

## **Topic 6: Fuzzy Inference**

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GitHub Repository: <https://github.com/JonathanPollyn/Machine-Learning-for-Data-Science>

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## Theory – Part 1

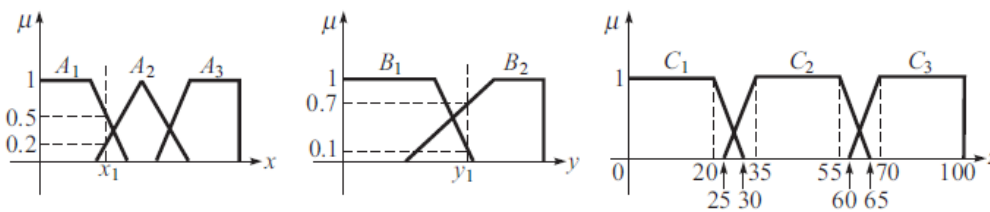
Consider the following fuzzy model of a system with inputs  $x$  and  $y$  and output  $z$

Rule 1: If  $x$  is  $A_3$  OR  $y$  is  $B_1$  THEN  $z$  is  $C_1$

Rule 2: If  $x$  is  $A_2$  AND  $y$  is  $B_2$  THEN  $z$  is  $C_2$

Rule 3: If  $x$  is  $A_1$  THEN  $z$  is  $C_3$

The membership functions of the input and output variables are given in the graphs below



Actual inputs are  $x_1$  and  $y_1$ . Find the output  $z$  by applying standard fuzzy operation:

*min* for AND

*max* for OR.

### Rule1

Rule 1: If  $x$  is  $A_3$  OR  $y$  is  $B_1$  THEN  $z$  is  $C_1$

$X = A_3 = \text{high}$

$Y = B_1 = \text{Low}$

$C_1 = \text{Low}$

Rule 1: IF  $x = A_3 = \text{high}$  AND  $y = B_1 = \text{low}$  THEN  $z = C_1 = \text{low}$

Rule 1: IF  $x = \text{high}$  AND  $y = \text{low}$  THEN  $z = \text{low}$

### Rule 2

Rule 2: If  $x$  is  $A_2$  AND  $y$  is  $B_2$  THEN  $z$  is  $C_2$

$X = A_2 = \text{High}$

$Y = B_2 = \text{High}$

$C_2 = \text{High}$

Rule 2: IF  $A_2 = \text{high}$  AND  $Y = B_2 = \text{High}$  THEN  $Z = C_2 = \text{High}$

Rule 2: If  $x = \text{high}$  AND  $y = \text{high}$  THEN  $z = \text{high}$

### Rule 3

Rule 3: If  $x$  is  $A_1$  THEN  $z$  is  $C_3$

$X = A_1 = \text{Low}$

$Y = ?$

$Z = C_3 = \text{high}$

Then

To determine  $y = \text{Low}$

Rule3: IF  $x = A_1 = \text{Low}$  AND  $y = \text{Low}$  THEN  $z = C_3 = \text{high}$

Rule 3: IF  $A_1 = \text{Low}$  AND  $y = \text{Low}$  THEN  $z = \text{High}$

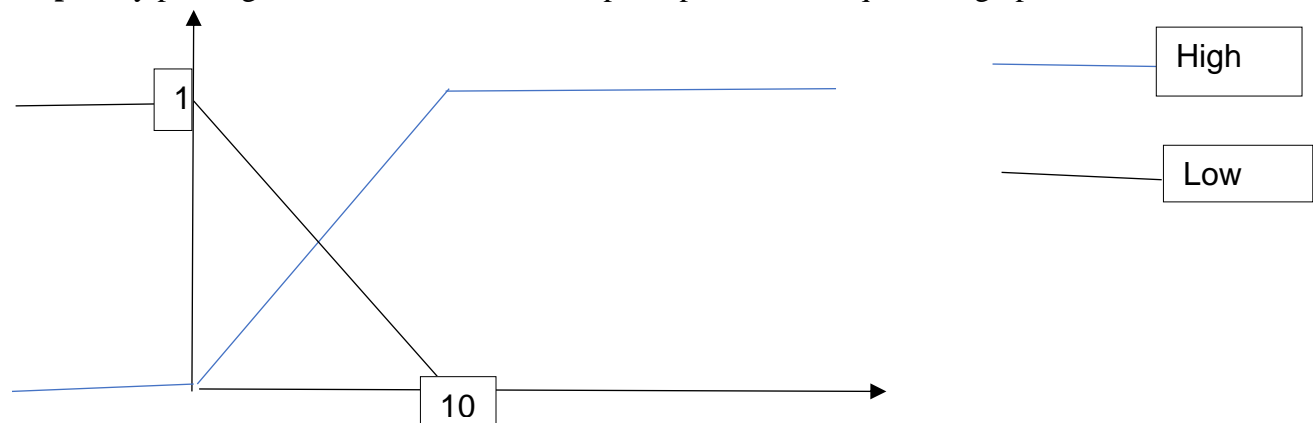
The function Membership of high and low is:

(t) high = (t) 0.1

(t) low = (t) 1 - 0.1

The system inputs are  $y = 3.2$  and  $x = 0$  require is the output of the system

**Step 1.** By plotting the function of membership, it's possible to acquire the graph below.



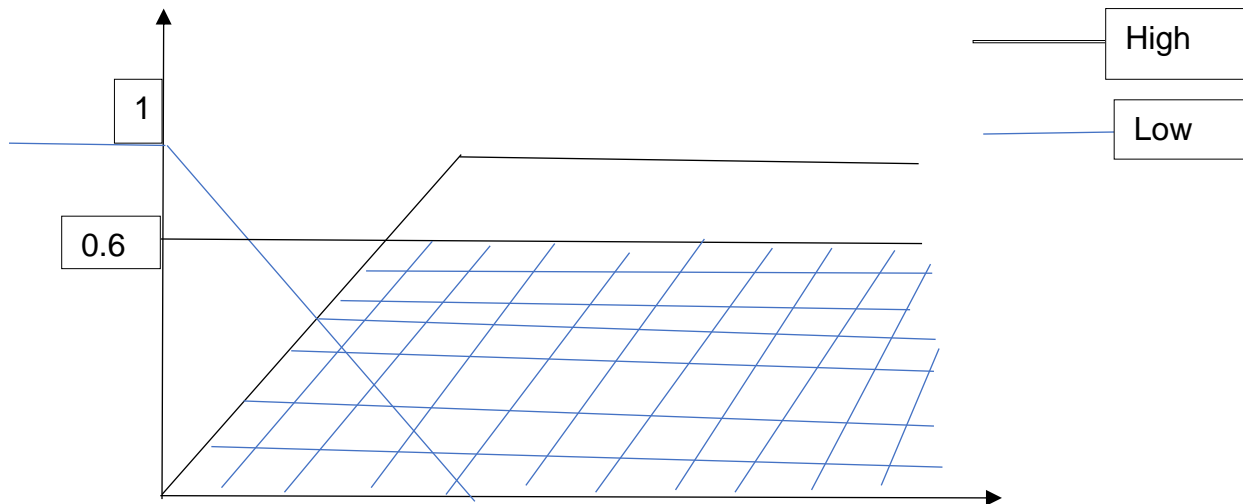
x	y	High x	Low x	High y	Low y	Rule 1	Rule2	Rule 3	
0	3.2	0	1	0.32	0.68	0*	0*	0.68	

Through the use of  $\mu_{A \cap B} [x] = \text{minimum} \{ \mu_A [x], \mu_B [x] \}$

**Step 2.** Output that uses Minimum Inferencing (also referred as Clipping)

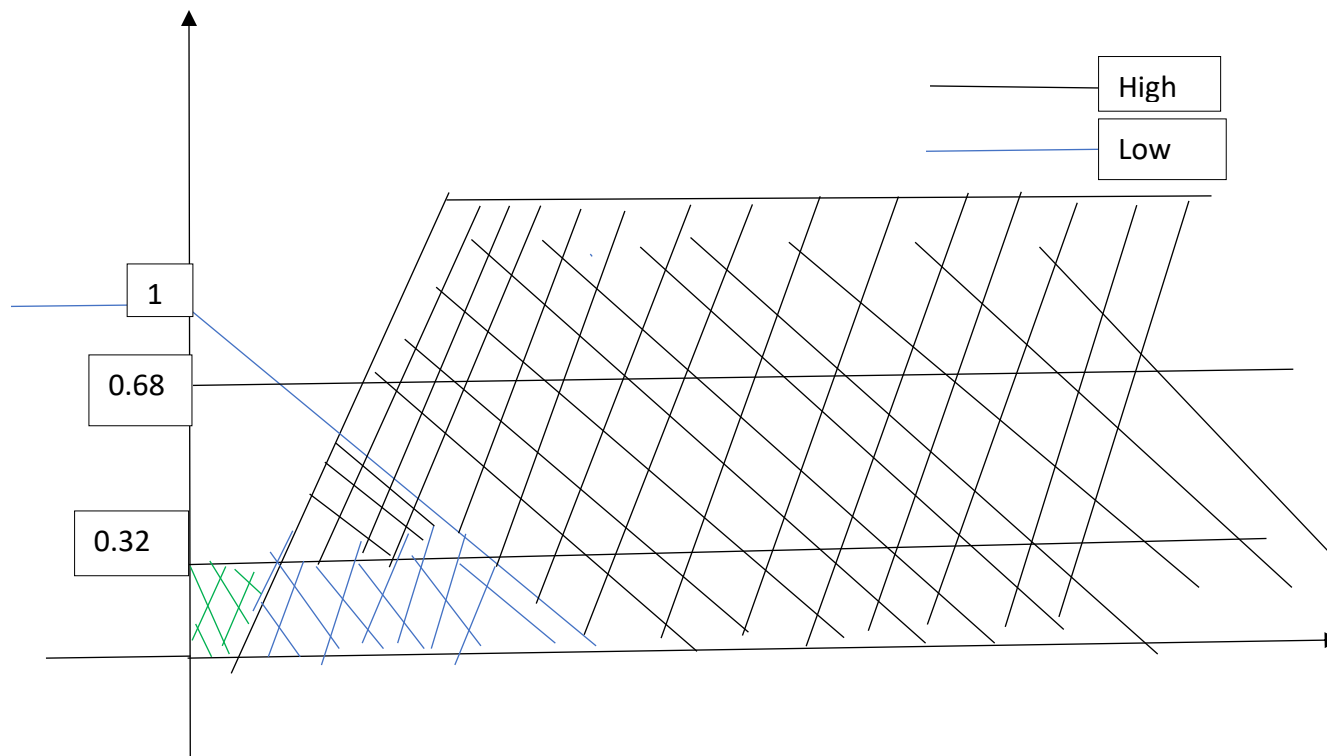
Rule 3: IF x = low AND y = low THEN z = high

Rule 3: where z= 0.68



- Rule 1, Z=0
- Rule 2, Z=0

**Step3:** Defuzzification



$$\text{fuzzy}(Z) = \begin{cases} 0.32, & z \leq 3.2 \\ 0.1z, & 3.2 < z < 6.8 \\ 0.68, & z \geq 6.8 \end{cases}$$

Maxi                      Maxi

$$c = \frac{\int_{\text{Mini}} z f(z) dz}{\int_{\text{Mini}} f(z) dz} \text{ divide by } \frac{\int_{\text{Maxi}} f(z) dz}{\int_{\text{Maxi}} 1 dz}$$

$$= \frac{0.32 \int_0^{3.2} dz + 6.8 \int_{3.2}^{6.8} 0.1z dz + 10 \int_{6.8}^{\infty} 0.68 dz}{\int_0^{3.2} 0.32 dz + 6.8 \int_{3.2}^{6.8} 0.1z^2 dz + 10 \int_{6.8}^{\infty} 0.68z dz} \text{ divide by } \int_{32}^{\infty} 1 dz$$

$$C = 29.3 \text{ divide by } 5$$

5.56

## **Part 2**

### **Solving the Ocean Color Inverse Problems by Using Evolutionary Multi-Objective Optimization of Neuro-Fuzzy Systems” from your topic resources.**

The efficiency of energy in the building industry is the energy amount relationship that is got through the saving and improvement of thermal. Therefore, it is significant to get the potential of energy-saving mechanisms of upgrading the thermal to determine whether the energy saved is got after correcting measures implementation. In the residential building of real estate's when initiation the energy calculation s challenges are encountered through incomplete architectural documents or data inaccuracy through object characterization when terming to partitions of thermal resistance and inhabitant number which are usable. Therefore, there is a need for technical measures that suit the measurement of analysis that improve energy efficiency in modern buildings. The project's objectives are testing the use of the fuzzy model of Takagi-Sugeno of models of inference that predict the efficiency energy real residential and habitant building that has taken thermal improvement (Jacquin et al.,2006). The group of building such as 109 and its powerful characterization of variables have already been analyzed using standards of statistical calibration developed and implemented by the society of refrigerating heat and engineering of air conditioning (ASHRAE). The results obtained were then estimated with other models of predictions through the basis of similar data set imputes with the help of modern rough and network sets of theories (Lohani et al.,2016).

## **Application of ANFIS**

The inference system of adaptive neuro-fuzzy or adaptive inference of fuzzy systems on network-based (ANFIS) is atypical of network artificial neural made through inference models of fuzzy Takagi–Sugeno. This is because, in offered integration of the two neural networks & principles of fuzzy logic, it contains the potential to hold the importance of the two in one framework (Stojčić, 2018). The inference system corresponds to fuzzy set rules (IF and THEN) that offer the capability of learning to approximate the nonlinear function. Therefore, ANFIS is supposed to be estimator universal. To use ANFIS, it is essential to be very optimal or efficient; this is to utilize parameters that are best obtained through a genetic algorithm (Angelov and Filev, 2004).

### **Application in ANFIS**

#### **The Representation of models of Fuzzy in Takagi-Sugeno**

Through simple terms, it is assumed that the system of inference in fuzzy is in consideration that has about two different inputs  $y$  and  $x$ , and only single output  $z$ . The initial model of Fuzzy in Takagi-Sugeno has the set of common rules of (IF and Then) (Naderloo et al., 2012).

Rule 1:

If  $x = A1$  &  $y = B1$  then  $f1$  is equal to  $r1 + p1x + q1y$

Rule 2:

If  $x = A2$  &  $y = B2$  then  $f2$  is equal to  $r2 + p2x + q2y$

#### **The Representation of models of Tsukamoto Fuzzy**

The TS ANFIS extension in Tsukamoto ANFIS, Since the output of every rule ( $F_i$  and  $i=1$  and 2) is jointly induced by a membership consequent of function and strengths of firing (Wahyun and Mahmudy 2017).

## **Representation of model of Mamdani Fuzzy**

The inference system of Mamdani fuzzy with the composition of max-min, an ANFIS corresponding, is constructed through the approximations to replace the centroids integrals in schemes of defuzzification introduced in the system. However, the resulting ANFIS is very complicated compared to Tsukamoto's ANFIS and TS ANFIS. The complexity of extra structure complexity in and Mamdani ANFIS computation with the assistance of max-min composition doesn't imply the capability of better learning or power approximation. The adoption of product sum of centroid defuzzification and composition for a model of Mamdani fuzzy, an ANFIS corresponding is easily constructed with the base of direct Theorem without approximation use at any place (Tung, et al 2009).

In other areas, the ANFIS controller is highly used in non-linear control systems. This is because it is the best of all the controllers when the comparison is made to the PID conventional controller, plus another known system controller. This controller is indeed utilized together with the water bath temperature controller. Another place where controllers are required is in planes to offer their control. Today studies indicated that intelligent planes used in learning contain this control in the research field, where it helps when the plains take off and when they land.

## **Part 2 – Fuzzy Models**

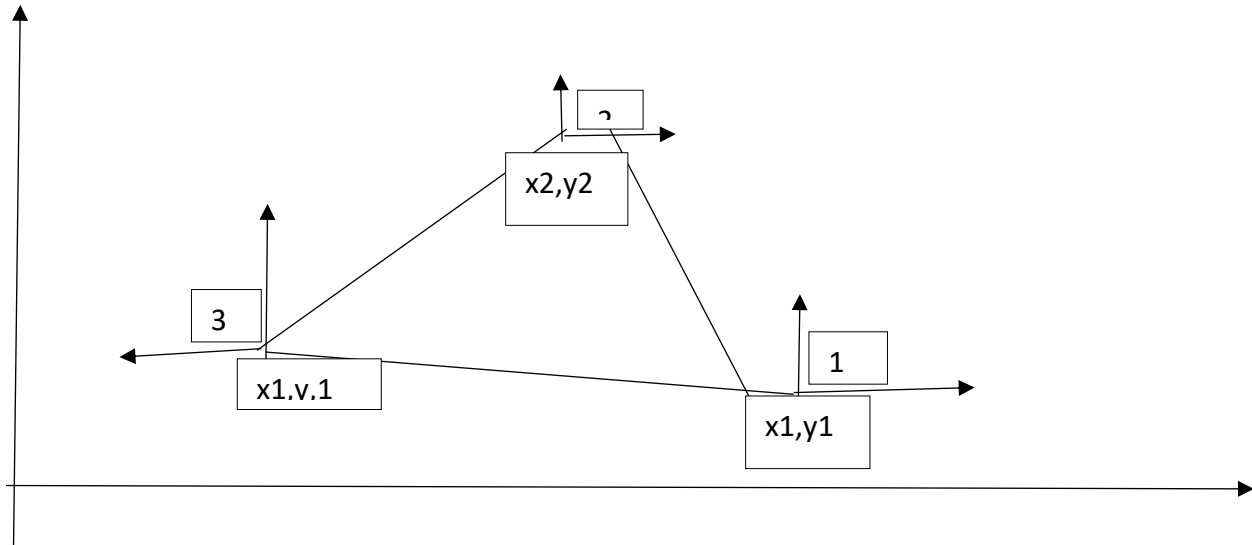
### **Task1.Two-dimensional equation on sinc definition by**

$Y = \text{sinc}(x_1, x_2) = \frac{\sin(x_1)}{x_1} \cdot \frac{\sin(x_2)}{x_2}$

input range between  $[-10, 10] \times [-10, 10]$ . two symmetric functions of triangular membership are assigned to every variable of input to the construction of Fuzzy models in Takagi-Sugeno for the function of Sinc. Required to offer definition determination of equations of the consequent and premise model parameters.



**Solution.**



Input data as  $x1, y1 = (np$

Input data as  $x2, y2 = (plt$

$\text{sinc2d}$  is equal to  $np=\text{zeros} = (50, 50)$

Therefore for

$x, x1$  will enumerate to the following  $\{np: \text{linspace} = (-10; 10; 50)\}$

$y, x2$  will enumerate to the following  $\{np: \text{linspace which is equal to } (-10; 10; 50)\}$

$=\text{sinc2d}\{x,y\} = np:\sin\{x1\}$  multiply by  $np:\sin\{x2\}$  divided by  $\{x1$  multiply with  $x2\}$

Equivalently the functions are as follows.

$x1$  is equal to  $np:\text{linspace}=(-10, 10, 50)$

$x2$  is equal to  $np:\text{linspace}=(-10, 10, 50)$

$\text{sinc2d}$  is equal to  $np:\text{outer}=(np:\sin\{x1\}$  multiply by  $np:\sin\{x2\}$  divide with  $np:\text{outer} = \{x1, x2\}$

## **Task 2. Identification of non-linear system**

$$y = (1 + (x_1)^{0.5} + (x_2)^{-1} + (x_3)^{-1.5})^2$$

Training data =  $[1, 6] \times [1, 6] \times [1, 6]$

Test data =  $[1.5, 5.5] \times [1.5, 5.5] \times [1.5, 5.5]$

Therefore, required to

Through explanation and listing all the rules.

According to Xiong (2001), the used rules are as indicated below.

**Rule 1** In the case of A & B are both sets of fuzzy in the antecedent, then  $z = f(x, y)$  becomes a crisp function in the consequent. For instance, take the  $Z = f(1, 6)$  training data, where  $f$  is the antecedent function.

### **Rule 2**

Where  $(x, y)$  is an input polynomial of variables  $y$  &  $x$ . However, it is considered as any function.

For instance, the data from the training and test are indicated below. Data from training  $x=1$  while  $y=6$  on the other hand data from the test  $x=1.5$  while  $y=5.5$

### **Rule 3**

Where  $f(x, y)$  occurs as the polynomial of first order, the resulting fuzzy system of inference is referred to as the model of fuzzy first-order in Sugeno, for instance,

$$z = c + ax + by.$$

In our case the Training data  $= Z = c + a(1) + b(6)$

$$\text{So, } Z = c + 1a + 6b$$

Other hand the test data  $= Z = c + a(1.5) + b(5.5)$

$$\text{So, } Z = c + 1.5a + 5.5b$$

### **Rule 4**

When constant  $f$  is present, then the model of fuzzy zero-order in Sugeno is  $z = c + ax + by$  and where  $(a$  is equal to  $b$  which is also equal to  $0)$  it is seen as a case specialty of the System interference of Mamdani Fuzzy where all rules in consequent offer the specification by singleton fuzzy. For example

Training data

$$Z=c+ax+by$$

$$Z=c+0(x)+0(y)$$

$$Z=c+0(1)+0(6)$$

$$Z=c$$

Test data

$$Z=c+ax+by$$

$$Z=c+0(x)+0(y)$$

$$Z=c+0(1.5)+0(5.5)$$

$$Z=c$$

## References List

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