

RMS voltage calculations

ENCE361

For a periodic waveform, period T ,

$$V_{RMS} = \left[\frac{1}{T} \int_0^T v(t)^2 dt \right]^{\frac{1}{2}} \quad V$$

For $v(t) = A \sin 2\pi ft$ ($f = \frac{1}{T}$),

$$V_{RMS} = \frac{A}{\sqrt{2}} \quad V$$

However, for $v(t) = A(1 + \sin 2\pi ft)$, i.e. an offset sinusoid with minimum value 0 and maximum value $2A$, the RMS value is significantly different.

$$\begin{aligned} V_{RMS} &= \left[\frac{A^2}{T} \int_0^T (1 + \sin 2\pi t/T)^2 dt \right]^{\frac{1}{2}} \\ &= \left[\frac{A^2}{T} \int_0^T (1 + 2\sin 2\pi t/T + \sin^2 2\pi t/T) dt \right]^{\frac{1}{2}} \\ &= \left[\frac{A^2}{T} \int_0^T \left(1 + 2\sin 2\pi t/T + \frac{1}{2} + \frac{1}{2} \cos 4\pi t/T \right) dt \right]^{\frac{1}{2}} \\ &= \left[\frac{A^2}{T} \left[t + t/2 \right]_0^T \right]^{\frac{1}{2}} \\ &= \frac{\sqrt{3} A}{\sqrt{2}} \quad V \end{aligned}$$

What about V_{RMS} for an offset pulsed waveform?