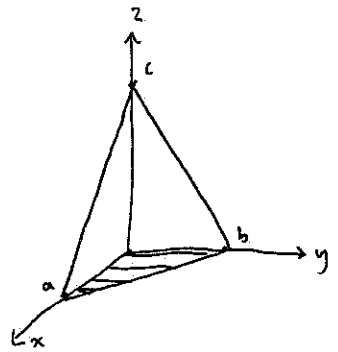
1. $$\int_0^1\int_0^1\int_0^1 yz^2\,dx\,dy\,dz = \int_0^1 dx\int_0^1\int_0^1 y z^2\,dy\,dz$$

$$= \int_0^1 \left[\frac{yz^3}{3}\right]_0^1 dy = \int_0^1 \frac{y}{3}\,dy = \frac{y^2}{6}\Big|_0^1 = \frac{1}{6}$$

2. 

$$V = \int_0^a dx \int_0^{b-\frac{b}{a}x} dy \int_0^{c-x\frac{c}{a}-y\frac{c}{b}} dz$$

$$= \int_0^a dx \int_0^{b-\frac{b}{a}x} c\left(1-\frac{x}{a}-\frac{y}{b}\right) dy$$

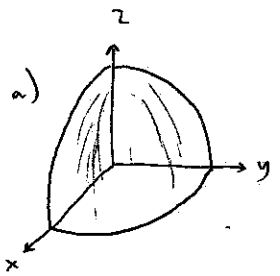
$$= \int_0^a c\left[y - \frac{xy}{a} - \frac{y^2}{2b}\right]_0^{b-\frac{b}{a}x} dx$$

$$= c\int_0^a \left(b - \frac{b}{a}x - \frac{bx}{a} + \frac{bx^2}{a^2} - \frac{b}{2} + \frac{bx}{a} - \frac{b}{2a^2}x^2\right) dx$$

$$= c\left[bx - \frac{bx^2}{2a} - \frac{bx^2}{2a} + \frac{bx^3}{3a^2} - \frac{bx}{2} + \frac{bx^2}{2a} - \frac{b}{6a^2}x^3\right]_0^a$$

$$= c\left[ba - \frac{ba}{2} - \frac{ba}{2} + \frac{ba}{3} - \frac{ba}{2} + \frac{ba}{2} - \frac{ba}{6}\right]$$

$$= \frac{abc}{6}$$

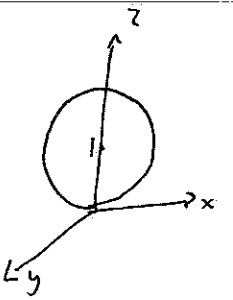
3. a) 

$$\int_0^{\frac{\pi}{2}} d\phi \int_0^{\frac{\pi}{2}} d\theta \int_0^a r\cdot r^2\sin\theta\,dr$$

$$= \frac{\pi}{2}\int_0^{\frac{\pi}{2}} \left[\frac{r^4}{4}\right]_0^a \sin\theta\,d\theta = \frac{\pi a^4}{8}\int_0^{\frac{\pi}{2}} \sin\theta\,d\theta$$

$$= \frac{\pi a^4}{8}$$

3. b) $$\int_0^{2\pi} d\phi \int_0^{\frac{\pi}{2}} d\theta \int_0^{2\cos\theta} \frac{r\cos\theta}{r} \cdot r^2 \sin\theta \, dr$$

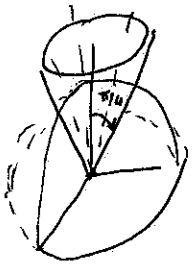


$$= 2\pi \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \left[\frac{r^3}{3}\right]_0^{2\cos\theta} d\theta$$

$$= 2\pi \int_0^{\frac{\pi}{2}} \cos^4\theta \sin\theta \cdot \frac{8}{3} \, d\theta$$

$$= \frac{16\pi}{3} \cdot \frac{1 \cdot 3 \cdot 1}{5 \cdot 3 \cdot 1} = \frac{16\pi}{15}$$

4.



$$\int_0^{2\pi} d\phi \int_0^{\frac{\pi}{4}} d\theta \int_0^{a} r^2 \cos^2\theta \, r^2 \sin\theta \, dr$$

$$= 2\pi \int_0^{\frac{\pi}{4}} \frac{a^5}{5} \cos^2\theta \sin\theta \, d\theta$$

$$= \frac{2\pi a^5}{5} \int_0^{\frac{\pi}{4}} \cos^2\theta \sin\theta \, d\theta$$

$u = \cos\theta$
$du = -\sin\theta \, d\theta$

$$= -\frac{2\pi a^5}{5} \int u^2 \, du = -\frac{2\pi a^5}{5} \left[\frac{\cos^3\theta}{3}\right]_0^{\pi/4}$$

$$= -\frac{2\pi a^5}{15} \left[\frac{1}{2\sqrt{2}} - 1\right]$$

$$= \frac{\pi a^5 (4 - \sqrt{2})}{30}$$