Examination

Mathematical Foundations for Software Engineering

Course code DIT022

Date:	2019-01-07
Time:	08:30-12:30
Place:	Lindholmen
Teacher:	Christian Berger Michel Chaudron Richard Torkar
Visit to e*xam hall:*	09:30 and 11:00
Questions:	6
Results:	Will be posted by 2019-01-28.
Grade Limits:	Pass (G) 50%, Pass with honors (VG) 90%
Allowed aids:	Calculators: Casio FX-82..., Texas TI-30... and Sharp EL-W531... Included appendix.

Please observe the following:

- DO NOT write your name on any answer sheet or exam sheet – write the anonymized code instead.
- Write in legible English (unreadable responses mean no points!).
- Motivate your answers and clearly state any assumptions made.
- Start each part of the exam on a new sheet!
- Write only on one side of the paper!
- Only answers written on the answer sheets will be graded, do not write on the exam sheets!
- Before handing in your exam, number and sort the sheets in task order!

NOTE:

Not following these instructions may result in the deduction of points!

Question 1 (1 + 1 + 1 + 1 + 3 + 4 = 11 pt)
"Languages"

1.1 What language is generated by the given grammar G1? (1pt)
Note that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, "choosing multiple solutions when only one is correct" will not be assessed as correct. Mark your choice(s) on your answer sheets, not this exam sheet.

G1 = (V, T, S, P) and V = {S, X}, where S is the start variable, T = {0,1} set of terminals and rules:

S → X1
X→ 0 | X0

a) {w∈ {0,1} * | w starts with a 0 and ends with a 1}
b) {w∈ {0,1} * | w contains only 0s except for the last symbol which is a 1}
c) {w∈ {0,1} * | w contains 1s and 0s}
d) {w∈ {0,1} * | w contains at least one 1 and at least one 0}
e) {w∈ {0,1} * | w contains firstly a random series of 1s and 0s, and a 1 as the last symbol}

1.2 What language is generated by the given grammar G2? (1pt)
Note that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, "choosing multiple solutions when only one is correct" will not be assessed as correct. Mark your choice(s) on your answer sheets, not this exam sheet.

G2 = (V, T, S, P) and V = {S, C, D}, where S is the start variable, T = {a,b, λ} set of terminals and rules:

S → aCb | bDa
C→ bDa | λ
D→ aCb | λ

a) {w∈ {a,b} * | w is a random series of `b's and `a's}
b) {w∈ {a,b} * | the length of w is even and the middle symbol is `b'}
c) {w∈ {a,b} * | w contains the same amount of `a's and `b's in alternating order} (Example: ababab, bababa, …)
d) {w∈ {a,b} * | the length of w is odd and the middle symbol is `a'}
e) {w∈ {a,b} * | w contains `a's followed by the same amount of `b's, or vice-versa} (Example: aaabbb, bbbaaa, …)

1.3 What language is generated by the given grammar G3? (1pt)
Note that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, "choosing multiple solutions when only one is correct" will not be assessed as correct. Mark your choice(s) on your answer sheets, not this exam sheet.

Note, a^m stands for a series which consist of a in m times, or " $\underbrace{a \ldots a}_{m\ times}$ *".*

G3 = (V, T, S, P) and V = {S, A}, where S is the start variable, T = {a, b, c, d} set of terminals and rules:

S→aSdd | A
A→bAc | bc

a) $\{a^{\frac{n}{2}}b^n c^m d^n$ | where $n \geq 0$ and $m > 0\}$
b) $\{a^{2n}b^m c^m d^{2n}$ | where $n, m \geq 0\}$
c) $\{a^m b^m c^m d^m$ | where $m > 0\}$
d) $\{a^n b^m c^m d^{2n}$ | where $n \geq 0$ and $m > 0\}$
e) $\{a^n b^n c^n d^{2n}$ | where $n \geq 0\}$

1.4 What language is generated by the given grammar G4? (1pt)
Note that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, "choosing multiple solutions when only one is correct" will not be assessed as correct. Mark your choice(s) on your answer sheets, not this exam sheet.

G4 = (V, T, S, P) and V = {a, b, …, z, λ, S}, where S is the start variable, T = {a, b, …, z, λ} set of terminals and rules P:

S→ aSa | bSb | cSc | … | zSz
S→ λ | a | b | c | … | z

a) The language defines words of odd length.
b) The language defines all words in the English language.
c) The language defines all words containing an arbitrary number of identical characters.
d) The language defines all words in the English language of even length.
e) The language defines all palindromes[1] over the English alphabet.

[1] A palindrome is any string which reads the same backwards and forwards.

1.5 Provide **three** different non-empty words that are generated by the given grammar G5. (3pt)

G5 = (V, T, S, P) and V = {postalcode, forward_or_stationarea, space, localdeliveryunit, provarea, loctype, letter, digit, rural, urban, A, B, C, E, G, H, J, K, L, M, N, P, R, S, T, V, X, Y, W, Z, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, where postalcode is the start variable, T ={"A", "B", "C", "E", "G", "H", "J", "K", "L", "M", "N", "P", "R", "S", "T", "V", "X", "Y", "W", "Z", "0", "1", "2", "3", "4", "5", "6", "7", "8", "9", "_"} set of terminals and rules P:

<postalcode> ::= <forward_or_stationarea> < underscore > <localdeliveryunit>
<forward_or_stationarea> ::= <provarea> <loctype> <letter>
<localdeliveryunit> ::= <digit> <letter> <digit>
<provarea> ::= A | B | C | E | G | H | J | K | L | M | N | P | R | S | T | V | X | Y
<loctype> ::= <rural> | <urban>
<rural> ::= 0
<urban> ::= 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
<letter> ::= A | B | C | E | G | H | J | K | L | M | N | P | R | S | T | V | W | X | Y | Z
<digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
<underscore> ::= _

1.6 What production rules need to be added to grammar below so that it describes the syntax of expressions in postfix notation (ie., in the so called "reverse Polish notation")? The grammar **must** support expressions with operators for addition, subtraction, multiplication and division. (4pt)

Reverse Polish notation (RPN) is a mathematical notation in which *operators* follow their *operands*. For instance, to add 3 and 4, one would write 3 4 + rather than 3 + 4. If there are multiple operations, operators are given immediately after their second operands; so, the expression written 3 − 4 + 5 in conventional notation would be written 3 4 − 5 + in reverse Polish notation: 4 is first subtracted from 3, then 5 is added to it.

Given:

<expression> ::= <term> | <term> <term> <addsubOperator>
<addsubOperator> ::= + | -
<digit> ::= 0|1|2|...|9

Question 2 (1 + 1 + 7 + 15 = 24 pt)
"Automata"

2.1 What does automaton A1 do? (1pt)
Note that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, "choosing multiple solutions when only one is correct" will not be assessed as correct. Mark your choice(s) on your answer sheets, not this exam sheet.

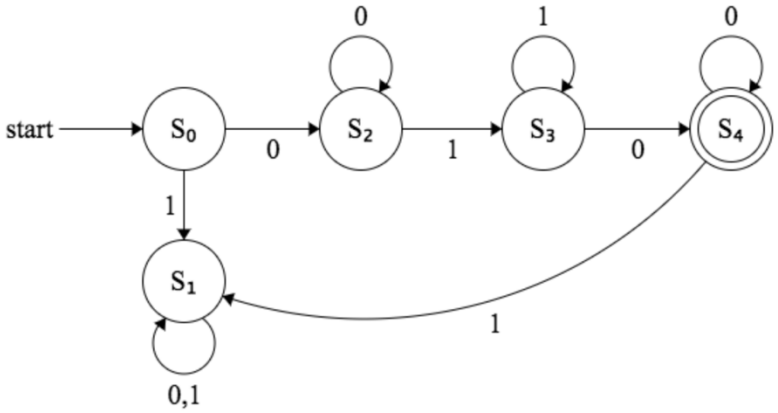
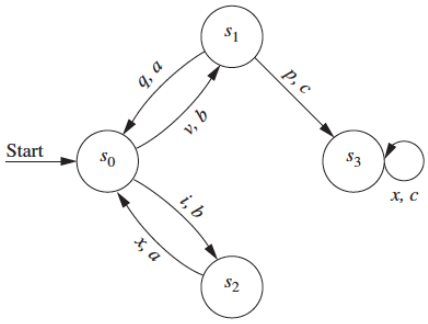


Figure 2.1: A1

a) It only recognizes the bit strings which contain only '1's.
b) It only recognizes the bit strings which contain at least one '0' and one '1'.
c) It only recognizes the bit strings which contain the sub-string '010'.
d) It only recognizes the bit strings which contain a series of '0's, followed by a series of '1's and then another series of '0's, each series contains at least one digit.
e) It only recognizes the bit strings which contain a series of odd number of '0's, followed by a series of odd number of '1's and then another series of odd number of '0's.

2.2 What does automaton A2 do? (1pt)
Note that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, "choosing multiple solutions when only one is correct" will not be assessed as correct. Mark your choice(s) on your answer sheets, not this exam sheet.



v = Valid ID
i = Invalid ID
p = Valid password
q = Invalid password
a = "Enter user ID"
b = "Enter password"
c = Prompt
x = Any input

Figure 2.2: A2

a) It represents the log-on procedure for a computer, where the user logs on by entering a user identification number, which is considered to be a single input, and then a password, which is considered to be a single input. If the password is incorrect, the procedure ends.
b) It represents the log-on procedure for a computer, where the user logs on by entering a user identification number, which is considered to be a single input, and then a password, which is considered to be a single input. If the user ID is incorrect, the user has to re-enter the user ID, no matter what password was. However, if the user ID is correct and the password is incorrect, the user is asked for the user identification number again.
c) It represents the log-on procedure for a computer, where the user logs on by entering a user identification number, which is considered to be a single input, and then a password, which is considered to be a single input. If the password is correct or incorrect, the procedure remains in state s3.
d) It represents the log-on procedure for a computer, where the user logs on by entering a user identification number, which is considered to be a single input, and then a password, which is considered to be a single input. If the user identification number is incorrect, the procedure stops.
e) It represents the log-on procedure for a computer, where the user logs on by entering a user identification number, which is considered to be a single input, and then a password, which is considered to be a single input. If both the user identification number and the password are incorrect, the procedure stops.

2.3 Add missing elements (states, transitions, or labels) or remove incorrect elements (states, transitions, or labels) in the deterministic finite-state automaton A3 so that it recognizes the set of all bit strings that contain EXACTLY the set S below. (7pt)

$$S = \{10^n \mid n \geq 0\} \cup \{10^n 10^m \mid n, m \geq 0\}$$

Note: The union of two sets A and B is the set of elements which are in A, in B, or in both A and B. For example:

$$A = \{x \text{ is an even integer larger than } 1\}$$
$$B = \{x \text{ is an odd integer larger than } 1\}$$
$$A \cup B = \{2, 3, 4, 5, 6, \dots\}$$

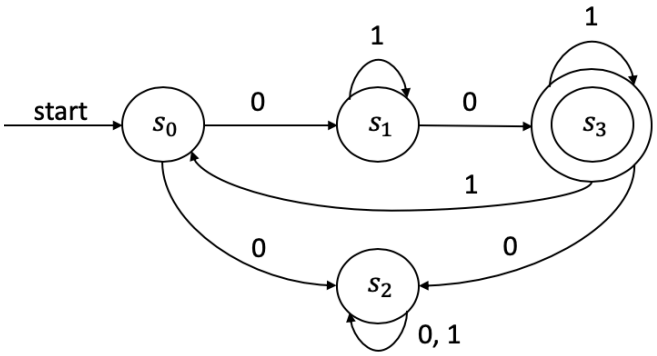


Figure 2.3: A3

2.4 Construct a finite-state machine that gives an output of "1" if the number of input symbols read so far is divisible by 3 and an output of "0" otherwise. (15pt)
Note that unreadable drawings will be awarded with 0 points.

Question 3 (8 + 1 + 1 + 10 = 20 pt)
"Logic"

3.1 Provide the complete truth table for the given compound preposition. (8pt)

$$\neg(p \wedge (q \vee r) \wedge ((p \wedge q) \rightarrow r))$$

3.2 In the following, the universe of discourse is {people}. One-place predicates are defined as follows: *cheats* is "... cheats at cards", *punk* is "… has punk hair", *scout* is "... is a Boy Scout". Express this statement using predicates and quantifiers. (1pt)

"Some people who cheat at cards don't have punk hair."

Note that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, "choosing multiple solutions when only one is correct" will not be assessed as correct. Mark your choice(s) on your answer sheets, not this exam sheet.

a) $\forall x, scout(x) \rightarrow \neg(punk(x) \vee cheats(x))$
b) $\forall x, punk(x) \rightarrow cheats(x)$
c) $\exists x, cheats(x) \wedge \neg punk(x)$
d) $\exists x, scout(x) \wedge cheats(x)$
e) $\exists x, \neg cheats(x) \leftrightarrow \neg punk(x)$

3.3 Let D(x) be "x is a day", S(x) be "x is a student" and E(x,y) be "x needs to attend the examination of the course DIT-022 on day y". Use quantifiers to express the following statement. (1pt)

"There would be someday that no student needs to attend the examination of the course DIT-022."

Note that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, choosing multiple solutions, when only one is correct will not be assessed as correct. Mark clearly the correct solution.

a) $\exists x(D(x)) \rightarrow \forall y(S(y) \rightarrow \neg E(y,x))$
b) $\forall x(D(x)) \rightarrow \forall y(S(y) \rightarrow \neg E(y,x))$
c) $\exists x(D(x)) \leftrightarrow \forall y(S(y) \rightarrow \neg E(y,x))$
d) $\forall x(D(x)) \leftrightarrow \forall y(\neg S(y) \rightarrow E(y,x))$
e) $\exists x(D(x)) \rightarrow \forall y(\neg S(y) \rightarrow E(y,x))$

3.4 Look at the following compound proposition and answer the questions below.

$$\neg((p \vee \neg q) \vee (r \rightarrow (p \vee \neg q))) \wedge \neg(p \wedge q)$$

i) What is a compound proposition? (1pt)
ii) What is a tautology? (1pt)
iii) What is a contradiction? (1pt)
iv) What is a contingency? (1pt)
v) Why is the given compound proposition not a contradiction? (4pt)
vi) Turn the given compound proposition into a tautology by making changes to the compound proposition (in other words, changing ∧, ∨, → and/or ↔, but not removing individual propositions p, q or r). (2pt)

Question 4 (6 + 4 = 10 pt)
"Proofs"

4.1 For all $n, n \geq 1$ prove that $(1 + x)^n \geq 1 + nx$, where x is any real number greater than -1. Note that this means that $(1 + x) > 0$. (6pt)

Note: In mathematics, a real number is a value of a continuous quantity that can represent a distance along a line. For example, -3.2, 1/3, $\sqrt{2}$, ...

4.2 Prove that $2 + 4 + 6 + \cdots + 2n = n(n + 1)$ for all $n > 0$. (4pt)

Question 5 (1 + 1 + 11 + 2 = 15 pt)
"Complexity and Graph Theory"

5.1 What is the complexity of the following code snippet (in terms of Big-O notation)? (1pt)

```
int count = 0;
for (i = n; i > 0; i = i/2) {
  for (j = 0; j < i; j++) {
    count += 1;
  }
}
```

5.2 What is the complexity of the following code snippet (in terms of Big-O notation)? Assume that "*doWork()*" has the complexity of O(1). (1pt)

```
for (i = 0; i < n; i++) {
  for (j = i+1; j < n+1; j++) {
    for (k = 0; k < i; k++) {
      doWork();
    }
  }
}
```

5.3 Look at the array below and consider how the array changes at each step of going through a selection-sort program, where the values of the array are being sorted in an increasing order (i.e. 1 2 3). Write down the state of the array AFTER each loop iteration of the program. What does the array look like after 6 iterations of selection-sort? (11pt)

Note that "after 6 iterations" means, when i=5, before entering the for loop and increasing i to 6. Assume that i begins with 0.

Array: -50 69 5 0 -20 32 1 8 90 103 76

5.4 Determine whether the graph in Figure 5.4 has an Euler circuit. If it does, construct the circuit. If it does not, determine whether the given graph has an Euler path. If it does, construct the path. (2pt)

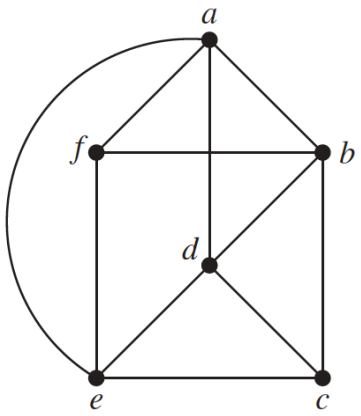


Figure 5.4

Question 6 (2 + 8 + 10 = 20 pt)
"Statistics"

6.1 Permutations and combinations are two of the basic concepts for counting different ways of doing things.

a) How many ways can we award 1st, 2nd, and 3rd place price among eight contestants? (1pt)

b) In how many ways can I give 3 presents to 8 people? (1pt)

6.2 Perform **calculations and find** the linear regression $Y = \alpha + \beta x$ for the data in the table below. **Account** for how you deal with missing y values! **Explain** what α, β means if you use them to plot a line, and if you have the time you might be able to calculate the error ε. (8pt)

X	Y
648	4.0
649	4.4
650	4.5
651	-
652	4.9
653	5.1
654	-
655	5.6

- : No data registered.

6.3 We have collected data from five groups and we would now like to see if there is a difference between the groups. Yes, we have more than 3 groups! ☺ Don't worry, you do exactly the same as you do with 3 groups, but your calculations, of course, considers all five groups.

	Counting	Rhyming	Adjective	Imagery	Intentional	Total
	9	7	11	12	10	
	8	9	13	11	19	
	6	6	8	16	14	
	8	6	6	11	5	
	10	6	14	9	10	
	4	11	11	23	11	
	6	6	13	12	14	
	5	3	13	10	15	
	7	8	10	19	11	
	7	7	11	11	11	
Mean	7.00	6.90	11.00	13.40	12.00	10.06
St. dev.	1.83	2.13	2.49	4.50	3.74	4.01
Variance	3.33	4.54	6.22	20.27	14.00	16.058

a) What are the two hypotheses you are using for your ANOVA? (1pt)
b) What is SST (calculate)? (1pt)
c) What is SSB (calculate)? (1pt)
d) What is SSW (calculate)? (1pt)
e) What are the degrees of freedom for SST, SSB, and SSW, respectively? (1pt)
f) What is the *F*-value? (1pt)
g) What is the critical value? (1pt)
h) Can you reject the null hypothesis? (1pt)

i) What is your opinion of the sample size of this study? Do you think how the sample was collected could skew the result? (1pt)
j) Can you use an ANOVA when the sample size differs between the groups? (1pt)

Appendix

Table of F-statistics P=0.05

t-statistics
F-statistics with other P-values: P=0.01 | P=0.001
Chi-square statistics

df2\df1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	22	24	26	28	30	35	40	45	50	60	70	80	100	200	500	1000	>1000	df1/df2
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.73	8.71	8.70	8.69	8.68	8.67	8.67	8.66	8.65	8.64	8.63	8.62	8.62	8.60	8.59	8.59	8.58	8.57	8.57	8.56	8.55	8.54	8.53	8.53	8.54	3
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.89	5.87	5.86	5.84	5.83	5.82	5.81	5.80	5.79	5.77	5.76	5.75	5.75	5.73	5.72	5.71	5.70	5.69	5.68	5.67	5.66	5.65	5.64	5.63	5.63	4
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.66	4.64	4.62	4.60	4.59	4.58	4.57	4.56	4.54	4.53	4.52	4.50	4.50	4.48	4.46	4.45	4.44	4.43	4.42	4.42	4.41	4.39	4.37	4.37	4.36	5
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.98	3.96	3.94	3.92	3.91	3.90	3.88	3.87	3.86	3.84	3.83	3.82	3.81	3.79	3.77	3.76	3.75	3.74	3.73	3.72	3.71	3.69	3.68	3.67	3.67	6
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57	3.55	3.53	3.51	3.49	3.48	3.47	3.46	3.44	3.43	3.41	3.40	3.39	3.38	3.36	3.34	3.33	3.32	3.30	3.29	3.29	3.27	3.25	3.24	3.23	3.23	7
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28	3.26	3.24	3.22	3.20	3.19	3.17	3.16	3.15	3.13	3.12	3.10	3.09	3.08	3.06	3.04	3.03	3.02	3.01	2.99	2.99	2.97	2.95	2.94	2.93	2.93	8
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07	3.05	3.03	3.01	2.99	2.97	2.96	2.95	2.94	2.92	2.90	2.89	2.87	2.86	2.84	2.83	2.81	2.80	2.79	2.78	2.77	2.76	2.73	2.72	2.71	2.71	9
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.89	2.86	2.85	2.83	2.81	2.80	2.79	2.77	2.75	2.74	2.72	2.71	2.70	2.68	2.66	2.65	2.64	2.62	2.61	2.60	2.59	2.56	2.55	2.54	2.54	10
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79	2.76	2.74	2.72	2.70	2.69	2.67	2.66	2.65	2.63	2.61	2.59	2.58	2.57	2.55	2.53	2.52	2.51	2.49	2.48	2.47	2.46	2.43	2.42	2.41	2.41	11
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69	2.66	2.64	2.62	2.60	2.58	2.57	2.56	2.54	2.52	2.51	2.49	2.48	2.47	2.44	2.43	2.41	2.40	2.38	2.37	2.36	2.35	2.32	2.31	2.30	2.30	12
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60	2.58	2.55	2.53	2.51	2.50	2.48	2.47	2.46	2.44	2.42	2.41	2.39	2.38	2.36	2.34	2.33	2.31	2.30	2.28	2.27	2.26	2.23	2.22	2.21	2.21	13
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53	2.51	2.48	2.46	2.44	2.43	2.41	2.40	2.39	2.37	2.35	2.33	2.32	2.31	2.28	2.27	2.25	2.24	2.22	2.21	2.20	2.19	2.16	2.14	2.14	2.13	14
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.45	2.42	2.40	2.38	2.37	2.35	2.34	2.33	2.31	2.29	2.27	2.26	2.25	2.22	2.20	2.19	2.18	2.16	2.15	2.14	2.12	2.10	2.08	2.07	2.07	15
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42	2.40	2.37	2.35	2.33	2.32	2.30	2.29	2.28	2.25	2.24	2.22	2.21	2.19	2.17	2.15	2.14	2.12	2.11	2.09	2.08	2.07	2.04	2.02	2.02	2.01	16
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38	2.35	2.33	2.31	2.29	2.27	2.26	2.24	2.23	2.21	2.19	2.17	2.16	2.15	2.12	2.10	2.09	2.08	2.06	2.05	2.03	2.02	1.99	1.97	1.97	1.96	17
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.31	2.29	2.27	2.25	2.23	2.22	2.20	2.19	2.17	2.15	2.13	2.12	2.11	2.08	2.06	2.05	2.04	2.02	2.00	1.99	1.98	1.95	1.93	1.92	1.92	18
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31	2.28	2.26	2.23	2.21	2.20	2.18	2.17	2.16	2.13	2.11	2.10	2.08	2.07	2.05	2.03	2.01	2.00	1.98	1.97	1.96	1.94	1.91	1.89	1.88	1.88	19
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28	2.25	2.23	2.20	2.18	2.17	2.15	2.14	2.12	2.10	2.08	2.07	2.05	2.04	2.01	1.99	1.98	1.97	1.95	1.93	1.92	1.91	1.88	1.86	1.85	1.84	20
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23	2.20	2.17	2.15	2.13	2.11	2.10	2.08	2.07	2.05	2.03	2.01	2.00	1.98	1.96	1.94	1.92	1.91	1.89	1.88	1.86	1.85	1.82	1.80	1.79	1.78	22
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18	2.15	2.13	2.11	2.09	2.07	2.05	2.04	2.03	2.00	1.98	1.97	1.95	1.94	1.91	1.89	1.88	1.86	1.84	1.83	1.82	1.80	1.77	1.75	1.74	1.73	24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15	2.12	2.09	2.07	2.05	2.03	2.02	2.00	1.99	1.97	1.95	1.93	1.91	1.90	1.87	1.85	1.84	1.82	1.80	1.79	1.78	1.76	1.73	1.71	1.70	1.69	26
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.15	2.12	2.09	2.06	2.04	2.02	2.00	1.99	1.97	1.96	1.93	1.91	1.90	1.88	1.87	1.84	1.82	1.80	1.79	1.77	1.75	1.74	1.73	1.69	1.67	1.66	1.66	28
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	2.06	2.04	2.01	1.99	1.98	1.96	1.95	1.93	1.91	1.89	1.87	1.85	1.84	1.81	1.79	1.77	1.76	1.74	1.72	1.71	1.70	1.66	1.64	1.63	1.62	30
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.08	2.04	2.01	1.99	1.96	1.94	1.92	1.91	1.89	1.88	1.85	1.83	1.82	1.80	1.79	1.76	1.74	1.72	1.70	1.68	1.66	1.65	1.63	1.60	1.57	1.57	1.56	35
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00	1.97	1.95	1.92	1.90	1.89	1.87	1.85	1.84	1.81	1.79	1.77	1.76	1.74	1.72	1.69	1.67	1.66	1.64	1.62	1.61	1.59	1.55	1.53	1.52	1.51	40
45	4.06	3.20	2.81	2.58	2.42	2.31	2.22	2.15	2.10	2.05	2.01	1.97	1.94	1.92	1.89	1.87	1.86	1.84	1.82	1.81	1.78	1.76	1.74	1.73	1.71	1.68	1.66	1.64	1.63	1.60	1.59	1.57	1.55	1.51	1.49	1.48	1.47	45
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.99	1.95	1.92	1.89	1.87	1.85	1.83	1.81	1.80	1.78	1.76	1.74	1.72	1.70	1.69	1.66	1.63	1.61	1.60	1.58	1.56	1.54	1.52	1.48	1.46	1.45	1.44	50