Jonathan Quang
Period 9
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HW#1

Part 2

The theorem that was trying to be proved was that there is no largest prime. It assumes that every set of natural numbers that has a largest number can have all its unique elements multiplied together to form another natural number. It also assumes that every natural number greater than 1 has at least one factor that is prime.

The proof starts by temporarily assuming that there is a largest prime number greater than any other prime numbers. This largest prime number is temporarily assigned the variable ***z***. If ***z*** exists, then there must exist the variable ***r*** where ***r*** is the product of all prime factors.

A variable ***w*** can be created where ***w*** = ***r*** + 1. Since ***r*** is a product of all prime numbers, ***r*** ≥ 1. Adding 1 to both sides yields ***r + 1*** ≥ 2. Substituting w for r+1 results in ***w*** ≥ 2. Going back to ***r***, there exists a number, ***v***, where ***v*** is a prime factor of ***r***, seeing as ***r*** is the product of all prime factors. If it is assumed that there ***v*** is also a factor of ***w.*** If ***v*** is both a factor of ***r*** and ***w***, then ***v*** must be a factor of (***w-r***) because if one number divides 2 numbers, then it can divide their difference. ***w - r*** is equal to 1, so ***v*** must be a factor of 1 by the assumption introduced in this paragraph. However, the only factor of 1 is 1, so ***v*** = 1. By the definition of ***v*** (a prime factor of ***r***), ***v*** must be prime, but 1 is not a prime number. This contradiction leads to the conclusion that the assumption that ***w*** is divisible by a prime number to be false.

Similarly, if ***t*** is a prime number, then it cannot be a prime factor of ***w***. The negation of this statement is that there is no prime number, ***t***, such that ***t*** is a factor of ***w***. Lets assign the variable ***s*** to be a greater than 2. If ***s*** is greater than 2, it contains a prime factor according to the assumption made in the first paragraph. This prime factor is ***t***. It is known that ***w*** is greater than 2, which also means that ***w*** contains a prime factor, which could be ***t***. This leads to the conclusion that there is a prime factor of ***w***.

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The previous paragraph's conclusion contradicts with that paragraph's preceding paragraph's conclusion. A number cannot both have a prime factor and not have a prime factor. Since ***w*** can only exist if ***r*** exists. ***r*** can only exist if there is a largest prime number. Since the existence of ***w*** is contradictory, there cannot exist a largest prime number.

Part 3

Universal introduction in the proof is used to introduce a new rule in line 18. The previous lines proved that for the product of all primes + 1, there exists no prime factor for that final sum. Lines 1-17 helped proved via contradiction that ***w*** does not have a prime factor, which line 18, the universal introduction, succinctly restates. What universal introduction seems to do is that for all values for which a property holds true for all values of y, and y contains all values of x, then for all x, that property is true.

Existential elimination is used in lines 25 and 26 to say that a certain number simultaneously has and does not have a prime factor. Existential elimination was used to say that provided for a value of x, there exists a y, such that y has the same property that the value of x had.