

# Extraction of Airways from CT Data

## Using Bayesian Smoothing

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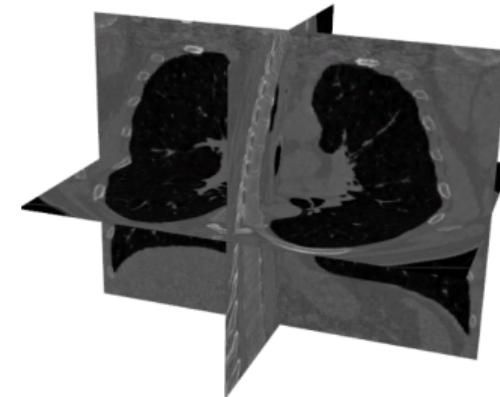
# Diagnosis

- Spirometry
  - Lung function tests
  - Volume of air inhaled & exhaled
    - Patient dependent
    - Low reproducibility
    - Mild cases go unnoticed



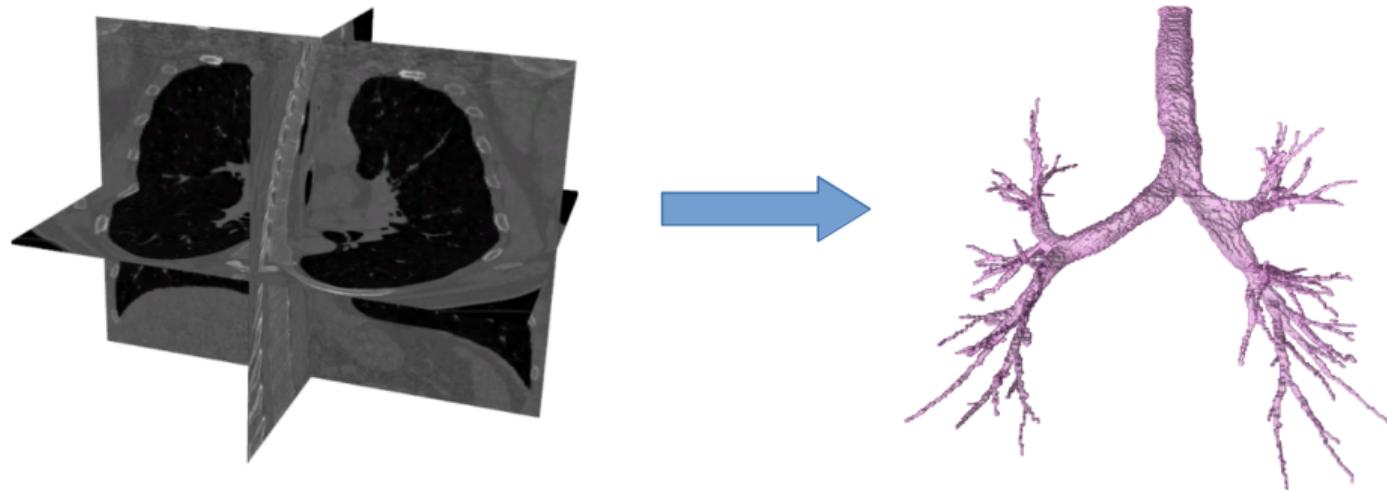
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- Spirometry
  - Lung function tests
  - Volume of air inhaled & exhaled
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  - Low reproducibility
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- 3D Chest computed tomography (CT)
  - Lot more information
  - Arduous to read the data; even for experts
  - Low inter-observer agreement



# Objective

Airway tree segmentation from CT to obtain useful COPD biomarkers



# Existing Methods

- **Sequential in nature**
  - Like Region-growing based methods
  - Susceptible to local anomalies
  - Acquisition noise / mucous plugs / artifacts



## Desired Properties

A stochastic and *exploratory* algorithm that is computationally tractable



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A stochastic and *exploratory* algorithm that is computationally tractable

- Probability densities to capture uncertainties
- Incorporate rich domain knowledge (*priors*)
- Particle filters do exist, but
  - Computationally expensive (volume data, several hundred branches)
  - Track sequentially

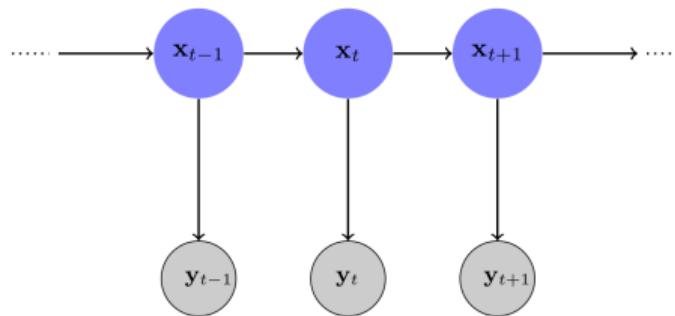


## Idea

Formulate tree *segmentation* as *Bayesian Tracking* of individual branches



# Bayesian Tracking



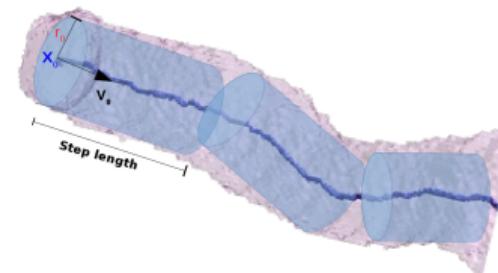
- Probabilistic state-space approach to tracking
- Strong model-based methods (almost) readily applicable
- State estimation from *a posteriori* density
- Batch data (like CT images) → Bayesian Smoothing



# Probabilistic State-space Model

## Model

- Airway tree as a set of *independent* branches  
 $\mathbf{X} = \{\mathbf{X}_1, \dots, \mathbf{X}_T\}$
- Each branch as a sequence of state vectors  
 $\mathbf{X}_i = [\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_{I_i}]$
- State vector at each step  
 $\mathbf{x}_k = [x, y, z, r, v_x, v_y, v_z]^T$
- Vectorised image data  $\mathbf{Y} = [\mathbf{y}_0, \dots, \mathbf{y}_T]$ ;  
 $\mathbf{y}_k = [x, y, z, r]^T$



## Objective

To estimate the *a posteriori* distribution:

$$p(\mathbf{X}|\mathbf{Y})$$



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$$p(\mathbf{X}|\mathbf{Y}) \approx \prod_i^T p(\mathbf{X}_i|\mathbf{Y})$$



# RTS (Kalman) Smoother

Density per branch,  $p(\mathbf{X}_i|\mathbf{Y})$ , approximated using RTS<sup>1</sup> Smoother.

- Recursively estimate posterior density
- Closed form, simple-to-compute expressions
- Provides Gaussian density estimates at each step
- Inherent uncertainty estimates

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<sup>1</sup>Rauch-Tung-Striebel



# Probabilistic State-space Models

## Process Model (Transition Density)

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{q} \equiv p(\mathbf{x}_k | \mathbf{x}_{k-1}) \quad (1)$$

$\mathbf{F}$ : State transition function,  $\mathbf{q} \sim N(\mathbf{0}, \mathbf{Q})$ : Process noise,  
 $\mathbf{x}_k = [x, y, z, r, v_x, v_y, v_z]^T$



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## Measurement Model (Measurement Likelihood)

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{m} \equiv p(\mathbf{y}_k | \mathbf{x}_k) \quad (2)$$

**H:** Measurement function,  $\mathbf{m} \sim N(\mathbf{0}, \mathbf{R})$ : Measurement noise,  
 $\mathbf{y}_k = [x, y, z, r]^T$



# Validation of Tracked branches

- Exploratory algorithm → Several Candidate branches
- Select likely candidates based on tracking confidence

$$\mu_i = \frac{\sum_{k=1}^{l_i} \text{Tr}(\mathbf{P}_{k|k})}{l_i}. \quad (3)$$

$\mathbf{P}_{k|k}$  is posterior covariance matrix at step  $k$ .

- Measure of how well tracked branches fit process & meas. models

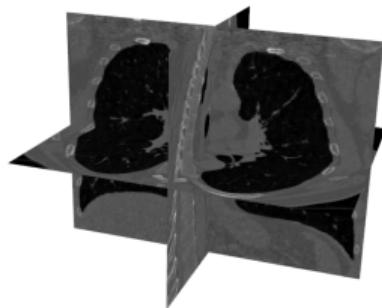


# Data

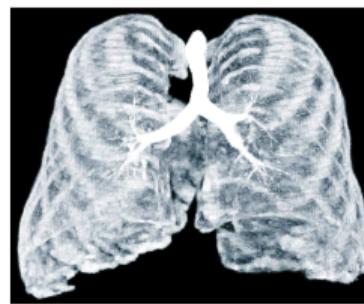
- Danish Lung Cancer Screening Trial
- Low-dose Chest CT scans
- 32 scans split into Training and Test sets
- Reference consists of expert verified union of results from two previous methods



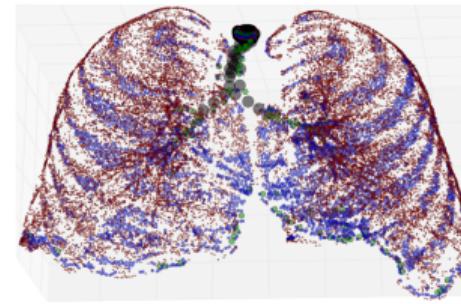
# Application to Airways



(a) Intensity image



(b) Probability image



(c) Multi-scale blob image

- Intensity to Probability Image using trained voxel classifier
- Normalised multi-scale blob detection (Laplacian of Gaussians)
- 3-D Volume Data  $\rightarrow$  4-D Measurement Vector ( $\mathbf{y}_k = [x, y, z, r]^T$ )



# Evaluation: CT Chest Data



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**Figure:** Input Data, Segmented results before and after thresholding to remove false positive branches.

In the segmented results, pink surface is the reference segmentation and blue lines correspond to the tracked centerlines.



# Results

- Error Measure based on centerline distances
- Average of two distances,  $d_{err} = (d_{FP} + d_{FN})/2$
- Compared with Region Growing (RG) on Probability image
- RTS+RG: Results from proposed method (RTS) merged with RG

Table 2: Performance comparison on the test set

Method	$d_{FP}$ (mm)	$d_{FN}$ (mm)	$d_{err}$ (mm)	Std.Dev. (mm)
RG	0.423	3.579	2.001	0.208
(RTS+RG) <sub>1</sub>	0.449	2.102	1.276	0.187
(RTS+RG) <sub>2</sub>	0.401	2.658	1.529	0.165

- (RTS+RG)<sub>1</sub>: Large improvement in  $d_{err}$  for a small increase in  $d_{FP}$
- (RTS+RG<sub>2</sub>): Simultaneous improvement in  $d_{FP}$ ,  $d_{FN}$



# Summary

- Medical images present interesting challenges
- Segmentation of Airways for COPD diagnosis
- Bayesian Tracking presented as a useful approach
- Exploratory algorithm; can overcome local anomalies
- Extract tree as set of branches
- Uncertainty estimates to qualify branches
- Broader applications beyond airways (vessels, neurons etc.)



# Questions/ Comments



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<sup>2</sup>[https://en.wikipedia.org/wiki/Tobacco\\_\(Last\\_Week\\_Tonight\)](https://en.wikipedia.org/wiki/Tobacco_(Last_Week_Tonight))

# Recursive estimation of Mean, Cov.

Table 1: Standard RTS Smoother Equations

## Forward Filtering

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}\hat{\mathbf{x}}_{k-1|k-1} \quad (6)$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}\mathbf{P}_{k-1|k-1}\mathbf{F}^T + \mathbf{Q} \quad (7)$$

$$\mathbf{v}_k = \mathbf{y}_k - \mathbf{H}\hat{\mathbf{x}}_{k|k-1} \quad (8)$$

$$\mathbf{S}_k = \mathbf{H}\mathbf{P}_{k|k-1}\mathbf{H}^T + \mathbf{R} \quad (9)$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}^T\mathbf{S}_k^{-1} \quad (10)$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \mathbf{v}_k \quad (11)$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T \quad (12)$$

## Backward Smoothing

$$\mathbf{G}_k = \mathbf{P}_{k|k}\mathbf{F}^T\mathbf{P}_{k+1|k}^{-1} \quad (13)$$

$$\hat{\mathbf{x}}_{k|L} = \hat{\mathbf{x}}_{k|k} + \mathbf{G}_k(\hat{\mathbf{x}}_{k+1|L} - \hat{\mathbf{x}}_{k+1|k}) \quad (14)$$

$$\mathbf{P}_{k|L} = \mathbf{P}_{k|k} - \mathbf{G}_k(\mathbf{P}_{k+1|k} - \mathbf{P}_{k+1|L})\mathbf{G}_k^T \quad (15)$$

