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Statistics of Points Scored in NBA Basketball

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Abstract

The distribution of points scored in NBA basketball is studied with the victorious Chicago Bulls of the 1990's as an example. The average points scored by the players of a team followed an exponential pattern. The frequency distribution of points scored by an individual player is normally Gaussian. In the six NBA finals won by the Chicago Bulls, the distribution of points scored by their two leading players Michael Jordan and Scottie Pippen showed interesting deviations from the normal Gaussian patterns. The distribution of points scored by Jordan was significantly skewed to the right and slightly peaked. The distribution of points scored by Pippen, on the other hand, was slightly skewed to the left and slightly flattened. Interestingly, the sum of the two distributions was nearly normal or Gaussian. The points scored by Jordan and Pippen were negatively correlated in the high scoring games (of 100 or more points) and in games won by the Bulls, and positively correlated in low-scoring games (of under 100 points) and in games lost by the Bulls. Overall, the Chicago Bulls were the most successful, when Jordan and Pippen scored well but when their scores were negatively correlated.

1. INTRODUCTION

With the possible exception of Cricket, the game of Basketball is perhaps unsurpassed in terms of the statistical data recorded. The National Basketball Association (NBA), for example, records points scored, rebounds taken, assists dished out, blocked shots, steals, fouls and turnovers, among others. However, the points scored by a team is the most important quantity upon which the outcome of a game is determined. In this study, we analyze the points scored by a team with the victorious ***Chicago Bulls*** of the 1990's as an example. We further analyze the distribution of points scored by the two leading players of that team − ***Michael Jordan*** and ***Scottie Pippen***.

Michael Jordan was, by acclamation, the greatest Basketball player in the history of the game [1]. Amongst the numerous records set by him in the NBA were: (1) a record 10-time scoring titles; (2) all-time regular season scoring average (33.12 points per game);

and (3) all-time playoffs scoring average (33.45 points per game) [1, 2]. Scottie Pippen was a great all-round player [3] who played 10 seasons with Michael Jordan winning six NBA titles together for the Chicago Bulls. This paper analyzes the distribution of points scored by the Chicago Bulls, and Jordan and Pippen in particular − in the NBA finals of the 1990's.

2. CHICAGO BULL'S NBA FINALS STATISTICS OF THE 1990'S

In the 1990's, the Chicago Bulls won six NBA Championships led by Michael Jordan and Scottie Pippen among others by dispatching their opponents in six or fewer games in best of seven series. Table I shows the points scored in each of those games by the Bulls, Jordan and Pippen (from [4 − 9]). It also shows their average points per game. Interestingly, Jordan alone scored 34.76% or over one third of the total points and Pippen was a distant second with 19.63%. The two players together thus scored 54.38% or over half of the total points. Jordan was the first option in the Bulls offense, who more often scored, even going around two defenders. When he was crowded by the defense, the team went to Pippen, their second option. This explains the scoring patterns of the Bulls and their two leading players.

3. DISTRIBUTION OF SCORING AVERAGES WITHIN A TEAM

In an NBA game, the team roster consists of up to 12 players, including the 5 starters and 7 reserves. In a modern playoff game, both teams score around 100 points, and the outcome is normally decided by only a few points. In an average game, the 100 points are contributed by the players who played, and the contributions naturally differ greatly, with the best offensive player scoring a lion's share of the points, and the reserve players in the tail-end of the roster scoring only a few points or none at all with the time they get on the floor. It is a law of nature that the scoring averages of the players follow an ***exponentially diminishing trend***. An exponential distribution is given by a two-parameter function

$$y = ae^{-\alpha x} \tag{1}$$

where a is the amplitude and α is the decay constant (in analogy with radioactive decay phenomenon). This trend is shown in Fig.1, where the histogram represents the scoring averages of the 12 players in a team. Since the x variable is discrete, the leading column is situated at x = 0.5; the second leading column is located at x = 1.5; and so, on up to x = 11.5. Integrating Eq. (1) from x = 0.5 to x = 11.5, we get

$$A = \int_{0.5}^{11.5} y dx = \frac{a}{\alpha}\left[e^{-0.5} - e^{-11.5}\right] \tag{2}$$

Equation (2) is recognized as the total average score of the team.

Table I. Points scored in NBA Finals by Chicago Bulls, Jordan & Pippen

Year	Opponents	Game	Win/Loss	Points scored		
				Chicago	Jordan	Pippen
1991	Los Angeles Lakers	1	Loss	91	36	19
		2	Win	107	33	20
		3	Win	104	29	19
		4	Win	97	28	14
		5	Win	108	30	32
1992	Portland Trailblazers	1	Loss	122	39	24
		2	Win	104	39	16
		3	Loss	94	26	18
		4	Win	88	32	17
		5	Win	119	46	24
		6	Win	97	33	26
1993	Phoenix Suns	1	Win	100	31	27
		2	Win	111	42	15
		3	Loss	121	44	26
		4	Win	111	55	14
		5	Loss	98	41	22
		6	Win	99	33	23
1996	Seattle Supersonics	1	Win	107	28	21
		2	Win	92	29	21
		3	Win	108	36	12
		4	Loss	86	23	9
		5	Loss	78	26	14
		6	Win	87	22	17
1997	Utah Jazz	1	Win	84	31	27
		2	Win	96	38	10
		3	Loss	93	26	27
		4	Loss	73	22	16
		5	Win	90	38	17
		6	Win	90	39	23
1998	Utah Jazz	1	Loss	85	33	21
		2	Win	93	37	21
		3	Win	96	24	10
		4	Win	86	34	28
		5	Loss	81	28	6
		6	Win	87	45	8
Average points/game →				96.66	33.60	18.97

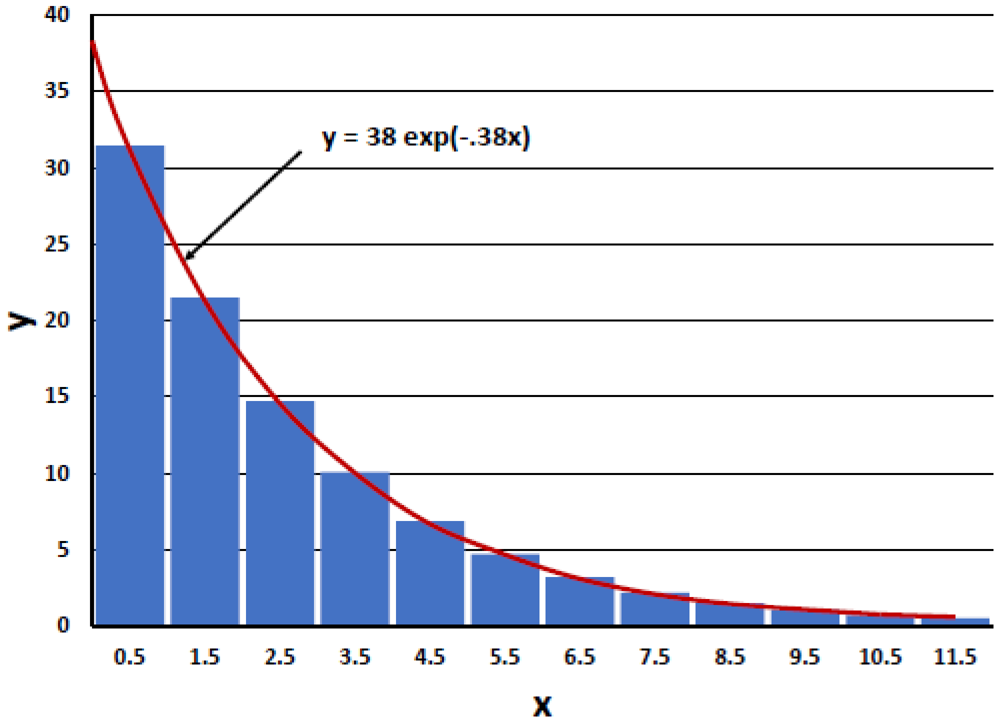


Fig. 1. Exponential trend of scoring averages within a team.

As applied to our problem detailed in Table I, we have: (1) $y_{0.5}$ = 33.60 (Jordan scoring average); (2) $y_{1.5}$ = 18.97 (Pippen scoring average); and (3) A = 96.66 (Chicago Bulls average score). Now, we have three equations to determine the two parameters a and α in Eq. (1). The problem is thus over-determined. In principle, one can hold any one condition as known and determine the two constants from the two remaining conditions. However, it is found that no solution satisfies all three conditions exactly. This means that the data, in fact, deviates slightly from that given by the exponential trend. Alternatively, one can find an approximate solution by ignoring the second term on the right hand side of Eq. (2), and iterating the solution. The best result was approximately given as: a = 38; and α = 0.38. Figure 1 has been plotted with these values. It was found that the solution slightly under-estimated the Jordan scoring average (31.42 instead of 33.60) and slightly over-estimated the Pippen scoring average (21.49 instead of 18.97).

4. FREQUENCY DISTRIBUTIONS OF POINTS SCORED BY JORDAN & PIPPEN

It is a law of nature that the random distribution of the scores of any individual player tends to be a bell-shaped curve known as the ***Gaussian or normal distribution***. If the variable x denotes the scoring of a player, the ***mean*** $\bar{x}$, **variance** σ^2 and ***standard deviation*** σ are defined by the following (cf. [10]):

$$\bar{x} = \frac{\sum x_i}{n} \tag{3}$$

$$\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n} \tag{4}$$

and

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} \tag{5}$$

where the summations run from 1 to n, the number of scores. The ***third and fourth moments*** are defined respectively as:

$$m_3 = \frac{\sum (x_i - \bar{x})^3}{n} \tag{6}$$

and

$$m_4 = \frac{\sum (x_i - \bar{x})^4}{n} \tag{7}$$

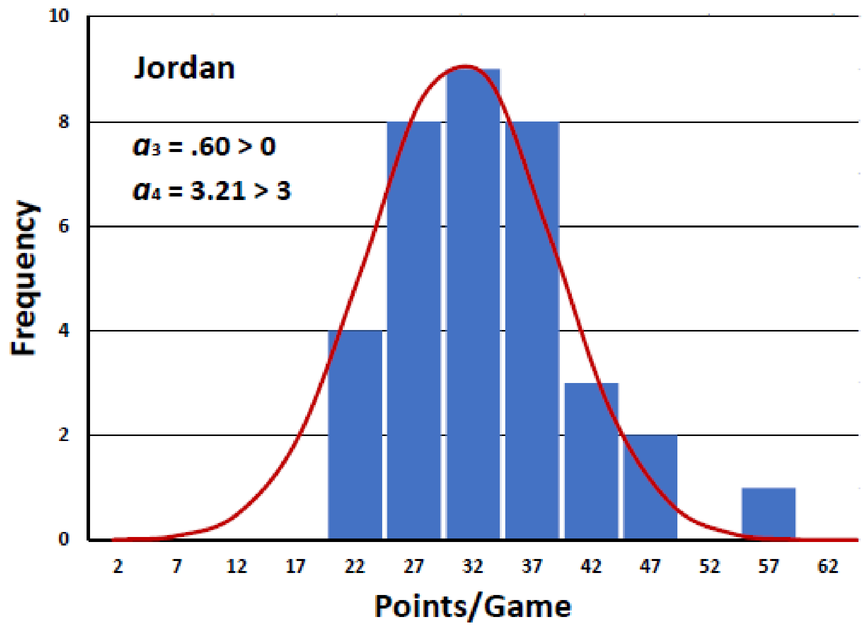
The ***coefficient of skewness*** a_3 and the ***coefficient of kurtosis*** a_4 are then defined respectively by (cf. [10]):

$$a_3 = \frac{m_3}{\sigma^3} \tag{8}$$

and

$$a_4 = \frac{m_4}{\sigma^4} \tag{9}$$

Skewness refers to the asymmetry of a distribution about the peak. For a symmetrical Gaussian distribution, $a_3 = 0$. If for a distribution, $a_3 < 0$, it is skewed to the left; and if $a_3 > 0$, the distribution is skewed to the right. Kurtosis, on the other hand, refers to the narrowness or flatness of the peak. For a Gaussian distribution, $a_4 = 3$. For a flatter peak than the Gaussian, $a_4 < 3$; whereas, for a narrower peak, $a_4 > 3$.



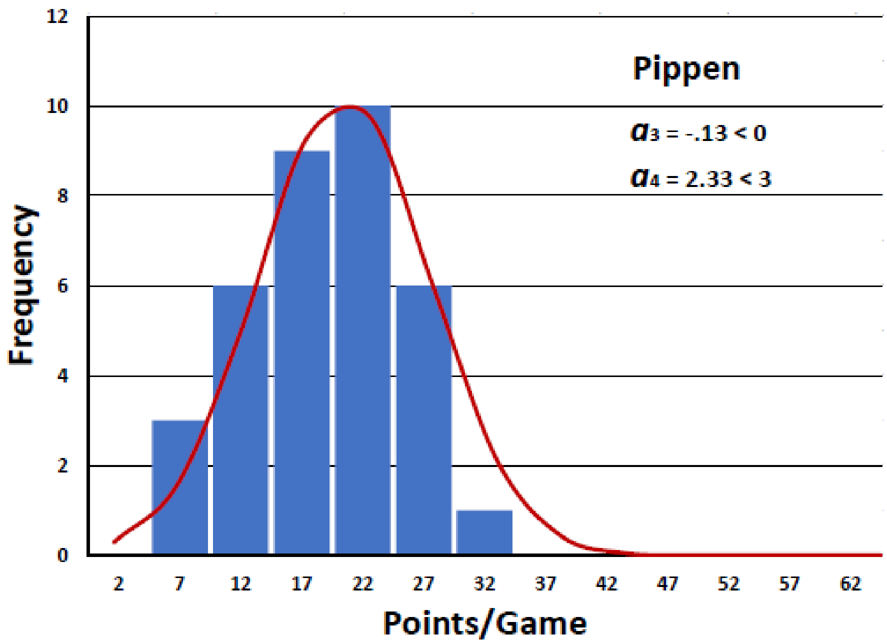


Fig. 2. Frequency distribution of Jordan & Pippen scores

Figure 2 shows the frequency distributions of Jordan and Pippen scores. The least-squares fit Gaussian envelopes are also shown by the red lines. Clearly, the two distributions are of the opposite types, referred to both the skewness and sharpness of the peak. The Jordan distribution is visibly skewed to the right (a_3 = 0.60 > 0), whereas as the Pippen distribution skewed to the left ($a_3 = -0.13 < 0$). The Jordan distribution was slightly narrower than the Gaussian ($a_4 = 3.21 > 3$), whereas the Pippen distribution was slightly flatter ($a_4 = 2.33 < 3$), even though this is difficult to discern visually. The skewness or asymmetry of the Jordan and Pippen distributions is traced to the fact that Jordan never scored fewer than 20 points in a game whereas Pippen scored over 30 points only once. Consequently, the left end of the Jordan distribution was clipped as was the right end of the Pippen distribution curve. This explains the positive / negative skewness of the two distributions. Interestingly, if one adds the two distribution into one, the combined distribution is very close to a normal bell-shaped curve as is shown in Fig. 3. This indicates that Jordan and Pippen together acted as one unit. When Jordan was in form, Pippen played second fiddle to him. And when Jordan was defended tightly, Pippen often picked up the slack.

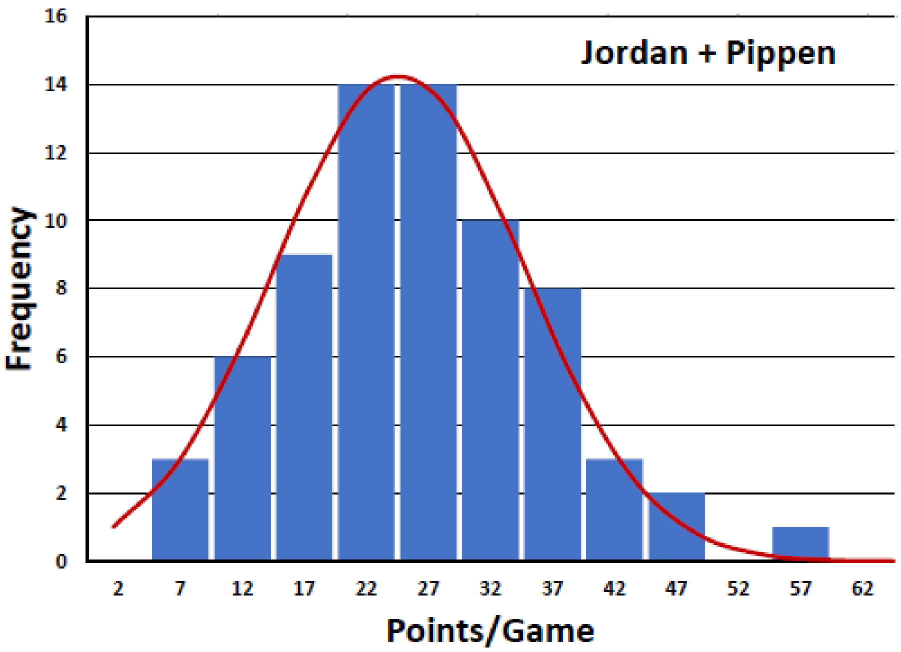


Fig. 3. Combined Jordan and Pippen frequency distributions

5. CORRELATION BETWEEN JORDAN AND PIPPEN SCORES

The last statement seemed to suggest that the Jordan scores and Pippen scores were perhaps correlated. In order to find the correlation between two variables x and y, the ***Pearson's correlation coefficient*** r_{xy} is often employed as defined (cf. [10]):

$$r_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 (y_i - \bar{y})^2}} \tag{10}$$

The value of the correlation coefficient ranges from −1 to 1. A correlation coefficient of 1 represents the limiting case where the two variables are completely correlated. In that case the data points on a scatterplot of the x and y would lie perfectly on a straight line having positive slope. On the other hand, a correlation coefficient of −1 would indicate a complete anti-correlation, where the data points on the scatterplot would lie on a straight line having negative slope. If there is no correlation between the variables, the correlation coefficient is 0. The data points in that case would have no discernible trend.

In most cases, a ***linear trend line*** sufficiently describes the inter-relationship between the two variables x and y:

$$y = mx + c \tag{11}$$

where the slope m and y-intercept c are given, respectively, by:

$$m = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^{\,2} - \left(\sum x_i\right)^2} \tag{12}$$

and

$$c = \frac{\sum y_i \sum x_i^{\,2} - \sum x_i \sum x_i y_i}{n\sum x_i^{\,2} - \left(\sum x_i\right)^2} \tag{13}$$

In this paper, we have calculated the correlation coefficients between the Jordan and Pippen points in four categories of the finals games: (**A**) high-scoring games of team scores of 100 or more points; (**B**) low-scoring games of team scores of under 100 points; (**C**) games won by the Bulls; and (**D**) games lost by the Bulls. The results are shown in Table II. The highest correlation between the Jordan and Pippen points were found in the 11 category **D** games which were lost by the Bulls. In those games, both Jordan and Pippen scored the fewest points together, which accounts for the high correlation coefficient of .5022. The lowest correlation between the Jordan and Pippen were found in the category **A** games (correlation coefficient −.3365) where both players (especially Jordan) had their highest averages. In the category **C** games won by the Bulls, both players achieved their next highest averages and the correlation coefficient was also negative. The similarity between the category **A** and **C** games is traced to the fact that 10 of the 12 category **A** games were won by the Bulls, which therefore, also belonged to the **C** category. In summary, the Chicago Bulls were most victorious in high scoring games when there was (surprisingly) negative correlation between Jordan and Pippen scores. Overall, there was a slight positive correlation between the points scored by the two (correlation coefficient .0556).

Table II. Correlation Coefficients between Jordan points and Pippen points

	No. of Games	Team average	Jordan average	Pippen average	Corr. Coeff.
Team score ≥ 100 pts	12	110.17	37.67	20.83	−.3365
Team score < 100 pts	23	89.61	29.91	17.17	.1462
Games Won	24	99.38	34.42	19.63	−.1749
Games Lost	11	90.73	28.55	15.82	.5022
All games	35	96.66	33.60	18.97	.0556

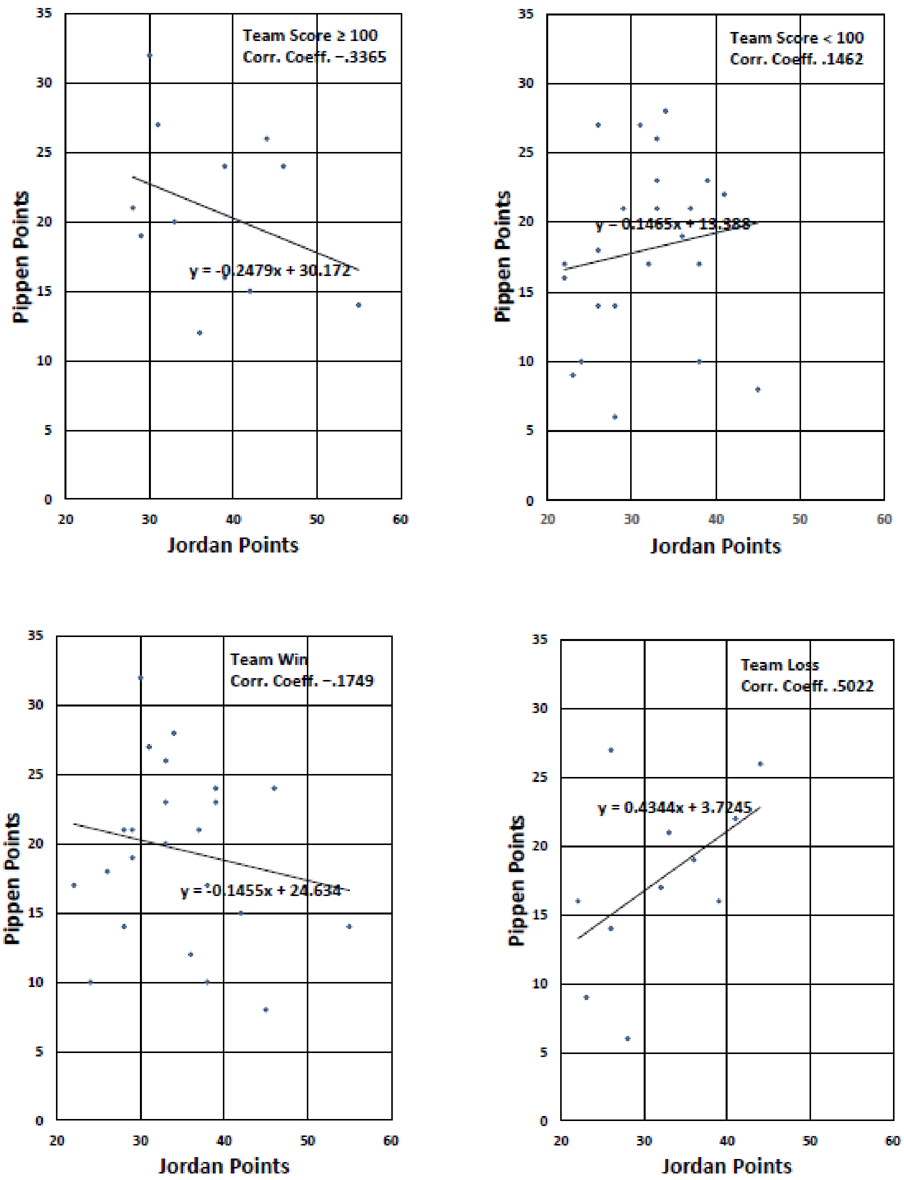


Fig. 4. Correlation between Jordan and Pippen Points

Figure 4 shows the scatterplots of the Jordan and Pippen points in the four categories of games mentioned above. In that figure, category **A** games are shown in the upper left panel; category **B** games in the upper right panel; category **C** games in the lower left panel; and category **D** games in the lower right panel. Also shown are the least-squares linear trend lines given by Eq. (11). As mentioned earlier, in the categories on the left hand side panels, the team scores (as well as Jordan and Pippen scores) were the highest; the correlations between the Jordan and Pippen scores were negative; and the Bulls were mostly victorious. On the opposite end, in the categories on the right hand side panels, the team and individual scores were the lowest; the correlation between Jordan and Pippen point were positive; and the outcome of the games less favorable to the Bulls. In a nutshell, the Chicago Bulls were the most successful, when Jordan and Pippen scored well but when their scores were negatively correlated.

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