CHALMERS
EXAMINATION / TENTAMEN

Course code/kurskod	Course name/kursnamn			
DIT 023	Mathematical Foundations for Software Engineering			
Anonymous code Anonym kod		Examination date Tentamensdatum	Number of pages Antal blad	Grade Betyg
1367		26.10.2021	12	

* I confirm that I've no mobile or other similar electronic equipment available during the examination.
Jag intygar att jag inte har mobiltelefon eller annan liknande elektronisk utrustning tillgänglig under eximinationen.

Solved task Behandlade uppgifter No/nr		Points per task Poäng på uppgiften	Observe: Areas with bold contour are to completed by the teacher. Anmärkning: Rutor inom bred kontur ifylles av lärare.
1	✓	5	
2	✓	~~22~~ 23	
3	✓	20	
4	✓	6	
5	✓	~~10~~ 8	
6	✓	19	
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
Bonus poäng			
Total examination points Summa poäng		91	

2

Q3

3.1)

p	q	r	p ∧ ¬q	¬q	¬p	r ∨ p	¬r	¬r ⟷ ¬p	(r∨p) ∧ (¬r → ¬p)
0	0	0	0	1	1	0	1	1	0
0	0	1	0	1	1	1	0	0	0
0	1	0	0	0	1	0	1	1	0
0	1	1	0	0	1	1	0	0	0
1	0	0	1	1	0	1	1	0	0
1	0	1	1	1	0	1	0	1	1
1	1	0	0	0	0	1	1	0	0
1	1	1	0	0	0	1	0	1	1

p	q	r	(p ∧ ¬q) → ((r ∨ p) ∧ (¬r ⟷ ¬p))
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

8p

full expression

3.2) (c) $\exists x \exists y ((x \neq y) \wedge V(x) \wedge V(y) \wedge \forall z (V(z) \rightarrow (z = x \vee z = y)))$

1p

x and y are not the same person

x and y are both vegan

if a group member is vegan, it is x or y. thus there are no other vegans.

3.3) (e) $\exists x (D(x)) \rightarrow \forall y (S(y) \rightarrow \neg E(y,x))$

1p

there would be someday

that every student

does not need to attend the examination

// no student attends the examination

2

3.4)

i) A tautology is a compound proposition which, no matter the input, always results in true. e.g. $(p \vee \neg p)$ 1p

ii) A compound proposition is not satisfiable when there exists no combination of inputs which make the compound proposition true.
Therefore it is a contradiction and always results in false. 1p

iii) Contingencies can result in true or false depending on the input.
Therefore when a compound preposition is a Tautology (always true) or a Contradiction (always false), it is not a contingency. 1p

iv)

p	q	r	$p \wedge \neg q$	$r \vee p$	$\neg r \vee \neg p$	$(\neg r \vee \neg p) \wedge p$	$(r \vee p) \wedge ((\neg r \vee \neg p) \wedge p)$	full expression
0	0	0	0	0	1	0	0	1
0	0	1	0	1	1	0	0	1
0	1	0	0	0	1	0	0	1
0	1	1	0	1	1	0	0	1
1	0	0	1	1	1	1	1	1
1	0	1	1	1	0	0	0	0
1	1	0	0	1	1	1	1	1
1	1	1	0	1	0	0	0	1

6 pts

this means there exists a combination which leads to false.
thus this compound proposition is NOT always true. 1p

v) $\big((p \wedge \neg q) \rightarrow ((r \vee p) \underline{\vee} ((\neg r \vee \neg p) \wedge p))\big)$

switches the 0 to a 1

1pt 2pts

total 3 / 20

CHALMERS	Anonymous code	Points for question (to be filled in by teacher)	Consecutive page no. Löpande sid nr 1
	Anonym kod 1367	Poäng på uppgiften (ifylles av lärare)	Question no. Uppgift nr 1

Q1

1.1) (c) ✓ w contains only 1s except for the last symbol which is a 0. 1

1.2) (b) ✓ w contains the same amount of 0s and 1s in alternating order. 1

1.3) 1SW3 → 1W3 → 123

~~1SW3 → 11SW33 → 11W33 → 1123~~

1SW3 → 11SW3W3 → 11W3W3 → 112W33 → 112233

(b) ✓ $1^n 2^n 3^n$ | where $n \geq 0$ 1

1.4) Help, Math ✓ 2 ✓

totalt 5

2

Q2

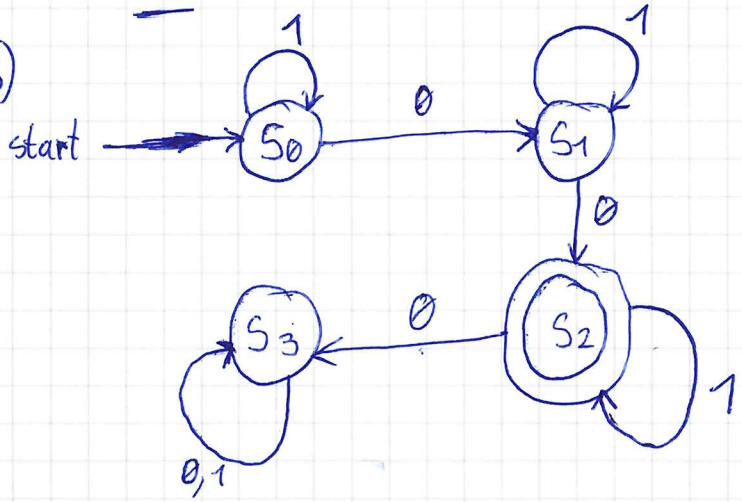
2.1) 1101 → 001101 ✓ (not c, d, or e) 11000 → 00110 (not b)

1100 → 001100 ✓

@ it recognizes when the output equals the input delayed by two 0s.

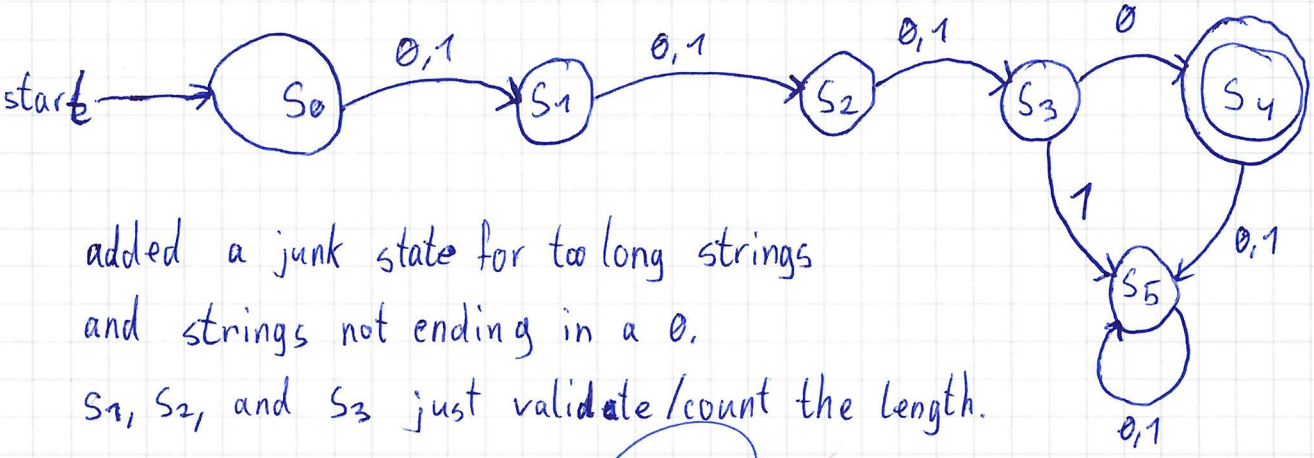
2.2) b bit strings with exactly one transition between 0 and 1

2.3)



allowed trailing 1s, added a junk state for too many 0s

2.4)



added a junk state for too long strings and strings not ending in a 0.
S_1, S_2, and S_3 just validate/count the length.

total 23

2

Q4

4.1) $1+2+\ldots+n = \frac{3n^2+3n}{6}$ for $n>0$

Mathematical Induction:

base step) $n=1$

$$1 = \frac{3\cdot 1^2+3\cdot 1}{6}$$

$$1 = \frac{3+3}{6}$$

$$1 = 1 \quad \checkmark$$

7/8

inductive hypothesis)

$P(k) \rightarrow p(k+1)$

$p(k)$: $1+2+\ldots+k = \frac{3k^2+3k}{6}$ for $k=n$

7/8

inductive step)

$p(k+1)$: $1+2+\ldots+k+(k+1) = \frac{3(k+1)^2+3(k+1)}{6}$

missing case I.H.

$$\frac{3k^2+3k}{6_2} + (k+1) = \frac{3(k+1)^2+3(k+1)}{6_2}$$

$$\frac{k^2+k}{2} + (k+1) = \frac{(k+1)^2+(k+1)}{2} \quad /\cdot 2$$

$$k^2+k+2k+2 = k^2+2k+1+k+1$$

$$k^2+3k+2 = k^2+3k+2 \quad \checkmark$$

7/8

2

4.2)

$2k$ ... even $\quad 2l+1$ ... odd

contradiction

there is ~~a~~ a largest even number (1pt)

$2k + x = 2l+1$ ~~largest even~~ largest even number + anything = odd number

let x be an even number as well (1pt)

$2k + 2m = 2l+1$

$2(k+m) = 2l+1 \qquad 2(k+m) > 2k$

$2(k+m)$ ← even number

thus every time we add an even number to our "largest even number" we get a new bigger even number.

As a result no "largest even number" exists.

3p

Total 4) 6

2

Q5

5.1) $O(1 \cdot n \cdot (\frac{1}{2} \cdot \frac{n}{2}))$

$O(n \cdot \frac{1}{4} n)$

$O(n^2)$ ✓ 2p ✓

5.2) $O(\log n \cdot n)$

$O(n \log n)$ ✓ 2p

5.3) $O(n \cdot \frac{n}{2})$

$O(n^2)$ ✓ 1p ✓

5.4)

initial:	3	15	-8	5	13	-1	9	19	-4
1. iteration:	-8	15	3	5	13	-1	9	19	-4
2. ...:	-8	-4	3	5	13	-1	9	19	15
3.:	-8	-4	-1	5	13	3	9	19	15
4.:	-8	-4	-1	3	13	5	9	19	15
5.:	-8	-4	-1	3	5	13	9	19	15
6.:	-8	-4	-1	3	5	9	13	19	15
7.:	-8	-4	-1	3	5	9	13	19	15
8.:	-8	-4	-1	3	5	9	13	15	19

8p ✓

5.5) both are $O(n^2)$ ✓ since the algorithm does not break nor detect when the array is sorted already. Therefore it will always run the full algorithm with its worst-case complexity.

2p ✓

5.6) best-case: O(n) since it halts when no swaps were needed which means the array is already sorted.
collection

average-case: $O(n^2)$ 2p

5.7) 1) in = 2 + 2 + 2 + 1 = 7

out = 1 + 2 + 3 + 1 = 7 2p

2) 2 euler paths exist 2p

C → B → D → C → C → A → B → A

C → C → B → D → C → A → B → A

3) C → B → D → C → C → A → B → A 1p

4) true the path A → B → D → C → A provides a circuit which can be started/ended at any vertice 1p

total 5) 5

2

Q6

6.1)

1) $\hat{y} = \alpha + \beta x$

$\hat{\beta} = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n \cdot \bar{x}^2}$

$\bar{x} = 1{,}5$ $\quad \bar{y} = 2{,}5$ $\quad n = 4$

$\Sigma xy = 2+6+12 = 20$ $\quad \Sigma x^2 = 1+4+9 = 14$

$\hat{\beta} = \frac{20 - 4 \cdot 1{,}5 \cdot 2{,}5}{14 - 4 \cdot 1{,}5^2}$

$\hat{\beta} = \frac{20-15}{14-9} = \frac{5}{5} = 1$

$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} = 2{,}5 - 1 \cdot 1{,}5 = 1$

$\hat{y} = 1 + 1 \cdot x$

2) $\Sigma (y - \hat{y})^2 = (1-1)^2 + (2-2)^2 + (3-3)^2 + (4-4)^2 = 0$

3) yes, the values match perfectly with a linear function.
Therefore the regression error = 0

4) yes, the values match perfectly with a linear function.
Therefore the correlation coefficient is 1

perfect! 4pt

5) $\hat{y}(\text{year } 2021) = \hat{y}(4) = 1 + 1 \cdot 4 = 5$

6.2)

$p(x) = e^{-\lambda} \cdot \frac{\lambda^x}{x!}$

$p(0 \text{ number} | \text{beneficial}) = 0{,}95 \cdot e^{-2} = 0{,}12057$

$p(0 \text{ number}) = 0.95 \cdot e^{-2} + 0{,}05 \cdot e^{-4} = 0{,}12948$

95% beneficial → e^{-2} 0

5% not beneficial → e^{-4} 0

~~P(beneficial | 0~~

$p(\text{beneficial} | 0 \text{ number}) = \frac{0{,}12057 \cdot 0{,}95}{0{,}12948} = 0{,}94332 \approx 94{,}332\ \%$

sajur! 3.5pt

totally 7.5pt

CHALMERS | Anonymous code: 1367 | Points for question (to be filled in by teacher) | Consecutive page no. Löpande sid nr: 10 | Question no. Uppgift nr: 6

6.3) $p = 1/5$, $n = 10$

$$P(x=5) = \binom{n}{x} \cdot p^x \cdot q^{n-x}$$

$$= \binom{10}{5} \cdot 0{,}2^5 \cdot 0{,}8^5$$

$$= 252 \cdot 0{,}00032 \cdot 0{,}32768$$

$$P(x=5) = 0{,}02642 \approx 2{,}642\%$$

very good! / 1.5pt

6.4)

2) Null hypothesis: the mean values are equal for the groups

$$\bar{x}_1 = \bar{x}_2 = \bar{x}_3$$

Alternative hypothesis: the mean values are NOT equal for the groups

$$\bar{x}_1 \neq \bar{x}_2 \neq \bar{x}_3$$

3) ~~SST = SSB + SSW = 390,6 + 3462,89 = 3853,49~~

~~$MSTR = \frac{SSB}{k-1} = \frac{3462,89}{2} = 1731,445$~~

~~$MSE = \frac{SSW}{(kn-k)} = 390,6$~~

$$MSTR = \frac{SSB}{k-1} = \frac{390,6}{2} = 195,3$$

$$MSE = \frac{SSW}{(kn-k)} = \frac{3462,89}{87} = 39,803$$

$$F = \frac{MSTR}{MSE} = \frac{195,3}{39,803} = 4,906 \approx 4,91$$

great! / 4.5pt

4) critical value $F_{2,87}$ for $\alpha = 1\%$:

critical value $= 4,86$

5) $4,91 > 4,86 \rightarrow$ reject null hypothesis

CHALMERS | Anonymous code: 1367 | Points for question (to be filled in by teacher) | Consecutive page no. Löpande sid nr: 11 | Question no. Uppgift nr: 6

6.5) $\frac{50}{\cancel{106}} = 7,25$ individual lines x^2 ... 1 main line

$\alpha = 0,05$ (5%)

1) null hypothesis = a single line of the food delivery service causes lower variation among waiting times

alternative hypothesis = a single line of the food delivery service does NOT cause lower variation among waiting times

2)

$$x^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

$$x^2 = \frac{142,08889}{7,25^2}$$

$$x^2 = 2,7032 \approx 2,703$$

$$(n-1)s^2 = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$= 584,49 - \frac{63,1^2}{9}$$

$$(n-1)s^2 = 142,08889$$

3) critical value = 15,507

4) $2,703 < 15,507$, we accept the null hypothesis

very good sp#

5/5

2

6.6)

1) null hypothesis: the new video is effective in increasing the children's knowledge about asthma.

alternative hypothesis: the new video is NOT effective in increasing the children's knowledge about asthma.

2) ~~$t = \frac{\bar{d} - \mu_0}{\frac{s}{\sqrt{n}}}$~~ ~~$\bar{d} = 1+1+1-1+1/7 = 3/7 = 0{,}42857$~~ ~~$\sqrt{n} = \sqrt{7} = 2{,}64575$~~

$$t = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n} + \frac{1}{m}}} \qquad s = \sqrt{\frac{(n-1)\cdot s_x^2 + (m-1)\cdot s_y^2}{(n-1)+(m-1)}}$$

$n = m = 7$

$s = \sqrt{\frac{6\cdot s_x^2 + 6\cdot s_y^2}{12}}$

x ... before $s_x = 2{,}16025$ $\bar{x} = 5$

y ... after $s_y = 1{,}71825$ $\bar{y} = 5{,}42857$

$s = \sqrt{1{,}93925}$

$s = 1{,}39257$

$$t = \left|\frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n} + \frac{1}{m}}}\right| = |-0{,}53305|$$

$t = 0{,}53305$

3) critical value = 1,895

4) $0{,}53305 < 1{,}895$, we accept the null hypothesis

→ 1pt

1pt + 1pt for 2+ test!

totally 2pt