Upper Atmosphere and Ionosphere Assignment 5

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1 Introduction

In order to improve upon the existing ionosphere structure developed in HW4, I attempted to add electron and ion cooling rates. I calculate these terms to develop a more accurate equation for energy dissipation in the previously derived tridiagonal matrix for electron temperature. I also add a tridiagonal matrix for ion temperature and similarly add/subtract terms to/from the ion energy dissipation.

2 Background

2.1 Tridiagonal Matrix for Ion Temperature

I essentially used the same tridiagonal matrix for the ions as I did for the electrons. The only major difference is that I used the following thermal conductivity for ions provided in class

$$\lambda_i = \frac{4.6 \times 10^4 T_i^{5/2}}{A_i^{1/2}} \tag{1}$$

Where A_i is the atomic weight given in units of [AMU].

2.2 Developing the Appropriate Momentum Transfer Collision Frequencies

I began by identifying the equations for calculating the appropriate loss terms. Using equation 4.129c in Schunk & Nagy, one of the linear collision terms for the 13 moment approximation.

$$\frac{\delta E_s}{\delta t} = -\Sigma \frac{n_s m_s \upsilon_{st}}{m_s + m_t} 3k(T_s - T_t) \tag{2}$$

2.3 Calculating Collision Frequencies

Equation 4.129c in Schunk & Nagy is dependent upon collision frequency. I identified the appropriate collision frequency based on the nature of the interaction (ie. electron-ion, electron-neutral, etc.). I considered the necessary interactions and evaluate the given equations for each species taken from Shunk & Nagy.
For the electron(cool)-neutral(heat) interaction we have

$$\upsilon_{eN_2} = 2.33 \times 10^{-11} n(N_2)(1 - 1.21 \times 10^{-4} T_e) T_e \tag{3}$$

$$\upsilon_{eO_2} = 1.82 \times 10^{-10} n(O_2)(1 + 3.6 \times 10^{-2} T_e^{1/2}) T_e^{1/2} \tag{4}$$

$$\upsilon_{eO} = 1.82 \times 10^{-10} n(O)(1 + 3.57 \times 10_{-4} T_e) T_e^{1/2} \tag{5}$$

For the neutral(cool)-ion(heat) interactions we have

$$\upsilon_{O^+O} = 3.67 \times 10^{-11} n(O) T_r^{1/2} (1 - 0.064 log_{10} T_r)^2 T_e^{1/2} \tag{6}$$

$$\upsilon_{N_2^+N_2} = 5.14 \times 10^{-11} n(N_2) T_r^{1/2} (1 - 0.069 log_{10} T_r)^2 T_e^{1/2} \tag{7}$$

$$\upsilon_{O_2^+O_2} = 2.59 \times 10^{-11} n(O_2) T_r^{1/2} (1 - 0.073 log_{10} T_r)^2 T_e^{1/2} \tag{8}$$

In these equations the average temperature, T_r, is

$$T_r = \frac{T_i + T_n}{2} \tag{9}$$

For the electron-ion interaction the following equation applies

$$\upsilon_{ei} = 54.5 \frac{n_i Z_i^2}{T_e^{3/2}} \tag{10}$$

Z, particle charge number, is equal to one in this instance because all ions are singly charged.

When calculating the ion-neutral interactions we must consider both resonant and non-resonant collision frequencies. They are taken from Schunk & Nagy Table 4.5 and equation 4.146 respectively.

$$\upsilon_{in} = C_{in} n_n \tag{11}$$

The coefficients are taken from Schunk & Nagy Table 4.4.

2.4 Electron Cooling Rates

Chapter 9 in Schunk & Nagy provides electron cooling rates. I calculated these values for the N_2 rotation, O_2 rotation, O_2 vibration, and O fine structure contributions. It should be noted that I attempted to calculate the N_2 vibration contribution with no success. The above mentioned equations are

N_2 rotation

$$L_e(N_2) = \frac{3.5 \times 10^{-14} n_e n(N_2)(T_e - T_n)}{T_e^{1/2}} \tag{12}$$

O_2 rotation

$$L_e(O_2) = \frac{5.2 \times 10^{-15} n_e n(O_2)(T_e - T_n)}{T_e^{1/2}} \tag{13}$$

O_2 vibration

$$L_e(O_2) = n_e n(O_2) Q(T_e)\{1 - exp[2239(T_e^{-1} - T_n^{-1})] \tag{14}$$

Where

$$\begin{aligned}\log_{10}[Q(T_e)] = \Big(&-19.9171 + 0.0267T_e - 3.9960 \times 10^{-5} T_e^2 \\ &+3.5187 \times 10^{-8} T_e^3 - 1.9228 \times 10^{-11} T_e^4 \\ &+6.6865 \times 10^{-15} T_e^5 - 1.4791 \times 10^{-18} T_e^6 \\ &+2.0127 \times 10^{-22} T_e^7 - 1.5346 \times 10^{-26} T_e^8 \\ &+5.0148 \times 10^{-31} T_e^9\Big)\end{aligned} \tag{15}$$

Oxygen fine structure

$$\begin{aligned}L_e(O) =& n_e n(O) D^{-1} \big(S_{10}\{1 - exp[98.9(T_e^{-1} - T_n^{-1})]\} \\ &+S_{20}\{1 - exp[326.6(T_e^{-1} - T_n^{-1})]\} \\ &+S_{21}\{1 - exp[227.7(T_e^{-1} - T_n^{-1})]\}\big)\end{aligned} \tag{16}$$

Where

$$D = 5 + exp - 326.6T_n^{-1} \tag{17}$$

$$S_{21} = 1.863 \times 10^{-11} \tag{18}$$

$$S_{21} = 1.191 \times 10^{-11} \tag{19}$$

$$S_{21} = 8.249 \times 10^{-11} \tag{20}$$

2.5 Energy Dissipation

Upon calculating all quantities we add or subtract these terms from the already developed equations for energy dissipation, Q, in the tridiagonal matrix. Subtraction or addition of a given term is determined by the source of the exchange (ie. loss or gain).

3 Results

The following plots are provided with captions discussing the results.

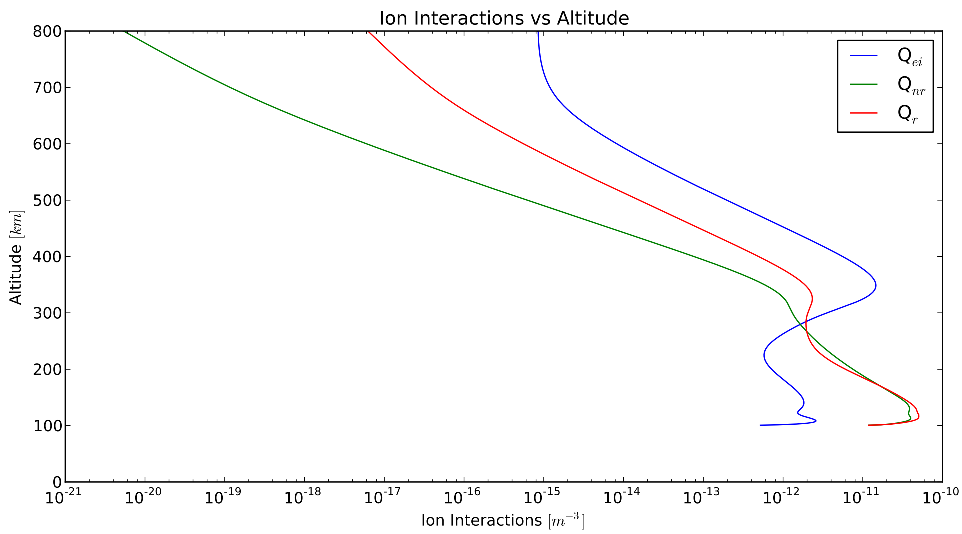

Figure 1: This is a plot of the Ion interactions vs Altitude. I don't have much to say about this because I have no physical intuition regarding what the behavior of these loss terms should look like.

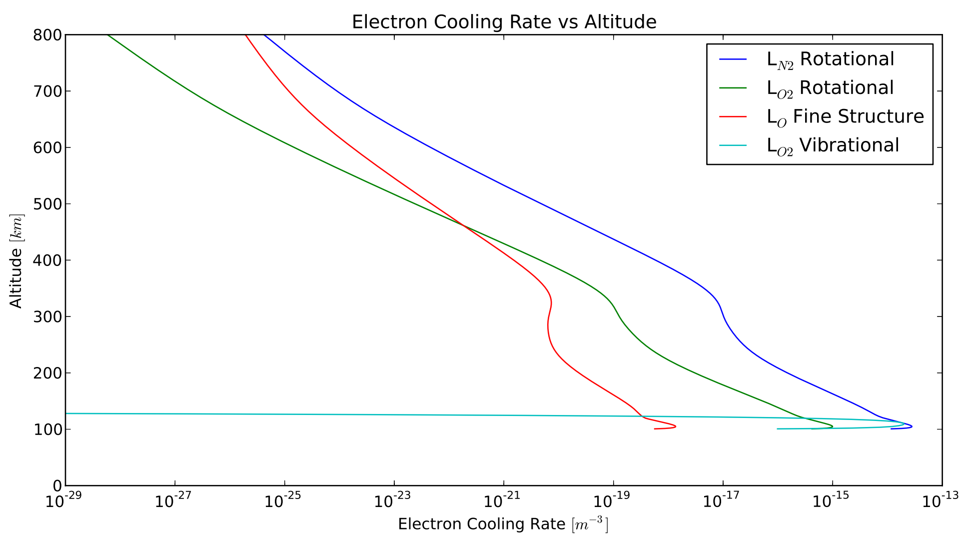

Figure 2: These are the electron coolins rates. Again I don't have much to say about this because I have no physical intuition regarding what the behavior of these loss terms should look like. I find it interesting the the L_{O2} term is so small compared to the rest. But this could just mean that the term isn't very important with respect to cooling.

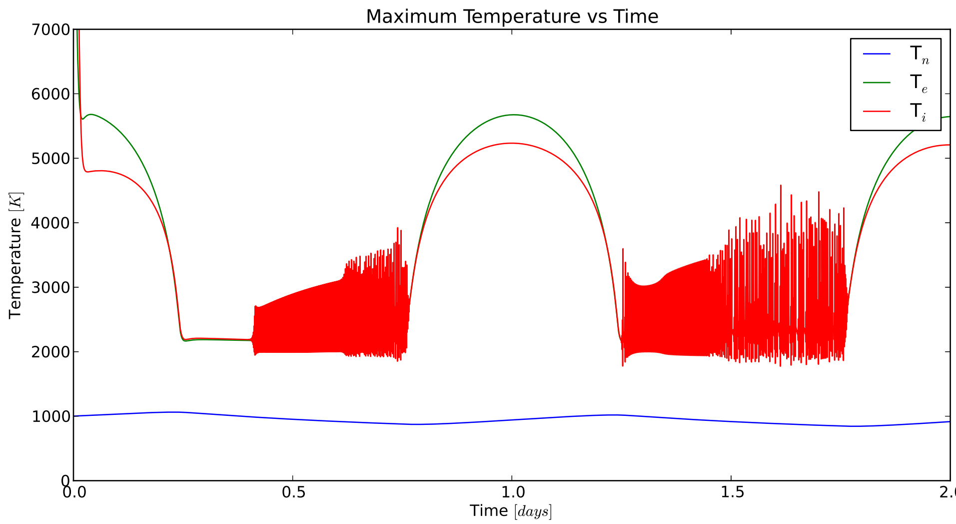

Figure 3: This is a plot of Maximum Temperature vs Time. I believe that this plot is the most important regarding my results. As you can see I ran my model over a period of two days in an attempt to flush out any numerical quirks. For some reason the ion temperature is not behaving very well. This is clearly due to a numerics problem.

4 Discussion & Conclusion

This was a difficult assignment. I must admit that I did not get things working how they should. In order to get the ion temperature to do something somewhat well behaved I had to fudge a few orders of magnitude in the ion interaction terms. Even with this fudge factor they still do not behave how they should. As previously mentioned I also couldn't get the N_2 vibration contribution to work. My intuition tells me that the elevtron temperatures are close to what they should be. By adding the N_2 vibration contribution I cannot predict how much this will alter the results. On the other hand the ion temperatures shown above are far too hot. If I were able to find what was going wrong with my ion temperature calculation I believe that they would be much cooler than the electrons (yet still hotter than the neutrals).