Uncertainty Modelling in Intelligent Systems Coursework

EMATM0060: An Investigation into Fuzzy Systems

Q1

Fuzzy control is the most successful application area of fuzzy set theory and has been embedded in systems as diverse as video cameras, washing machines, and metro trains. Write an essay (no more than three typed pages) giving an overview of fuzzy control. The essay should:

- Be well written with clear explanations including appropriate and consistent use of mathematical notation accompanied by an explanation of any symbols used.
- Give a discussions of the main motivations for fuzzy controllers. This could include that they are useful for highly non-linear systems where traditional controllers are hard to formulate. Also that they facilitate the elicitation of control rules from human experts.
- Provide an overview of the main approaches to fuzzy control. This should include the steps of fuzzification, inference and defuzzification. It should include details of the main inference methods such as extended modus ponens. It should also describe some of the different types of fuzzy controller.
- Discuss existing applications of fuzzy control. This should include at least one detailed case-study.
- Discuss whether other statistical based approaches can be just as effective as fuzzy control.

Key resources to look at are:

- R. Babuska, H. B Verbruggen, 'An Overview of Fuzzy Modelling for Control', *Control Eng. Practice*, Vol.4, No. 11, 1996, pp1593-1606
  `https://www.sciencedirect.com/science/article/pii/096706619600175X`
- M. Laviolette, J.W. Seaman, J Douglas Barrett, W.H. Woodall, 'A Probabilistic and Statistical view of Fuzzy Methods', *Technometrics*, Vol. 37, No.3, 1995, 249-261 `https://www.jstor.org/stable/1269905`
- T. Takagi and M. Sugeno, 'Fuzzy identification of systems and its applications to modeling and control,' *IEEE Trans. Syst., Man, Cybern.*, vol. 15, pp. 116–132, 1985 `https://ieeexplore.ieee.org/document/6313399`
- H. Takagi, 'Application of neural networks and fuzzy logic to consumer products' in *Proceedings of the 1992 International Conference on Industrial Electronics, Control, Instrumentation, and Automation*, 1992, pp. 1629-1633 vol.3 `https://ieeexplore.ieee.org/document/254355`
- Wikipedia, Fuzzy Control System

40 marks

Q2

For this question provide the fully documented code for each part together with an explanation of what each element does and why.

Q2(a)

Write some documented python code that inputs a (discrete) fuzzy set and outputs the corresponding α-cuts as a piecewise function of α for $\alpha \in (0, 1]$. Give an example applying your code to determine the α-cut of a discrete fuzzy sets with at least 10 elements with non-zero membership value.
15 marks

Q2(b)

Write some documented code that inputs the α-cuts of a fuzzy set as a piecewise function of α and outputs the original fuzzy set. Show that this code is consistent with the code from part Q2(a).
10 marks

Q2(c)

Write code that inputs a function $f : \mathbb{R} \to \mathbb{R}$ and a set of real numbers S (represented as a list), and outputs the set $\{f(x) : x \in S\}$. Use this together with the code from Q2(a) and Q2(b) to write a python function which inputs a function $f : \mathbb{R} \to \mathbb{R}$ and a fuzzy number $\tilde{A}$ and outputs $f(\tilde{A})$ as determined using the $\alpha - cut$ method.
15 marks

Q2(d)

Consider the following definition of a conditional probability distribution given a fuzzy proposition i.e. a fuzzy set on possible worlds: Let P be a probability distribution on W and $\tilde{A}$ be a fuzzy set also on W then we define the conditional probability distribution $P(w|\tilde{A})$ using α-cuts such that for all $w \in W$,

$$P(w|\tilde{A}) = \int_0^1 P(w|\tilde{A}_\alpha) d\alpha$$

Using your answer to part Q2(a) write some code that inputs a probability distribution P and fuzzy set $\tilde{A}$ and outputs the conditional probability distribution $P(w|\tilde{A})$. Demonstrate your code on several examples with different P and $\tilde{A}$. Are there constraints on P and $\tilde{A}$ required to ensure that this conditional probability is well defined? How is this type of conditional probability related to conditional probability given a mass function in Dempster-Shafer Theory?
20 marks