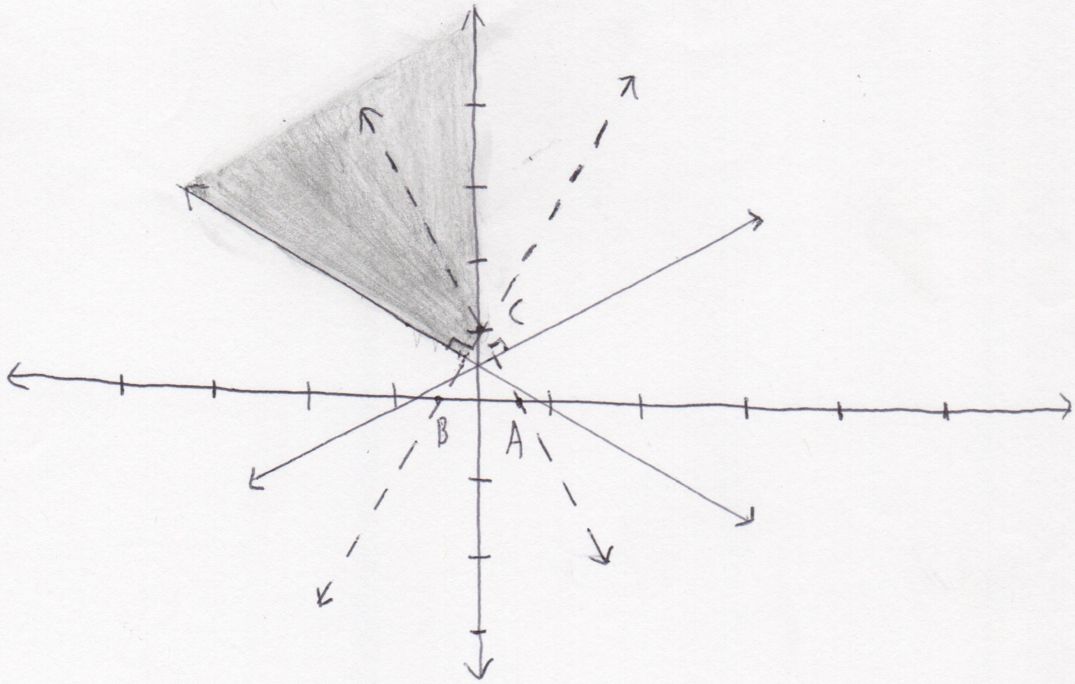
A.



The signal is coming from the North-West [redacted] direction $(-x, +y)$.

B. Since the wavefront is circular and the wave hits C first, the sound source is between the perpendicular bisectors of segments BC and AC. Also, because the sound hits B before A, the source must be to the left of the y-axis. The shaded area above shows the region.

C. Let the coordinate of the source be (x, y). Then,

$$1481\left(2.63474\times10^{-4}\right)=\sqrt{(x+0.5)^2+(y)^2}-\sqrt{(x)^2+(y-1)^2}$$

by the distance formula. Notice that this equation is homogenous, and thus, not a parabola. [redacted] Squaring twice, we can easily see that the x^2 and y^2 terms are subtracted. This is [redacted] therefore a hyperbola.

D. The formulas for TDoA between hydrophones C, B and C, A are:

$$1481(2.63474\times10^{-4}) = \sqrt{(x+0.5)^2+y^2} - \sqrt{x^2+(y-1)^2}$$

and

$$1481(7.07023\times10^{-4}) = \sqrt{(x-0.5)^2+y^2} - \sqrt{x^2+(y-1)^2}$$

respectively.

To make calculations simpler, we can temporarily shift the coordinate system so the new origin is at $(0,1)$. Then, hydrophone B is at $(-0.5,-1)$ and hydrophone A is at $(0.5,-1)$. We then get:

$$1481(2.63474\times10^{-4}) = \sqrt{(x+0.5)^2+(y+1)^2} - \sqrt{x^2+y^2}$$

and

$$1481(7.07023\times10^{-4}) = \sqrt{(x-0.5)^2+(y+1)^2} - \sqrt{x^2+y^2}$$

To generalize, if hydrophone C is at (x_1,y_1), hydrophone B is at (x_2,y_2), and hydrophone A is at (x_3,y_3), then shifting the coordinate system so hydrophone C is at the origin will cause hydrophone B to be at (x_2-x_1, y_2-y_1) and hydrophone A to be at (x_3-x_1, y_3-y_1).

To solve this system of non-linear equations, ~~an~~ intuitive first step to make is to solve for y.

We want to solve

$$k = \sqrt{(x-a)^2+(y-b)^2} - \sqrt{x^2+y^2} \quad \text{for } y.$$

Squaring, we get

$$k^2 = (x-a)^2+(y-b)^2+x^2+y^2-2\sqrt{x^2+y^2}\,\sqrt{(x-a)^2+(y-b)^2}$$

$$k^2 = x^2+(x-a)^2+y^2+(y-b)^2-2\sqrt{(x^2+y^2)((x-a)^2+(y-b)^2)}$$

Isolating the square root, we get

$$-2\sqrt{((x-a)^2+(y-b)^2)(x^2+y^2)} = k^2-x^2-(x-a)^2-y^2-(y-b)^2$$

Squaring again,

$$4((x-a)^2+(y-b)^2)(x^2+y^2) = 4a^2x^2+4b^2x^2-8ax^3+4x^4-8bx^2+y^2(4a^2+4b^2-8ax+8x^2)-8by^3+4y^4$$

Expanding the LHS and collecting in terms of y,

$$-a^4-2a^2b^2-b^4+2a^2k^2+2b^2k^2-k^4+4a^3x+4ab^2x-4ak^2x-4a^2x^2+4k^2x^2$$
$$+y(4a^2b+4b^3-4bk^2-8abx)+y^2(4k^2-4b^2)=0$$

Dividing by the leading coefficient,

$$\frac{y(4a^2b+4b^3-8abx)}{4k^2-4b^2}+y^2 = -\frac{-a^4-2a^2b^2-b^4+2a^2k^2+2b^2k^2-k^4+4a^3x+4ab^2x-4ak^2x}{4k^2-4b^2}$$

$$+\frac{-4a^2x^2+4k^2x^2}{4k^2-4b^2}$$

Completing the square,

$$\left(\frac{4a^2b+4b^3-4bk^2-8abx}{2(4k^2-4b^2)}+y\right)^2 = \frac{k^2(a^2+b^2-k^2)(a^2+b^2-k^2-4ax+4x^2)}{4(b-k)^2(b+k)^2}$$

Finally, taking the square root and subtracting,

$$y = \pm\frac{1}{2}\sqrt{\frac{k^2(a^2+b^2-k^2)(a^2+b^2-k^2-4ax+4x^2)}{(b-k)^2(b+k)^2}} - \frac{4a^2b+4b^3-4bk^2-8abx}{2(4k^2-4b^2)}$$

[redacted]

Applying this general formula for the hydrophones, we get

$$y = \pm\frac{1}{2}\sqrt{0.9217x^2+0.4609x+0.2531} - (0.589x+0.64725)$$

$$y = \pm\frac{1}{2}\sqrt{72.4486x^2-36.2243x+2.78165} - (5.18561x-0.796403)$$

To solve this, we must approximate the roots of the second equation subtracted from the first. To do this, we can use Newton's Method. Taking the derivative,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{1}{2}\sqrt{\frac{k^2(a^2+b^2-k^2)(a^2+b^2-k^2-4ax+4x^2)}{(b-k)^2(b+k)^2}}\right] - \frac{d}{dx}\left[\frac{4a^2b+4b^3-4bk^2-8abx}{2(4k^2-4b^2)}\right]$$

$$= \left[\frac{-1(a-2x)\sqrt{\frac{k^2(a^2+b^2-k^2)((a-2x)^2+(b-k)(b+k))}{(b^2-k^2)^2}}}{(a-2x)^2+(b-k)(b+k)} - \frac{ab}{(b-k)(b+k)}\right]$$

We now need to draw a line at a point on the curve and keep using the slopes to find better lines.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

To account for the $\pm$, we need 4 functions:

$$(+)-(+),\ (+)-(-),\ (-)-(+),\ (-)-(-)$$

Where (+) or (-) refers to the sign of the square root term.

Starting with $x_0 = 0$, we get

	(+) - (+)	(+) - (-)	(-) - (+)	(-) - (-)
x_0	0	0	0	0
x_1	0.197462	-0.591649	0.258323	-0.810545
x_2	—	-17.4271	—	0.771164
x_3	—	-3.51585	—	0.344281
x_4	—	-3.50348	—	—
	⋮	⋮	⋮	⋮

We see that the answer is $x = -3.50348$. We can plug this value into the equation with y in terms of x to get:

$$(-3.50348,\ 4.00356)$$

E. To generalize, we can hard-code the functions and derivatives into separate functions and iterate the process until the answer is found.