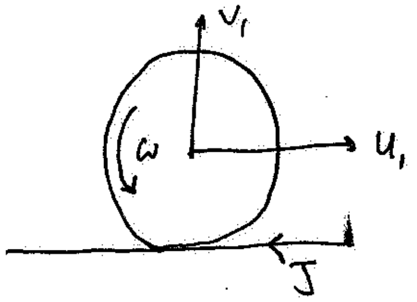
Elastic impact of rigid bodies

1\. 

During impact with plane

Angular momentum:

$$\frac{2}{5} ma^2 \left(\frac{5u}{2a} - \omega \right) = Ja \qquad -(1)$$

No slipping $\therefore$ $u_1 = -a\omega$ $\qquad -(2)$

Newton's exp. rule vertically: $v_1 = ev$ $\qquad -(3)$

Horizontal momentum equation: $-J = m(u_1 - u)$ $\qquad -(4)$

(1) & (4) give $\quad \frac{2}{5} ma \left(\frac{5u}{2a} - \omega \right) = mu - mu_1$

$$\therefore \quad mu_1 = \frac{2}{5} ma\omega$$

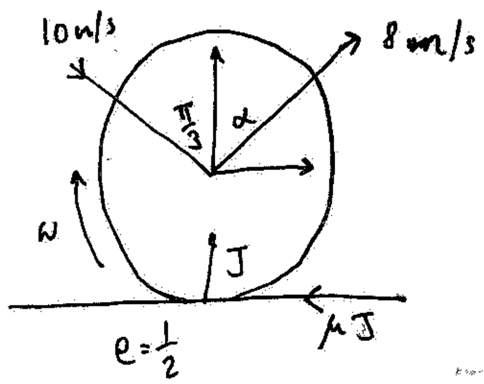
(2) gives $\quad \frac{2}{5} a\omega = -a\omega$, i.e. $\omega = 0$ so no spin after impact

Also, (2) gives $u_1 = 0$, so sphere rises vertically.

$$\text{Loss in KE} = \frac{1}{2}m(u^2+v^2-v_1^2) + \frac{1}{2}\cdot\frac{2}{5}ma^2\left(\frac{5u}{2a}\right)^2$$

$$= \frac{1}{2}m(u^2+v^2-ev^2) + \frac{5}{4}mu^2 \quad \text{(using (3))}$$

$$= \frac{7}{4}mu^2 + \frac{1}{2}mv^2(1-e^2).$$

2. 

Let ω, α and J be as shown

$\perp$ plane :

Newton's exp. rule : $8\cos\alpha = e\cdot 10\cos\frac{\pi}{3}$

$$\therefore \cos\alpha = \frac{1}{8}\cdot\frac{1}{2}\cdot 5 = \frac{5}{16} \qquad -(1)$$

Momentum eqn. : $J = 1\cdot 8\cos\alpha + 1\cdot 10\cos\frac{\pi}{3}$

$$= 2.5 + 5$$

$$\therefore J = \frac{15}{2}\ \text{Ns} \qquad -(2)$$

Horizontal momentum eqn:

$$\mu J = 1 \cdot 10 \sin\tfrac{\pi}{3} - 1 \cdot 8 \sin\alpha$$

and (1), (2) give $\quad 7.5\mu = \dfrac{10\sqrt{3}}{2} - 8\sqrt{1 - \dfrac{25}{16^2}}$

$$\therefore \quad \mu \approx 0.141$$

Angular momentum:

$$\tfrac{1}{2} m a^2 (\omega - 3) = \mu J a$$

$$\therefore \quad \tfrac{1}{4}(\omega - 3) = 0.141 \cdot 7.5$$

$$\therefore \quad \omega \approx 7.2 \text{ rads/sec}$$

[Note that, before impact, velocity of point of contact is $10 \sin\frac{\pi}{3} - 0.5 \cdot 3 > 0$ and so slips to right. Hence μJ is in direction as shown].

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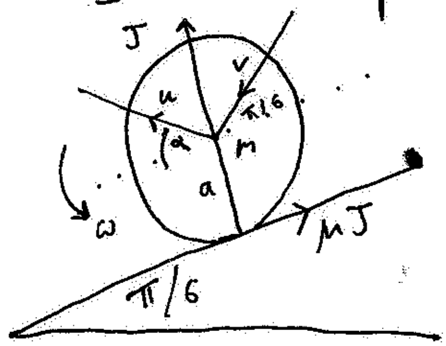
3.



J = impulse plane gives disc at impact

α = angle of u (vel. c.o.m.) to plane

ω: ang. speed of disc after impact

Newton's exp. rule $\perp$ plane:

$$u \sin\alpha = e v \sin\frac{\pi}{6} \qquad \left(e = \frac{1}{4}\right)$$

$$\therefore \quad u \sin\alpha = \frac{v}{8} \qquad -(1)$$

Linear momentum // plane:

$$mv \cos\frac{\pi}{6} - mu\cos\alpha = \mu J \qquad \left(\mu = \frac{1}{2}\right)$$

$$\therefore \quad mv\frac{\sqrt{3}}{2} - mu\cos\alpha = \frac{J}{2} \qquad -(2)$$

Linear momentum $\perp$ plane:

$$J = mu\sin\alpha + mv\sin\frac{\pi}{6}$$

$$= mu\sin\alpha + \frac{mv}{2} \qquad -(3)$$

(2) & (3) give

$$mv\sqrt{3} - 2mu\cos\alpha = mu\sin\alpha + \frac{mv}{2}$$

and (1) gives $\quad v\left(\sqrt{3} - \frac{1}{2}\right) = \frac{v}{8} + 2\cos\alpha \frac{v}{8\sin\alpha}$

$$\therefore \quad \frac{\cos\alpha}{4\sin\alpha} = \sqrt{3} - \frac{1}{2} - \frac{1}{8} = \frac{8\sqrt{3} - 5}{8}$$

(i) $\quad \therefore \tan\alpha = \frac{2}{8\sqrt{3}-5} \quad \therefore \alpha = 12.73^\circ$

(1) & (3) give $\quad J = \frac{mv}{8} + \frac{mv}{2} = \frac{5mv}{8} \quad$ (ii)

Ang. mom. : $\quad \frac{1}{2} ma^2\omega = \mu Ja$

$$\therefore \quad \cancel{4}\ \frac{1}{2} ma\omega = \frac{1}{2} \cdot \frac{5}{8} mv$$

$$\therefore \quad \omega = \frac{5v}{8a} \quad \text{(iii)}$$

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4. The linear momentum equations are

$$\left.\begin{aligned} S_1 : \quad & m_1(\underline{v}_1 - \underline{u}_1) = -I\,\underline{n} \\ S_2 : \quad & m_2(\underline{v}_2 - \underline{u}_2) = I\,\underline{n} \end{aligned}\right\} \quad -(1)$$

Newton's law of impact:

$$(\underline{v}_2 - \underline{v}_1)\cdot\underline{n} = -e(\underline{u}_2 - \underline{u}_1)\cdot\underline{n}$$

Rearranging (1) gives

$$\underline{v}_1 = \underline{u}_1 - \frac{I}{m_1}\underline{n}, \quad \underline{v}_2 = \underline{u}_2 + \frac{I}{m_2}\underline{n}$$

Putting these into (2) gives

$$I = -\frac{m_1 m_2(1+e)}{m_1+m_2}(\underline{u}_2 - \underline{u}_1)\cdot\underline{n}$$

$$\therefore \quad \underline{v}_1 = \underline{u}_1 + \frac{m_2(1+e)}{m_1+m_2}\left[(\underline{u}_2 - \underline{u}_1)\cdot\underline{n}\right]\underline{n}$$

$$= \underline{u}_1 - \frac{2m_2}{m_1+m_2}(\underline{u}_1\cdot\underline{n})\,\underline{n}$$

as $\underline{u}_2 = \underline{0}$ and $e = 1$

Now

$$\left.\begin{aligned} \underline{v}_1 \cdot \underline{u}_1 &= \underline{u}_1 \cdot \underline{u}_1 - \frac{2m_2}{m_1+m_2}(\underline{u}_1 \cdot \underline{n})^2 \\ \underline{v}_1 \times \underline{u}_1 &= \frac{2m_2}{m_1+m_2}(\underline{u}_1 \cdot \underline{n})\, \underline{u}_1 \times \underline{n} \end{aligned}\right\} \quad (*)$$

From the definitions of θ and ϕ,

$$\underline{u}_1 \cdot \underline{n} = u_1 \cos\theta \qquad \underline{v}_1 \cdot \underline{u}_1 = u_1 v_1 \cos\phi$$

$$|\underline{u}_1 \times \underline{n}| = u_1 \sin\theta \qquad |\underline{v}_1 \times \underline{u}_1| = u_1 v_1 \sin\phi$$

where $|\underline{u}_1| = u_1$, $|\underline{v}_1| = v_1$

Eqns. $(*)$ gives

$$v_1 \cos\phi = u_1 \left(1 - \frac{2m_2}{m_1+m_2}\cos^2\theta\right)$$

$$v_1 \sin\phi = u_1 \frac{2m_2}{m_1+m_2}\sin\theta\cos\theta$$

Eliminating $\frac{v_1}{u_1}$,

$$\tan\phi = \frac{\sin 2\theta}{(m_1/m_2) - \cos 2\theta}$$