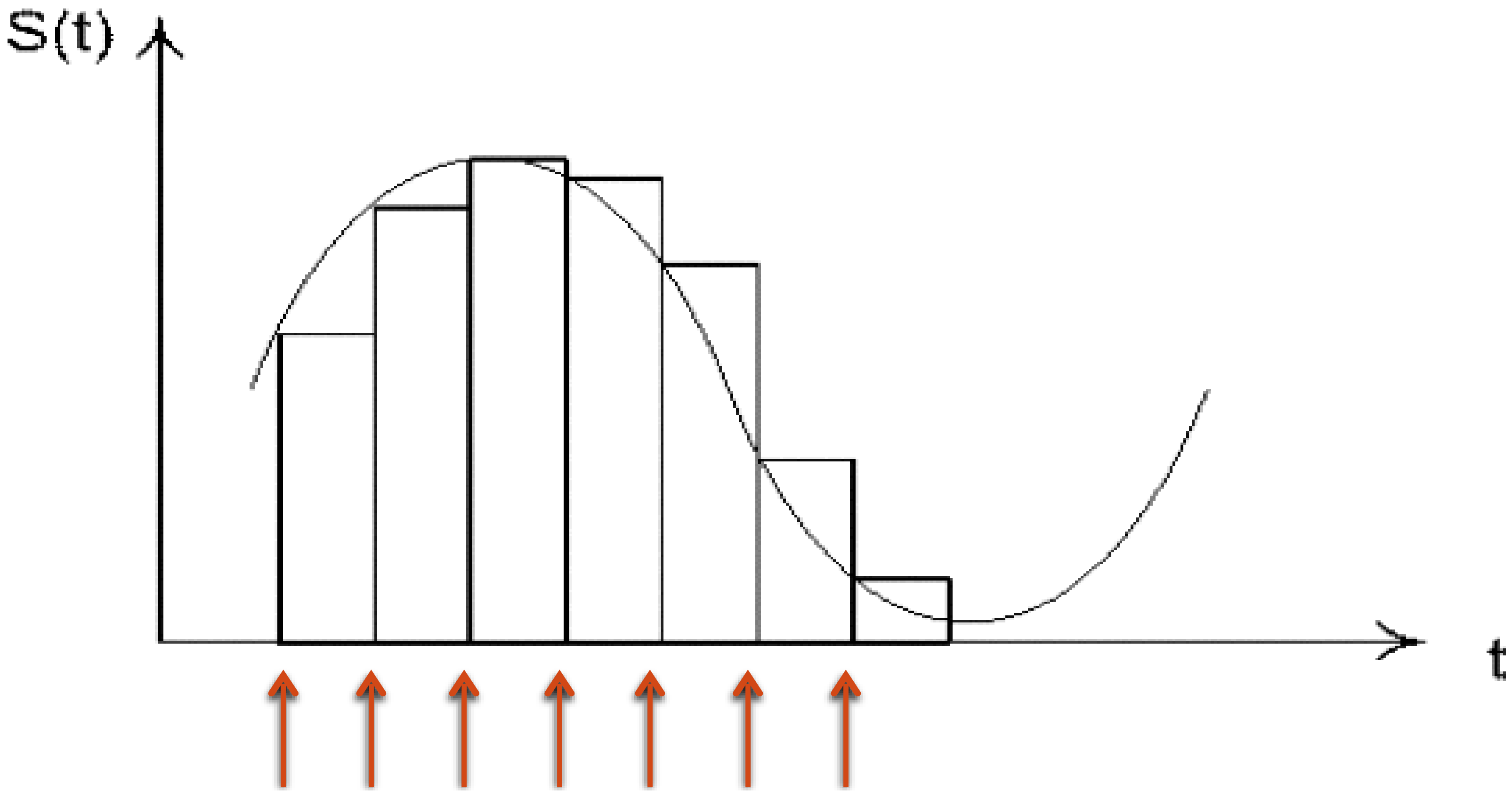
1. According to Shannon-Nyquist theorem, we must sample at more than twice the highest frequency to reproduce the original waveform. Why not sampling **at** twice the highest frequency?

2. What is the condition called that refers to the distortion caused by sampling below the Nyquist rate?

3. Slide 9 outlines and mathematically defines ideal sampling. What does $1/T_s$ represent and what units does it have?

4. If signal averaging (see slide 14) is used for sampling an input signal of with a highest frequency f_{max}, what is the relationship between M, f_{max} and the sampling rate f_s to ensure that the signal averaging operation does not introduce significant aliasing?

Zero-Order Hold (ZOH) Sampling



- Sampler <u>holds</u> the input value at the conversion trigger for quantization until the next conversion trigger arrives