WORKING TITLE: UTILIZING INVARIANT MANIFOLDS OF CISLUNAR PERIODIC ORBITS FOR EFFICIENT DEEP SPACE TRANSFERS

by

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TABLE OF CONTENTS

LIST OF TABLES . . . 6

LIST OF FIGURES . . . 7

LIST OF SYMBOLS . . . 8

ABBREVIATIONS . . . 9

ABSTRACT . . . 10

1 INTRODUCTION . . . 11
- 1.1 Subcaption / Cleveref Testing . . . 11
  - 1.1.1 Important Math . . . 11
  - 1.1.2 Numbers/Units . . . 11
    - A subsubsection . . . 13
      - A paragraph . . . 13
  - 1.1.3 Custom variables . . . 13
  - 1.1.4 Custom colors . . . 13
  - 1.1.5 Acronyms . . . 13

2 DYNAMICAL MODELS . . . 14
- 2.1 Coordinate Frames . . . 14
  - 2.1.1 Barycentric Rotating and Inertial Frames . . . 14
  - 2.1.2 The Ecliptic J2000 Primary-Centered Inertial Frame . . . 15
- 2.2 The Two-Body Problem . . . 16
- 2.3 The Circular Restricted Three-Body Problem . . . 16
- 2.4 The 2BP-CR3BP Patched Model . . . 16
- 2.5 Coordinate Frame Transformations . . . 16

3 CR3BP DYNAMICAL STRUCTURES . . . 17
- 3.1 Differential Corrections . . . 17
- 3.2 Periodic Orbits . . . 17
- 3.3 Invariant Manifolds . . . 17

4 TRAJECTORY CONSTRUCTION . . . 18
- 4.1 2BP Lambert Arcs . . . 18
- 4.2 The Moon-to-Moon Analytical Transfer Method . . . 18

REFERENCES . . . 19

LIST OF TABLES

1.1 Sample Table 12

LIST OF FIGURES

1.1 Two images of Orion: (a) and (b). 12

2.1 Barycentric rotating and inertial frames in a CR3BP system. 15

2.2 Earth-centered Ecliptic J2000 inertial frame. 16

LIST OF SYMBOLS

Variables

P	Primary

Coordinate Frames

$\{\hat{X}, \hat{Y}, \hat{Z}\}$	Arbitrary inertial coordinate frame
$\{\hat{X}_{Ec}, \hat{Y}_{Ec}, \hat{Z}_{Ec}\}$	Ecliptic J2000 inertial coordinate frame
$\{\hat{x}, \hat{y}, \hat{z}\}$	Rotating coordinate frame

ABBREVIATIONS

2BP	Two-Body Problem
CR3BP	Crircular Restricted Three-Body Problem
NAIF	Navigation and Ancillary Information Facility

ABSTRACT

ADD ABSTRACT

1. INTRODUCTION

Experimenting with the available typographic conventions defined in the Purdue file: `pa-typographic-conventions.sty`: these include *Emph* *First* *Title* Keys `Literal` Menu Open menu › Preferences **`Shell.sh`**. Now let's try out a footnote[1], one of the fancy TODO notes , and more scary TODO , as well as a a todo error as well as a citation [1]. Note the TODO comments currently only show up in `quick` or `debug` modes (for now).

1.1 Subcaption / Cleveref Testing

Here is a very important and informative figure for Orion. You can see in Figure 1.1 that there is both Figure 1.1(a) and Figure 1.1(b)! There is also important information in Table 1.1. If you're confused, then Equation (1.1) should clarify things. Some other ways to put it: Equations (1.1) and (1.2) and Equations (1.1) to (1.3).

1.1.1 Important Math

$$e^{i\pi} + 1 = 0 \tag{1.1}$$

$$a^2 + b^2 = c^2 \tag{1.2}$$

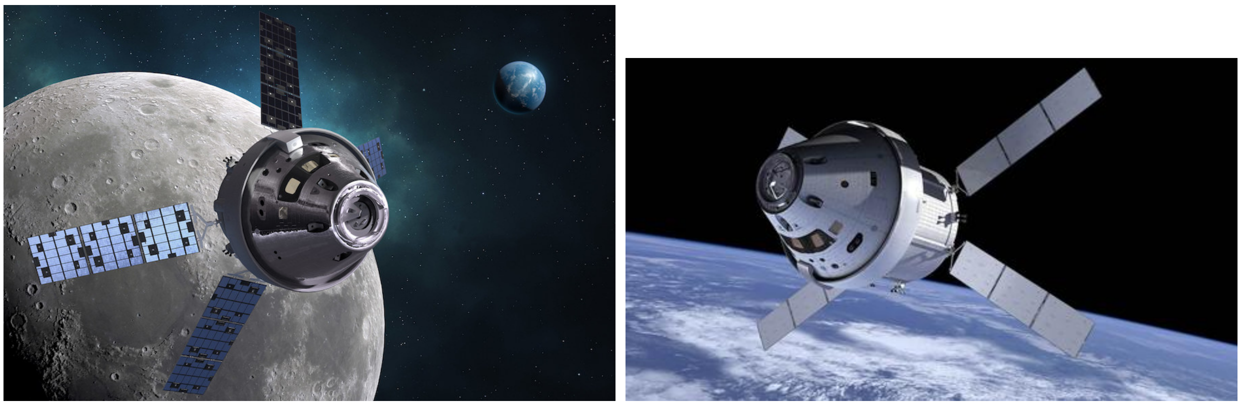
$$\frac{df}{dt} = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h} \tag{1.3}$$

1.1.2 Numbers/Units

Some of the number formats available: -10^{10}. 2×4. 10 to 11. 12.3°.

Experimenting with the siunits package: $8\ \mathrm{kg\,m\,s^{-2}}$. 9N. $2.3 \times 10^{27}\,\mathrm{kg}$. $1.345\,\frac{\mathrm{C}}{\mathrm{mol}}$.

[1] ↑I'm a footnote!



(a) Orion 1 (b) Orion 2

Figure 1.1. Two images of Orion: (a) and (b).

Table 1.1. Sample Table

Sample	Table
x	2

A subsubsection

A subsubsection for testing out the table of contents

A paragraph

What happens for a paragraph in the table of contents?

1.1.3 Custom variables

Variables can be defined as functions in t0-template > te4-custom-variables.tex

The rotating x axis is clearly the best of all axes. But even better is the $\boldsymbol{x}$ vector and the $\hat{x}$ direction! See the appenix in Debug mode for details

1.1.4 Custom colors

There are a variety of available colors from Purdue's branding[2] like: Boilermaker Gold, Rush. This example document also include the Tableau colors[3]. For example, tab-blue and tab-red.

1.1.5 Acronyms

Acronyms handled through `glossaries`, and defined in t0-template > te6-acronyms.tex. For example, the first time we will refer to the Circular Restricted Three Body Problem (CR3BP), and in the future only say CR3BP.

[2]↑see https://marcom.purdue.edu/our-brand/visual-identity/

[3]↑used in matplotlib - https://matplotlib.org/3.4.1/gallery/color/named_colors.html

2. DYNAMICAL MODELS

This analysis relies on the utilization of two primary dynamical models: The Two-Body Problem (2BP) and the Circular Restricted Three-Body Problem (CR3BP). The 2BP serves as a model for spacecraft dynamics when their motion is solely governed by the gravitational influence of a single body, primarily applied to the study of heliocentric arcs within this investigation. In cases where the dynamics are significantly influenced by the gravitational forces of two bodies, as exemplified in Sun-planet or the Earth-Moon systems, the CR3BP offers a more accurate description of the spacecraft's motion.

2.1 Coordinate Frames

In this investigation, Cartesian coordinate frames are employed to represent three-dimensional vector quantities. These frames may either remain fixed in space (inertial) or rotate about the origin at a constant angular rate (rotating). The choice of coordinate frame depends on the specific application as it can be advantageous to position the origin at the center of mass of the system (barycenter) or align it with a primary body of interest.

2.1.1 Barycentric Rotating and Inertial Frames

In a CR3BP system, the motion of a spacecraft is best depicted within a rotating frame with its origin at the system barycenter. The $\hat{x}$-axis is defined to extend from the barycenter toward the smaller primary body, while the $\hat{z}$-axis aligns with the sytem's angular momentum vector. Completing the triad, the $\hat{y}$-axis is established as $\hat{y} = \hat{z} \times \hat{x}$. This frame rotates about the barycenter at a constant angular rate identical to that of the primary bodies.

Additionally, an arbitrary barycentric inertial frame can be similarly defined using the rotating axes at a specific instance in time, denoted as $\hat{X}$, $\hat{Y}$, and $\hat{Z}$. As time progresses, the inertial frame remains fixed in space, whereas the rotating frame revolves around the shared origin with the primaries. In Figure 2.1, the barycentric $\{\hat{x}, \hat{y}, \hat{z}\}$ rotating frame and $\{\hat{X}, \hat{Y}, \hat{Z}\}$ inertial frames for an example CR3BP system are illustrated, with their common origin centered at the barycenter of the primaries, P_1 and P_2. The angle between the two frames is denoted by θ, and it increases at a rate of $\dot{\theta}$. It is important to note that both the $\hat{Z}$- and $\hat{z}$-axes adhere to the right-hand frame convention, pointing out of the page.

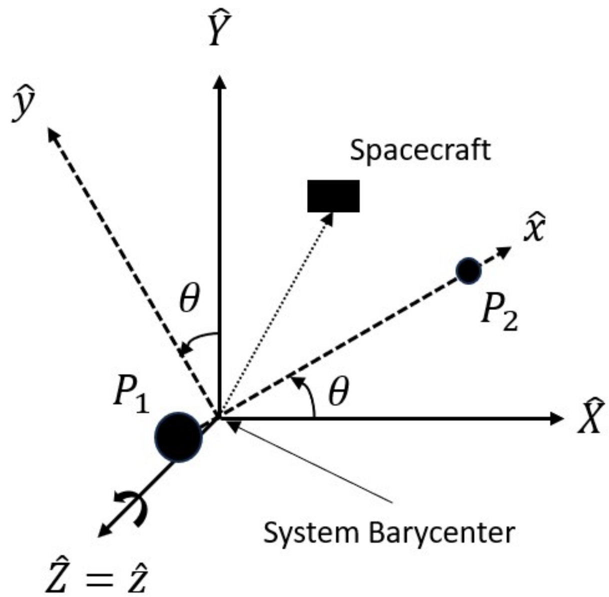


Figure 2.1. Barycentric rotating and inertial frames in a CR3BP system.

2.1.2 The Ecliptic J2000 Primary-Centered Inertial Frame

A commonly used primary-centered inertial frame is the Ecliptic J2000. As the name implies, this frame is established with its origin at the center of a primary body, and the Sun-Earth orbital plane on January 1, 2000 as the $\hat{X}_{Ec}\hat{Y}_{Ec}$-plane. The $\hat{X}_{Ec}$-axis is directed towards the vernal equinox, which is the line of intersection between the Earth's equatorial and ecliptic planes on January 1, 2000. The $\hat{Z}_{Ec}$-axis is orthogonal to the ecliptic plane, and the $\hat{Y}_{Ec}$-axis completes the triad, defined as $\hat{Y}_{Ec} = \hat{Z}_{Ec} \times \hat{X}_{Ec}$.

Since the frame is centered on a primary, it is applicable to both the 2BP and CR3BP, making it also valuable for patched dynamical models. The construction of this coordinate frame, as depicted in Figure 2.2, is computed using the Navigation and Ancillary Information Facility's (NAIF) SPICE ephemeris toolkit[2].

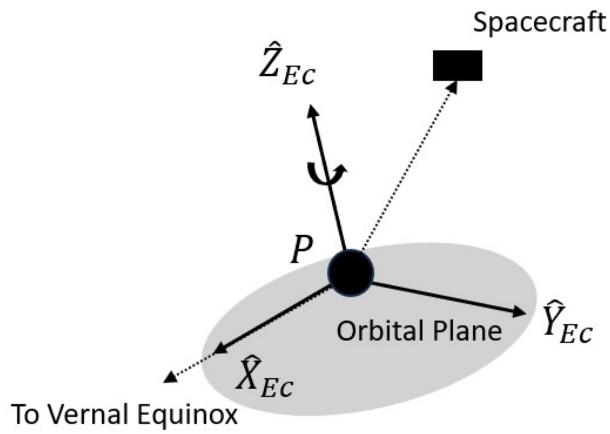


Figure 2.2. Earth-centered Ecliptic J2000 inertial frame.

2.2 The Two-Body Problem

2.3 The Circular Restricted Three-Body Problem

2.4 The 2BP-CR3BP Patched Model

2.5 Coordinate Frame Transformations

3. CR3BP DYNAMICAL STRUCTURES

3.1 Differential Corrections

3.2 Periodic Orbits

3.3 Invariant Manifolds

4. TRAJECTORY CONSTRUCTION

4.1 2BP Lambert Arcs

4.2 The Moon-to-Moon Analytical Transfer Method

REFERENCES

[1] K. C. Howell, "Three-dimensional, periodic, 'halo' orbits," *Celestial Mechanics*, vol. 32, no. 1, pp. 53–71, 1984. DOI: 10.1007/BF01358403.

[2] B. Semenov, *Spice: An observation geometry system for space science missions*, 2023. [Online]. Available: https://naif.jpl.nasa.gov/naif/.