THREE DIMENSIONAL FORCES AND ENERGY COMPUTATION SOFTWARE PACKAGE "FEMAN" AND ITS APPLICATIONS

B. Azzerboni (member IEEE), G. Tina

Dipartimento Elettrico, Elettronico e Sistemistico
Universita' di Catania
viale A. Doria, 6 - 95125 Catania.

E. Cardelli (member IEEE)

Dipartimento Sistemi Elettrici ed Automazione
Universita' di Pisa
via Diotisalvi, 2 - 56100 Pisa.

Abstract-This paper deals with a numerical computer program, called FEMAN, which may be successfully employed to calculate the magnetic field, energy, forces and coupling coefficients in iron free media, due to a collection of rectangular busbars or annular arc shape conductors, in which flows a constant current density, arbitrarily oriented in space.

Systems of this type can well simulate electromagnetic launchers, flux compression and homopolar generators.

The program is based on the integral formulation of the Biot-Savart law, evaluated in closed form for conductors having elementary shape, and is built-up in order to permit the successive addition of routines that allow consideration of other conductor shapes.

The software package has been checked against known results found in the literature.

I. INTRODUCTION

The design of electromagnetic launch systems requires the analysis of systems with massive and moving conductors. A three-dimensional study of systems of this type involves the use of numerical codes with a long series of input data that can operate only in large size hardware tools.

The need of methodologies that allow a reduction of this charge suggested the development of integral formulations of the problem.

Analytic expressions for the calculation of the magnetic field and vector potential due to conductors in elementary shape have been derived in previous papers [1] [2] [3].

These expressions can be usefully employed to evaluate the magnetic field, the forces and the energy, in systems with massive conductors, like coilguns and railguns, compression flux and homopolar generators etc., taking into account that whatever conductor geometry are considered, they can be well approximated with a suitable number of elementary conductors, in each of which it is possible to suppose constant current density.

Moreover the magnetic coupling coefficients of each elementary conductor can be computed by means of the same relations, and they are useful for a circuital approach to the analysis of the skin-effect in the massive conductor parts of these devices.

In this paper a numerical code, called FEMAN, is presented which utilises exact analytical expressions and allows the computation of the magnetic field and vector potential in iron-free media due to a collection of rectangular busbars and annular arc shape conductors, carrying current distributed with constant density and arbitrarily oriented in space.

The program FEMAN is structured to add to existing ones other expressions, with regard to other elementary conductors, that are currently under investigation.

II. ANALYTIC EXPRESSIONS USED IN THE PROGRAM

A. Magnetic vector potential and induction

The vector potential of a finite current slab with current density (see Fig. 1a) is evaluated by:

$$A_z = \frac{\mu_0 J_z}{4\pi}\int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{z_1}^{z_2}\frac{dx\,dy\,dz}{[(x-x_p)^2+(y-y_p)^2(z-z_p)^2]^{1/2}} \quad (1)$$

The integration is made in closed form by means of suitable variable transformations, integration by parts and other similar techniques. The resulting analytical expression is a sum of polynomial, logarithmic and inverse trigonometric functions of the point in which the vector potential is evaluated [5].

The potential vector of a finite annular ring of rectangular cross section in which a z-axial current distributed with constant density flows (see Fig.1b) is evaluated by:

$$A_z = \frac{\mu_0 J_z}{4\pi}\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\int_{z_1}^{z_2}\frac{r\,d\theta\,dr\,dz}{[r^2+r_p^2-2rr_p\cos(\theta-\theta_p)+(z-z_p)^2]^{1/2}} \quad (2)$$

In this case the integration gives an analytical expression constituted by the sum of logarithmic, inverse trigonometric and Legendre's Elliptical functions of first, second and third kind, and a one-dimensional integral of the type:

$$\int_{\Phi_1}^{\Phi_2} (z_1-z_p)r_p \ln(r_2{}^2+r_p{}^2 + (z_1-z_p)^2-r_p\cos 2\Phi-r_2)d\Phi \quad (3)$$

that is the only integral to be computed numerically.

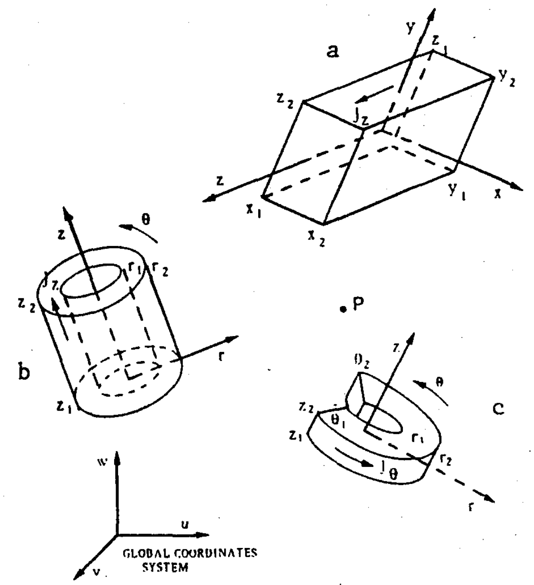

Fig. 1 - Elementary conductor shapes considered in FEMAN, with their local coordinate systems and the global Cartesian frame.

The expressions of the A-components of a finite annular ring of rectangular cross section in which a current flows in azimuthal direction (see Fig. 1c) are evaluated by means of the following expressions:

$$A_\theta = \frac{\mu_0 J_\theta}{4\pi} \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\int_{z_1}^{z_2} \frac{r\cos(\theta-\theta_p)\ d\theta\ dr\ dz}{[r^2+r_p{}^2-2rr_p\cos(\theta-\theta_p)+(z-z_p)^2]^{1/2}} \quad (4)$$

$$A_r = \frac{\mu_0 J_\theta}{4\pi} \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\int_{z_1}^{z_2} \frac{-r\sin(\theta-\theta_p)d\theta\ dr\ dz}{[r^2+r_p{}^2-2rr_p\cos(\theta-\theta_p)+(z-z_p)^2]^{1/2}} \quad (5)$$

and, integrated in analogy with the previous case, gives results in similar analytic shape.

The components of the magnetic induction are evaluated still by the Biot-Savart law.

The expression derived always verified that B is the curl of A.

B. Magnetic energy

The magnetic energy is evaluated by the relation:

$$W_m = \frac{1}{2}\int_V A\cdot J\ dV \quad (6)$$

where V is the volume of the conductor. In the case of conductors in slab shape (6) is integrated in closed form and gives resulting analytical expression in shape of the sum of polynomial, logarithmic and inverse trigonometric functions [7].

For conductors in annular arc shape the integration is currently made by Gauss's numerical routines, but the possibility to obtain results in closed form is under investigation .

C. Self- and mutual inductance coefficients

Once the magnetic vector potential components are explicitated, the magnetic coupling coefficients are evaluated by the relation:

$$M_{ij} = \frac{\int_{V_i} A_{ij}\cdot J_j\ dV_i}{I_i\ I_j} \quad (7)$$

where A_{ij} is the vector potential in the i-th conductor due to the current I_j that flows in the j-th conductor.

D. Forces

The forces acting on the conductors are evaluated by means of the expression:

$$F_m = \frac{1}{2}\,\mathrm{grad}_{xyz}\left(\int_V A\cdot J\ dV\right) \quad (8)$$

where $grad_{xyz}$ is the gradient operator in Cartesian coördinates.

When the conductor volume is a slab the results of (8) are again given in closed form [7].

III. COMPUTER PROGRAM

A numerical code, called FEMAN, uses the relations presented in the previous sections and can perform the electromagnetic analysis of three-dimensional structures formed by massive conductors, like railguns and coilguns, homopolar and flux compression generators.

The structure under analysis must be discretized in a series of slabs and annular sectors.

The input data identifies each elementary conductor by means of:

- a number that identifies its form (slab, annular arc with azimuthal or axial current);
- the value of the current in each conductor;
- its dimensions, given relative to its local coordinate system;
- the position of its local coordinates frame with respect to the global one.

The input data can be prepared in a short time even without graphic preprocessors. However, the availability of the three-dimensional graphic software package allows a easier preparation and an efficient verification of the input data.

The use of slabs in the discretization is, if possible, preferred, because, since the integration is made in closed form, a strong reduction of the computational time is obtained. If ring conductors are used in the discretization the integration of (3) (6) (7) and (8) must be computed numerically, by means of well known routines, and this necessitates a computing time from 10 to 100 times greater, depending on the required degree of precision.

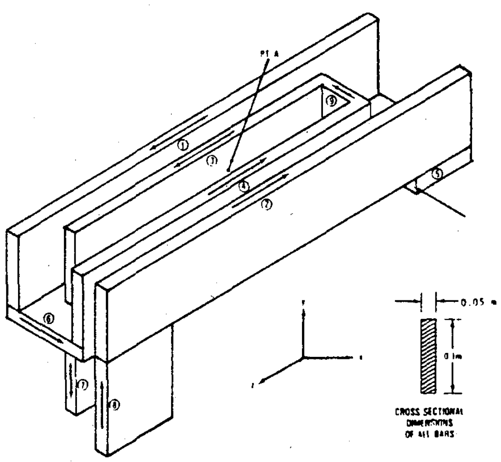

Fig. 2 - Augmented railgun configuration with turnarounds of ref.[11].

The program was checked against standard results as, for instance, may be found in Grover [9] and Hayt [10]. In all cases a good agreement was obtained.

As an example, the results of the analysis of two railgun geometries are reported, they can be simulated only by means of slab conductors.

Fig 2 shows the example of an augmented railgun configuration with turnarounds [11].

A comparison between self and mutual inductances and force values obtained by FEMAN and the corresponding ones reported on [11], is shown in table 1. A good agreement between the two types of results arises from this comparison.

TABLE I

RESULTS OF THE ANALYSIS OF AUGMENTED RAILGUNS

	Ref. [11]	FEMAN
Magnetic induction components in the point A [Wb/m^2].		
B_x	.13E-2	.15E-2
B_y	11.13	10.26
B_z	.13E-3	.21E-3
Self-inductance of turnarounds and breech connections bars 5-8 [H]	.110E-5	.115E-5
Self-inductance of rail bars 3 and 4	.333E-5	.365E-5
Self inductance of augmenting bars 1 and 2	.692E-5	.714E-5
Mutual inductance of rails 1 and 2 with augmenting bars 3 and 4	.264E-5	.264E-5
X-component of the force in bar #1 [N]	1.73E6	1.37E6
Y-component of the force in bar #1	-3.09E6	-3.03E6
X-component of the force in bar #2	-3.59E6	-3.04E6
Y-component of the force in bar #2	2.19E6	2.37E6
X-component of the force in bar #3	-5.23E7	-5.27E7
Y-component of the force in bar #3	1.35E5	1.72E5
X-component of the force in bar #4	5.51E7	5.55E7
Y-component of the force in bar #4	1.82E5	1.64E5

Y-component of the force in bar #5	1.81E2	1.81E2
Z-component of the force in bar #5	-3.15E5	-3.17E5
Y-component of the force in bar #6	-1.96E5	-1.96E5
Z-component of the force in bar #6	3.97E5	3.99E5
X-component of the force in bar #7	-1.73E4	-1.74E4
Z-component of the force in bar #7	-3.34E4	-3.67E4
X-component of the force in bar #8	1.01E5	-1.14E5
Z-component of the force in bar #8	6.59E5	6.34E5
Y-component of the force in bar #9	-1.76E2	-1.76E2
Z-component of the force in bar #9	-2.54E5	-2.55E5

Another example is shown in fig. 3 which represents a model of a plasma-driven railgun with primary and secondary arcs.

The forces exerted on each arc by the system for different distances between the arcs and the corresponding mutual inductances, calculated by FEMAN, are reported in fig. 4 and 5.

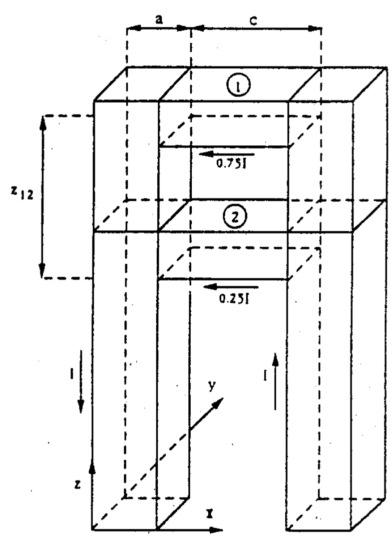

Fig. 3 - Modelling of a plasma-driven railgun with primary and secondary arcs by FEMAN.

The skin effect, in the electromagnetic analysis of FEMAN, can be taken into account with sufficient accuracy, supposing that the currents are distributed with constant density along the skin depth, evaluated versus the time by means of the extension of the one-dimensional model of the magnetic penetration [12].

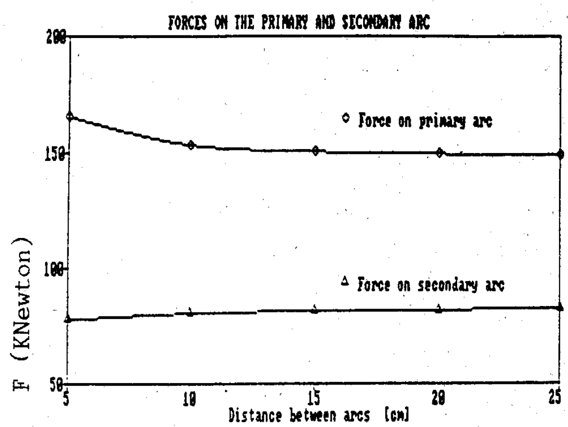

Fig. 4 - Forces acting on main and parasitic arcs by of a plasma-driven railgun, against the distance between the arcs. I = 1 MA, a = 5 cm, c = 4.3 cm.

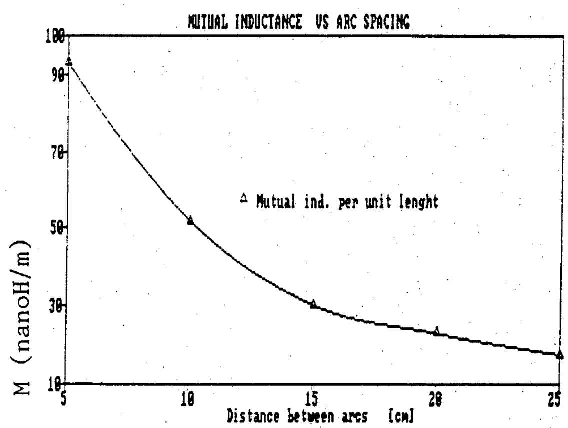

Fig. 5 - Mutual inductance coefficient between main and parasitic arcs of a plasma-driven railguns, against the distance between the arcs.

IV. CONCLUSIONS

The program FEMAN is an useful code in the analysis of electromagnetic accelerator systems, because it is able to perform the calculations of the potential vector, magnetic field, self and mutual inductances and forces for systems composed by structures formed by one or more massive conductors.

The option of analytical calculus of forces and inductances gives accurate results, in a short time, to analyze the railgun launchers also in the presence of transient skin effect, because this effect can be taken into account for rectangular bars by approximating the areas through which most of the current is expected to flow as thin rectangular slabs.

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