Examination

Mathematical Foundations for Software Engineering

Course code DIT022

Date:	2019-10-29
Time:	14:00-18:00
Place:	Lindholmen
Teacher:	Christian Berger Richard Torkar
Visit to e*xam hall:*	15:00 and 16:30
Questions:	6
Results:	Will be posted by 2019-11-19.
Grade Limits:	Pass (G) 50%, Pass with honors (VG) 90%
Allowed aids:	Casio FX-82..., Texas TI-30... and Sharp EL-W531...

Please observe the following:

- Write in legible English (unreadable means no points!).
- Motivate your answers, and clearly state any assumptions made.
- Start each task on a new sheet!
- Write only on one side of the paper!
- Before handing in your exam, number and sort the sheets in task order!
- Write your complete name one every sheet!

NOTE:

Not following these instructions may result in the deduction of points!

Written Examination

Question 1 (1 + 1 + 1 + 1 + 2 + 4 = 10 pt)
"Languages"

1.1 What language is generated by the given grammar G1? (1pt)
Note, that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, choosing multiple solutions, when only one is correct will not be assessed as correct.

G1 = (V, T, S, P) and V = {S, X}, where S is the start variable, T = {'a', 'b', 'AB', 'C', 'DEF'} set of terminals and production rules:

S → X
X → aX | bX | AB | C | DEF

a) w∈ {a, b, AB, C, DEF} * | w contains words beginning with a string of only 'a's or a string of only 'b's, then followed by one or more uppercase terminals 'AB','C','DEF'.
b) w∈ {a, b, AB, C, DEF} * | w contains words beginning with a random string of lowercase terminal symbols 'a' and/or 'b', then followed by one of the uppercase terminals 'AB','C','DEF'.
c) w∈ {a, b, AB, C, DEF} * | w contains words beginning with a string of lowercase terminal symbols 'a' and 'b' of even length, then followed by one or more uppercase terminals 'AB','C','DEF'.
d) w∈ {a, b, AB, C, DEF} * | w possibly contains any combination of all terminals as long as it begins with either 'a' or 'b'.
e) w∈ {a, b, AB, C, DEF} * | w contains words of even length, with any combination of all terminals which begins with either 'a' or 'b'.

1.2 What language is generated by the given grammar G2? (1pt)
Note, that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, choosing multiple solutions, when only one is correct will not be assessed as correct.

G2 = (V, T, S, P) and V = {S}, where S is the start variable, T = {a, b, λ} set of terminals ($\lambda = empty\ string$) and production rules:

S → aaSb | λ

a) $\{a^{2n}b^{n} \mid n \geq 0\}$
b) $\{a^{2n}b^{n} \mid n \geq 1\}$
c) $\{a^{n}b^{2n} \mid n \geq 0\}$
d) $\{a^{n}b^{2n} \mid n \geq 1\}$
e) $\{a^{2n}b^{2n} \mid n \geq 0\}$

Written Examination

1.3 What language is generated by the given grammar G3? (1pt)
Note, that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, choosing multiple solutions, when only one is correct will not be assessed as correct.

G3 = (V, T, S, P) and V = {S, A}, where S is the start variable, T = {0, 1, λ} set of terminals and production rules:

$S \rightarrow 1S$
$S \rightarrow 1A$
$A \rightarrow 00A$
$A \rightarrow 0$

a) $\{\{1\}^m\{0\}^n \mid m, n \geq 1\}$
b) $\{\{1\}^m\{00\}^n \mid m \geq 1, n \geq 0\}$
c) $\{\{11\}^m\{00\}^n 0 \mid m, n \geq 1\}$
d) $\{\{1\}^m\{00\}^n 0 \mid m \geq 1, n \geq 0\}$
e) $\{\{1\}^m\{00\}^n 0 \mid m, n \geq 0\}$

1.4 What language is generated by the given grammar G4? (1pt)
Note, that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, choosing multiple solutions, when only one is correct will not be assessed as correct.

G4 = (V, T, S, P) and V = {a, b, c, …, z, λ, S}, where S is the start variable, T = {a, b, c, …, z, λ} set of terminals and production rules:

S→ aSa | bSb | cSc | … | zSz
S→ λ | a | b | c | … | z

a) The language defines words of odd length.
b) The language defines all words in the English language.
c) The language defines all words containing an arbitrary number of identical characters.
d) The language defines all words in the English language of even length.
e) The language defines all palindromes[1] over the English alphabet.

1 A palindrome is any string which reads the same backwards and forwards.

Written Examination

1.5 Provide two different non-empty words that are generated by the given grammar G5. (2pt)

G5 = (V, T, S, P) and V = {expr, term, factor, const}, where expr is the start variable, T = {set of all integers} set of terminals and production rules:

$$expr \rightarrow term - expr$$
$$expr \rightarrow term$$
$$term \rightarrow term/factor$$
$$term \rightarrow factor$$
$$factor \rightarrow (expr)$$
$$factor \rightarrow const$$
$$const \rightarrow integer$$
$$integer \rightarrow 0 \mid 1 \mid 2 \mid \ldots \mid 9$$

1.6 What production rule needs to be added to grammar G6 so that the following language is created? (4pt)

G6 = A grammar describing polynomials. A polynomial is defined by this grammar as an expression consisting of variables (or indeterminates) and coefficients, that involve only the operations of addition, subtraction, and non-negative integer exponents of variables. An example of a polynomial of a single indeterminate x is “x^2−4x+7”. Complete the Backus-Naur form of a grammar defining polynomials of one indeterminant ‘x’.

Given:

<polynomial> ::= <term> | <sign><term> | <polynomial><sign><term>
<digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
| 0 <digit> | 1 <digit> | 2 <digit> | 3 <digit> | 4 <digit> | 5 <digit>
| 6 <digit> | 7 <digit> | 8 <digit> | 9 <digit>

Question 2 (1 + 1 + 4 + 14 = 20 pt)
"Automata"

2.1 What does the deterministic finite state automaton A1 do? (1pt)
Note, that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, choosing multiple solutions, when only one is correct will not be assessed as correct.

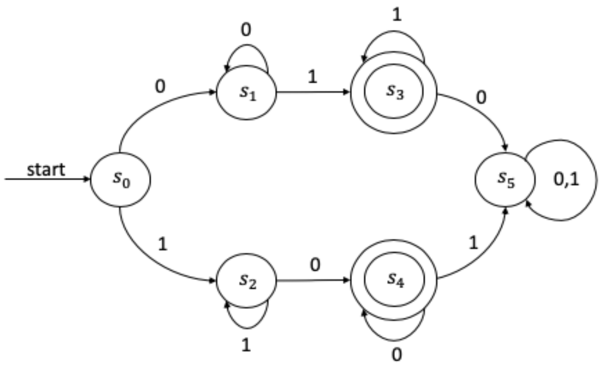

Figure 1: A1

a) It describes the language defined by grammar G = (V, S, T, P), where V = {0, 1, $s_0, s_1, s_2, s_3, s_4, s_5$, T}, s_0 is the starting state, T = {0, 1} and the production rules P define the acceptance of only bit strings '01', '001', '10', and '110'.
b) It describes the language defined by grammar G = (V, S, T, P), where V = {0, 1, $s_0, s_1, s_2, s_3, s_4, s_5$, T}, s_0 is the starting state, T = {0, 1} and the production rules P define the acceptance of bit strings with exactly one transition between 0 and 1 (transition either from 0 to 1 or from 1 to 0).
c) It describes the language defined by grammar G = (V, S, T, P), where V = {0, 1, $s_0, s_1, s_2, s_3, s_4, s_5$, T}, s_0 is the starting state, T = {0, 1} and the production rules P define the acceptance of all bit strings containing an odd number of '1's.
d) It describes the language defined by grammar G = (V, S, T, P), where V = {0, 1, $s_0, s_1, s_2, s_3, s_4, s_5$, T}, s_0 is the starting state, T = {0, 1} and the production rules P define the acceptance of all bit strings containing an even number of '0's.
e) It describes the language defined by grammar G = (V, S, T, P), where V = {0, 1, $s_0, s_1, s_2, s_3, s_4, s_5$, T}, s_0 is the starting state, T = {0, 1} and the production rules P define the acceptance of all bit strings starting with '01' or '10'.

2.2 What does automaton A2 do if the input characters are all letters of the English alphabet? Assume that transitions with missing labels occur upon any input character (i.e., the transitions $s_0 - s_0, s_1 - s_0, s_2 - s_0, s_3 - s_3, s_4 - s_0, s_5 - s_0$). (1pt)
Note, that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, choosing multiple solutions, when only one is correct will not be assessed as correct.

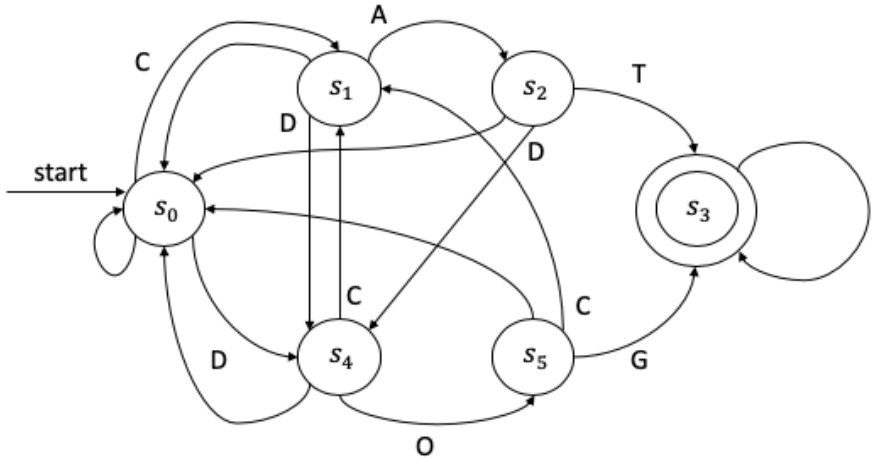

Figure 2: Automaton A2.

a) It accepts strings that begins either with ‘CAT’ or with ‘DOG’.
b) It accepts strings of odd length that begin with ‘C’.
c) It accepts strings that begin with either ‘C’ or ‘D’ and continue with either ‘A’ or ‘O’, respectively.
d) It accepts strings containing ‘CAT’ or ‘DOG’ anywhere in the string.
e) None of the above.

2.3 Add missing elements (states, transitions, or labels) or remove incorrect elements (states, transitions, or labels) in a deterministic finite-state automaton A3 so that it accepts the string ‘FROG’ with any prefix (G = (V, S, T), where V = {‘F’, ‘G’,’O’,’R’, s_0, s_1, s_2, s_3, s_4, T}, $S = \{s_0\}$, T = {‘F’,’G’,’O’,’R’}). (4pt)

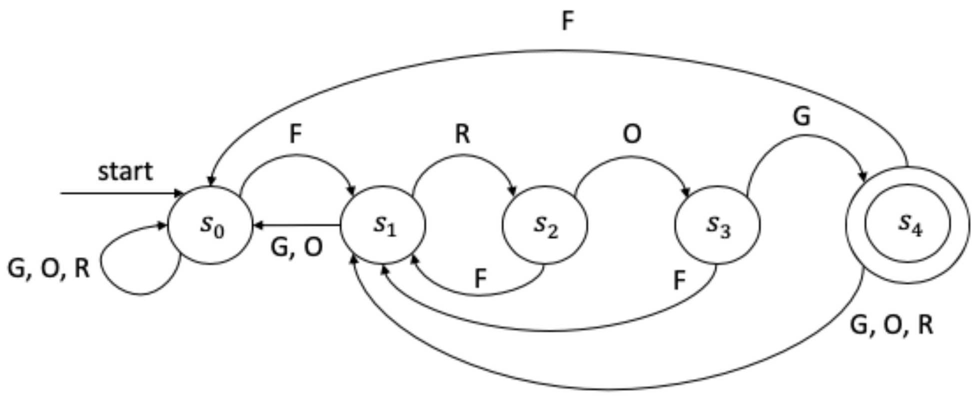

Figure 3: Automaton A3.

2.4 A bar code represents bits as bands of alternating light and dark stripes. The light bands are of uniform width while the dark bands may have a width, which is either equal or double the width of the light bands. In the following Figure, you find an example of such a bar code; the additional tick marks on top depict the single widths.

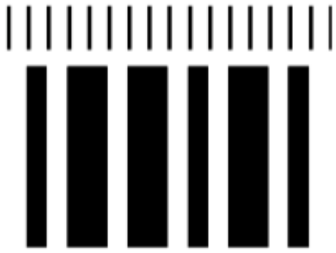

Figure 4: A bar code scheme.

A hand-held bar code reader is translating such bands into a sequence of ***L*** representing a light band and ***D*** representing a dark band with one symbol per single width. The aforementioned example would result in:

`L D L D D L D D L D L D D L D L`

A bar code is transformed into a binary sequence of 0s and 1s by a finite-state *transducer* M, which is translating the sequence of light and dark bands as follows:

- The output alphabet for the transducer M is {0, 1, _, end}
- A sequence of ***LD*** is representing 0 and a sequence of ***LDD*** is representing 1
- When started in its initial state, the transducer will be “idle” giving out _
- When the transducer reads the first ***D*** it knows that the bar code has started
- The transducer will give out 0 or 1 as soon as it has determined the *next* bit from the bar code

The transducer will return the following binary pattern for the aforementioned bar code:

L	D	L	D	D	L	D	D	L	D	L	D	D	L	D	L
_	_	0	_	1	_	_	1	_	_	0	_	1	_	_	0

The transducer will produce the symbol *end* when it reads two ***L***'s; in addition, it will also return to its initial state. It will also produce *end* when it reads ***LDDD*** and return to its initial state.

Draw a <u>deterministic finite state machine</u> for the transducer M. (14 pt)

Question 3 (8 + 1 + 1 + 10 = 20 pt)
"Logic"

3.1 Provide the complete truth table for the given compound preposition? (8pt)

$$((((p \land \neg p) \lor \neg r) \to (\neg q \land \neg r)) \leftrightarrow p)$$

3.2 When planning a party, you want to know whom to invite. Among the people you would like to invite are three touchy friends. You know that if Jasmine attends, she will become unhappy if Samir is there, Samir will attend only if Kanti will be there, and Kanti will not attend unless Jasmine also does. Which combinations of these three friends can you invite so as not to make someone unhappy? (1pt)
Note, that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, choosing multiple solutions, when only one is correct will not be assessed as correct.

a) Jasmine and Kanti and Samir
b) Samir and Kanti
c) Only Jasmine, none, or Jasmine and Kanti
d) Only Kanti, none, or Jasmine and Kanti

3.3 Let Q(x) be the statement "x + 1 > 2x". If the domain consists of all integers, which of the following nested propositions are true? (1pt)
Note, that we are not asking for partially correct solution(s) but for the fully correct one(s). Also, choosing multiple solutions, when only one is correct will not be assessed as correct.

a) $\exists x \neg Q(x), Q(1), \exists x Q(x)$
b) $\forall x \neg Q(x), Q(0), \exists x Q(x)$
c) $\exists x \neg Q(x), Q(0), \forall x Q(x)$
d) $\forall x \neg Q(x), Q(1), \exists x Q(x)$
e) $\exists x \neg Q(x), Q(-1), \forall x Q(x)$
f) $\exists x \neg Q(x), Q(-1), \exists x Q(x)$

3.4 Which changes need to be made (e.g., changing an $\land$ to $\lor$ is one change, multiple changes including additions but not removals are fine) in the following conditional statement for it to become a tautology? What about a contradiction? (10pt)

$$((p \to r) \lor (q \to r)) \land ((p \land q) \to r)$$

Question 4 (2 + 3 + 5 = 10pt)
"Proofs"

4.1 Explain why the following proofs are wrong. (2pt)

i) Suppose $k \geq 1$ and $k \in \mathbb{Z}$. Prove if $(2^k \ mod \ 3 = 0) \ then \ (8^k \ mod \ 3 = 1)$.

Proof:
The claim is false.
Here are two counter-examples:
If $k = 1$ then $8 \ mod \ 3 = (2 * 3 + 2) \ mod \ 8 = 2 \neq 1$.
If $k = 3$ then $8^3 \ mod \ 3 = 512 \ mod \ 3 = (170 * 3 + 2) \ mod \ 8 = 2 \neq 1$.
In fact, whenever *k* is odd, $8^k \ mod \ 3 = 2$.

ii) Prove that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ for all $n > 0$.

Proof:
Take $n = 3$ as an example for direct proof:
$Left = 1 + 3 + 5 = 9$
$Right = 3^2 = 9$
Thus $Left = Right$, so the claim is true.

4.2 Prove by contradiction that there is no "largest even number". (3pt)

4.3 Prove that $\frac{1}{1\times3} + \frac{1}{3\times5} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$ for all $n > 0$. (5pt)

Question 5 (1 + 1 + 1 + 7 + 5 = 15pt)
"Complexity and Graph Theory"

5.1 What is the complexity of the following code snippet (in terms of Big-O notation)? (1pt)

```
for (int i = 0; i < n; i++) {
    if (i % 2 ==0) {
        for (int j = i; j < n; j++) { /*some functions of O(1)*/  }
    } else {
        for (int j = 0; j < i; j++) { /*some functions of O(1)*/ }
    }
}
```

5.2 What is the complexity of the following code snippet (in terms of Big-O notation)? (1pt)

```
int Fibonacci(int n)
{
    if (n <= 1)
        return n;
    else
        return Fibonacci(n - 1) + Fibonacci(n - 2);
}
```

5.3 What is the complexity of the following code snippet (in terms of Big-O notation)? (1pt)

```
While (n > 0) {
    Some_function(); // This is a function of O(n)
    N = n / 2;
}
```

5.4 Look at the following array of integer and answer the questions below.

[6, -4, 9, 13, 31, 7, -1, 27, 42]

i) Trace the execution of the Bubble sort algorithm over the array and show the status of the array **after each swap of values**. (5pt)

ii) What is the **best-case** and the **average-case** complexity (in terms of Big-O) of the Bubble sort algorithm? (2pt)

5.5 Look at the graph in Figure 5 and answer the questions below.

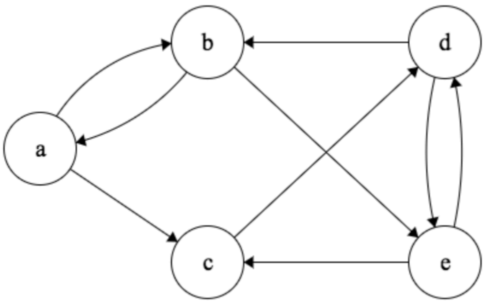

Figure 5: A directed graph.

i) What is the in-degree and out-degree of vertex c in the graph? (1pt)

ii) Find the adjacency matrix of the graph. (2pt)

iii) Does the graph contain an Euler path? If so, find it and write it in a sequence of vertices. You do not need to list all possible Euler paths if there are many. (1pt)

iv) Does the graph contain an Euler circuit? If so, find it and write it in a sequence of vertices. You do not need to list all possible Euler paths if there are many. (1pt)

Question 6 (6 + 4 + 10 + 5 = 25pt)

6.1 Perform calculations and find the linear regression $Y = \alpha + \beta x$ for the following data:

X	Y
5	0.1
10	0.3
15	7.8
17	9.9
21	10.4
24	14.2
28	19.5
32	20

In addition, please report the standard error of the estimate! (5 pt)

Is correlation statistically significant for $\alpha = 0.5\%$ (1 pt)
The table of critical values is enclosed.

6.2 Suppose that a blood test detects a number of vampires with the probability which follows Poisson distribution with the parameter $\lambda = 4$. Suppose that a new blood test (detection method) has been introduced. It is stated by the manufacturer that the probability where a blood test detects vampirism is Poisson random variable with the parameter $\lambda = 2$ for 85% of the test cases. For other 15% a new test is not effective.
How likely that a new method is beneficial for detection of 0 (zero) vampires ? (4 pt)
The probabilities are defined as follows:

$P(0\ vampires|beneficial)$ is Poisson random variable ($X = 0$, where X is the number of vampires) with the parameter $\lambda = 2$

$P(0\ vampires|not\ beneficial)$ is Poisson random variable ($X = 0$) with the parameter $\lambda = 4$

$P(beneficial) = 0.85$
$P(not\ beneficial) = 0.15$

Use Bayes theorem for probability inversion $P(A|B) = P(B|A)P(A)/P(B)$ or in the vampire world
$P(benefical|0\ vampires) = P(0\ vampires|beneficial)P(beneficial)/P(0\ vampires)$

6.3 Suppose that we have three groups of data, where the mean values $\overline{X}$ and standard deviations S are calculated for each group :

	Group 1	Group 2	Group 3
	10	20	34
	11	17	45
	13	11	35
	25	26	21
	33	8	34
$\overline{X}$	18.4	16.4	33.8
S	10.1	7.16	8.53

1) Test whether the mean values are equal for the groups, please use $\alpha = 5\%$ (totally 10 points, see the distribution below:)
2) State the null and alternative hypothesis. (2 pt)
3) Calculate the appropriate test statistic. (4 pt)
4) Find the critical value. (2 pt)
The table for critical values is enclosed.

5) What is the decision rule? (1 pt)
6) What is your interpretation of the findings? (1 pt)

6.4 Test whether the mean values are equal for two populations presented in the table for $\alpha = 20\%$ and $\alpha = 10\%$, (4 pt). What is your interpretation of the findings? (1 pt).
The table for critical values is enclosed.

Statistical Quantities	Population 1	Population 2
Sample Mean Value	22.8	19.2
Sample Standard Deviation	6.1	7.5
Sample Size	11	19

Written Examination

Formulas and Tables

1 Linear Regression

1.1 Pearson Correlation Coefficient and Variances:

$$
\begin{aligned}
r &= \frac{1}{s_x s_y} \frac{1}{(n-1)} \sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y}) \\
&= \frac{\sum_{i=1}^{n} (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \overline{x})^2 \sum_{i=1}^{n} (y_i - \overline{y})^2}} \\
s_x^2 &= \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2 \\
s_y^2 &= \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \overline{y})^2
\end{aligned}
$$

1.2 Equation for Linear Regression:

$$
\begin{aligned}
y - \overline{y} &= r \frac{s_y}{s_x} (x - \overline{x}) \\
y - \overline{y} &= b(x - \overline{x}) \\
b &= \frac{\sum_{i=1}^{n} x_i y_i - n\, \overline{x}\, \overline{y}}{\sum_{i=1}^{n} x_i^2 - n(\overline{x})^2}
\end{aligned}
$$

1.3 Regression Error Estimation:

$$
\epsilon = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n-2}}
$$

2 Probability and Distributions

2.1 Bayes Theorem and Total Probablity

$$P(B|A) = \frac{P(B)P(A|B)}{P(A)}$$

$$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$$

2.2 Poisson Distribution

$$P\{X = i\} = e^{-\lambda}\frac{\lambda^i}{i!}$$

$$E(x) = \lambda, \quad Var(X) = \lambda$$

3 ANOVA

$$\begin{aligned} MSTR &= \frac{n\sum_{i=1}^{k}(\bar{x}_i - \bar{X})^2}{(k-1)} \\ MSE &= \frac{(n-1)\sum_{i=1}^{k} s_i^2}{(n_t - k)} \\ F &= \frac{MSTR}{MSE} \\ & \text{critical value,} \quad F_{k-1,\ n_t - k,\ \alpha} \end{aligned}$$

4 Two Sample t-Test

$$\begin{aligned} t &= \frac{\overline{x} - \overline{y}}{S\sqrt{1/n + 1/m}} \\ S &= \sqrt{\frac{(n-1)S_x^2 + (m-1)S_y^2}{(n-1) + (m-1)}} \\ df &= n + m - 2 \end{aligned}$$

Written Examination

Table of critical values for the F distribution (for use with ANOVA):

How to use this table:

**There are two tables here. The first one gives critical values of F at the p = 0.05 level of significance.
The second table gives critical values of F at the p = 0.01 level of significance.**
1. Obtain your F-ratio. This has (x,y) degrees of freedom associated with it.
2. Go along x columns, and down y rows. The point of intersection is your critical F-ratio.
3. If your obtained value of F is equal to or larger than this critical F-value, then your result is significant at that level of probability.
**An example: I obtain an F ratio of 3.96 with (2, 24) degrees of freedom.
I go along 2 columns and down 24 rows. The critical value of F is 3.40. My obtained F-ratio is larger than this, and so I conclude that my obtained F-ratio is likely to occur by chance with a p<.05.**

Critical values of F for the 0.05 significance level:

	1	2	3	4	5	6	7	8	9	10
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.39	19.40
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.97	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.10	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.97	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.56	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.33	3.47	3.07	2.84	2.69	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.38	2.32	2.28
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.26
25	4.24	3.39	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.17
31	4.16	3.31	2.91	2.68	2.52	2.41	2.32	2.26	2.20	2.15
32	4.15	3.30	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14
33	4.14	3.29	2.89	2.66	2.50	2.39	2.30	2.24	2.18	2.13
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11

Table of Critical Values for Pearson's r

	Level of Significance for a One-Tailed Test					
	.10	.05	.025	.01	.005	.0005
	Level of Significance for a Two-Tailed Test					
df	.20	.10	.05	.02	.01	.001
1	0.951	0.988	0.997	0.9995	0.9999	0.99999
2	0.800	0.900	0.950	0.980	0.990	0.999
3	0.687	0.805	0.878	0.934	0.959	0.991
4	0.608	0.729	0.811	0.882	0.917	0.974
5	0.551	0.669	0.755	0.833	0.875	0.951
6	0.507	0.621	0.707	0.789	0.834	0.925
7	0.472	0.582	0.666	0.750	0.798	0.898
8	0.443	0.549	0.632	0.715	0.765	0.872
9	0.419	0.521	0.602	0.685	0.735	0.847
10	0.398	0.497	0.576	0.658	0.708	0.823
11	0.380	0.476	0.553	0.634	0.684	0.801
12	0.365	0.457	0.532	0.612	0.661	0.780
13	0.351	0.441	0.514	0.592	0.641	0.760
14	0.338	0.426	0.497	0.574	0.623	0.742
15	0.327	0.412	0.482	0.558	0.606	0.725
16	0.317	0.400	0.468	0.542	0.590	0.708
17	0.308	0.389	0.456	0.529	0.575	0.693
18	0.299	0.378	0.444	0.515	0.561	0.679
19	0.291	0.369	0.433	0.503	0.549	0.665
20	0.284	0.360	0.423	0.492	0.537	0.652
21	0.277	0.352	0.413	0.482	0.526	0.640
22	0.271	0.344	0.404	0.472	0.515	0.629
23	0.265	0.337	0.396	0.462	0.505	0.618
24	0.260	0.330	0.388	0.453	0.496	0.607
25	0.255	0.323	0.381	0.445	0.487	0.597
26	0.250	0.317	0.374	0.437	0.479	0.588
27	0.245	0.311	0.367	0.430	0.471	0.579
28	0.241	0.306	0.361	0.423	0.463	0.570
29	0.237	0.301	0.355	0.416	0.456	0.562
30	0.233	0.296	0.349	0.409	0.449	0.554
40	0.202	0.257	0.304	0.358	0.393	0.490
60	0.165	0.211	0.250	0.295	0.325	0.408
120	0.117	0.150	0.178	0.210	0.232	0.294
∞	0.057	0.073	0.087	0.103	0.114	0.146

Adapted from Appendix 2 (Critical Values of t) using the square root of $[t^2/(t^2 + df)]$
Note: Critical values for Infinite df actually calculated for $df = 500$.

Written Examination

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										