Solusi soal SPL

1. OBE

$$\left[\begin{array}{ccc|c} 2 & -5 & 2 & 7 \\ 1 & 2 & -4 & 3 \\ 3 & -4 & -6 & 5 \end{array}\right] \begin{array}{l} b_1 - 2b_2 \\ \\ b_3 - 3b_2 \end{array} \left[\begin{array}{ccc|c} 0 & -9 & 10 & 1 \\ 1 & 2 & -4 & 3 \\ 0 & -10 & 6 & -4 \end{array}\right] \begin{array}{l} b_1 - b_3 \\ \\ \\ \end{array} \left[\begin{array}{ccc|c} 0 & 1 & 4 & 5 \\ 1 & 2 & -4 & 3 \\ 0 & -10 & 6 & -4 \end{array}\right] \begin{array}{l} \\ \\ b_3 + 10b_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 4 & 5 \\ 1 & 2 & -4 & 3 \\ 0 & 0 & 46 & 46 \end{array}\right]$$

$\to 46z = 46 \to z = 1$

$\to y + 4z = 5 \to y = 5 - 4\cdot 1 = 1$

$\to x + 2y - 4z = 3 \to x = 3 + 4\cdot 1 - 2\cdot 1 = 5$.

Crammer

$$A = \begin{bmatrix} 2 & -5 & 2 \\ 1 & 2 & -4 \\ 3 & -4 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 7 \\ 3 \\ 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -5 & 2 \\ 1 & 2 & -4 \\ 3 & -4 & -6 \end{vmatrix} = -24 + 60 - 8 - 12 - 32 - 30 = -46 \neq 0$$

$$x = \frac{1}{|A|} \begin{vmatrix} 7 & -5 & 2 \\ 3 & 2 & -4 \\ 5 & -4 & -6 \end{vmatrix} = -\frac{1}{46}\left[-84 + 100 - 24 - 20 - 112 - 90\right]$$

$$= -\frac{1}{46}(-230) = 5$$

$$y = \frac{1}{|A|} \begin{vmatrix} 2 & 7 & 2 \\ 1 & 3 & -4 \\ 3 & 5 & -6 \end{vmatrix} = -\frac{1}{46}\left[-36 - 84 + 10 - 18 + 40 - 42\right]$$

$$= -\frac{1}{46}(-46) = 1$$

$$z = \frac{1}{|A|} \begin{vmatrix} 2 & -5 & 7 \\ 1 & 2 & 3 \\ 3 & -4 & 5 \end{vmatrix} = -\frac{1}{46}\left[20 - 45 - 28 - 42 + 24 + 25\right]$$

$$= -\frac{1}{46}(-46) = 1$$

Invers

terlebih dahulu dicari A^{-1} (ini pasti ada karena $|A| \neq 0$)

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A) \quad , \quad \operatorname{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$A_{11} = \begin{vmatrix} 2 & -4 \\ -4 & -6 \end{vmatrix} = -28 \qquad A_{12} = -\begin{vmatrix} 1 & -4 \\ 3 & -6 \end{vmatrix} = -6 \qquad A_{13} = \begin{vmatrix} 1 & 2 \\ 3 & -4 \end{vmatrix} = -10$$

$$A_{21} = -\begin{vmatrix} -5 & 2 \\ -4 & -6 \end{vmatrix} = -38 \qquad A_{22} = \begin{vmatrix} 2 & 2 \\ 3 & -6 \end{vmatrix} = -18 \qquad A_{23} = -\begin{vmatrix} 2 & -5 \\ 3 & -4 \end{vmatrix} = -7$$

$$A_{31} = \begin{vmatrix} -5 & 2 \\ 2 & -4 \end{vmatrix} = 16 \qquad A_{32} = -\begin{vmatrix} 2 & 2 \\ 1 & -4 \end{vmatrix} = 10 \qquad A_{33} = \begin{vmatrix} 2 & -5 \\ 1 & 2 \end{vmatrix} = 9$$

$$\operatorname{adj}(A) = \begin{bmatrix} -28 & -6 & -10 \\ -38 & -18 & -7 \\ 16 & 10 & 9 \end{bmatrix}^T = \begin{bmatrix} -28 & -38 & 16 \\ -6 & -18 & 10 \\ -10 & -7 & 9 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-46} \begin{bmatrix} -28 & -38 & 16 \\ -6 & -18 & 10 \\ -10 & -7 & 9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} B = \frac{1}{-46} \begin{bmatrix} -28 & -38 & 16 \\ -6 & -18 & 10 \\ -10 & -7 & 9 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 5 \end{bmatrix}$$

$$= -\frac{1}{46} \begin{bmatrix} -196 - 114 + 80 \\ -42 - 54 + 50 \\ -70 - 21 + 45 \end{bmatrix} = -\frac{1}{46} \begin{bmatrix} -230 \\ -46 \\ -46 \end{bmatrix}$$

$$x = -\frac{1}{46} \cdot (-230) = 5$$

$$y = -\frac{1}{46} (-46) = 1$$

$$z = -\frac{1}{46} (-46) = 1$$

2. Diperbaiki dulu bentuk SPL nya.

$$3x + y - 2z = 3$$
$$x - 2y - 3z = 1$$
$$2x + 3y + z = 2$$

$$\rightarrow \underset{A}{\begin{bmatrix} 3 & 1 & -2 \\ 1 & -2 & -3 \\ 2 & 3 & 1 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}}$$

OBE

$$\left[\begin{array}{ccc|c} 3 & 1 & -2 & 3 \\ 1 & -2 & -3 & 1 \\ 2 & 3 & 1 & 2 \end{array}\right] \begin{array}{l} b_1 - 3b_2 \\ \\ b_3 - 2b_2 \end{array} \left[\begin{array}{ccc|c} 0 & 7 & 7 & 0 \\ 1 & -2 & -3 & 1 \\ 0 & 7 & 7 & 0 \end{array}\right] b_3 - b_1 \left[\begin{array}{ccc|c} 0 & 7 & 7 & 0 \\ 1 & -2 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \rightarrow 1 \\ \rightarrow 2 \\ \\ \end{array}$$

1 $7y + 7z = 0 \rightarrow y = -z$

2 $x - 2y - 3z = 1 \rightarrow x = 1 + 2y + 3z = 1 + 2(-z) + 3z)$

$= 1 + z$

$z = t$ (untuk suatu $t \in \mathbb{R}$)

Jadi $x = 1 + t$; $y = -t$; $z = t$.

Crammer

$$|A| = \begin{vmatrix} 3 & 1 & -2 \\ 1 & -2 & -3 \\ 2 & 3 & 1 \end{vmatrix} = 0 \quad \text{(cek ya !!)}$$

karena $|A| = 0$, SPL tidak bisa diselesaikan dengan metoda Crammer maupun Invers.

(A^{-1} tidak ada).

3:
$$\underset{A}{\begin{bmatrix} 2 & -3 & 2 & 5 \\ 1 & -1 & 1 & 2 \\ 3 & 2 & 2 & 1 \\ 1 & 1 & -3 & -1 \end{bmatrix}} \underset{X}{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}} = \underset{B}{\begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}}$$

<u>OBE</u>

$$\left[\begin{array}{cccc|c} 2 & -3 & 2 & 5 & 3 \\ 1 & -1 & 1 & 2 & 1 \\ 3 & 2 & 2 & 1 & 0 \\ 1 & 1 & -3 & -1 & 0 \end{array}\right] \begin{array}{l} b_1 - 2b_2 \\ \\ b_3 - 3b_2 \\ b_4 - b_2 \end{array} \left[\begin{array}{cccc|c} 0 & -1 & 0 & 1 & 1 \\ 1 & -1 & 1 & 2 & 1 \\ 0 & 5 & -1 & -5 & -3 \\ 0 & 2 & -4 & -3 & -1 \end{array}\right] \begin{array}{l} \\ \\ b_3 + 5b_1 \\ b_4 + 2b_1 \end{array}$$

$$\left[\begin{array}{cccc|c} 0 & -1 & 0 & 1 & 1 \\ 1 & -1 & 1 & 2 & 1 \\ 0 & 0 & -1 & 0 & 2 \\ 0 & 0 & -4 & -1 & 1 \end{array}\right] \begin{array}{l} \\ \\ \\ b_4 - 4b_3 \end{array} \left[\begin{array}{cccc|c} 0 & -1 & 0 & 1 & 1 \\ 1 & -1 & 1 & 2 & 1 \\ 0 & 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & -1 & -7 \end{array}\right] \begin{array}{l} \to ① \\ \to ② \\ \to -c = 2 \to c = -2 \\ \to -d = -7 \\ \quad d = 7 \end{array}$$

① $-b + d = 1 \to -b = 1 - d = 1 - 7 = -6$
$b = 6$

② $a - b + c + 2d = 1 \to a = 1 + b - c - 2d$
$= 1 + 6 - (-2) - 2(7) = -5$

$a = -5$; $b = 6$; $c = -2$; $d = 7$

<u>crammer</u>

$$|A| = \begin{vmatrix} 2 & -3 & 2 & 5 \\ 1 & -1 & 1 & 2 \\ 3 & 2 & 2 & 1 \\ 1 & 1 & -3 & -1 \end{vmatrix} \begin{array}{l} b_1 - 2b_2 \\ = \\ b_3 - 3b_2 \\ b_4 - b_2 \end{array} \begin{vmatrix} 0 & -1 & 0 & 1 \\ 1 & -1 & 1 & 2 \\ 0 & 5 & -1 & -5 \\ 0 & 2 & -4 & -3 \end{vmatrix} \begin{array}{l} \\ \\ b_3 + 5b_1 \\ b_4 + 2b_1 \end{array}$$

$$= \begin{vmatrix} 0 & -1 & 0 & 1 \\ 1 & -1 & 1 & 2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & -4 & -1 \end{vmatrix} \underset{b_4 - 4b_3}{=} \begin{vmatrix} 0 & -1 & 0 & 1 \\ 1 & -1 & 1 & 2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} \overset{b_1 \leftrightarrow b_2}{=} - \begin{vmatrix} 1 & -1 & 1 & 2 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}$$

$= -\cdot 1 \cdot (-1) \cdot (-1) \cdot (-1)$
$= 1 \neq 0$

$$a = \frac{1}{|A|}\begin{vmatrix} 3 & -3 & 2 & 5 \\ 1 & -1 & 1 & 2 \\ 0 & 2 & 2 & 1 \\ 0 & 1 & -3 & -1 \end{vmatrix} \begin{matrix} b_1 - 3b_2 \\ = \\ b_3 - 2b_3 \end{matrix} \begin{vmatrix} 0 & 0 & -1 & -1 \\ 1 & -1 & 1 & 2 \\ 0 & 0 & 8 & 3 \\ 0 & 1 & -3 & -1 \end{vmatrix} b_3 + 8b_1$$

$$= \begin{vmatrix} 0 & 0 & -1 & -1 \\ 1 & -1 & 1 & 2 \\ 0 & 0 & 0 & -5 \\ 0 & 1 & -3 & -1 \end{vmatrix} b_1 \leftrightarrow b_4$$

$$= - \begin{vmatrix} 0 & 1 & -3 & -1 \\ 1 & -1 & 1 & 2 \\ 0 & 0 & 0 & -5 \\ 0 & 0 & -1 & -1 \end{vmatrix} \begin{matrix} b_1 \leftrightarrow b_2 \\ b_3 \leftrightarrow b_4 \end{matrix}$$

$$= (-1)(-1)(-1) \begin{vmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -5 \end{vmatrix}$$

$$= - \cdot 1 \cdot 1 \cdot (-1) \cdot (-5) = -5$$

$$b = \frac{1}{|A|}\begin{vmatrix} 2 & 3 & 2 & 5 \\ 1 & 1 & 1 & 2 \\ 3 & 0 & 2 & 1 \\ 1 & 0 & -3 & -1 \end{vmatrix} \begin{matrix} b_1 - 2b_2 \\ = \\ b_3 - 3b_2 \\ b_4 - b_2 \end{matrix} \begin{vmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 2 \\ 0 & -3 & -1 & -5 \\ 0 & -1 & -4 & -3 \end{vmatrix} \begin{matrix} \\ \\ b_3 + 3b_1 \\ b_4 + b_1 \end{matrix}$$

$$= \begin{vmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & -4 & -2 \end{vmatrix} \begin{matrix} b_1 \leftrightarrow b_2 \\ \\ \\ b_4 - 4b_3 \end{matrix}$$

$$= - \begin{vmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 6 \end{vmatrix} = - \cdot 1 \cdot 1 \cdot (-1) \cdot 6$$

$$= 6$$

$$c = \frac{1}{|A|}\left|\begin{array}{cccc}2 & -3 & 3 & 5\\ 1 & -1 & 1 & 2\\ 3 & 2 & 0 & 1\\ 1 & 1 & 0 & -1\end{array}\right|\begin{array}{l}b_1-2b_2\\ \\ b_3-3b_2\\ b_4-b_2\end{array} = \left|\begin{array}{cccc}0 & -1 & 1 & 1\\ 1 & -1 & 1 & 2\\ 0 & 5 & -3 & -5\\ 0 & 2 & -1 & -3\end{array}\right|\begin{array}{l}\\ \\ b_3+5b_1\\ b_4+2b_1\end{array}$$

$$= \left|\begin{array}{cccc}0 & -1 & 1 & 1\\ 1 & -1 & 1 & 2\\ 0 & 0 & 2 & 0\\ 0 & 0 & 1 & -1\end{array}\right|\begin{array}{l}b_1 \leftrightarrow b_2\\ \\ \\ 2b_4-b_3\end{array} = -\cdot\frac{1}{2}\left|\begin{array}{cccc}1 & -1 & 1 & 2\\ 0 & -1 & 1 & 1\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & -2\end{array}\right|$$

$$= -\cdot\frac{1}{2}\cdot 1\cdot(-1)\cdot 2\cdot(-2) = -2$$

$$d = \frac{1}{|A|}\left|\begin{array}{cccc}2 & -3 & 2 & 3\\ 1 & -1 & 1 & 1\\ 3 & 2 & 2 & 0\\ 1 & 1 & -3 & 0\end{array}\right|\begin{array}{l}b_1-2b_2\\ \\ b_3-3b_2\\ b_4-b_2\end{array} = \left|\begin{array}{cccc}0 & -1 & 0 & 1\\ 1 & -1 & 1 & 1\\ 0 & 5 & -1 & -3\\ 0 & 2 & -4 & -1\end{array}\right|\begin{array}{l}\\ \\ b_3+5b_1\\ b_2+2b_1\end{array}$$

$$= \left|\begin{array}{cccc}0 & -1 & 0 & 1\\ 1 & -1 & 1 & 1\\ 0 & 0 & -1 & 2\\ 0 & 0 & -4 & 1\end{array}\right|\begin{array}{l}b_1 \leftrightarrow b_2\\ \\ \\ b_4-4b_3\end{array}$$

$$= -\left|\begin{array}{cccc}1 & -1 & 1 & 1\\ 0 & -1 & 0 & 1\\ 0 & 0 & -1 & 2\\ 0 & 0 & 0 & -7\end{array}\right| = -\cdot 1\cdot(-1)\cdot(-1)\cdot(-7)$$

$$= 7$$

<u>Invers</u> Terlebih dahulu dicari A^{-1}

$$A^{-1} = \frac{1}{|A|}\operatorname{adj}(A), \quad \begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix} = A^{-1}\begin{bmatrix}3\\ 1\\ 0\\ 0\end{bmatrix}$$

4. $x + 2y - z = 0$

$2x + 5y + 2z = 0$

$x + 4y + 7z = 0$

$x + 3y + 3z = 0$

$$\begin{bmatrix}1&2&-1\\2&5&2\\1&4&7\\1&3&3\end{bmatrix}\begin{array}{l}\\b_2-2b_1\\b_3-b_1\\b_4-b_1\end{array}\begin{bmatrix}1&2&-1\\0&1&4\\0&2&8\\0&1&4\end{bmatrix}\begin{array}{l}\\ \\b_3-2b_2\\b_4-b_2\end{array}\begin{bmatrix}1&2&-1\\0&1&4\\0&0&0\\0&0&0\end{bmatrix}\begin{array}{l}\to ②\\ \to ①\\ \\ \\ \end{array}$$

① $y + 4z = 0 \rightarrow y = 4z$

② $x + 2y - z = 0 \rightarrow x = -2y + z = -2(4z) + z = -7z$

$z = t$ (suatu $t \in R$)

$x = -7t$; $y = 4t$; $z = t$.

5.
$$\left[\begin{array}{ccc|c}1&2&-1&a\\2&5&4&b\\3&7&4&c\end{array}\right]\begin{array}{l}\\b_2-2b_1\\b_3-3b_1\end{array}\left[\begin{array}{ccc|c}1&2&-1&a\\0&1&6&b-2a\\0&1&7&c-3a\end{array}\right]\begin{array}{l}\\ \\b_3-b_2\end{array}$$

$$\left[\begin{array}{ccc|c}1&2&-1&a\\0&1&6&b-2a\\0&0&1&c-3a-(b-2a)\end{array}\right]$$

karena semua kolom punya satu utama, maka SPL punya solusi tunggal tidak bergantung nilai a, b, c

6.

$$\left[\begin{array}{cccccc|c} 1 & -3 & 2 & 1 & -2 & -2 & 0 \\ 0 & -2 & -1 & 4 & -3 & 0 & -1 \\ -3 & 7 & -7 & 1 & 3 & 6 & -1 \\ 2 & -6 & 4 & 2 & -4 & -4 & 0 \\ -1 & 1 & -3 & 3 & -1 & 2 & -1 \\ 1 & -5 & 1 & 5 & -5 & -2 & -1 \end{array}\right]\begin{array}{l} \\ \\ b_3+3b_1 \\ b_4-2b_1 \\ b_5+b_1 \\ b_6-b_1 \end{array}$$

$$\longrightarrow \left[\begin{array}{cccccc|c} 1 & -3 & 2 & 1 & -2 & -2 & 0 \\ 0 & -2 & -1 & 4 & -3 & 0 & -1 \\ 0 & -2 & -1 & 4 & -3 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & -1 & 4 & -3 & 0 & -1 \\ 0 & -2 & -1 & 4 & -3 & 0 & -1 \end{array}\right]\begin{array}{l} \\ \\ b_3-b_2 \\ \\ b_5-b_2 \\ b_6-b_2 \end{array}$$

$$\longrightarrow \left[\begin{array}{cccccc|c} 1 & -3 & 2 & 1 & -2 & -2 & 0 \\ 0 & -2 & -1 & 4 & -3 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]\begin{array}{l} \to ② \\ \to ① \\ \\ \\ \\ \\ \end{array}$$

① $-2x_2 - x_3 + 4x_4 - 3x_5 = -1$

$x_3 = 1 - 2x_2 + 4x_4 - 3x_5$

②. $x_1 - 3x_2 + 2x_3 + x_4 - 2x_5 - 2x_6 = 0$

$x_1 = 3x_2 - 2x_3 - x_4 + 2x_5 + 2x_6$

$= -2 + 7x_2 - 9x_4 + 8x_5 + 2x_6$

$x_2 = a$, $x_4 = b$, $x_5 = c$, $x_6 = d$

$x_3 = 1 - 2a + 4b - 3c$

$x_1 = -2 + 7a - 9b + 8c + 2d$

a, b, c, d sebarang bil. real.

7. Karena SPL di soal no 6 solusi banyak, maka $|A| = 0$ dan A^{-1} tidak ada.

8. A dikatakan non singular jika A^{-1} ada, yaitu jika $|A| \neq 0$

$$|A| = \begin{vmatrix} -3 & 4 & 6 & -2 \\ 2 & 6 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & -8 & 3 & 1 \end{vmatrix} = a_{31} A_{31} = 1 \cdot \begin{vmatrix} 4 & 6 & -2 \\ 6 & 0 & 0 \\ -8 & 3 & 1 \end{vmatrix}$$

$$= 1 \cdot 6 \cdot (-1) \begin{vmatrix} 6 & -2 \\ 3 & 1 \end{vmatrix} = -6 \cdot 12 = -72 \neq 0$$

Jadi A non singular.

Menentukan A^{-1}

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A).$$

$$A_{11} = \begin{vmatrix} 6 & 0 & 0 \\ 0 & 0 & 0 \\ -8 & 3 & 1 \end{vmatrix} = 0 \qquad A_{12} = -\begin{vmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$$A_{13} = \begin{vmatrix} 2 & 6 & 0 \\ 1 & 0 & 0 \\ 2 & -8 & 1 \end{vmatrix} = -6 \qquad A_{14} = -\begin{vmatrix} 2 & 6 & 0 \\ 1 & 0 & 0 \\ 2 & -8 & 3 \end{vmatrix} = 18$$

$$A_{21} = -\begin{vmatrix} 4 & 6 & -2 \\ 0 & 0 & 0 \\ -8 & 3 & 1 \end{vmatrix} = 0 \qquad A_{22} = \begin{vmatrix} -3 & 6 & -2 \\ 1 & 0 & 0 \\ 2 & 3 & 1 \end{vmatrix} = -12$$

$$A_{23} = -\begin{vmatrix} -3 & 4 & -2 \\ 1 & 0 & 0 \\ 2 & -8 & 1 \end{vmatrix} = 12 \qquad A_{24} = \begin{vmatrix} -3 & 4 & 6 \\ 1 & 0 & 0 \\ 2 & -8 & 3 \end{vmatrix} = -60$$

$$A_{31} = \begin{vmatrix} 4 & 6 & -2 \\ 6 & 0 & 0 \\ -8 & 3 & 1 \end{vmatrix} = -72 \qquad A_{32} = -\begin{vmatrix} -3 & 6 & -2 \\ 2 & 0 & 0 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= 24$$

$$A_{33} = \begin{vmatrix} -3 & 4 & -2 \\ 2 & 6 & 0 \\ 2 & -8 & 1 \end{vmatrix} = 30 \qquad A_{34} = -\begin{vmatrix} -3 & 4 & 6 \\ 2 & 6 & 0 \\ 2 & -8 & 3 \end{vmatrix} = 246$$

$$A_{41} = -\begin{vmatrix} 4 & 6 & -2 \\ 6 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0 \qquad A_{42} = \begin{vmatrix} -3 & 6 & -2 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$$A_{43} = -\begin{vmatrix} -3 & 4 & 2 \\ 2 & 6 & 0 \\ 1 & 0 & 0 \end{vmatrix} = 12 \qquad A_{44} = \begin{vmatrix} -3 & 4 & 6 \\ 2 & 6 & 0 \\ 1 & 0 & 0 \end{vmatrix} = -36$$

$$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 0 & -6 & 18 \\ 0 & -12 & -12 & -60 \\ -72 & 24 & 30 & 246 \\ 0 & 0 & 12 & -36 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 0 & -72 & 0 \\ 0 & -12 & 24 & 0 \\ -6 & -12 & 30 & 12 \\ 18 & -60 & 246 & -36 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$= \frac{1}{-72} \begin{bmatrix} 0 & 0 & -72 & 0 \\ 0 & -12 & 24 & 0 \\ -6 & -12 & 30 & 12 \\ 18 & -60 & 246 & -36 \end{bmatrix}$$