EXAMINATION AM1	SETTER RBP
FOR DIET May 2005	CHECKER TNL PAGE NO

COMMENTS	SOLUTION	MARK
QUESTION NO. A1 (a)	$y' + \frac{2}{x}y = 1 - \frac{2}{x}$ Integrating factor $= e^{2\int \frac{dx}{x}} = e^{2\ln x} = x^2$	2
	Hence $x^2y' + 2xy = x^2 - 2x \Rightarrow (x^2y)' = x^2 - 2x$	
	so that $x^2y = \int (x^2 - 2x)\,dx = \frac{x^3}{3} - x^2 + C$	2
	When $x = 1, y = 2 \Rightarrow 2 = \frac{1}{3} - 1 + C \quad \therefore C = \frac{8}{3}$	
	$\therefore$ Solution is $y = \frac{x}{3} - 1 + \frac{8}{3x^2}$.	2
(b)	$y'' + 3y' + 2y = 0$ Auxiliary equation: $m^2 + 3m + 2 = 0$ i.e. $m = -1$ and -2.	
	$\therefore$ General solution is $y = Ae^{-x} + Be^{-2x}$	3
(c)	$\ddot{y} + 2\dot{y} + 5y = 1 + 10t, \; y(0) = 1, \; \dot{y}(0) = 0$	
	Auxiliary equation is $m^2 + 2m + 5 = 0 \Rightarrow m = -1 \pm 2i$	
	$\therefore$ CF is $e^{-t}(A\cos 2t + B\sin 2t)$.	3
	For the PI, try $y = Ct + D$ to yield	
	$2C + 5(Ct + D) = 1 + 10t$	
	i.e. $C = 2$ and $2C + 5D = 1 \quad \therefore D = -\frac{3}{5}$	3
	$\therefore$ General solution is $y = e^{-t}(A\cos 2t + B\sin 2t) + 2t - \frac{3}{5}$	1
	$\dot{y} = -e^{-t}(A\cos 2t + B\sin 2t) + e^{-t}(-2A\sin 2t + 2B\cos 2t) + 2$	
	Then $y(0) = 1 = A - \frac{3}{5} \quad \therefore A = \frac{8}{5}$	
	$\dot{y}(0) = 0 = -A + 2B + 2 \Rightarrow B = -\frac{1}{5}$	
	$\therefore$ Particular solution is	4
	$y = e^{-t}\left(\frac{8}{5}\cos 2t - \frac{1}{5}\sin 2t\right) + 2t - \frac{3}{5}$.	

EXAMINATION AM3	SETTER RBP
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QUESTION NO. A2 (a)

$$u_{r+2} - \tfrac{5}{2}u_{r+1} + u_r = 2r, \quad u_1 = 1,\ u_2 = -3$$

Characteristic equation is $\lambda^2 - \frac{5}{2}\lambda + 1 = (\lambda - \frac{1}{2})(\lambda - 2) = 0$

$\therefore$ CF is $A\left(\frac{1}{2}\right)^r + B2^r$.

For the inhomogeneous equation try $u_r = Cr + D$

$\Rightarrow C(r+2) + D - \frac{5}{2}(C(r+1)+D) + Cr + D = 2r$

i.e. $-\frac{1}{2}Cr - \frac{1}{2}C - \frac{1}{2}D = 2r \quad \therefore C = -4$

and $C + D = 0 \Rightarrow D = 4$

General solution is $u_r = A\left(\frac{1}{2}\right)^r + B2^r + 4 - 4r$

$u_1 = 1 = \frac{1}{2}A + 2B \qquad \Rightarrow \quad \frac{1}{2}A + 2B = 1$

$u_2 = -3 = \frac{1}{4}A + 4B - 4 \qquad \frac{1}{2}A + 8B = 2$

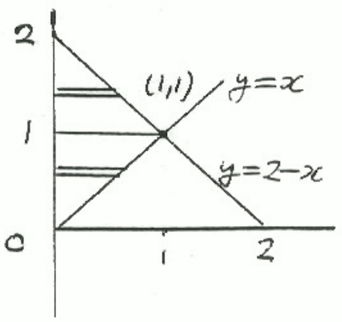
so that $B = \frac{1}{6}$ and $A = \frac{4}{3}$

$\therefore$ Particular solution is $u_r = \frac{4}{3}2^{-r} + \frac{1}{6}2^r + 4 - 4r$. 11

As $r \to \infty$ $2^{-r} \to 0$ and $2^r \gg r$.

$\therefore$ Dominant term is $u_r \sim \frac{2^r}{6}$ as $r \to \infty$. 1

(b) $I = \int_0^1 dx \int_x^{2-x} \frac{x}{y}\,dy = \int_0^1 dy \int_0^y \frac{x}{y}\,dx + \int_1^2 dy \int_0^{2-y} \frac{x}{y}\,dx$



$= \frac{1}{2}\int_0^1 y\,dy + \frac{1}{2}\int_1^2 \frac{(2-y)^2}{y}\,dy$

$= \frac{1}{4} + \frac{1}{2}\int_1^2 \left(\frac{4}{y} - 4 + y\right)dy = \frac{1}{4} + \frac{1}{2}\left[4\ln y - 4y + \frac{y^2}{2}\right]_1^2$

$= \frac{1}{4} + 2\ln 2 - \frac{8}{2} + \frac{2}{2} - \frac{1}{2}\left(-4 + \frac{1}{2}\right) = 2\ln 2 - 1$. 8

EXAMINATION AM3	SETTER RBP
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QUESTION NO. A3 (a)

$$\nabla\times\underline{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = i(x+1-x-1)+j(y-y) + \underline{k}\,(z+2x\cos y - z - 2x\cos y) \equiv 0.$$

Hence $\underline{F}$ is irrotational so that $F=\nabla\phi$ (since $\nabla\times\nabla\phi\equiv 0$) **2**

$$\frac{\partial\phi}{\partial x} = F_x = yz+2x\sin y \Rightarrow \phi = xyz + x^2\sin y + f(y,z)$$

$$\frac{\partial\phi}{\partial y} = xz + x^2\cos y + \frac{\partial f}{\partial y} = F_y = xz + x^2\cos y + z \Rightarrow \frac{\partial f}{\partial y} = z \quad \text{i.e. } f = zy + g(z)$$

$$\frac{\partial\phi}{\partial z} = xy + y + g' = F_z = xy+y+1 \Rightarrow g'=1 \quad \text{i.e. } g = z + \text{const.}$$

$$\phi(0,0,0)=0 \Rightarrow \text{const}=0$$

$\therefore$ Potential $\underline{\phi = xyz + x^2\sin y + zy + z.}$ **5**

Then $\int_C \underline{F}.d\underline{r} = \int_C \nabla\phi . d\underline{r} = \phi(1,\tfrac{1}{2}\pi,1) - \phi(0,0,0)$

$$= \tfrac{1}{2}\pi + 1 + \tfrac{1}{2}\pi + 1 - 0 = \underline{\pi+2.}$$ **2**

(b)

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy \qquad P = x - \frac{y^5}{5},\ \frac{\partial P}{\partial y} = -y^4$$

$$Q = \tfrac{1}{3}x^3y^2,\ \frac{\partial Q}{\partial x} = x^2y^2$$

$$= \iint_R (x^2y^2 + y^4)\,dx\,dy \qquad R:\ x^2+y^2=a^2$$

$$= \int_0^{2\pi}\int_0^a r^4(\cos^2\theta\sin^2\theta + \sin^4\theta)\,r\,dr\,d\theta \qquad \begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}$$

$$= \frac{a^6}{6}\int_0^{2\pi} \sin^2\theta(\sin^2 + \cos^2\theta)\,d\theta = \frac{a^6}{6}\int_0^{2\pi}\sin^2\theta\,d\theta = \underline{\frac{\pi a^6}{6}}.$$ **6**

(c)

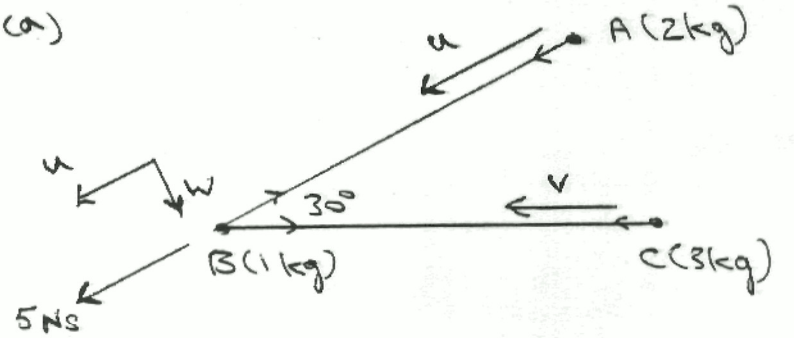
$$\nabla.\underline{F} = \frac{\partial}{\partial x}F_x + \frac{\partial}{\partial y}F_y + \frac{\partial}{\partial z}F_z = 2z - 2z + 3 = 3.$$

By Gauss' theorem $\iint_S \underline{F}.\underline{n}\,dS = \iiint_V \nabla.\underline{F}\,dV = 3\iiint_V dV$

$$= 3\times\tfrac{4}{3}\pi a^3 = \underline{4\pi a^3.}$$ **5**

AR0970a - Applied Maths 3 Solutions (Dynamics) 2004/05.

B1. (a)



u, v, w = component velocities as shown (A and C move along AB and CB directions)

Momentum eqⁿ along AB direction:

$$5 = 1\times u + 2\times u + 3\times v\cos 30^\circ$$

$$\therefore \quad 5 = 3u + \frac{3\sqrt{3}}{2}v \quad --- \quad (1)$$ 2

Momentum eqⁿ perp. to AB direction (conserved):

$$1\times w = 3\times v\sin 30^\circ$$

$$\therefore \quad w = \frac{3v}{2} \quad --- \quad (2)$$ 1

Also component of velocity of B along CB direction = vel. of C

$$\therefore \quad u\cos 30^\circ - w\sin 30^\circ = v$$

or $\quad u\sqrt{3} - w = 2v \quad --- \quad (3)$ 2

(2) & (3) give $u\sqrt{3} = \frac{7v}{2}$, and substituting this into (1) gives

$$5 = 3 . \frac{7v}{2\sqrt{3}} + \frac{3\sqrt{3}}{2}v$$

$$\therefore \quad 5 = \frac{\sqrt{3}}{2}v(7+3) \Rightarrow \underline{\underline{v = \frac{1}{\sqrt{3}}}} \text{ m/s. (speed of C)}$$ 2

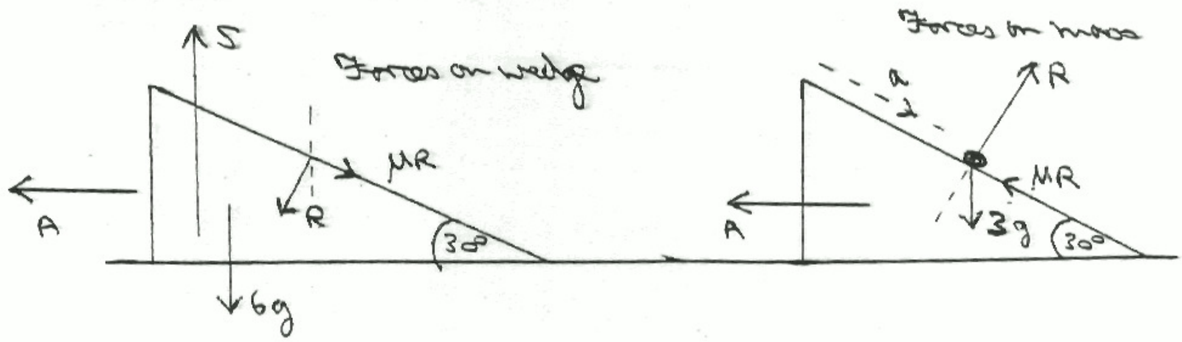
Hence, $u = \frac{7}{2\sqrt{3}} . \frac{1}{\sqrt{3}} = \underline{\underline{\frac{7}{6}}}$ m/s (speed of A) 1

and $w = \frac{3}{2} . \frac{1}{\sqrt{3}}$ (from (2)) $= \frac{\sqrt{3}}{2}$ m/s.

So speed of B $= \sqrt{u^2 + w^2} = \frac{1}{2}\sqrt{\left(\frac{7}{3}\right)^2 + 3} = \frac{1}{6}\sqrt{49+27}$

$$= \frac{1}{6}\sqrt{76} = \underline{\underline{\frac{\sqrt{19}}{3}}} \text{ m/s.}$$ 2

81-(b)



3

A = acceln of wedge (a = acceln of mass relative to wedge)

Other forces as shown. Frictional forces = μR (moving).

Eqn of motion of wedge:

$$6A = R\sin 30^\circ - \mu R\cos 30^\circ$$

$$\therefore \quad 6A = \frac{R}{2} - \frac{1}{2\sqrt{3}} R \frac{\sqrt{3}}{2} = \frac{R}{4}$$

$$\therefore \quad R = 24A \quad \cdots \quad (1)$$ 2

Eqn of motion of mass perp. to plane:

$$3A\sin 30^\circ = 3g\cos 30^\circ - R \quad \cdots \quad (2)$$ 2

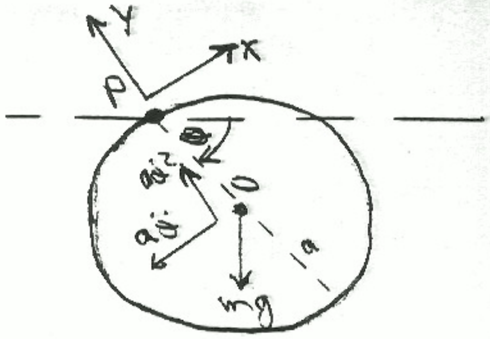
Using (1) in (2): $$\frac{3A}{2} = 3g\frac{\sqrt{3}}{2} - 24A$$

$$\therefore \quad 51A = 3g\sqrt{3}$$

$$\therefore \quad A = \frac{3g\sqrt{3}}{51} = \frac{g\sqrt{3}}{17}$$

$$\simeq \underline{\underline{1 \text{ m/s}^2}} \quad \text{(as required)}$$ 3

B2. (a)



X and Y = components of thrust on axis perp. to and along OP respectively.

Eqⁿs of motion:

Along OP $-$ $\quad ma\dot{\theta}^2 = Y - mg\sin\theta \quad \cdots\cdots (1)$

Perp. to OP $-$ $\quad ma\ddot{\theta} = mg\cos\theta - X \quad \cdots\cdots (2)$ 3

Rotational eqⁿ of motion about P: $\quad (I\ddot{\theta} = \text{sum of moments})$

$$\frac{3}{2}ma^2\ddot{\theta} = mga\cos\theta$$

$$\therefore \; a\ddot{\theta} = \frac{2}{3}g\cos\theta \quad \cdots (3)$$ 1

and integrating (3) gives $\quad a\dfrac{\dot{\theta}^2}{2} = \dfrac{2}{3}g\sin\theta \quad (\dot{\theta} = 0 \text{ at } \theta = 0)$
$\cdots (4)$ 1

(1) & (4) $\Rightarrow \quad m \cdot \frac{4}{3}g\sin\theta = Y - mg\sin\theta$

$$\therefore Y = \frac{7}{3}mg\sin\theta = \frac{7mg}{3\sqrt{2}} \text{ at } \theta = 45^\circ$$ 2

(2) & (3) $\Rightarrow \quad \frac{2}{3}mg\cos\theta = mg\cos\theta - X$

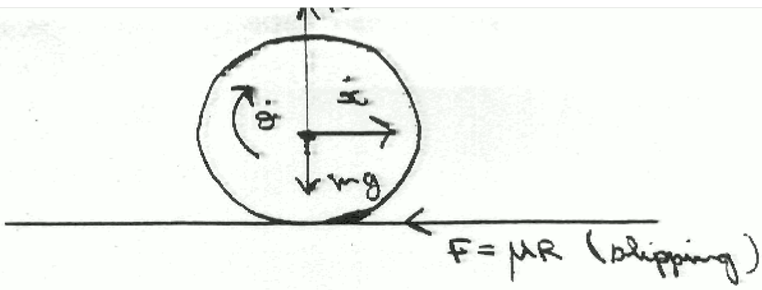
$$\therefore \; X = \frac{1}{3}mg\cos\theta = \frac{mg}{3\sqrt{2}} \text{ at } \theta = 45^\circ.$$ 2

$\therefore$ Resultant thrust magnitude $= \sqrt{X^2+Y^2}$

$$= \frac{mg}{3\sqrt{2}}\sqrt{1+49}$$

$$= \underline{\underline{\frac{5}{3}mg}} \quad \text{(as required)}$$ 1

B2.(b)



$\dot x, \dot\theta$ are velocity and angular velocity resp. of sphere.
R = normal reaction force = mg (no vertical motion)
$F = \mu R = \mu m g$ is frictional force as slipping occurs. 1

Linear eqⁿ of motion of sphere: $m\ddot x = -F$

$$\therefore \ddot x = -\frac{1}{2} g \quad (\mu = \tfrac{1}{2})$$

$$\therefore \dot x = -\frac{gt}{2} + V \quad (\dot x = V \text{ at } t = 0) \quad \ldots (1)$$ 3

Rotational eqⁿ of motion: (moments about centre)

$$\frac{2}{5} m a^2 \ddot\theta = F a$$

$$\therefore \frac{2}{5} m a \ddot\theta = \frac{1}{2} m g \Rightarrow a\ddot\theta = \frac{5}{4} g$$

$$\therefore a\dot\theta = \frac{5gt}{4} \quad (\dot\theta = 0 \text{ at } t = 0) \quad \ldots (2)$$ 3

Sphere rolls when $\dot x = a\dot\theta$, so equating (1) and (2) gives

$$\frac{5}{4} gt = -\frac{gt}{2} + V$$

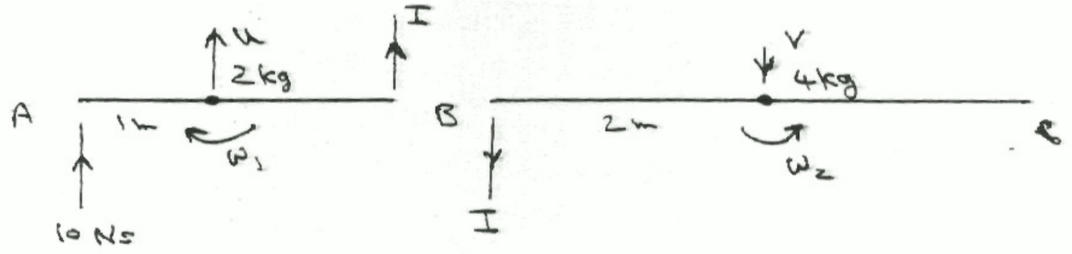
$$\therefore \left(\frac{5}{4} + \frac{1}{2}\right) gt = V$$ 2

i.e. $t = \dfrac{4V}{7g}$ (time before rolling starts)

Rolling velocity given by using this t in (1), i.e.

$$\dot x = -\frac{2V}{7} + V = \frac{5V}{7}.$$ 1

3. (a)



2

u, v = velocities of c. of g. of rods AB and BC resp. } after
ω_1, ω_2 = angular velocities of AB and BC resp. } impulse
I = impulsive reaction at hinge B (must be perp. to rods)

For AB, linear momentum eqⁿ: $10 + I = 2u$ --- (1) 1

ang. mom. eqⁿ: $10 \times 1 - I \times 1 = \frac{1}{3} \times 2 \times 1^2 \times \omega_1$
(moments about centre)

$\therefore 10 - I = \frac{2}{3}\omega_1$ --- (2) 1

For BC, linear mom.: $I = 4v$ ---- (3) 1

ang. mom.: $2I = \frac{1}{3} \times 4 \times 2^2 \times \omega_2$

$\therefore I = \frac{8}{3}\omega_2$ --- (4) 1

Velocity of B is common to both rods:

$\therefore u - 1 \times \omega_1 = -(v + 2\omega_2)$

or $\omega_1 - u = v + 2\omega_2$ --- (5) 2

Substituting (1)-(4) into (5) gives

$$\frac{3}{2}(10 - I) - \frac{(10+I)}{2} = \frac{I}{4} + \frac{3I}{4}$$

$\therefore 30 - 3I - 10 - I = 2I$

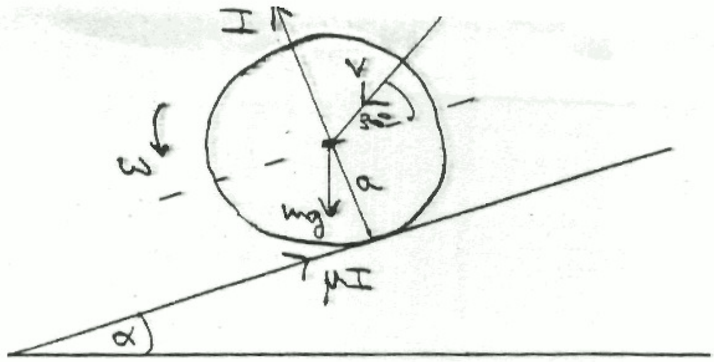
$\therefore 6I = 20 \Rightarrow I = \frac{10}{3}$ Ns. 2

Now, velocity of C $= v - 2\omega_2$ (direction of v)

$= \frac{I}{4} - \frac{3I}{4} = -\frac{I}{2}$

i.e. speed of C $= \frac{5}{3}$ m/s (as required) 2

B3. (b)



I = impulsive force of plane on disc on impact

μI = " frictional force " " " (slips) 1

ω = angular velocity of disc after impact.

Angular momentum eq^n (moment about centre of disc):

$$\frac{1}{2} m a^2 . \omega = \mu I . a$$

$$\therefore \quad m a \omega = I \quad (\mu = \tfrac{1}{2}) \quad ---- \quad (1)$$ 2

If u = vel of disc perpendicular to plane after impact then Newton's restitution law gives:

$$u = e V \sin 30°$$

$$\therefore \quad u = \frac{V}{8} \quad ---- \quad (2)$$ 1

Linear momentum eq^n perpendicular to plane:

$$I = m(u + V \sin 30°) \quad --- \quad (3)$$ 2

(1), (2) & (3) give

$$m a \omega = \left(\frac{V}{8} + \frac{V}{2}\right) m$$

$$\therefore \quad \omega = \frac{5V}{8a}$$