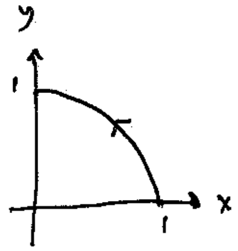
1.



$$x = \cos t \qquad y = \sin t \qquad t \in [0, \tfrac{\pi}{2}]$$

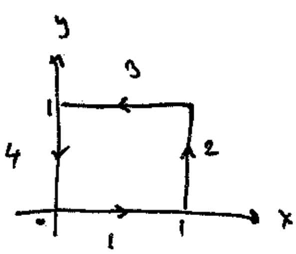
$$\therefore \int_C 3xy^2\,dx + x^2y\,dy$$

$$= \int_0^{\pi/2} 3\cos t \sin^2 t\cdot(-\sin t)\,dt + \cos^2 t \sin t\cdot\cos t\,dt$$

$$= \int_0^{\pi/2} (-3\cos t\sin^3 t)\,dt + \int_0^{\pi/2}\cos^3 t\sin t\,dt$$

$$= -3\cdot\frac{2}{4\cdot 2\cdot 1} + \frac{2}{4\cdot 2} = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$$

2.



1: $x = t$, $t \in [0,1]$ $\quad \therefore I_1 = 0$
$y = 0$

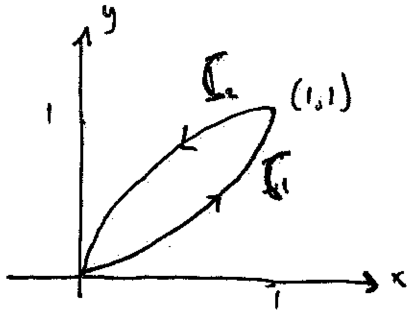
2: $x = 1$
$y = t$, $t \in [0,1]$ $\quad \therefore I_2 = \int_0^1 2\,dt = 2$

3: $y = 1$
$x = t$, $t \in [1,0]$ $\quad \therefore I_3 = -\int_0^1 2t\,dt = -2\frac{t^2}{2}\Big|_0^1 = -1$

4: $x = 0$
$y = t$, $t \in [1,0]$ $\quad \therefore I_4 = 0$

$$\therefore I = 2 - 1 = 1$$

3.



C_1 : $y = t,\ x = t^2,\ t \in [0,1]$

$$\therefore I_1 = \int_{C_1} (x-y)\,dx + (x+y)\,dy$$

$$= \int_0^1 (t^2 - t)\cdot 2t\,dt + (t^2 + t)\,dt$$

$$= 2\frac{t^4}{4} - 2\frac{t^3}{3} + \frac{t^3}{3} + \frac{t^2}{2}\Big|_0^1$$

$$= \frac{2}{4} - \frac{2}{3} + \frac{1}{3} + \frac{1}{2} = 1 - \frac{1}{3} = \frac{2}{3}$$

C_2 : $x = t,\ y = t^2,\ t \in [1,0]$

$$\therefore I_2 = \int_1^0 (t - t^2)\,dt + (t + t^2)\cdot 2t\,dt$$

$$= \frac{t^2}{2} - \frac{t^3}{3} + 2\frac{t^3}{3} + \frac{2t^4}{4}\Big|_1^0$$

$$= -\frac{1}{2} + \frac{1}{3} - \frac{2}{3} - \frac{1}{2} = -1 - \frac{1}{3}$$

$$= -\frac{4}{3}$$

$$\therefore I = \frac{2}{3} - \frac{4}{3} = -\frac{2}{3}$$

4. $$\int_C \underline{F} \cdot d\underline{r} = \int_C 3x^2\,dx + (2xz - y)\,dy + z\,dz$$

$$= \int_0^1 3(2t^2)^2 \cdot 4t\,dt + \left(2 \cdot 2t^2(4t^2 - t) - t\right)dt + (4t^2 - t)\,\cancel{d}\,(8t - 1)\,dt$$

$$= \int_0^1 \cancel{24}\,48\,t^5\,dt + \int_0^1 (16t^4 - 4t^3 - t)\,dt + \int_0^1 (32t^3 - 4t^2 - 8t^2 + t)\,dt$$

$$= \frac{48}{6} + \frac{16}{5} - \frac{4}{4} - \frac{1}{2} + \frac{32}{4} - \frac{4}{3} - \frac{8}{3} + \frac{1}{2}$$

$$= \frac{71}{5}$$

5. $$\nabla \times \underline{F} = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ 3x^2 + 2y^2 & 4xy + z^2 - 2z & 2y(z-1) \end{vmatrix}$$

$$= \left(2(z-1) - (2z - 2),\ 0 - 0,\ 4y - 4y\right) = \underline{0}$$

$\therefore$ $\underline{F}$ is conservative

Let $\underline{F} = \nabla\phi$

Then
$$\frac{\partial\phi}{\partial x} = 3x^2 + 2y^2$$
$$\frac{\partial\phi}{\partial y} = 4xy + z^2 - 2z$$
$$\frac{\partial\phi}{\partial z} = 2y(z-1)$$

$$\therefore \phi = x^3 + 2y^2x + f(y,z)$$

$$\therefore 4yx + \frac{\partial f}{\partial y} = 4xy + z^2 - 2z$$

$$\text{ie. } \frac{\partial f}{\partial y} = z^2 - 2z$$

$$\therefore f = (z^2-2z)y + g(z)$$

$$\therefore (2z-2)y + g'(z) = 2y(z-1)$$

$$\text{ie. } g'(z) = 1$$

$$\text{ie. } g = \text{const.}$$

$$\therefore \text{take } \phi = x^3 + 2y^2x + (z^2-2z)y$$

$$\int_c \underline{F} \cdot d\underline{r} = \phi(2,2,0) - \phi(1,0,9)$$
$$= 23$$

6. $$\nabla \times \underline{F} = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ 2xz & 6yz & g(x,y,z) \end{vmatrix}$$

$$= \left(\frac{\partial g}{\partial y} - 6y, \ 2x - \frac{\partial g}{\partial x}, \ 0 \right)$$

$$\therefore \frac{\partial g}{\partial x} = 2x$$

$$\frac{\partial g}{\partial y} = 6y$$

$$\therefore g = x^2 + f(y) \qquad \therefore f'(y) = 6y \quad \text{ie } f = 3y^2 + C$$

Take $g = x^2 + 3y^2$ $\therefore$ $\underline{F}$ is conservative

6. Let $\underset{\sim}{F} = \nabla \phi$

$$\therefore \frac{\partial \phi}{\partial x} = 2xz$$

$$\frac{\partial \phi}{\partial y} = 6yz$$

$$\frac{\partial \phi}{\partial z} = x^2 + 3y^2$$

$$\therefore \phi = x^2 z + f(y,z)$$

$$\therefore \frac{\partial f}{\partial y} = 6yz \quad \text{ie.} \quad f = 3y^2 z + g(z)$$

$$\therefore x^2 + 3y^2 + g'(z) = x^2 + 3y^2$$

$$\therefore g = \text{const}$$

$$\therefore \phi = x^2 z + 3y^2 z$$