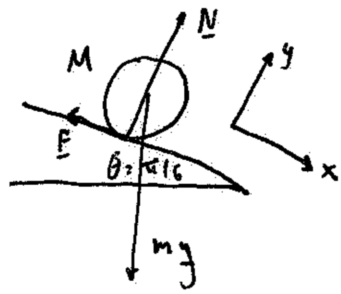
1.




$$x = a\theta$$

From force balance in the x-direction:

$$M\ddot{x} = -F + Mg\sin\theta$$

Rotational: $I\ddot{\theta} = aF$

$$\therefore \quad M\ddot{x} = -I\frac{\ddot{x}}{a^2} + Mg\sin\theta$$

$$\therefore \quad \ddot{x} = \frac{Ma^2}{(I + Ma^2)} g\sin\theta$$

For a sphere, $I = \frac{2}{5}Ma^2$

$$\therefore \quad \ddot{x} = \frac{Ma^2}{\frac{7}{5}Ma^2} g \cdot \frac{1}{2} = \frac{5g}{14}$$

2. Using the result from Q1.

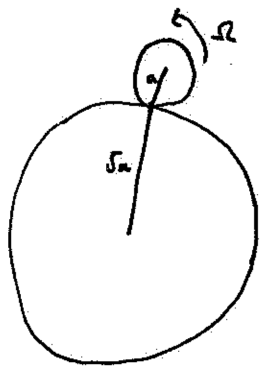
Cylinder: $\ddot{x} = \dfrac{Ma^2}{(I_c + Ma^2)} g\sin\alpha = \dfrac{Ma^2}{\frac{3}{2}Ma^2} g\sin\alpha = \dfrac{2}{3}g\sin\alpha$

Sphere: $\ddot{x} = \dfrac{Ma^2}{\frac{7}{5}Ma^2} g\sin\beta = \dfrac{5}{7}g\sin\beta$

Equating the accelerations gives $\frac{2}{3}\sin\alpha = \frac{5}{7}\sin\beta$

i.e. $14\sin\alpha = 15\sin\beta$

3.



$$\omega = \frac{5a + a}{a}\Omega = 6\Omega$$

4. In the x-direction, balance the work done over a distance x with the kinetic energy.

$$\therefore \quad Fx = 4 \cdot \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2$$

$$= 4 \cdot \frac{1}{2} \cdot \frac{1}{2} m r^2 \cdot \frac{v^2}{r^2} + \frac{1}{2} M v^2$$

$$\therefore \quad 2Fx = 2mv^2 + Mv^2$$

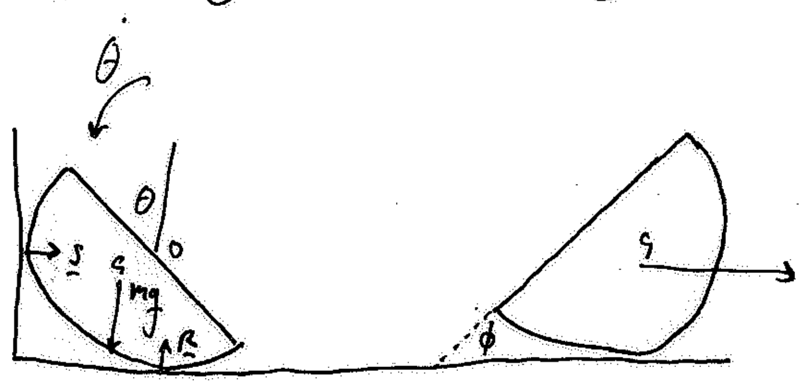
$$\therefore \quad v = \sqrt{\frac{2Fx}{M+2m}}$$

Now $\frac{d}{dt}(v^2) = 2v\dot{v} = 2va$

$$\therefore \quad 2va = \frac{2Fv}{M+2m}$$

$$\therefore \quad a = \frac{F}{M+2m}$$

5.





a) The energy equation

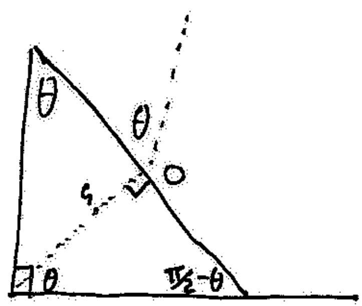
KE = change in pot. energy

$$\therefore \frac{1}{2}\left(\frac{2}{5}Ma^2\right)\dot\theta^2 = \underbrace{\left(\frac{3a}{8}\sin\theta\right)}_{\text{vertical height}}.Mg$$

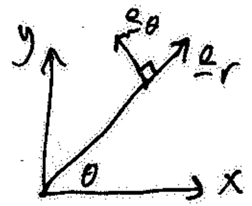
$$\therefore a\dot\theta^2 = \frac{15}{8}g\sin\theta$$

$\therefore$ upon integrating, $a\ddot\theta = \frac{15g}{16}\cos\theta$

b) Consider the geometry :




Hence, we can use the standard polar system




$$\underline{e}_r = \cos\theta\,\underline{i} + \sin\theta\,\underline{j}$$
$$\underline{e}_\theta = -\sin\theta\,\underline{i} + \cos\theta\,\underline{j}$$

Now $\quad M\ddot{\underline{r}} = M\underline{g} + \underline{R} + \underline{S}$

$\underline{S}$ is only in the horizontal direction so we aim to find the $\underline{i}$ components.

First $\quad M\ddot{\underline{r}} = M\left(-\left(\frac{3a}{8}\right)\dot{\theta}^2\underline{e}_r + \left(\frac{3a}{8}\right)\ddot{\theta}\,\underline{e}_\theta\right)$

Splitting $\underline{e}_r, \underline{e}_\theta$ into $\underline{i}, \underline{j}$ and substituting the expressions from part (a) gives, for the $\underline{i}$ component,

$$M\left(-\frac{3}{8}\cdot\frac{15}{8}g\sin\theta\cdot\cos\theta - \frac{3}{8}\cdot\frac{15g}{16}\cos\theta\cdot\sin\theta\right) = S$$

$$\therefore\ |S| = \frac{135}{128}Mg|\cos\theta\sin\theta|$$

After the solid is released, the reaction at the wall will first become zero when $\theta = \frac{\pi}{2}$, ie. when the face is horizontal.

c) At the instant when $S = 0$, G has a horizontal speed of $\frac{3a\dot{\theta}}{8}$ (comes from the vector product).

As $\theta = \frac{\pi}{2}$, the result from (a) gives

$$\dot{\theta} = \sqrt{\frac{15g}{8a}}$$

In the subsequent motion, there is no horizontal force and G retains this horizontal speed. If the angular speed vanishes when the plane face makes an angle $\phi$ with the horizontal (as shown), the body is then moving with pure translation. Its kinetic energy is equal to the work done by gravity since it left its position of rest.

$$\therefore \underbrace{\frac{3a}{8} Mg\cos\phi}_{PE} = \frac{1}{2} M \left( \frac{3}{8} a \cdot \sqrt{\frac{15g}{8a}} \right)^2$$

$$\therefore \cos\phi = \frac{45}{128} \qquad \therefore \phi = \cos^{-1}\left(\frac{45}{128}\right)$$