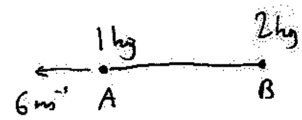
Connected Masses - Impulsive tensions in strings

1.



Simple 1D problem : Impulse $J = 6 \cdot 1 = 6\,Ns$

$$J = m_A(u-0) + m_B(u-0)$$

i.e. $6 = u + 2u$

$\therefore u = 2\,ms^{-1}$

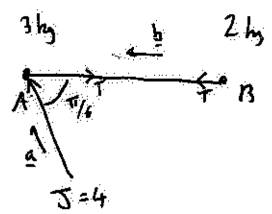
Impulsive tension in string for B : $2u = 4\,Ns$

Initial KE : $\frac{1}{2} m_A u_A^2 = \frac{1}{2} \cdot 1 \cdot 6^2 = 18\,J$

Final KE : $\frac{1}{2}(m_A + m_B)u^2 = \frac{1}{2} \cdot 3 \cdot 2^2 = 6\,J$

$\therefore$ KE loss $= 12\,J$

2.



Let $\underline{a}, \underline{b}$ be unit vectors as shown in the diagram.

The linear momentum equations are

$$m_A \underline{v}_A - J\underline{a} = -T\underline{b} \qquad -(1)$$

$$m_B \underline{v}_B = +T\underline{b} \qquad -(2)$$

The kinematic condition for the inelastic string is

$$(\underline{v}_A - \underline{v}_B) \cdot \underline{b} = 0 \qquad -(3)$$

Substitute (2) into (1): $m_A \underline{v}_A + m_B \underline{v}_B = J\underline{a}$

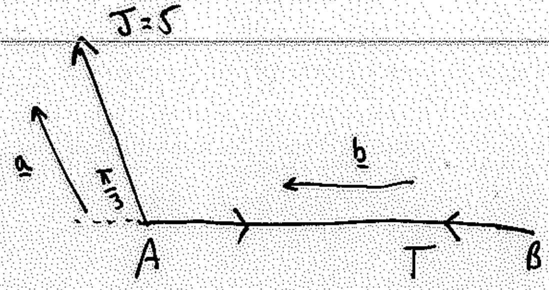
Take dot product with $\underline{b}$ and use (3):

$$m_A \underline{v}_A \cdot \underline{b} + m_B \underline{v}_A \cdot \underline{b} = J\underline{a}\cdot\underline{b}$$

i.e. $(m_A + m_B)v = J\cos\frac{\pi}{6}$ $\quad (\underline{v}_A \cdot \underline{b} = v)$

$\therefore v = \frac{4\sqrt{3}}{10}\,ms^{-1}$

Impulsive Tension $T = m_B \underline{v}_B \cdot \underline{b} = m_B \underline{v}_A \cdot \underline{b} = \frac{4\sqrt{3}}{5}\,Ns$

3. 

Let $\underline{a}, \underline{b}$ be unit vectors as shown.

The linear momentum equations are

$$m_A \underline{v}_A - J\underline{a} = -T\underline{b}$$

$$m_B \underline{v}_B = T\underline{b}$$

Kinematic condition

$$(\underline{v}_A - \underline{v}_B) \cdot \underline{b} = 0$$

It follows that

$$m_A \underline{v}_A \cdot \underline{b} + m_B \underline{v}_B \cdot \underline{b} = J\underline{a}\cdot\underline{b}$$

Then, with the kinematic condition,

$$3\underline{v}_B \cdot \underline{b} + \underline{v}_B \cdot \underline{b} = \frac{5}{2}$$

$$\therefore \quad \underline{v}_B \cdot \underline{b} = \frac{5}{8}\ \mathrm{ms}^{-1} \quad (= |\underline{v}_B|)$$

$$\therefore \quad T = m_B \underline{v}_B \cdot \underline{b} = \frac{5}{8}\ \mathrm{Ns}$$

By the kinematic condition,

$$\underline{v}_A \cdot \underline{b} = \frac{5}{8}\ \mathrm{ms}^{-1}$$

Let $\underline{c}$ be a unit vector $\perp \underline{b}$

$$\therefore \quad \underline{c} = \left(\underline{a} - \frac{(\underline{a}\cdot\underline{b})\,\underline{b}}{|\underline{b}|^2}\right) \Big/ \left|\underline{a} - \frac{(\underline{a}\cdot\underline{b})}{|\underline{b}|^2}\,\underline{b}\right|$$

(2.5)

$$\therefore \ \underline{c} = \frac{2}{\sqrt{3}}\left(\underline{a} - \frac{1}{2}\underline{b}\right)$$

Now $\quad m_A \underline{V}_A \cdot \underline{c} + m_B \underline{V}_B \cdot \underline{c} = J \underline{a} \cdot \underline{c}$

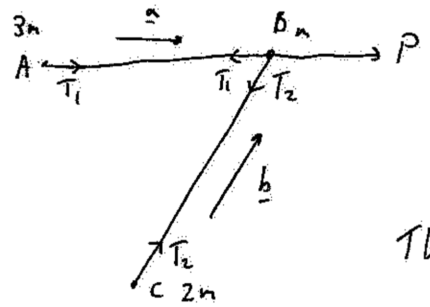
$$\therefore \quad \underline{V}_A \cdot \underline{c} = \frac{1}{3} \cdot 5 \cdot \frac{2}{\sqrt{3}} \cdot \frac{3}{4}$$

$$= \frac{5\sqrt{3}}{6}$$

$$\therefore \quad |\underline{V}_A| = \sqrt{\left(\frac{5}{8}\right)^2 + \left(\frac{5\sqrt{3}}{6}\right)^2} = \frac{5\sqrt{57}}{27} \text{ ms}^{-1}$$

$$= \frac{5}{8}\sqrt{\frac{19}{3}} \text{ ms}^{-1}$$

4.



Let $\underline{a}, \underline{b}$ be unit vectors as shown.

The linear momentum eqns. are

$$m_A \underline{v}_A = T_1 \underline{a}$$

$$m_B \underline{v}_B - P\underline{a} = -T_1 \underline{a} - T_2 \underline{b}$$

$$m_C \underline{v}_C = T_2 \underline{b}$$

The kinematic conditions are

$$(\underline{v}_B - \underline{v}_A)\cdot\underline{a} = 0 \rightarrow u = \underline{v}_A\cdot\underline{a} = \underline{v}_B\cdot\underline{a}$$ ie. speed is u along AB

$$(\underline{v}_B - \underline{v}_C)\cdot\underline{b} = 0 \rightarrow v = \underline{v}_C\cdot\underline{b} = \underline{v}_B\cdot\underline{b}$$

*

$$\therefore \left(\frac{1}{m_B}\left(P\underline{a} - T_1\underline{a} - T_2\underline{b}\right) - \frac{T_1}{m_A}\underline{a}\right)\cdot\underline{a} = 0$$

$$\left(\frac{1}{m_B}\left(P\underline{a} - T_1\underline{a} - T_2\underline{b}\right) - \frac{T_2}{m_C}\underline{b}\right)\cdot\underline{b} = 0$$

Using $\underline{a}\cdot\underline{b} = \cos\alpha$, $\underline{a}\cdot\underline{a} = \underline{b}\cdot\underline{b} = 1$, these eqns can be used to solve for T_1

ie. $$T_1(4 - 2\cos^2\alpha) = P(3 - 2\cos^2\alpha)$$

$$\therefore T_1 = \frac{P(1 + 2 - 2\cos^2\alpha)}{2(2 - \cos^2\alpha)} = \frac{P(1 + 2\sin^2\alpha)}{2(1 + \sin^2\alpha)}$$

Now $$u = \underline{v}_A\cdot\underline{a} = \frac{T_1}{m_A} = \frac{P(1 + 2\sin^2\alpha)}{6m(1 + \sin^2\alpha)}$$

* $\perp$ AB, $$\underline{v}_B\cdot\frac{(\underline{b} - (\underline{a}\cdot\underline{b})\underline{a})}{|\underline{b} - (\underline{a}\cdot\underline{b})\underline{a}|} = \frac{v - u\cos\alpha}{\sqrt{1 - \cos^2\alpha}} = \frac{v - u\cos\alpha}{\sin\alpha}$$

(using the relations from the kinematic conditions)