Estimating changes in velocity and source separation using coda wave interferometry: MATLAB code user guide

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1 Introduction

This guide accompanies Singh et al. (2018): *Coda Wave Interferometry for Velocity Monitoring and Acoustic Source Location in Experimental Rock Physics*, which can be downloaded from **https://edin.ac/2OhWHFP**. Contained in this package are a suite of well-commented MATLAB functions for estimating the changes in the bulk velocity of a medium, the small change in source or receiver locations, or the joint estimation of velocity and source/receiver location changes when both occur simultaneously. This code package can be used in conjunction with the codes of Zhao & Curtis (2018), available at **https://www.geos.ed.ac.uk/eip/codes.html**. These codes provide methods for the estimation of relative source locations in a cluster of sources. By combining these two code packages, estimates of the relative locations of a cluster of sources can be made as well as any velocity perturbations that occur in the medium, all using a single receiver. Users can exectute all function contained in the package by running a single script: *examples_running_script.m*, which demonstrates the implementation of all the functions and how they can edited to suit the users requirements. First we cover the theory of Coda Wave Interferometry (CWI) and how it can be used to estimated changes in the velocity of a medium, a change in the location of a source or receiver, and the random displacement of scatterer locations. We then provide an overview of the code package contents and explain each code with examples for their implementation.

2 Theory

CWI allows small changes in velocity, the displacement of source or receiver locations, or movement of scatterers to be monitored (Snieder et al., 2002; Sens-Schönfelder & Wegler, 2006; Snieder, 2006). These different perturbations and their effect on recorded signals are illustrated in Figure 1. First we consider the effect of a velocity perturbation (ΔV in Figure 1a), where the direct arriving wave between a source and receiver would only sample the perturbation once (or not at all), whereas the multiply reflected waves sample the perturbation many times. Therefore the change in arrival times for later arriving waves (t3,t4) is much larger than for the first arrival (t1,t2). The second perturbation type is a displacement of the source or receiver location (source displacement in Figure 1b): in this case, the difference in ray paths before and after the perturbation is the path

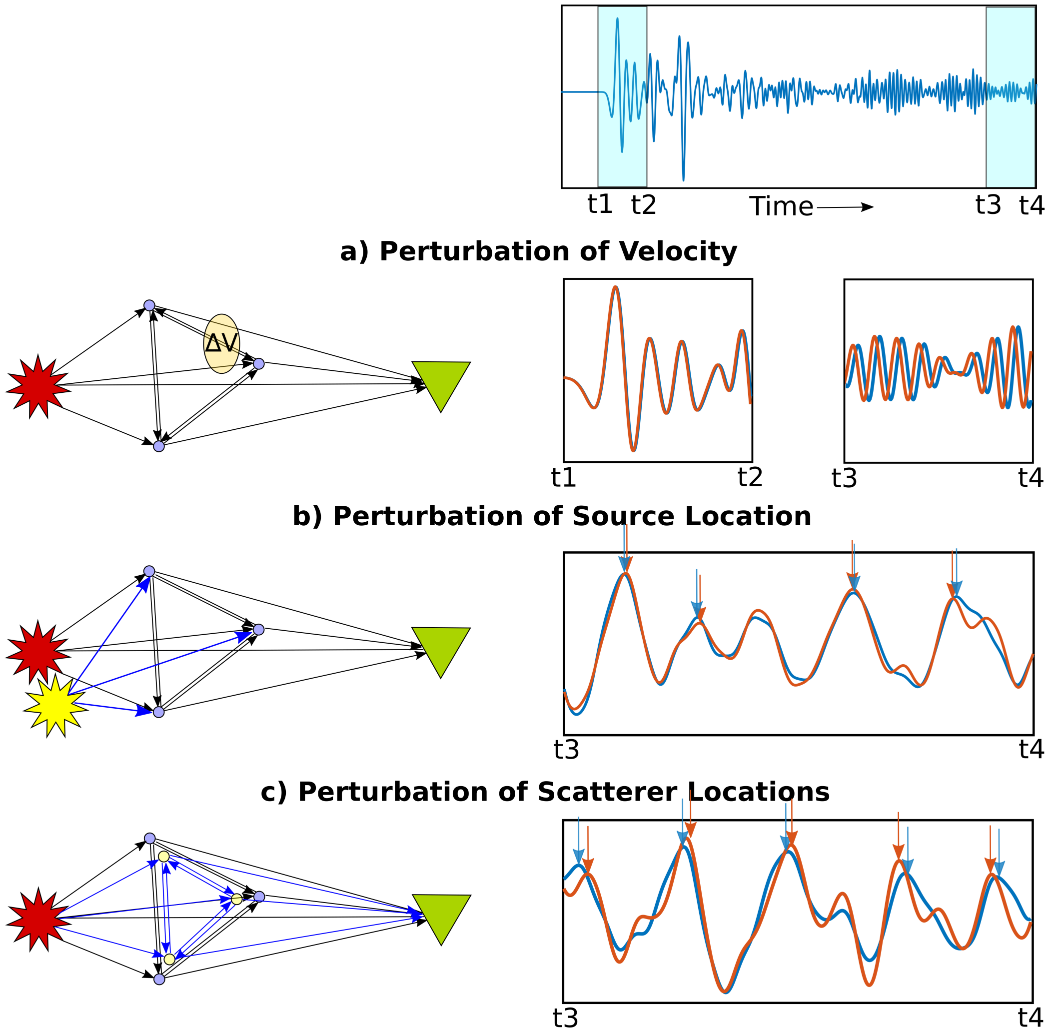

Figure 1: Illustrations of different perturbation types and their effects on coda waves. The cartoons (left) represent a scattering medium, with a source (star), receiver (triangle), and point scatterers (circles). Ray paths between the source and receiver, including multiple reverberations, are represented as black arrows. A velocity perturbation (a) is represented as a yellow ellipse, which has a velocity different to the background medium. New ray paths that are introduced due to source location (b) and scatterer location (c) perturbations are represented as blue arrows. Example recorded signals (right) at early (t1,t2) and late (t3,t4) time windows for each perturbation type are shown before and after each perturbation takes place (blue and red, respectively). Differences in travel times of arriving energy for b) and c) are highlighted with vertical arrows.

between the source and the first scattering point (blue arrows in Figure 1b). Paths would be both shortened and lengthened depending on the location of the first scatterer, which is reflected by the advancement and retardation of peaks highlighted by red and blue arrows. The extent to which these travel times are perturbed (their variance) is directly proportional to the (small) displacement of the source. The third perturbation type is the displacement of all scattering points (yellow circles in Figure 1c): in this case, all paths between scattering points are perturbed (both shortened and lengthened) and similarly to the previous case, the statistics of travel time perturbations are related to the displacement of scattering points. All three perturbation types can be monitored by using a cross correlation of the unperturbed (u_{unp}) and perturbed (u_{per}) waveforms - the waveforms from the source recorded by the receiver before and after the change or displacement takes place.

The method we use to estimate the change in velocity is known as trace stretching (Sens-Schönfelder & Wegler, 2006), where the perturbed waveform is assumed to be a time-stretched version of a reference waveform; this follows if one assumes that a velocity perturbation is uniform across the entire medium, so all arriving energy is perturbed at the same temporal rate. We stretch the time axis of the perturbed signal by a range of stretching factors (ϵ) and compute the correlation coefficient R between $u_{unp}(t)$ and the stretched version of the perturbed waveform $u_{per}(t[1+\epsilon])$ over a given time window (t_1, t_2):

$$R^{(t_1,t_2)}(\epsilon) = \frac{\int_{t_1}^{t_2} u_{unp}(t) u_{per}(t[1+\epsilon]) dt}{\sqrt{\int_{t_1}^{t_2} u_{unp}^2(t) dt \int_{t_1}^{t_2} u_{per}^2(t[1+\epsilon]) dt'}} \tag{1}$$

The optimum stretching factor ϵ_{max} that maximizes the correlation coefficient (for which $R = R_{max}$), is related to the ratio of the change in velocity ΔV to the original velocity V) by

$$\epsilon_{max} = -\frac{\Delta V}{V} \tag{2}$$

(Sens-Schönfelder & Wegler, 2006). That method also assumes that the velocity changes are small to avoid cycle skipping in the calculation of R in equation 1. In cases where the medium changes significantly, such as during material deformation where new scattering paths are introduced due to fracturing, it may not be appropriate to use a constant reference trace (u_{unp}) for all recorded waveforms during deformation. We propose the use of a moving reference trace, where the optimum stretching factor from the initial reference trace (u_0) to any other recorded waveform during deformation (u_n) can be calculated as

$$\epsilon_{u_0 u_n} = \epsilon_{u_0 u_s} + \epsilon_{u_s u_n} \tag{3}$$

where $s = k\lfloor n/k \rfloor$, n is the trace number, k is the user-selected step size of the moving reference trace, and $\lfloor \ldots \rfloor$ denotes a floor function, which outputs the greatest integer less than or equal to the input.

CWI allows the joint estimation of both a velocity perturbation and the displacement r of the source/receiver location to be made from a single receiver. This is because velocity perturbation information is retrieved from the consistent phase information along the waveforms, whereas the source or receiver separation is related to the variance of inconsistent phase perturbations and hence to the maximum value of the cross correlation value (R_{max}) in equation 1 (Figure 1). Snieder (2006) derives the relationship between the maximum cross-correlation and the variance of the travel time perturbation (σ_τ^2) as

$$R_{max} = 1 - \frac{1}{2}\bar{\omega}^2 \sigma_\tau^2 \tag{4}$$

where $\bar{\omega}^2$ is the dominant mean squared frequency in the recorded waveform. When a source or receiver is displaced by distance r, one can estimate separation r from the variance of the travel time perturbation

$$\sigma_\tau^2 = \frac{(\frac{6}{\alpha^8} + \frac{7}{\beta^8})}{7(\frac{2}{\alpha^6} + \frac{3}{\beta^6})} r^2 \tag{5}$$

where α and β are estimates of the representative P- and S-wave velocities of the medium. In a two-dimensional acoustic medium this relationship simplifies to:

$$\sigma_\tau^2 = \frac{1}{2\alpha^2} r^2. \tag{6}$$

The estimates of source separation for a cluster of sources can be used to estimate relative locations of all sources within the cluster with a single receiver (Zhao et al., 2017). CWI is also able to resolve another type of perturbation on which we do not focus: the average displacement of all scatterers, δ, illustrated in Figure 1c (Snieder et al., 2002). This value is related to the variance of travel time perturbations by

$$\sigma_\tau^2 = \frac{2\delta^2 t}{v l_\star} \tag{7}$$

where $l_\star$ is the transport mean free path. An estimate for δ can be made using the output of the *cwi_sep.m* function and the equation above.

3 Package Contents

3.1 Functions

- **cwi_stretch_vel.m**: for estimating a velocity change, using the CWI stretching technique (Equation 1). The function initially searches through stretching factors equivalent to a range of 10% to -10% velocity changes, then searches at fine increments of 0.005% changes in velocity. If a wider search range is required, the user should edit the *s_factors* variable.

  Inputs:

  – *sig*1: Reference signal recorded prior to a perturbation
  – *sig*2: Signal recorded after a perturbation

  Output:

  – *epsilon*: Stretching factor which maximizes correlation (equal to $-\Delta V/V$)

- **cwi_sep.m**: for estimating the separation between a pair of sources or a pair of receivers using coda wave interferometry. The function uses Equation 4 to estimate and output the variance of travel time perturbations σ_τ^2, which is related to the displacement of source location by Equations 5 and 6.

  Inputs:

  – *sig*1: Reference signal recorded prior to a perturbation
  – *sig*2: Signal recorded after a perturbation
  – *dt*: Sampling interval time in seconds - must be the same for both signals
  – *win_start*: Index of sig1 and sig2 corresponding to the start of the desired time window
  – *win_end*: Index of sig1 and sig2 corresponding to the end of the desired time window

Output:

- *variance*: Variance of travel time perturbations. Proportional to the separation and the velocity of the medium (Equations 5 and 6).

- **cwi_stretch_vel_and_sep.m**: for joint estimation of changes in velocity and source location, when both occur simultaneously.

  Inputs:

  - *sig1*: Reference signal recorded prior to a perturbation
  - *sig2*: Signal recorded after a perturbation
  - *dt*: Sampling interval time in seconds - must be the same for both signals
  - *win_start*: Index of sig1 and sig2 corresponding to the start of the desired time window
  - *win_end*: Index of sig1 and sig2 corresponding to the end of the desired time window

  Output:

  - *variance*: Variance of travel time perturbations. Proportional to the separation and the velocity of the medium (Equations 5 and 6).
  - *epsilon*: Stretching factor which maximizes correlation (equal to $-\Delta V/V$)

- **mov_ref_trace.m**: a function for combining estimates of velocity change from coda wave interferometry (*epsilon* from *cwi_stretch_vel.m*), employing a moving reference trace (Equation 3)- with a user selected step size. Velocity changes are required to be combined into a NxN matrix, where N is the number of signals.

  Inputs:

  - *epsilon_matrix*:
  - *k*: Step size for moving reference trace

  Output:

  - *dv_mrt*: 1xN vector of cumulative velocity change.

3.2 Scripts

- **examples_running_script.m**: Full script for loading in example data and using all functions for changes in velocity, source location, both perturbation types occurring simultaneously, and the combination of multiple velocity changes using a moving reference trace. This script also generates a series of plots, all contained in this report.

3.3 Data

- **fluid_change_data.mat**: example data generated from finite difference simulation of a wavefield through a Tivoli Travertine, where pore fluid velocity is perturbed by 100 m/s.

- **source_change_data.mat**: example data generated at an array of source locations, represented a fracture plane occurring in the Tivoli Travertine digital rock.

- **source_and_vel_data.mat**: example generated for a range of source locations, and a range of velocity perturbations occurring simultaneously.

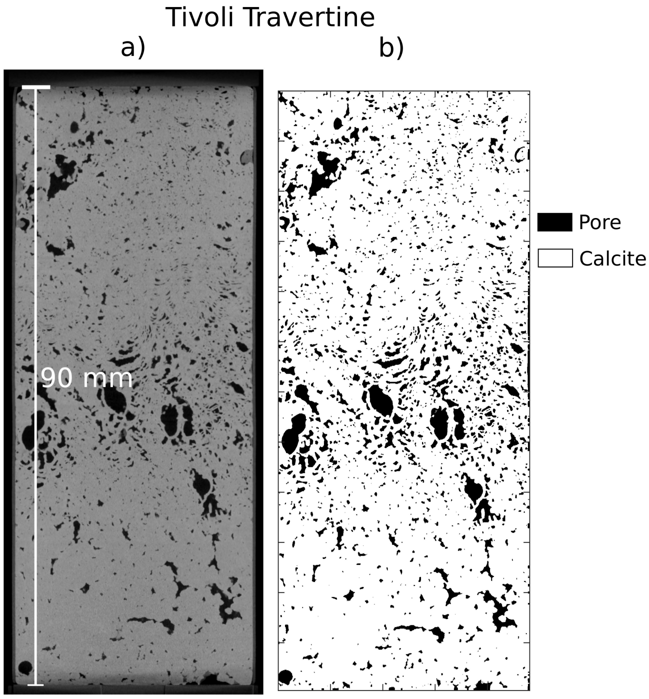

Figure 2: a) X-ray micro-tomography slice of a Tivoli Travertine core. b) The equivalent model of segmented phases, in this case we assume two phases, calcite and pore fluid. The elastic properties of calcite and water are used in the finite difference simulations of wave propogation to generate the example data sets used in this guide.

- **mrt_example_data.mat**: example CWI data from laboratory experiment of the deformation of a laminated carbonate core. Used for the demonstration of the moving reference trace method.

4 Codes and Example

We demonstrate the use of all function listed above in the *examples_running_script.m* script, using appropriate example data. We generate example data using finite difference simulation of wavefield propogation through a model based on a x-ray micro-tomography slice through a Tivoli Traverine core (shown in Figure 2). Here we demonstrate the use of each function and describe how they can be edited to be used different data sets. All figures shown in this user guide are also generated as part of the *examples_running_script.m* script.

4.1 Estimating a change in velocity

The first example data set was generated from finite difference simulation of a wavefield through a Tivoli Travertine, where pore fluid velocity is perturbed by range of velocities up to 100 m/s. The data can be loaded into MATLAB with the following lines

```
load('example_data/fluid_change_data.mat')
```

The example data is plotted in Figure 3, comparing signals before and after a velocity perturbation. The lower panels highlight how the coda waves are more sensitive to small changes in velocity when compared against first arriving waves. CWI can be performed on the example data set with the by computing the following:

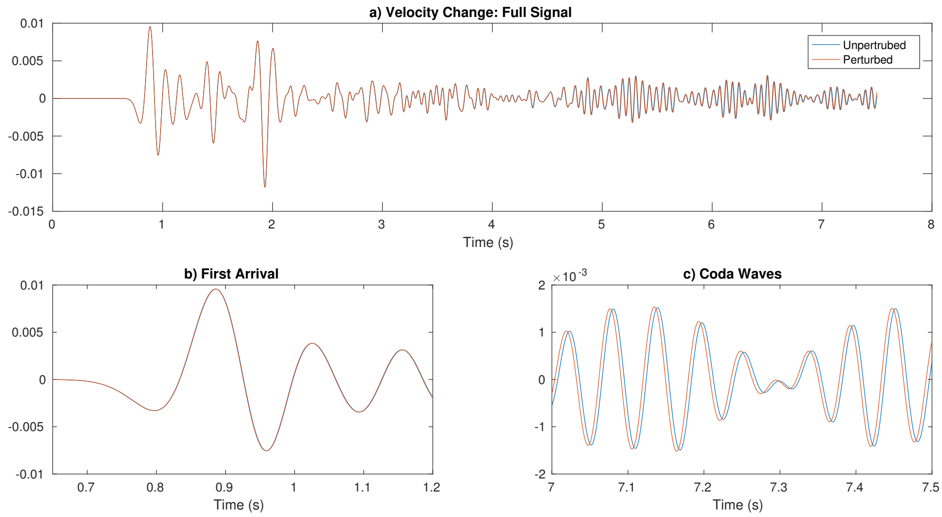

Figure 3: Example data generated from finite difference simulation of a wavefield through a Tivoli Travertine, before (blue) and after pore fluid velocity is perturbed by 1.5 m/s (red). Comparison is shown for a) the full signal, b) the first arriving waves and c) the coda waves.

```
% Unperturbed signal:
sig1 = fluid_change_data(:,1);
% Loop through all signals performing CWI
for i = 1:size(fluid_change_data,2)
    % Perturbed signal:
    sig2 = fluid_change_data(:,i);
    % CWI stretching method for velocity change
    epsilon=cwi_stretch_vel(sig1,sig2);
    % Velocity change dV/V = - epsilon:
    dV(i) = -epsilon;
end
```

The output **dV** is a vector of velocity change estimates between the reference signal (sig1) and every other signal in the data set (plotted in Figure 4). If the user requires to apply this method on different data sets, the $fluid\_change\_data$ variable can replaced with an $l \times N$ matrix, where l is the length of the signals and N is the number of signals.

4.2 Estimating a source location perturbation

The second example data set is for the estimation of a change in source location, i.e., inter source distance. Example data can be loaded from $source\_change\_data.m$, a pair of signals are compared in Figure 5, from this we see that the perturbation causes a change in the correlation of the coda waves, but is not a coherent shift of the travel times as seen for the velocity perturbation in Figure 3. The data is from an array of source locations occurring along a plane in the Tivoli Traverine digital rock sample (shown in Figure 6). To perform CWI to estimate the source separation, first find the variance of travel time perturbations, then the source location perturbation can be estimated using Equation 6. This is achieved be executing the following:

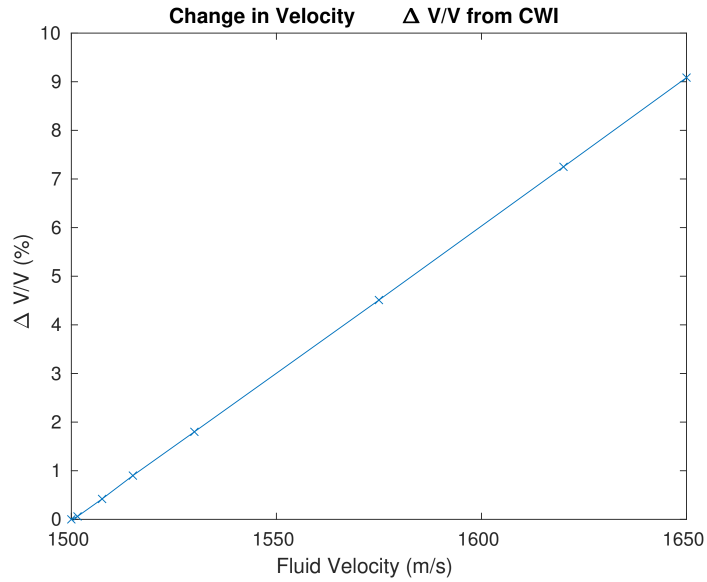

Figure 4: Estimates of velocity change ($\Delta V/V$) using CWI, as a function of pore fluid velocity in a Tivoli Travertine digital rock.

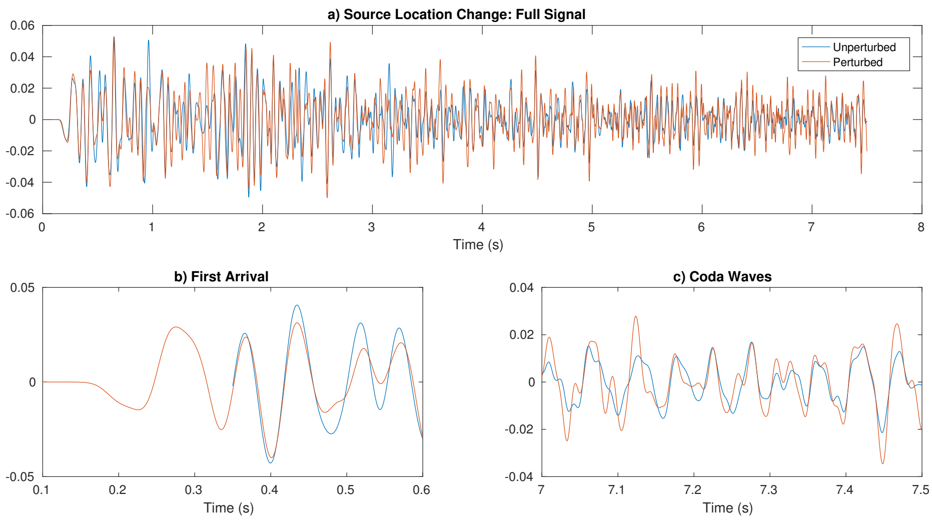

Figure 5: Example data generated from finite difference simulation of a wavefield through a Tivoli Travertine, before (blue) and after the source location is perturbed by 0.001λ (red). Comparison is shown for a) the full signal, b) the first arriving waves and c) the coda waves.

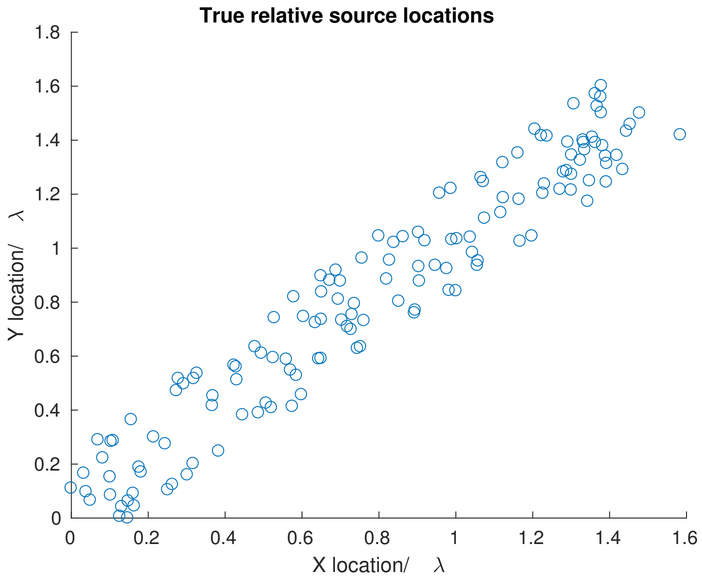

Figure 6: Relative locations of a cluster of sources used to generate the example data for estimating source separation using CWI.

```
% Required parameters:
N = size(source_change_data,1); % Number of signals
win_start = 100000; % Index for start time window
win_end = 150000; % Index for end of time window
dt = 5e-5; % Sampling interval
vel = 2200; % velocity of the medium
lambda = 67; % dominant wavelength in the signal
% Assign reference signal:
sig1 = source_change_data(1,:);
% Loop through all sources:
for i = 1:N
    % Perturbed signal for varying source location
    sig2 = source_change_data(i,:);
    % Perform CWI to estimate variance of travel time perturbations
    [var] = cwi_sep(sig1,sig2,dt,win_start,win_end);
    % Calculate inter-source separation using relationship between
    % velocity and variance of travel time perturbations.
    sep_cwi(i) = sqrt(2*vel^2.*var)/lambda; % Normalised by dominant wavelength
end
```

The output *sep_cwi* is a vector of estimated inter source separations, normalised by the dominant wavelength in the signal. The resulting estimates are plotted as a function of true separation in Figure 7. The variance *var* can also be used to estimate the average displacement of scatterers (using Equation 7) though we do not illustrate this use in the package. To use a different data set, change the *source_change_data* variable to an $N \times l$ matrix, where l is the length of the signals and N is the number of signals.

Note: the output from this function can be used in conjunction with publicly available codes of Zhao & Curtis (2018), for the relocation of relative source locations, using a single receiver.

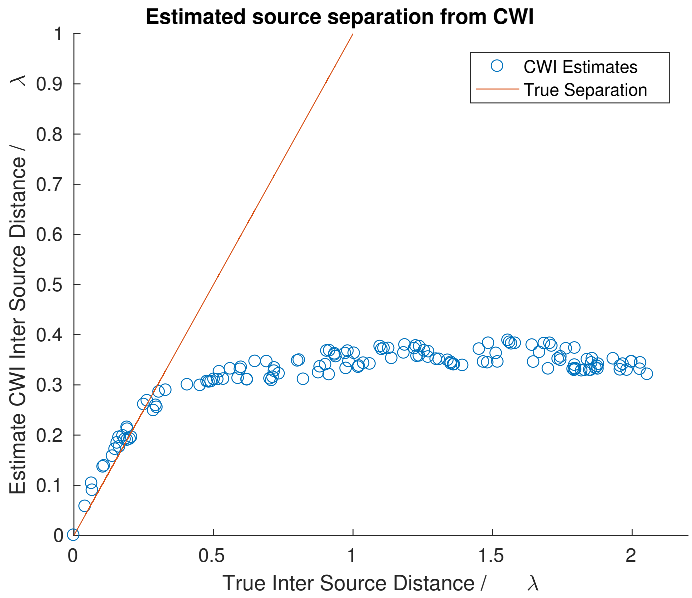

Figure 7: Estimated source separation between a single source, and every other source in the cluster (circles) plotted as a function of the true source separation. The red line shows the true separations for reference.

4.3 Estimating simultaneous source location and velocity perturbations

The third example data set is synthetically generated using finite difference wavefield simulation, where signals are generated at an array of source locations, and source locations survey are repeated with a range of velocity perturbations occurring simultaneously. Example signals are compared in Figure 8, where a source location perturbation of 0.04λ and a velocity perturbation of 0.2% have occurred. The data is loaded from $source\_and\_vel\_data.m$, and is a 6×10 matrix, representing the 6 different velocity models, and 10 different source locations used. To estimate both the change in velocity and source location, use the $cwi\_stretch\_vel\_and\_sep.m$ function, demonstrated below:

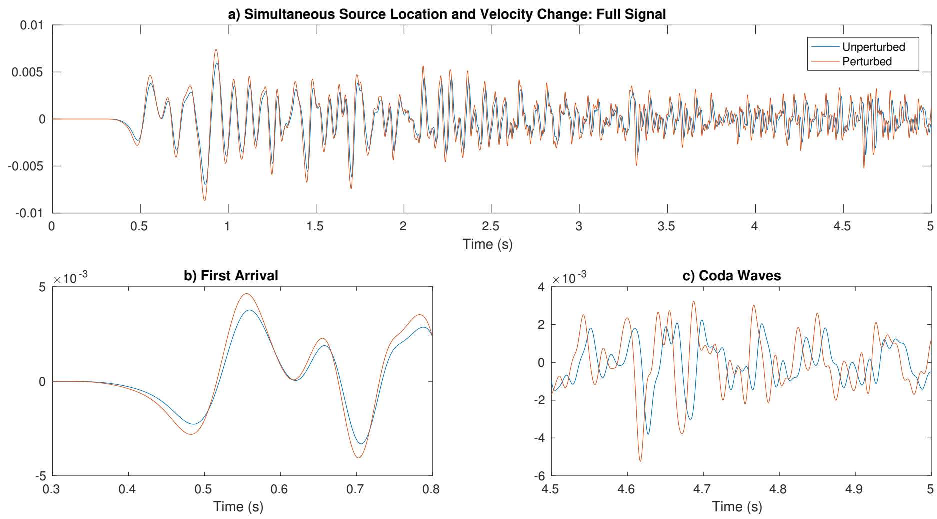

Figure 8: Example data generated from finite difference simulation, before (blue) and after the simultaneous perturbation of source location by 0.05λ and velocity by 0.2 % (red). Comparison is shown for a) the full signal, b) the first arriving waves and c) the coda waves.

```
% Required parameters:
dt = 5e-5; % Sampling interval
win_start = 500; % Index for start time window
win_end = 100000; % Index for end of time window
dt = 5e-5; % Sampling interval
vel = 3500; % velocity of the medium
% Assign reference signal:
sig1 = source_and_vel_data(:,1,1);
% Loop through all all velocity perturbations and source location changes
for i = 1:6 % Number of velocity models
    for j = 1:10 % Number of source locations
        % Assign perturbed signal
        sig2 = source_and_vel_data(:,i,j);
        % CWI for simultaneous changes in velocity and source location
         [eps,var]=cwi_stretch_vel_and_sep(sig1,sig2,dt,win_start,win_end);
        % Velocity change = -epsilon:
        vel_change_cwi(i,j) = eps;
        % Calculate inter-source separation using relationship between
        % velocity and variance of travel time perturbations:
        sep_cwi(i,j) = sqrt(2*vel^2.*var)/67;
    end
end
```

The output is a pair of matrices, one represents the velocity change estimates $vel\_change\_cwi\_mat$ and the other source location change estimates sep_cwi for all combinations of velocity and source location changes. The estimates of both source location and velocity changes are plotted against their true changes in Figure 9.

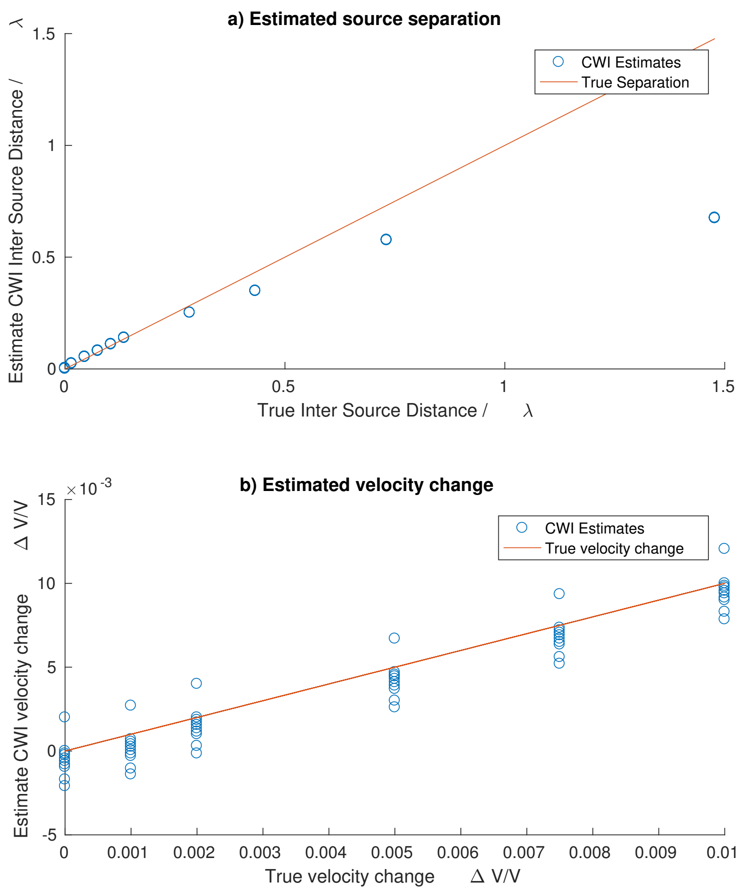

Figure 9: a) Estimates of source separation with simultaneous velocity perturbation. b) Estimates of velocity perturbation with simultaneous source location perturbations. CWI estimates plotted as circles, and the true solutions are represented as red lines.

4.4 Combining estimates of velocity change using the moving reference trace function

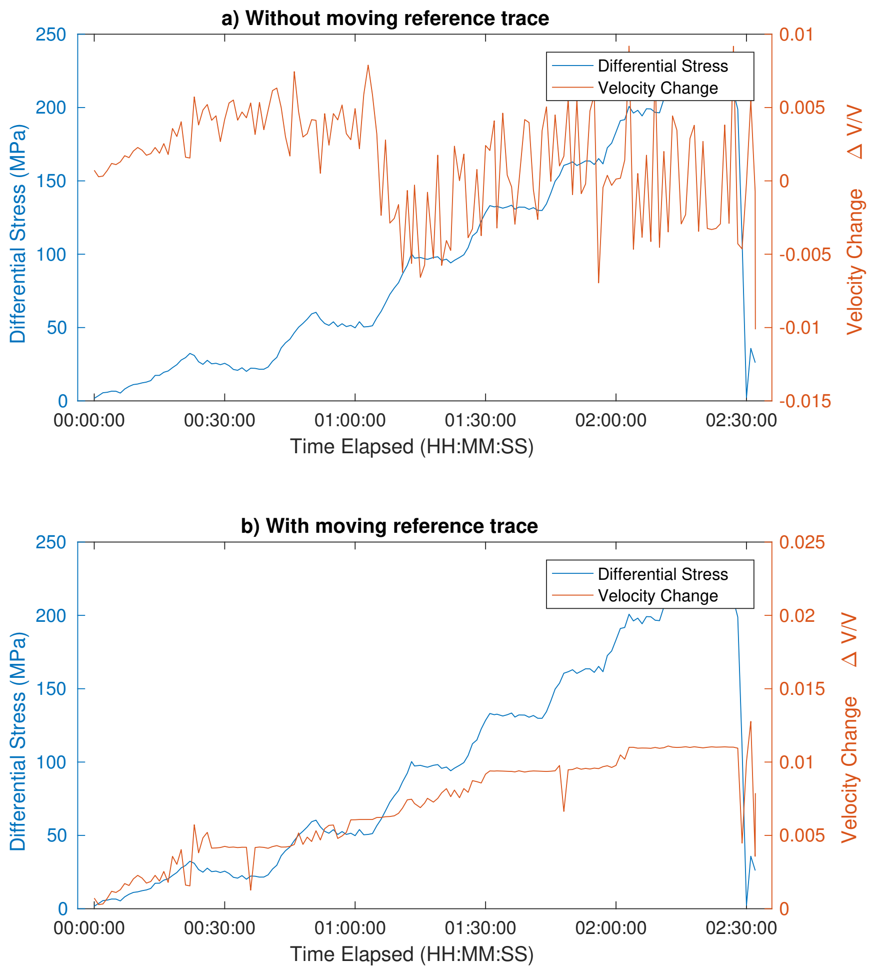

Figure 10: Velocity change $\Delta V/V$ measured by CWI (red lines) for a finely laminated carbonate rock during experimental deformation by increasing differential stress (blue lines), with corresponding stress values labelled on the left axes. a) Estimates of velocity change from CWI are calculated using a single reference signal measured prior to deformation. b) Estimates of velocity change from CWI use multiple reference signals. In this case, the reference signal is reassigned every 32 surveys. This ensures that the velocity changes remain within the working range of CWI.

The final example is for implementing the moving reference trace method. The function $mov\_ref\_trace.m$ requires an $N \times N$ matrix of estimates of velocity change. Where the row index indicates the reference trace used for CWI, and the column index indicates the index of the perturbed signal for CWI. This matrix can be generated from the following script:

```
% Take the output of cwi_stretch_vel in a nested for loop
% signals:Nxl matrix, where N is the number of signals and l is the signal length
 for i = 1:N
     for j = i:N % only cells where j>i are required, as matrix is symmetric
         % estimate velocity change using CWI
         epsilon=cwi_stretch_vel(signals(i,:),signals(j,:));
         % write into an NxN matrix
         epsilon_matrix(i,j)=epsilon;
     end
 end
```

An example matrix can be loaded from *mrt_example_data.mat*, which is laboratory data collected during the deformation of a laminated carbonate core sample. The step size k indicates the step size of the moving reference trace, and depends on the rate of change between the signals. Example data is loaded, and moving reference trace function is executed with the following script:

```
% Load epsilon matrix: Experimental deformation of a laminated carbonate
load('example_data/mrt_example_data.mat')
% Step size for moving reference trace
k = 32;
% Use mov_ref_trace.m function
[dv_mrt] = mov_ref_trace(epsilon_matrix,k);
```

The output vector *dv_mrt* is the cumulative velocity change $\Delta V/V$, shown in Figure 10. To apply this method to a different data set, edit the *signals* variable, to be a $N \times l$ matrix, where N is the number if signals, and l is the length of the signals.

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