

# CHALMERS

## EXAMINATION / TENTAMEN

Course code/kurskod	Course name/kursnamn			*
DITO 22	matematisk foundation			Y
Anonymous code Anonym kod	768	Examination date Tentamensdatum	Number of pages Antal blad	Grade Betyg
			19	6

\* I confirm that I've no mobile or other similar electronic equipment available during the examination.  
Jag intygar att jag inte har mobiltelefon eller annan liknande elektronisk utrustning tillgänglig under examinationen.

Solved task Behandlade uppgifter	Points per task Poäng på uppgiften	Observe: Areas with bold contour are to completed by the teacher. Anmärkning: Rutor inom bred kontur ifylles av lärare.
No/nr		
1	✓	
2	✓	
3	✓	
4	✓	
5	✓	
6	✓	
7	✓	
8	✓	
9	✓	
10	✓	
11	✓	
12	✓	
13	✓	
14	✓	
15	✓	
16	✓	
17	✓	
Bonus credits/ poäng	5	
Total examination points Summa poäng	62+5=	67

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Poäng på uppgiften

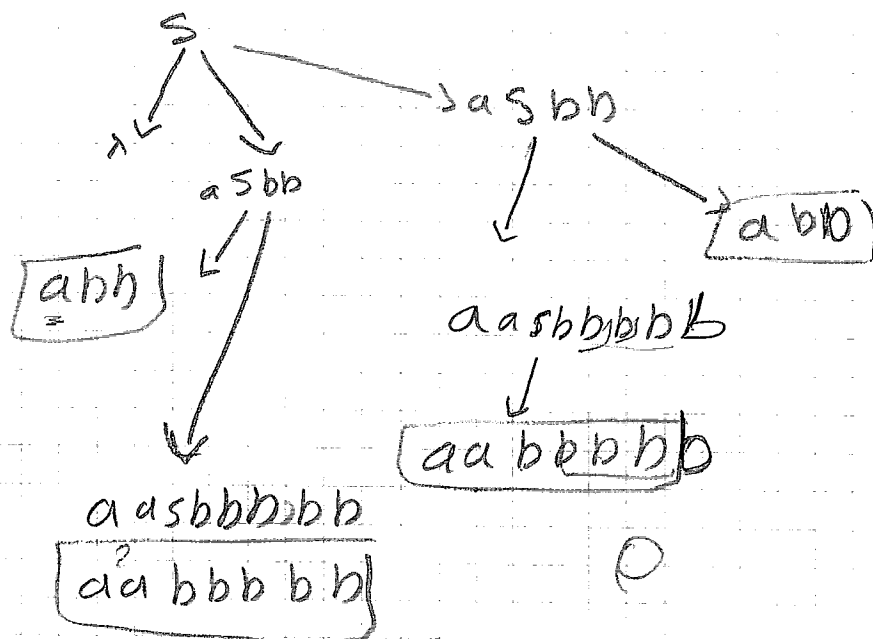
(if applicable)

Question no.

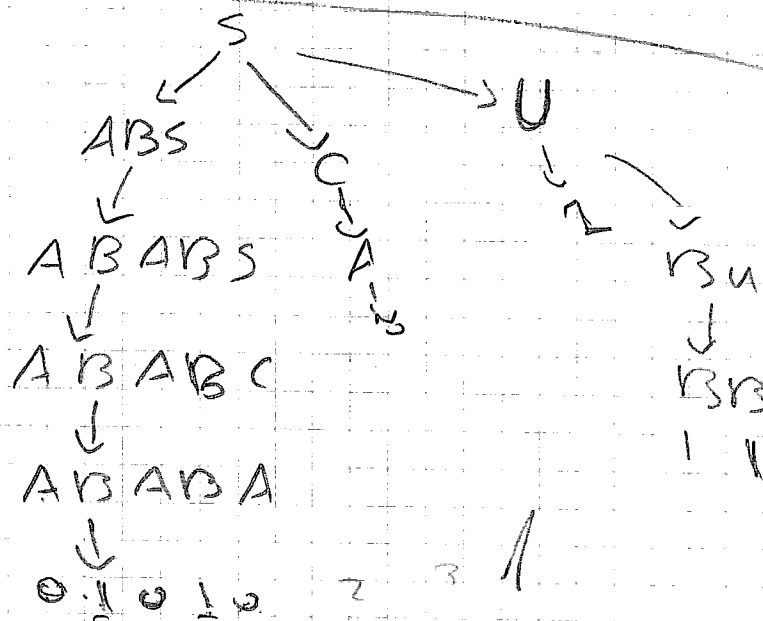
Uppgift nr

1

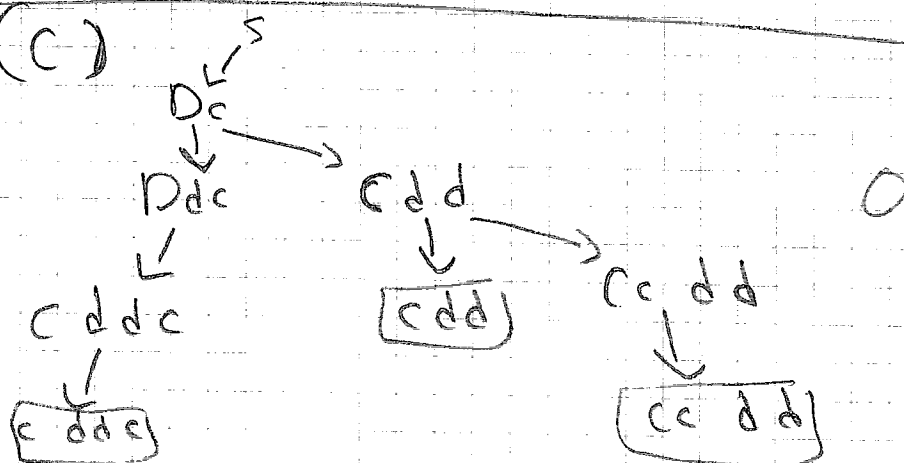
1.1(b)



1.2(c)



1.3(c)





1.5

Declaration

↓  
<type> <Declaration>

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int name

① — 1 ABO

Declaration

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<type> <Declaration>

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B <Decl> "number"

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B name "number"

B dergham "1"

② — B dergham 1

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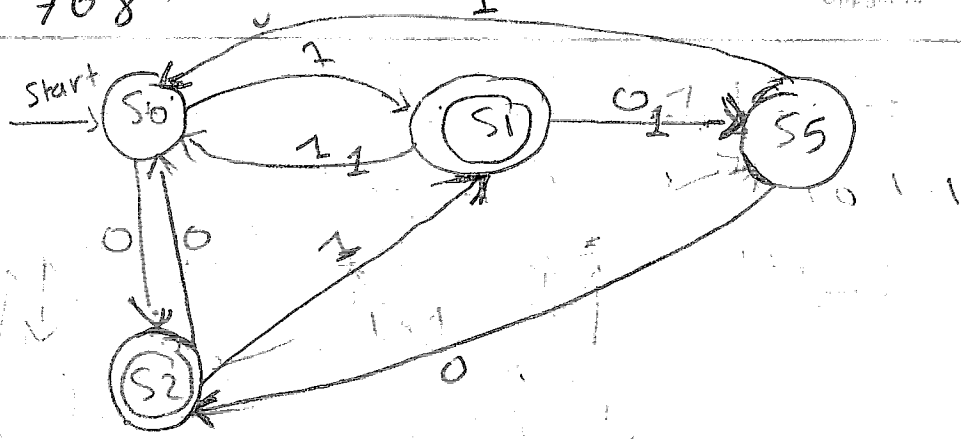
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1.6 1APP Expr  $\rightarrow$  Var | BoolVar  $\rightarrow$  and | why | whereBool  $\rightarrow$  true | false1, Atomic  $\rightarrow$  Number | VarNumber  $\rightarrow$  1 | 2 | 3 | 4 | ... ,

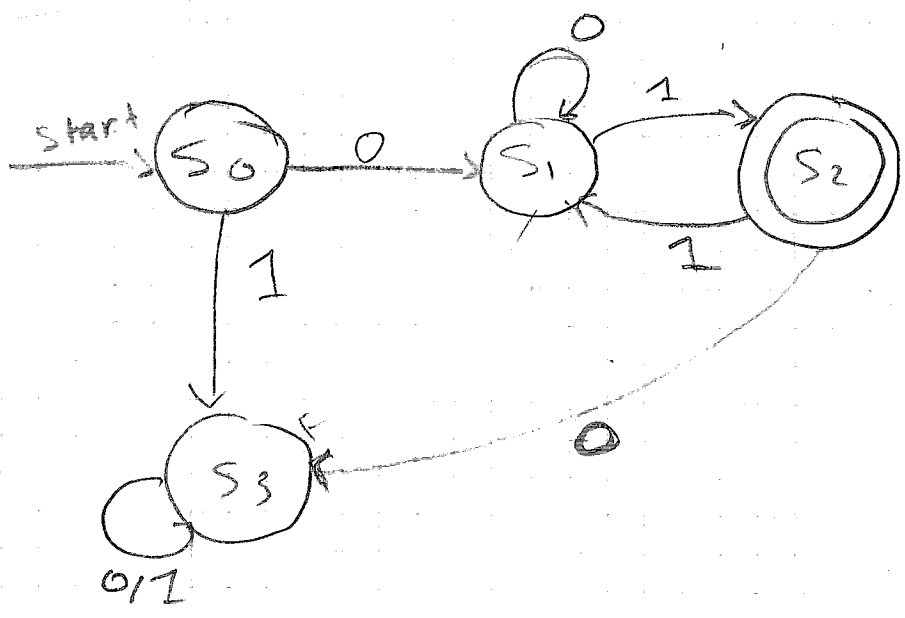
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2.3



010

2.4



0111

01111  
011

0110

010

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01011

2.5

11111111

18

011

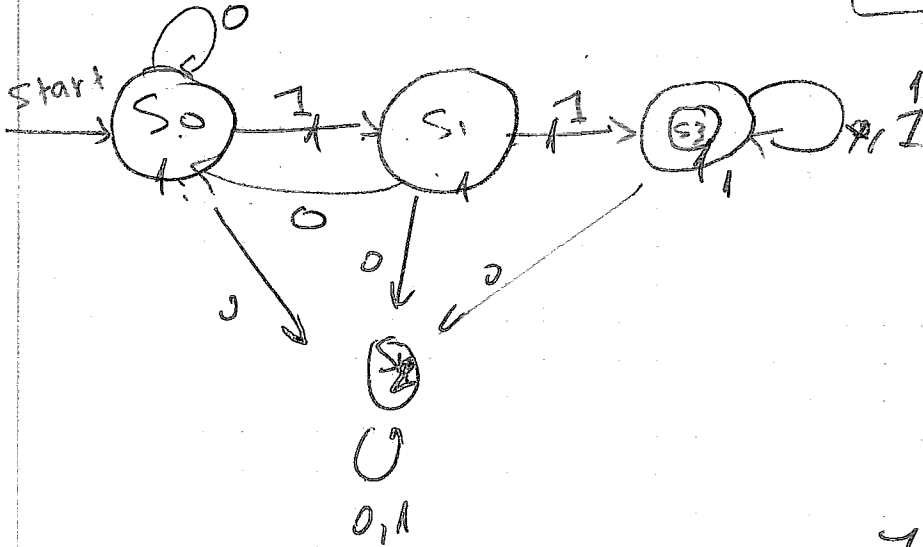
0101

0001100

11010

0101011

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3.4 6

i) the tautology is the compound proposition that give us always true  
1

ii).

p	q	$\neg p$	$\neg q$	r	$\neg r$	$(p \rightarrow q)$	$\neg q \rightarrow \neg p$	$(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$
1	1	0	0	0	1	1	1	1
1	0	0	1	1	0	0	0	1
0	1	1	0	0	1	1	1	1
0	0	1	1	1	0	1	1	1

$r \wedge \neg r$	$(\neg q \rightarrow \neg p) \wedge (r \wedge \neg r)$	$(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p) \wedge (r \wedge \neg r)$
0	0	0
0	0	1
0	0	0
0	0	0

We can see that compound proposition has a true Value which it can be contradiction (it should be all true value is False).

iii)  $(p \rightarrow q) \rightarrow (\neg p \rightarrow \neg q) \rightarrow (r \wedge \neg r)$



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Poeng på oppgiffen

if any other

Question no.

Oppgitt nr

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3.1. 3

2<sup>3</sup> =

$m$	$n$	$L$	$m \leftrightarrow n$	$\neg n$	$(\neg n \vee m)$
1	1	1	1	0	1
1	1	0	1	0	1
1	0	1	0	1	1
1	0	0	0	1	1
0	1	1	0	0	0
0	1	0	0	0	0
0	0	1	1	1	0
0	0	0	1	1	0

$(m \leftrightarrow n) \wedge (\neg n \vee m)$	$\neg m$	$(\neg m \rightarrow L)$	$(m \leftrightarrow n) \wedge ((\neg n \vee m) \rightarrow (\neg m \rightarrow L))$
1	0	1	1
1	0	1	1
0	0	1	0
0	0	1	0
0	1	1	0
0	1	0	0
0	1	1	0
0	1	0	0

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4.2

for basic case  $n = 2$ 

$$4 + 3 \leq 2^4 \quad // \quad n=4$$

$$7 \leq 16$$

let  $n = \text{nonnegative integers}$   $n = k$  ✓ 1

$$2k + 3 \leq 2^k$$

$$P(k+1) = 2(k+1) + 3 \leq 2^{(k+1)}$$

$$✓ 1 \quad 2k + 2 + 3 \leq 2^{k+1}$$

2 for induction

$$2k + 5 \leq 2^{k+1}$$

$$2 \cdot 2^k = 2^{k^2} //$$

$$2 \cdot [k] + 5 \leq 2^{[k^2]}$$

$$2k + 5 \leq 2 \cdot 2^k \quad / : 2$$

$$k + \frac{5}{2} \leq 2^k$$

We can see that  $k^2$  will always be bigger

$$\text{than } \cancel{2k+5} \leq 2^k = \boxed{k \leq k^2}$$

$$\cancel{2k+5} \leq 2^2$$

$$7 \leq 4$$

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4.1

Case ①  $P(1) = n = 1 \checkmark 1$

$$3^2 - 1 = 9 - 1 = 8$$

$$\frac{8x+1}{\sqrt{8x+1}}$$

Case  $P(k) = k^2 - 1 = 8x \rightarrow x \in \mathbb{Z}$

$$\checkmark 1 \quad k^2 = 8x + 1$$

$$k =$$

$$2$$

$$P(k+1) = (k+1)^2 - 1$$

$$\checkmark 1 \quad (\sqrt{8x+1} + 1)^2 - 1$$

$$= 8x+1 + 2\sqrt{8x+1} + 1$$

$$= 8x+1 + 2\sqrt{8x+1}$$

$$= 8x+1 + 2 \frac{8x+1}{\sqrt{8x+1}}$$

$$= 8x+1 + \frac{2 \cdot 8x+2}{\sqrt{8x+1}}$$

$$= 8x + \frac{\sqrt{8x+1} + 2 \cdot 8x+2}{\sqrt{8x+1}}$$

$$= 8x + \frac{\sqrt{8x+1} + 16x+2}{\sqrt{8x+1}}$$

We can see that the first term is displaying 8 and the second term is sum of 8.

3

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5.1.  $\Theta(n)$  0

1/

5.2 (

$\Theta(n)$

$\Theta(n-1)$

$\Theta(n^2 \times n) \Theta(1)$

$= \Theta(n^3)$

which gives

$\Theta(n-1) \Theta(n)$

$\Theta(n^3)$

$\Theta(n)$

$\Theta(n^3)$  1

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5.3

6

16 21 45 8 11 53 3 26

0 1 2 3 4 5 6 7

1) 3 21 45 8 11 53 16 26

2) 3 8 45 21 11 53 16 26

3) 3 8 11 21 45 53 16 26

4) 3 8 11 16 45 53 21 26

5) 3 8 11 16 21 53 45 26

6) 3, 8, 11, 16, 21, 26, 45, 53

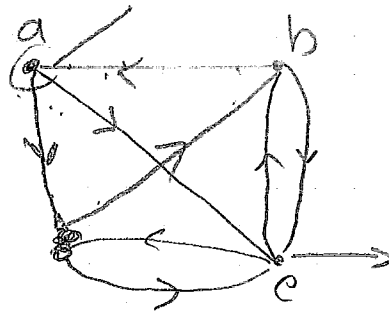
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5.4

3 edges = odd

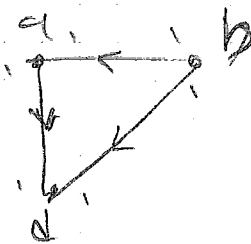
i)



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5 edges which is odd

ii)



all the edges are even

0/

iii)

The first one

$$\text{Matrix 1} = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \quad \checkmark$$

$$\text{Matrix 2} = \begin{matrix} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix} \end{matrix} \quad \checkmark$$

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6.1

$x_i$	$y_i$	$x_i^2$	$y_i^2$	$x_i y_i$
1.47	52.21	2.16	2725.88	76.74
1.50	53.12	2.25	2821.73	79.68
1.52	54.48	2.31	2968.07	82.80
1.55	55.84	2.40	3118.10	86.55
1.57	57.20	2.46	3271.84	89.80
1.60	58.57	2.56	3430.44	93.712
1.63	59.93	2.65	3591.60	97.68
1.65	61.29	2.72	3756.46	101.12
1.68	63.11	2.82	3982.87	106.02
1.70	64.47	2.89	4156.380	109.599
1.73	66.28	2.99	4393.03	114.66
1.75	68.09	3.062	4637.61	119.175
1.78	70.41	3.16	4888.80	124.45
1.80	71.46	3.24	5211.39	129.94
1.83		3.34	5544.29	136.21
$\bar{x} = 1.65$	$\bar{y} = 63.99$	$\sum x_i^2 = 40.01$	$\sum y_i^2 = 57598.39$	$\sum x_i y_i = 1627.26$

n = 15

$$r = \frac{\sum x y}{\sqrt{\sum x^2 \cdot \sum y^2}} = \frac{1627.26}{\sqrt{40.01 \times 57598.39}}$$

$$\sum x y = 1627.26$$

$$\sum x^2 =$$

$$r \approx 1.071$$

$$r = 0.99$$

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6.1

$$\beta = r \times \frac{s_y}{s_x} \checkmark$$

$$s_y = \sqrt{\frac{(y - \bar{y})^2}{n-1}}$$

$$s_y = \sqrt{\frac{(52.21 - 63.99)^2 + (53.12 - 63.99)^2 + (5. \dots)}{14}}$$

$$s_y = \sqrt{\frac{813.711}{14}} = 2.137 = 7.037$$

$$\bar{x} = 1.650$$

$$s_x = \sqrt{\frac{\dots}{14}}$$

$$s_x = \sqrt{\frac{26732.40}{14}} = 43.69 = 0.1142$$

$$\beta = 1.071 \times \frac{2.137}{43.69}$$

$$\beta \approx 0.052 \checkmark 61.54825$$

$$\alpha = \bar{y} - \beta \bar{x}$$

$$= 63.99 - 0.052 \times 1.650$$

$$63.99 \quad 0.085$$

$$= 63.905 \checkmark - 39.51765$$

$$y = 63.90 + 0.052x$$

1

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6.2  $\varphi$ 

2.85

9.7

12.55

$$P(A|B) = \frac{95 \times 3}{95 \times 3 + 97 \times 10}$$

$$= \frac{2.85}{2.85 + 9.7} = \boxed{0.227}$$

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6.3

- i) 1. We assume paired samples
2. We assume our samples <sup>are</sup> independent (since we have two different groups tested twice and not related to each others ( $X_M$ ),  $X_F$ , male female).
3. We assume that we have different sample size //
4. We assume that we don't have large sample size because  $n \leq 30$
5. Normally distributed. 1
6. We assume that we have unequal variances.  
 $\sigma_M \neq \sigma_F$  //

ii) It's two tailed test ✓ 1

iii)  $H_0 = \mu_M = \mu_F$   
 $H_0 \neq \mu_M = \mu_F$  ✓ 1

next page

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6.3

iv)

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SA}$$

$$n = 18$$

$$SA = \sqrt{\frac{12^2}{18} + \frac{12^2}{18}}$$

$$SA = 4$$

$$t = \frac{102.4 - 98.9}{4} = \frac{3.5}{4}$$

$$t = 0.875 \quad \checkmark 1$$

$$v) \text{ d.f.} = 18 - 1 = 17$$

$$\alpha = 0.05 \quad T =$$

$$T_c = 2.110 \quad T_c = 2.030$$

$$T_c > T_s$$

We accept the  $H_0$ 

$$H_0 = \mu_1 = \mu_2$$

$$vi) H_0 = \mu_1 = \mu_2$$

We accept the  $H_0$  //

vii) the sample is quite equal to each other //

//