Pre-trade Transaction Cost Analysis

Modeling Liquidating cost in HongKong Stock Exchange

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REFINITIV

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I-Star(I^*) model

Model Structure:

$$I^* = a_1 \cdot \left(\frac{Q}{ADV}\right)^{a_2} \cdot \sigma^{a_3}$$

$$MI = b_1 \cdot I^* \cdot POV^{a_4} + (1 - b_1) \cdot I^*$$

$$TR = \sigma \cdot \sqrt{\frac{1}{3} \cdot \frac{1}{250} \cdot \frac{Q}{ADV} \frac{1 - POV}{POV}} \cdot 10^4$$

The variables of the model are:

- I^* is the instantaneous impact
- $POV = Q/(Q + V)$ is the percentage of volume trading rate, where Q is the net imbalance, V is the expected volume excluding the order imbalance, $Q + V$ is the total number of shares that is the expected to trade in the market
- b_1 is the temporary impact parameter
- a_1, a_2, a_3, a_4 are the model parameters

Volume Profiling

- Step 1 : Determine the lookback for ADV or MDV
  - Compute the ADV and MDV simple forecast measure for various look-back periods, i.e. $t = 1, 2, \ldots 30$
  - Compute the percentage error between the actual volume on the day and simple forecast measure
  - Calculate the standard deviation of the error term for each stock over the sample period
  - Calculate the average standard deviation across all stocks in the sample
  - Repeat the analysis for look-back periods from 1 to 30 days
  - Plot the average standard deveiation across stocks for each day(1 to 30)
  - Based on the plots, one can decide whether to go with ADV or MDV? One can also decide the lookback period to consider
- Step 2: Estimate the *Day of the week*(t) parameter
  - For each stock, compute the percentage of volume traded on the day compared to the average volume in the week
  - Exclude the special event days that are historically associated with higher traded volume(ex: month end)
  - Compute the average percentage traded on each day across all stocks in the sample(Use atleast one year of trading data)

Volume Forecasting

- Step 3 : Estimate ARMA parameters

$$\hat{V}(t) = \alpha + \overline{V}_t(n) \cdot \text{Day of the week}(t) + \hat{\beta} \cdot e(t-1)$$

where $\hat{V}(t)$ is $n-$day ADV or MDV, and $e(t-1)$ is the previous day's volume forecast error.

  - Estimate volume for the day based on the $n-$day ADV or MDV
  - Compute the forecast error term as the difference between the actual volume on the day and the estimated volume

$$\varepsilon(t) = V(t) - (\overline{V}_t(n) \cdot \text{Day of the week}(t))$$

  - Run a regression of the error term on its one-day lagged term

$$\varepsilon(t) = \alpha + \beta\varepsilon(t-1)$$

  - Compute the slope for large and small cap stocks
  - In the ARMA model, one needs to include all days. This means that it is better to use a dummy variable for special event days like month ends
  - Better to perform ARMA across all stocks in each of the market cap category

Training Data : Q, Size and Cost

- Consider 30 days data across N stocks for various trading sessions
- Sort the data in the ascending order by time
- Buy initiated trades are those trades that occurred on an up tic or zero up tic
- Sell initiated trades are those trades that occurred on a down tic or zero-down tic
- Trades are not designated as buy-initiated or sell-initiated until after the first price change

$$Q = \left| \sum \text{Buy Volume} - \sum \text{Sell Volume} \right|$$

- Size is expressed as a percentage of ADV

$$\text{Size} = \frac{Q}{ADV}$$

- POV, a proxy for trading strategy

$$\text{POV} = \frac{Q}{V(t)}$$

- Cost

$$Cost = \ln\left(\frac{VWAP}{P_0}\right) \cdot Side \cdot 10^4 bps$$

Parameter Estimation

- Filter the data to include only those data points with

$$\text{Daily Volume} \leq 3 \cdot ADV$$

- Remove all points whose log price change is more than 4 deviations away

$$\frac{-4\sigma}{\sqrt{250}} \leq \text{log price change} \leq \frac{-4\sigma}{\sqrt{250}}$$

- Categorize the points in the various combinations of Size, Volatility and POV rate
- Estimate parameters for the following categories
  - Large Cap - Buy
  - Large Cap - Sell
  - Small cap - Buy
  - Small cap - Sell
- Perform Error analysis to check for parameter stability

Risk Aversion parameters

Optimization Problem:

$$\min\left(MI + \lambda TR\right)$$

where

$$I^* = a_1 \cdot \left(\frac{Q}{ADV}\right)^{a_2} \cdot \sigma^{a_3}$$

$$MI = b_1 \cdot I^* \cdot POV^{a_4} + (1 - b_1) \cdot I^*$$

$$TR = \sigma \cdot \sqrt{\frac{1}{3} \cdot \frac{1}{250} \cdot \frac{X}{ADV} \frac{1 - POV}{POV}} \cdot 10^4$$

- Formulate the problem as an optimization problem
- Solve the optimization problem and estimate ETF(Efficient Trading frontier)
- Shorlist λ values for various risk aversion profiles

Model Input & Output

Model Input

- Stock symbol, Buy or Sell
- Number of shares in the transaction
- Execution Type
  - Percentage of Volume
    - Trade time window $(\%POV, t_{start})$ OR $(\%POV, t_{end})$ OR (t_{start}, t_{end})
  - VWAP
    - Trade time window: (t_{start}, t_{end})
  - Risk Aversion
    - Trade time window: (t_{start}, t_{end})
    - Risk aversion $\in$ { very low, low, normal, high, very high }
- Market Data Input
  - Last Trade Price , Average Daily Volume
  - Expected Market volume over the trade time window
  - Volatility of the stock
  - Calibrated Model parameters a_1, a_2, a_3, a_4, b_1

Model Output

- Trade time, Trading cost(bps, \$, \$/share), Timing Risk(bps, \$, \$/share)

Q&A