1. Let $Q = 4xy^2$, $P = y^3$

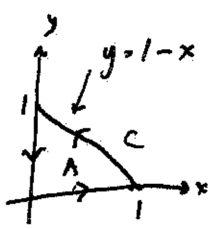
$$\therefore \int_c y^3 dx + 4xy^2 dy = \iint_A \left(\frac{\partial}{\partial x}(4xy^2) - \frac{\partial}{\partial y}(y^3) \right) dx\,dy$$

$$= \iint_A \left(4y^2 - 3y^2\right) dx\,dy$$

$$= \iint_A y^2 \, dx\,dy$$

$$= \int_0^{2\pi} d\theta \int_0^a r^2 \sin^2\theta \cdot r\,dr$$

$$= 4 \cdot \frac{1}{2} \cdot \frac{\pi}{2} \cdot \frac{a^4}{4} = \frac{\pi a^4}{4}$$

2. 

$$\int_c xy\,dx + 6(1+x)\,dy = \iint_A \left(\frac{\partial}{\partial x}\left(6(1+x)\right) - \frac{\partial}{\partial y}(xy) \right) dx\,dy$$

$$= \iint_A (6 - x)\,dx\,dy$$

$$= \int_0^1 dx \int_0^{1-x} (6-x)\,dy$$

$$= \cancel{\left[6x - \frac{x^2}{2}\right]_0^1}$$

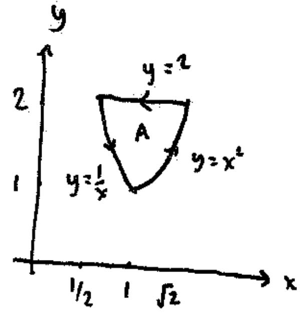
$$= \int_0^1 (6-x)(1-x)\,dx$$

$$= \int_0^1 (6 - 7x + x^2)\,dx$$

$$= 6 - \frac{7}{2} + \frac{1}{3} = \frac{36}{6} - \frac{21}{6} + \frac{2}{6}$$

$$= \frac{17}{6}$$

3.



$$\int_C y(1-xy)\,dx + x(3xy+1)\,dy$$

$$= \iint_A \left(\frac{\partial}{\partial x}\big(x(3xy+1)\big) - \frac{\partial}{\partial y}\big(y(1-xy)\big) \right) dx\,dy$$

$$= \iint_A \left[\frac{\partial}{\partial x}\left(3x^2y + x\right) - \frac{\partial}{\partial y}\left(y - xy^2\right) \right] dx\,dy$$

$$= \iint_A (6xy + 1 - 1 + 2xy)\,dx\,dy$$

$$= 8\iint_A xy\,dx\,dy$$

$$= 8\int_1^2 dy \int_{\frac{1}{y}}^{\sqrt{y}} xy\,dx$$

$$= 8\int_1^2 \frac{x^2y}{2}\Big|_{1/y}^{\sqrt{y}} dy = 8\int_1^2 \left(\frac{y^2}{2} - \frac{1}{2y}\right) dy$$

$$= 8\left[\frac{y^3}{6} - \frac{1}{2}\log y\right]_1^2$$

$$= 8\left[\frac{8}{6} - \frac{1}{2}\log 2 - \frac{1}{6} + 0\right]$$

$$= 8\cdot\frac{7}{6} - 4\log 2$$

$$= \frac{28}{3} - 4\log 2$$

4. Green's theorem cannot be used because $\frac{\partial P}{\partial y}$ is not continuous throughout the enclosed region. In fact, $\frac{\partial P}{\partial y} = \frac{2y}{x^2+y^2}$ is not even defined at $(0,0)$.