Introduction to Programming using R

Organizational Matters

Lecturers:

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Timetable:

- Tuesday Thursday, 9am 5pm; Friday, 9am 3pm
- Morning: Lectures and presenation of solutions
- Lunchbreak: 12-1pm
- Afternoon: Supervised learning

Preliminaries

Before starting the class make sure to install the required software. We will be using the programming language R (R Core Team 2019) and the development environment RStudio (RStudio Team 2015). We recommend first installing R and then RStudio (desktop).¹

- R: https://cran.rstudio.com
- RStudio: https://rstudio.com/products/rstudio/download

Introduction

"[...] computers and mathematics are like beer and potato chips: two fine tastes that are best enjoyed together. Mathematics provides the foundations of our models and of the algorithms we use to solve them. Computers are the engines that run these algorithms." – (Stachurski 2009)

In this section we will present the very basics of R. We will go through some arithmetic, variables, special numerical objects, comments and data types.²

Arithmetic

- 1 + 1
- [1] 2
- 1 1

¹ In case you have problems with the installation process press here or here.

² All statements are typed into the R console and the results are displayed after [1].

[1] 0

Decimals:3

- 2.5 * 4
- [1] 10
- 1 / 3
- [1] 0.3333333
- 2 ^ 2 ^ 3
- [1] 256
- 2 ** 2 ** 3
- [1] 256

Parentheses:4

- (2 ** 2) ** 3
- [1] 64

Variables

- x <- 10 + 5
- Х
- [1] 15
- x <- x + x
- X
- [1] 30
- x ** 2
- [1] 900
- y <- x
- [1] 30

³ Note that the decimal mark is denoted by a dot (.) and not a comma (,).

⁴ If in doubt use parentheses to ensure that R will compute the correct expression.

Special Numerical Objects

Constants, infinity and NaNs (Not a Number):

рi

[1] 3.141593

1/0

[1] Inf

0/0

[1] NaN

Data Types

```
• numeric: x <- 1.25
```

- integer: x <- 1L
- character: x <- "this_works"⁵
- logical: x <- TRUE, y <- FALSE
- complex: x <- 1 + 2i

Data Structures

In this section we consider the most common data structures used in R. This includes

- list
- (atomic) vector
- matrix

[1] 1.25

• data.frame

Lists

Lists are objects which can contain different objects of different data types.6,7

```
x <- list(1, 1.25, "this works?")
[[1]]
[1] 1
[[2]]
```

⁵ This data type is also known as a

⁶ list is a (built-in) function which takes as argument multiple objects and combines them to a list. See section function.print is a (built-in) function which prints its argument on the console.

⁷ length is a (built-in) function which returns the length of its argument.

```
[[3]]
[1] "this works?"
x < - list(x, 1 + 2i)
[[1]]
[[1]][[1]]
[1] 1
[[1]][[2]]
[1] 1.25
[[1]][[3]]
[1] "this works?"
[[2]]
[1] 1+2i
length(x)
[1] 2
```

Vectors

Vectors are similar to lists in that they can contain multiple objects, however, any vector can contain only objects of one data type.⁸

```
x < -c(1, 2, 3)
Х
[1] 1 2 3
x <- c("this", "actually", "works")</pre>
[1] "this"
            "actually" "works"
x <- c("wait?", 1)
Х
[1] "wait?" "1"
length(x)
[1] 2
```

⁸ c is a (built-in) function which takes as argument multiple objects of the same data type and combines them to a vector. c stands for combine.

Indexing of Lists and Vectors

We access elements of lists and vectors via their index. If x is a list (or vector) we get the i-th element as x[i]. Note that for a list (or vector) of length n we can of course only ask for elements 1 to n, otherwise R returns NA which stands for Not Available. If we supply a vector of indices we can access more than one element, i.e. x[c(1,3, 5)] will return the first, third and fifth element of x. Examples:10

```
x < -c(2, 4, 6, 8, 10)
x[1]
[1] 2
x[2]
[1] 4
x[6]
[1] NA
x[6] < -0
[1] 2 4 6 8 10 0
x[c(1, 3, 5)]
[1] 2 6 10
x[3] < 100
Х
[1]
        4 100
     2
                 8 10
                         0
x[c(2, 4)] < -100
Х
[1]
      2 -100 100 -100
                         10
                               0
x[-1]
[1] -100 100 -100
                     10
                          0
x[-c(1, 2)]
[1] 100 -100
               10
```

- ⁹ This is slightly different to many other programming languages which start indexing at 0 instead of 1.
- ¹⁰ Note the difference between y[1] and y[[1]] for lists.

```
y <- list(1, 1.25, "this works?")</pre>
y[1]
[[1]]
[1] 1
y[[1]]
[1] 1
Useful commands:11,12
x <- 1:10
Х
[1] 1 2 3 4 5 6 7 8 9 10
x \leftarrow seq(from=1, to=10, by=2)
Х
[1] 1 3 5 7 9
x <- seq(from=0, to=1, length.out=10)</pre>
round(x, digits = 2)
 [1] 0.00 0.11 0.22 0.33 0.44 0.56 0.67 0.78 0.89 1.00
Calculating with Vectors
x < -1:10
Х
[1] 1 2 3 4 5 6 7 8 9 10
y < - -5:4
У
 [1] -5 -4 -3 -2 -1 0 1 2 3 4
x + y
[1] -4 -2 0 2 4 6 8 10 12 14
x * y
 [1] -5 -8 -9 -8 -5 0 7 16 27 40
round(x ** y, digits = 2)
                  0.06
                                    0.06
 [1]
        1.00
                           0.04
                                             0.20
                                                      1.00
                                                               7.00
```

- 11 seq is a (built-in) function which produces sequences from a specified number to another; with the extra argument by one can specify the increment; with the extra argument length.out one can specify the desired length of the sequence.
- 12 A quick way to create sequences which increment by one is by using the syntax a:b to create the sequence (vector) c(a, a + 1, ..., b - 1, b).

729.00 10000.00 64.00

```
2 * x
[1] 2 4 6 8 10 12 14 16 18 20
10 + x
[1] 11 12 13 14 15 16 17 18 19 20
x ** 2
[1] 1 4 9 16 25 36 49 64 81 100
Recycling:
x < -1:4
y < -c(1, 5, 10)
x + y
Warning in x + y: longer object length is not a multiple of shorter object length
[1] 2 7 13 5
(Some) useful functions:13
x < -5:5
Х
[1] -5 -4 -3 -2 -1 0 1 2 3 4 5
sum(x)
[1] 0
mean(x)
[1] 0
sd(x)
[1] 3.316625
var(x)
[1] 11
cumsum(x)
```

[1] -5 -9 -12 -14 -15 -15 -14 -12 -9 -5 0

13 sum is a (built-in) function which sums all elements of its argument (also works on matrices). mean computes the mean of its argument, sd the (unbiased) standard deviation, var the variance and cumsum the cumulative sum.

Matrices

Matrices represent two dimensional arrays which works similar to vectors in that matrices can only contain objects of a single data type. To create a matrix we need to know how many rows and columns it should have and what data it should contain.14

```
data <- 1:9
rows <- 3
cols <- 3
x <- matrix(data, rows, cols)</pre>
Х
     [,1] [,2] [,3]
[1,]
         1
              4
                    7
         2
[2,]
              5
                    8
[3,]
         3
                    9
y <- matrix(data, rows, cols, byrow=TRUE)</pre>
     [,1] [,2] [,3]
              2
                    3
[1,]
         1
[2,]
         4
              5
                    6
[3,]
         7
              8
                    9
dim(x)
[1] 3 3
nrow(x)
[1] 3
ncol(x)
[1] 3
```

14 Note that 1:x is equivalent to c(1,2,...,x). Also note matrix(data, rows, cols) is not equal to matrix(data, cols, rows); if you do not know which argument comes when, simply ask R for help: ?matrix. (This works for any R function, just type ?function_name).

Combining Vectors and Matrices

When computing different intermediate results it is often useful to combine them to get an end result.15

```
x <- matrix(1:9, 3)
     [,1] [,2] [,3]
[1,]
        1
        2
              5
[2,]
                   8
[3,]
        3
              6
                   9
```

¹⁵ The (built-in) function rbind takes two matrices (or data frames) as input and stacks them on top of each other (on the rows). Similarly, cbind, stacks the two arrays next to each other (on the columns).

```
y <- matrix(1:6, 2)</pre>
У
      [,1] [,2] [,3]
[1,]
         1
              3
[2,]
         2
                    6
z < - rbind(x, y)
      [,1] [,2] [,3]
[1,]
                    7
[2,]
         2
              5
                    8
[3,]
                    9
         3
              6
[4,]
              3
                    5
         1
[5,]
         2
                    6
x < - cbind(1:3, 4:6)
      [,1] [,2]
[1,]
         1
              5
[2,]
         2
[3,]
         3
              6
(Some) useful functions:16
m <- matrix(1:9, 3)</pre>
      [,1] [,2] [,3]
[1,]
         1
                    7
         2
              5
                    8
[2,]
[3,]
         3
                    9
rowSums(m)
[1] 12 15 18
colSums(m)
[1] 6 15 24
rowMeans(m)
[1] 4 5 6
```

In more general settings we might wish to apply an arbitrary function to the rows or columns of a matrix. We can do this with the function apply.¹⁷ Example:

 $^{^{\}rm 16}$ rowSums computes the sum of each row of a matrix (or data frame) and returns the resulting vector. colSums , rowMean and colMeans work analogously.

¹⁷ apply(X, MARGIN, FUN), X = matrix of interest, MARGIN = 1 to apply the function over the rows and 2 to apply the function over the columns, FUN = the function of interest.

```
m \leftarrow matrix(1:9, 3)
apply(m, 1, sd)
[1] 3 3 3
apply(m, 2, sd)
[1] 1 1 1
Matrix algebra <sup>18</sup>
X <- matrix(1:9, 3)</pre>
y < -1:1
X * y
      [,1] [,2] [,3]
[1,]
        - 1
              -4
                    - 7
[2,]
         0
               0
                     0
[3,]
         3
               6
                     9
X %*% y
      [,1]
[1,]
[2,]
         6
         6
[3,]
X * X
      [,1] [,2] [,3]
[1,]
         1
              16
                    49
[2,]
         4
              25
                    64
[3,]
         9
              36
                    81
X %*% X
      [,1] [,2] [,3]
[1,]
        30
              66
                  102
[2,]
        36
              81
                  126
[3,]
        42
              96
                 150
  (Some) useful functions:19
t(X)
      [,1] [,2] [,3]
```

[1,]

[2,]

[3,]

1

4

7

5

8

3

6

9

¹⁸ Note the difference between X * y and X %*% y; the first multiples the ith element of y onto the ith row of X and the second computes the regular matrix product known from linear algebar.

¹⁹ t computes the inverse of its argument, which can be either a matrix or a data frame. diag returns the diagonal entries of its argument. solve can be used to either solve a linear system of equations or compute the inverse of its argument: if we provide solve with one matrix (or data frame) then it returns the inverse; if we supply a matrix (or data frame) and a vector, solve returns the solution to the system of linear equations. That is, $solve(X) = X^{-1}$ and solve(X, y) = b with Xb = y (if the system has a solution).

```
diag(X)
[1] 1 5 9
A \leftarrow matrix(c(1, 10, -2, 3), 2)
     [,1] [,2]
[1,]
     1 -2
[2,] 10
             3
solve(A)
           [,1]
                      [,2]
[1,] 0.1304348 0.08695652
[2,] -0.4347826 0.04347826
b < -c(-1, 1)
solve(A, b)
[1] -0.04347826 0.47826087
```

Data Frames

Data frames represent data sets. The difference to matrices is that different columns can have different data types. Note that there are many different ways of creating a data frame.

```
x <- c("Micheal", "Robin", "Jonah")</pre>
y < -c(1.0, 1.3, 3.0)
z <- c("a", "b", "c")
df <- data.frame(name=x, grades=y, type=z)</pre>
df
     name grades type
1 Micheal
             1.0
    Robin
             1.3
                     b
3
    Jonah
             3.0
                     С
m <- matrix(1:9, 3)</pre>
df <- as.data.frame(m)</pre>
df
  V1 V2 V3
1 1 4 7
2 2 5 8
3 3 6 9
```

The iris data set:20

head(iris)

Sepal.Length Sepal.Width Petal.Length Petal.Width Species 1.4 1 5.1 3.5 0.2 setosa 2 4.9 3.0 1.4 0.2 setosa 3 4.7 3.2 1.3 0.2 setosa 4 4.6 3.1 1.5 0.2 setosa 5 5.0 3.6 1.4 0.2 setosa 6 5.4 3.9 1.7 0.4 setosa

²⁰ The function head returns the first few rows of a data frame, which can be useful to have a quick glance at a data frame. str returns the internal structure of its argument and is particularly useful for data frames to output the key information in a data set.

```
str(iris)
```

```
150 obs. of 5 variables:
'data.frame':
$ Sepal.Length: num 5.1 4.9 4.7 4.6 5 5.4 4.6 5 4.4 4.9 ...
$ Sepal.Width : num 3.5 3 3.2 3.1 3.6 3.9 3.4 3.4 2.9 3.1 ...
$ Petal.Length: num 1.4 1.4 1.3 1.5 1.4 1.7 1.4 1.5 1.4 1.5 ...
$ Petal.Width : num    0.2    0.2    0.2    0.2    0.4    0.3    0.2    0.2    0.1    ...
                : Factor w/ 3 levels "setosa", "versicolor", ...: 1 1 1 1 1 1 1 1 1 1 1 ...
$ Species
```

Indexing of Matrices and Data Frames

Matrices and data frames constitute two dimensional objects, this means we can ask for submatrices, columns, rows or individual elements.

```
m <- matrix(1:9, 3)</pre>
m
      [,1] [,2] [,3]
[1,]
         1
               4
                    7
[2,]
         2
              5
                    8
[3,]
         3
              6
                    9
m[1, 1]
[1] 1
m[1,]
[1] 1 4 7
m[, 1]
[1] 1 2 3
m[c(1, 2), c(2, 3)]
```

```
[,1] [,2]
[1,]
              7
[2,]
         5
              8
```

When dealing with data frames we can also access the columns by their respective names.21

```
df <- data.frame(name=c("Thomas", "Susan"), grade=c(1, 2))</pre>
    name grade
1 Thomas
              1
2 Susan
             2
df$name
[1] "Thomas" "Susan"
df[["grade"]]
[1] 1 2
df["grade"]
  grade
      1
2
      2
```

Digression on asserting data types

WHEN BUILDING programs which handle data and objects that are unknown while developing the code it is often necessary to check of what type they are. Say we get an object (a variable) x from somewhere and we need to evaluate if its a number or data frame; and if it is a number, is it an integer or a real number. For these questions are supplies the many functions of the type is.vector, is.matrix, is.integer. We will not list them all and only provide a small example.

```
x <- 1:10
is.integer(x)
[1] TRUE
is.numeric(x)
[1] TRUE
```

²¹ To access a column of a data frame by name use df\$column_name. Note the different results of df[["grade"]] and df["grade"].

```
is.vector(x)
[1] TRUE
is.data.frame(x)
[1] FALSE
is.function(x)
[1] FALSE
```

Logical Operators

In this section we consider logical operators which form the direct equivalent to logical operators in mathematics. We first note that we can induce boolean values by comparison via relations (<, >, <=, >=) or (in)equalities (==, !=). On boolean values we may use logical operators as and (&), or (|), but also quantifier as \exists (any) and \forall (all).

```
x <- 1:10
x < 5
[1] TRUE TRUE TRUE TRUE FALSE FALSE FALSE FALSE FALSE
sum(x < 5)
[1] 4
x[x < 5]
[1] 1 2 3 4
x[!(x < 5)]
[1] 5 6 7 8 9 10
x[(x < 3) | (x > 7)]
[1] 1 2 8 9 10
x[(x < 8) & (x > 3)]
[1] 4 5 6 7
any(x < 8)
[1] TRUE
```

```
all(x < 8)
[1] FALSE
```

Often we are interested in the (indices of the) elements of a vector (matrix) that fulfill a certain condition.

```
x < -c(3, 2, -100, 400)
which(x > 100)
[1] 4
```

Conditional Expressions

IN MANY SCENARIOS our decisions depend on the specific state of the situation. For example, if it rains we will take the umbrella with us. Or a little more complex. If it rains we will take the umbrella, otherwise, if we fixed the flat bike tires already we will go by bike. (We illustrate the a fictional conditional decision tree on the blackboard.) This brings us to conditional expressions.

```
if
x <- 10
if (x < 10) {
  print("x is smaller than 10.")
}
else
x <- 10
if (x < 10) {
  print("x is smaller than 10.")
} else{
  print("x is *not* smaller than 10.")
}
[1] "x is *not* smaller than 10."
else if
x <- 10
if (x < 10) {
  print("x is smaller than 10.")
} else if (x > 0) {
  print("x is between 0 and 10.")
```

```
} else {
  print("x is either smaller than 0 or bigger than 10.")
}
[1] "x is between 0 and 10."
```

Short digression into User Input

Sometimes we want to write programs which work in many different scenarios that can be specified by the user of the program.^{22,23}

```
cat("Please choose which type of regression should be run:\n")
x <- readline(prompt="Linear regression (1); Polynomial regression argument an R object and tries to coerce
x <- as.integer(x)</pre>
if (x == 1) {
  print("Okay lets do linear regression!")
} else if (x == 2) {
  print("Oh no I hate polynomial regression :(")
  print("There were only two options what did you do?")
```

Control Flow Statements

When working on nearly any project we often find ourselves repeating simple tasks over and over again. If this happens with tasks that cannot be managed on a computer we hire research assistants; however, if it can be done on a computer there are cheaper ways.

For Loops

Let's say we want to create a list with 10 entries and the 1th entry is a matrix of dimension ixi filled with numbers 1 to i^2 . This can be achieved very easily with a for loop.

```
matrices <- list()</pre>
for (i in 1:10) {
  imatrix <- matrix(1:(i ** 2), nrow=i)</pre>
  matrices[[i]] <- imatrix</pre>
}
matrices[[5]]
```

- ²² Note that the (built-in) function readline reads input from the user in the R console and stores it as a string.
- the input to an integer if possible; For example the string "1" can be coerced to a 1 but the string "text" cannot.

```
[,1] [,2] [,3] [,4] [,5]
[1,]
         1
              6
                   11
                         16
                              21
         2
              7
[2,]
                   12
                         17
                              22
[3,]
         3
              8
                   13
                         18
                              23
[4,]
              9
                   14
                         19
                               24
[5,]
         5
             10
                   15
                         20
                              25
```

Short digression into Monte Carlo simulation

Say we have two uniform random variables on [0,1], i.e. $X,Y \sim$ $\mathcal{U}[0,1]$. And say we want to estimate $\mathbb{P}(X+Y\in[0.75,1.25])$ without doing any analytical mathematics. One solution to problems of this kind are so called Monte Carlo estimates, in which we simulate (in this case) two uniform random variables for many many times and each time we simply check if the sum of the realizations fulfills the statement. The frequency of times when the statement was fulfilled then approximates the probability.^{24,25}

```
count <- 0
nsim <- 10000
for (i in 1:nsim) {
  x <- runif(1, min=0, max=1)</pre>
  y <- runif(1, min=0, max=1)</pre>
  z \leftarrow x + y
  if (z \ge 0.75 \& z \le 1.25) {
    count <- count + 1</pre>
  }
}
count / nsim # analytical solution = 7/6 = 0.4375
[1] 0.4448
```

While Loops

For loops are very useful if we know exactly how many times we need to execute some statement. If we do not know the number of repetitions before starting the loop we can use while loops. Cherry picking results:

Once we introduced linear models and ordinary least squares regression we will show a simple example on how to cherry pick your data such that you can claim statistical significance even if there is none.

Example:

²⁴ In the latter chapters we will consider the function runif in more detail; for here only note that runif(1, 0, 1) evaluates to a realization of a uniform random variable on [0,1].

²⁵ Note the use of # which tells R to ignore the following statement; These are called comments and should be used to clarify ones code.

```
userinput <- NULL
while(is.null(userinput)) {
  input <- readline("Type in a number between 0 and 10. \n")</pre>
  input <- as.integer(input)</pre>
  if (is.numeric(input)) {
    if (input >= 0 \&\& input <= 10) {
      userinput <- input
    }
  }
}
userinput
```

Functions

FUNCTIONS ARE arguably the most important building block when writing large programs. We have already seen the use of many (builtin) functions. Functions, in general, allow us to use a piece of code multiple times in a program without repeating all of the code at every instance.

A normal example

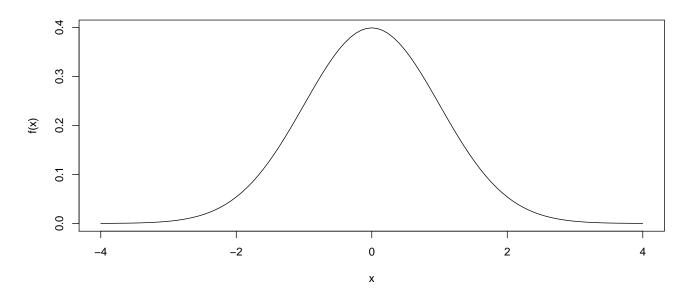
Say we need to compute the value of a normal density with mean mu and standard deviation sigma at some point x. In case we need to compute this value for many different means, variances or points we can save on time (and erros) by implementing a function once.^{26,27}

```
normaldensity <- function(x, mu, sigma) {</pre>
  constant <-1 / sqrt(2 * pi * sigma ** 2)
  exponential <-\exp(-(x - mu) ** 2 / (2 * sigma ** 2))
  return(constant * exponential)
}
normaldensity(x=0, mu=0, sigma=1)
[1] 0.3989423
normaldensity(x=1, mu=0, sigma=1)
[1] 0.2419707
standardnormaldensity <- function(x) {</pre>
  normaldensity(x, mu=0, sigma=1)
```

- ²⁶ Note the use of special (built-in) mathematical functions sgrt and exp, which compute the square root and exponential of their arguments, respectively.
- ²⁷ The (built-in) function curve displays the graph of a (mathematical) function.

}

curve(standardnormaldensity, from=-4, to=4, xlab="x", ylab="f(x)")



Recursive functions

The Fibonacci sequence is defined by the following (recursive) func-

$$f(n) = \begin{cases} 0, n = 0 \\ 1, n = 1 \\ f(n-1) + f(n-2), n > 1. \end{cases}$$

We can implement this function easily using an R function.²⁸

```
fibonacci <- function(n) {</pre>
  if (n == 0) {
    return(0)
  } else if (n == 1) {
    return(1)
  } else {
    return(fibonacci(n - 1) + fibonacci(n - 2))
  }
}
n <- 1:10
fib <- sapply(n, fibonacci)</pre>
rbind(n, fib)
```

²⁸ The (built-in) function sapply applies a function to each element of its first argument, which is typically a list or a vector.

```
[,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
n
                                   6
                                                        10
       1
                        3
                             5
                                       13
                                                  34
                                                        55
fib
                                            21
```

Importing and Exporting Data

This section shows you how to import data from different file types into R. Before we start, some remarks are in order.

- Variables should be kept in columns, observations in rows
- Missing values should be coded consistently (e.g. NA)
- Variable names must not begin with a number and must not contain #, % or spaces
- Don't ever replace the source file
- Check whether whether reading was successful before (!) starting your analysis

Importing .txt, .csv, and .dta files

For many file types, importing is facilitated by base-R functions. For example, importing a .txt file can be achieved with

read.table(file, header = FALSE, sep = "", dec = ".",...), where file is the location of the file to be imported²⁹, header indicates whether the first line of the file contains variable names, sep is the field separator character, and dec is the character used in the file for decimal points. Similarly, if your data comes in .csv format you would go for

read.csv(file, header = TRUE, sep = ",", dec = ".",...). 30 R also has an own file type, .RData. These files can be read using load(file)

As an example, let us look at the dataset that can be found in the GitHub repo of this course.

```
<sup>29</sup> Note that standard backslashes
(the Windows default) do not work
and need to be replaced by either
forwardslashes or double back-
slashes, i.e. "../data/mtcars.txt"
or "..\\data\\mtcars.txt" instead of
"..\data\mtcars.txt")
```

³⁰ R can also deal with the standard file types from Stata, SPSS, and SAS, among many others. Reading those might involve commands from additional R-packages.

```
dat <- read.table("../data/mtcars.txt", header = TRUE, sep = "")</pre>
head(dat)
```

	mpg	cyl	disp	hp	drat	wt	qsec	٧s	am	gear	carb
Mazda RX4	21.0	6	160	110	3.90	2.620	16.46	0	1	4	4
Mazda RX4 Wag	21.0	6	160	110	3.90	2.875	17.02	0	1	4	4
Datsun 710	22.8	4	108	93	3.85	2.320	18.61	1	1	4	1
Hornet 4 Drive	21.4	6	258	110	3.08	3.215	19.44	1	0	3	1
Hornet Sportabout	18.7	8	360	175	3.15	3.440	17.02	0	0	3	2
Valiant	18.1	6	225	105	2.76	3.460	20.22	1	0	3	1

In R-Studio you might want to use View(dat) to get an overview.

Exporting

Similar to the read-commands, you can save objects using the writecommands. For instance, we might save a dataframe in our workspace as a .csv using

```
write.csv(x, file = "",...),
```

where x is the object to be written and file is path where the object ought to be saved. If you would like to save the object as a .RData file, the syntax is

```
save(...),
```

where ... are the names of the objects to be saved.

To save the object in our workspace to a .csv, we execute the following command.

```
write.csv(dat, file = "../data/mtcars.csv")
```

Plotting Data

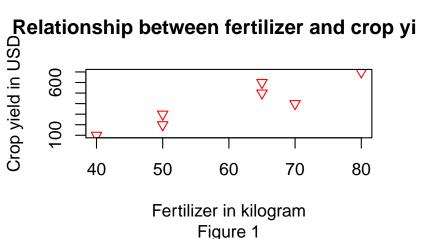
PLOTS can be powerful tool to summarize data succinctly. This section provides a short overview of plotting capabilities of base R.31

Generally, base-R distinguishes between two types of graphics commands, high- and low-level commands. While a high-level command creates a plot (and overwrites a previously displayed plot), low-level commands are used to add things to an existing plot. Highlevel commands include, among others, plot, hist, barplot, boxplot, qqnorm, and curve.

As an example, consider the following data, where *Y* is the crop yield of corn and *X* is the amount of fertilizer used at each farm, respectively.

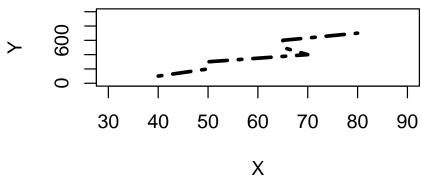
```
31 Advanced users usually create their
plots using the ggplot2 package de-
veloped by Hadley Wickham. If
you are interested, have a look here:
https://ggplot2.tidyverse.org/
```

```
Y <- c(100, 200, 300, 400, 500, 600, 700)
X \leftarrow c(40, 50, 50, 70, 65, 65, 80)
plot(X, Y, main = "Relationship between fertilizer and crop yield", sub = "Figure 1",
     xlab = "Fertilizer in kilogram", ylab = "Crop yield in USD",
     pch = 25, col = "red")
```



As is vividly illustrated in the previous example, the plot-command has various optional features. Let us try some more below.

plot(X, Y, type = "l", lwd = 3, lty = 6,
ylim =
$$c(0, 1000)$$
, $xlim = c(30, 90)$)



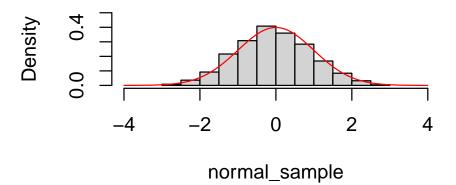
Digression - sampling from probability distributions

R provides pseudo-random sampling from many of the common probability distributions.³² Below we draw 500 realizations from a standard normal distribution. By plotting a histogram of the data, we can obtain at least suggestive evidence that we have indeed drawn from a standard normal distribution. To further corroborate our hypothesis, we add the density of the standard normal distribution to our plot.

³² See ?distribution for an overview.

```
normal_sample <- rnorm(500, mean = 0, sd = 1)</pre>
hist(normal_sample, freq = FALSE, xlim = c(-4, 4), ylim = c(0, 0.5))
curve(dnorm(x, mean = 0, sd = 1), col ="red", add = TRUE)
```

Histogram of normal_sample

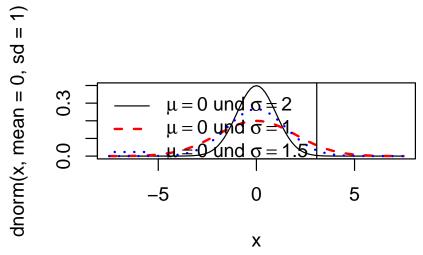


Low-level commands

To modify existing plots, R offers a variety of low-level commands. Some of them and their features are listed below.

- abline(a, b) adds a straight line with intercept a and slope b
- lines(x, y) joins the points of x and y and adds the line to the plot
- points(x, y) similar to lines, but with points
- text(x, y, "Text") adds "Text" at coordinates (x, y)
- legend(x, y, legend,...) adds a legend at coordinates (x, y) using the strings provided in legend

```
curve(dnorm(x, mean = 0, sd = 1), from = -7.5, to = 7.5, lty = 1)
curve(dnorm(x, mean = 0, sd = 2), add = TRUE, col = "red", lwd = 2, lty = 2)
lines(seq(-7.5, 7.5, length.out = 1000),
      dnorm(seq(-7.5, 7.5, length.out = 1000), sd = 1.5), col = "blue", lty = 3, lwd = 2)
legend("topleft",
      c(expression(paste(mu == 0," und ", sigma == 2)),
         expression(paste(mu == 0, " und ", sigma == 1)),
         expression(paste(mu == 0, und sigma == 1.5)),
      lwd = c(1, 2, 2), lty = 1:3, col = c("black", "red", "blue"))
```



To display multiple plots in the same window as a matrix, use par(mfrow = c(x1, x2))

to determine the number of rows (x1) and columns (x2) of your plot matrix. Use dev.off() to reset the setting.

Additionally, you might use RStudio to export your plot to a certain file type.

Additional Packages

So FAR we have been using built-in functions that are predefined by R, or have been writing functions on our own. In practice many functions that are not shipped with base R have already been implemented by someone else and are often available online. In particular we can download so called *packages* which provide a set of functions for a given topic.

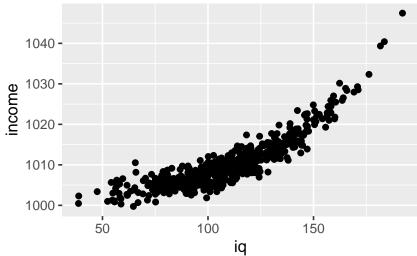
For example we could install the ggplot package, a package for creating fancy plots; or the stargazer package, which helps with creating LATEXtables from data frames.

Installing packages

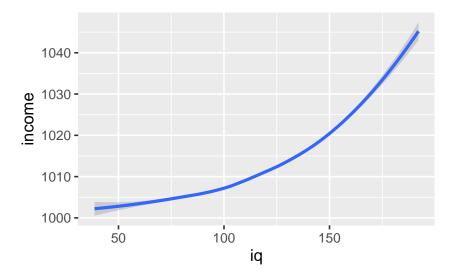
```
install.packages("ggplot2")
Error in install.packages : Updating loaded packages
install.packages("stargazer")
Error in install.packages : Updating loaded packages
```

Load packages

```
library("ggplot2")
library("stargazer")
ggplot2
n <- 500
iq <- rnorm(n, mean=110, sd=25)</pre>
income <- 1000 + \exp(iq / 50) + rnorm(n, sd=2)
df <- data.frame(iq=iq, income=income)</pre>
ggplot2::ggplot(df, aes(x=iq, y=income)) +
  geom_point()
```



```
ggplot2::ggplot(df, aes(x=iq, y=income)) +
  geom_smooth()
```



Statistical Analysis

Linear Regression (Ordinary Least Squares)

Assume we observe data $\{(y_i, X_i) : i = 1, ..., n\}$ with outcomes y_i and covariates X_i . Assume further that we impose a linear model on the data, i.e. we assume that

$$y_i = \beta^\top X_i + \epsilon_i$$

and we want to estimate β using the ordinary least squares method. Simulation:

```
n <- 100 # number of data points
x1 <- runif(n)</pre>
x2 <- rnorm(n)
x3 <- rchisq(n, df=3)
X <- cbind(x1, x2, x3) # covariate matrix
beta <- c(2, 0, -1) # true parameter
eps <- rcauchy(n) # error terms</pre>
y <- X %*% beta + eps # simulated outcomes
linear_model <- lm(y \sim X)
summary(linear_model)
Call:
```

```
lm(formula = y \sim X)
Residuals:
    Min
                                         Max
               10
                    Median
                                 30
-137.720
            0.008
                     1.856
                              3.856
                                      12.831
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)
            1.5224
                         3.5369
                                0.430 0.66784
Xx1
             -0.7663
                         5.2523 -0.146 0.88431
             -0.9042
Xx2
                       1.4992 -0.603 0.54786
Xx3
             -1.7022
                        0.4905 -3.470 0.00078 ***
- - -
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 15.37 on 96 degrees of freedom
Multiple R-squared: 0.1116,
                               Adjusted R-squared: 0.08379
F-statistic: 4.018 on 3 and 96 DF, p-value: 0.009689
Using data frames:
df <- data.frame(income=y, age=x1, edu=x2, nationality=x3)</pre>
linear_model <- lm(income ~ age + edu + I(edu**2), data=df)</pre>
summary(linear_model)
Call:
lm(formula = income \sim age + edu + I(edu^2), data = df)
Residuals:
    Min
               10
                    Median
                                 30
                                         Max
-144.177
            0.055
                    2.575
                              4.931
                                      14.947
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -5.86659
                        3.54771 -1.654
                                           0.101
age
            0.59897
                        5.59137
                                  0.107
                                           0.915
                                           0.979
edu
             0.04329
                        1.63112
                                  0.027
```

Residual standard error: 16.3 on 96 degrees of freedom Multiple R-squared: 0.001154, Adjusted R-squared: -0.03006 F-statistic: 0.03698 on 3 and 96 DF, p-value: 0.9904

0.319

0.751

1.34111

The broom package:

 $I(edu^2)$

0.42751

install.packages("broom")

Error in install.packages : Updating loaded packages

library("broom")

broom::tidy(linear_model)

A tibble: 4 x 5

term	estimate	std.error	${\tt statistic}$	p.value
<chr></chr>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>
1 (Intercept)	-5.87	3.55	-1.65	0.101
2 age	0.599	5.59	0.107	0.915
3 edu	0.0433	1.63	0.0265	0.979
4 I(edu^2)	0.428	1.34	0.319	0.751

The stargazer package:

See live coding (or internet).

R Core Team. 2019. R: A Language and Environment for Statistical Computing. Vienna, Austria: R Foundation for Statistical Computing. https://www.R-project.org/.

RStudio Team. 2015. RStudio: Integrated Development Environment for r. Boston, MA: RStudio, Inc. http://www.rstudio.com/.

Stachurski, John. 2009. Economic Dynamics: Theory and Computation. Vol. 1. MIT Press Books 0262012774. The MIT Press. https:// ideas.repec.org/b/mtp/titles/0262012774.html.