A PROGRAM TO COMPUTE MAGNETIC FIELDS, FORCES, AND INDUCTANCES DUE TO SOLID RECTANGULAR CONDUCTORS ARBITRARILY POSITIONED IN SPACE

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Abstract - The computer program BUS3D was developed to compute the magnetic field and vector potential produced by a collection of rectangular busbars carrying DC current and arbitrarily oriented in space, using exact analytic expressions. The field and vector potential expressions are used in numerical integrations to obtain forces on individual busbars and inductances or mutual inductances between collections of busbars. The program has been checked against known results found in the literature. BUS3D has been used in the design of electromagnetic launch system component interconnections and in the analysis of a rail gun. An example of a rail gun configuration is presented to show the various options BUS3D offers.

Introduction

The design of an electromagnetic launch system (EML) involves interconnecting various components such as the generator, the inductor, and the rails using solid rectangular busbars. Some of the components themselves such as a barrel, including rails and possibly augmentation turns, may also be constructed of solid rectangular busbars. It is therefore desirable to calculate the forces exerted by busbar systems on individual component busbars in order to brace mechanically such structures against the high forces which result from the large currents in typical EML systems. It is also desirable to calculate self- and mutual inductances produced by these systems in order that they may be properly taken into account in EML system simulation codes.

Urankar [1] presented analytic expressions for the magnetic field and vector potential produced by a busbar having a standard orientation. These expressions were used in conjunction with coordinate transformations to find the field and vector potential produced by a busbar arbitrarily positioned in space. These results were then used in a numerical integration scheme to find forces and mutual inductances between collections of busbars.

Theory

In our work, a busbar having the standard configuration of $x_1 = y_1 = z_1 = 0$ (see Fig. 1) is used but in Urankar's these quantities may be non-zero. The current is z - directed and of constant density, J. The expressions for the field and vector potential at a point $\vec{r} = (x,y,z)$ are given respectively by:

$$\vec{B}(\vec{r}) = \frac{\mu_0 J}{4\pi} \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} dx'dy'dz' \; \frac{\hat{k} \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \qquad (1)$$

$$\vec{A}(\vec{r}) = \frac{\mu_0 J}{4\pi} \int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} dx'dy'dz' \; \frac{\hat{k}}{|\vec{r} - \vec{r}'|} \qquad (2)$$

Manuscript received March 17, 1986.

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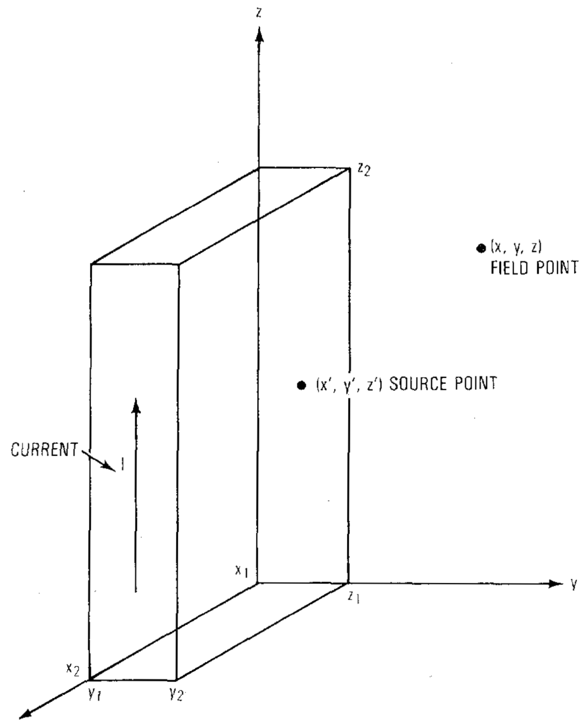

Fig. 1. Busbar in standard configuration

By defining the following quantities as:

$$v_k = x - x_k$$

$$n_j = y - y_j$$

$$s_i = z - z_i$$

$$r_{ijk} = \sqrt{v_k^2 + n_j^2 + s_i^2}$$

The analytic expressions resulting from performing the integrals in equations 1 and 2, are given by: [1]

$$\vec{B}(\vec{r}) = \frac{\mu_0 J}{4\pi} \sum_{i,j,k=1}^{2} (-1)^{i+j+k}$$

$$\Big\{ \hat{i} \Big[-v_k \sinh^{-1}(s_i/\sqrt{v_k^2 + n_j^2}) - s_i \sinh^{-1}(v_k/\sqrt{n_j^2 + s_i^2}) + n_j \tan^{-1}[s_i v_k/(n_j r_{ijk})] \Big] +$$

$$\hat{j} \Big[n_j \sinh^{-1}(s_i/\sqrt{v_k^2 + n_j^2}) + s_i \sinh^{-1}(n_j/\sqrt{v_k^2 + s_i^2}) - v_k \tan^{-1}[s_i n_j/(v_k r_{ijk})] \Big] \Big\} \qquad (3)$$

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$$\vec{A}(\vec{r}) = -\frac{\mu_o J}{4\pi}\hat{k}\sum_{i,j,k=1}^{2}(-1)^{i+j+k}$$

$$\Big\{n_j v_k \sinh^{-1}\left[s_i/\sqrt{v_k^2+n_j^2}\right]$$

$$n_j s_i \sinh^{-1}\left[v_k/\sqrt{n_j^2+s_i^2}\right] +$$

$$s_i v_k \sinh^{-1}\left[n_j/\sqrt{v_k^2+s_i^2}\right] -$$

$$(v_k^2/2)\tan^{-1}\left[s_i n_j/(v_k r_{ijk})\right] -$$

$$(n_j^2/2)\tan^{-1}\left[s_i v_k/(n_j r_{ijk})\right] -$$

$$(s_i^2/2)\tan^{-1}\left[n_j v_k/(s_i r_{ijk})\right]\Big\} \qquad (4)$$

Equations 3 and 4 include corrections made in a later paper by Urankar. [2] The notation used here differs slightly from Urankar's notation.

It is desirable to refer these local bar coordinates to a global system as shown in Fig. 2 in order to perform calculations for collections of busbars. Let $\bar{e}_i$ denote an orthonormal right-handed coordinate system oriented with respect to the bar coordinate system. Let $\hat{e}_j$ denote a global orthornormal coordinate system. Then using direction cosines, one has

$$\bar{e}_i = \sum_{j=1}^{3}\alpha_{ij}\,\hat{e}_j\,, \qquad i = 1,2,3 \qquad (5)$$

The α_{ij} form an orthogonal matrix $M = (\alpha_{ij})$ ($MM^T = I$, where I = unit matrix and T denotes transpose). Let $\vec{r}_o$ be the position vector of the origin of the bar's coordinate system and $\vec{r}_p$ the position of the field point, both referred to the global system. Then it can be shown that

$$\vec{B}_{global}(\vec{r}_p) = M^T\,\vec{B}_{bar}\,(M(\vec{r}_p - \vec{r}_o)) \qquad (6)$$

where $\vec{B}_{global}$ is the magnetic field referred to the global system and $\vec{B}_{bar}$ the magnetic field in the bar's system. A similar expression is used to obtain the vector potential.

These fields are then used to determine forces on individual bars via

$$\vec{F}_{bar\ i} = \iiint_{bar\ i} \vec{J}_{bar\ i} \times \vec{B}_{\substack{remaining\\bars}}\,dV \qquad (7)$$

where all the above quantities are referred to the global system. Force densities are obtained from

$$\frac{d\vec{F}}{dV_{bar\ i}} = \vec{J}_{bar\ i} \times \vec{B}_{\substack{all\\bars}}(\vec{r}_p) \qquad (8)$$

Here the $\vec{B}$ field is that due to all the bars and $\vec{r}_p$ is located in the ith bar. By symmetry, there is no contribution to B due to the ith bar in equation 7. the integral was evaluated using Simpson's rule in 3-D.

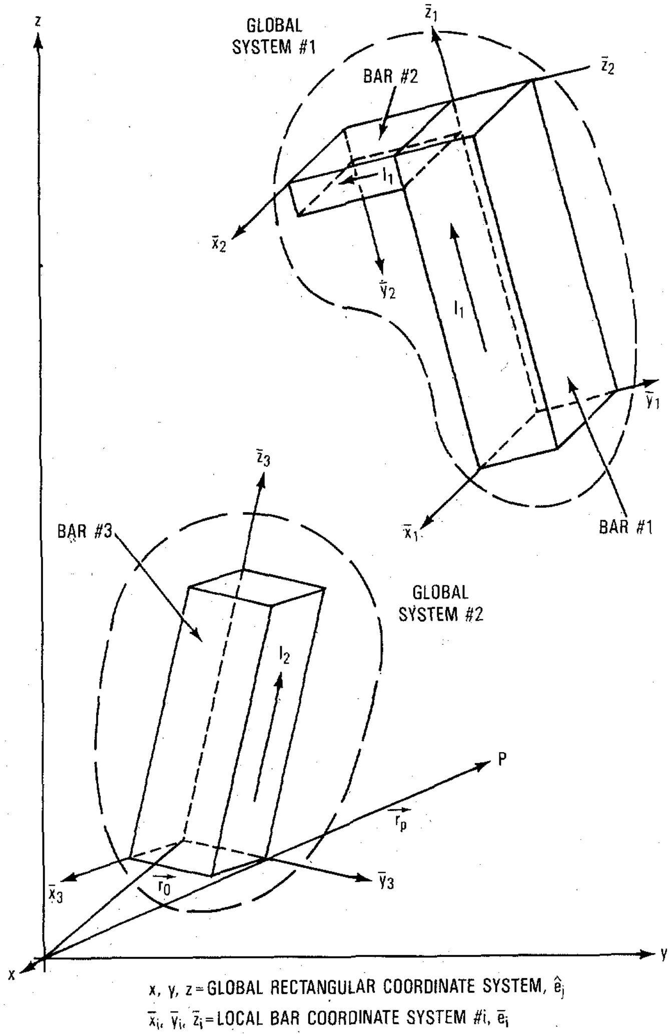

Fig. 2. Coordinate systems

Given one or more collections of busbars called systems or global systems as shown in Fig. 2, it can be shown that the mutual induction between any two systems is given by

$$M_{ij} = \frac{1}{I_i I_j}\iiint \vec{J}_i \cdot \vec{A}_j\,dV_i \qquad (9)$$

where the integral is over all the bars in system i and A_j is the vector potential due to all the bars in system j. It is assumed that all bars in system i carry current I_i and similarly for system j. M_{ij} is really independent of I_i and I_j. The self inductance of system i is given by

$$L_i = \frac{1}{I_i^2}\iiint \vec{J}_i \cdot \vec{A}_i\,dV_i \qquad (10)$$

where A_i is due to all the bars in system i and the integral is over all the bars in i. The above integrals were evaluated by Simpson's rule in 3 D.

Computer Program

A computer program called BUS3D was written to perform the calculations described here. Some of its salient features are: 1) It uses MKS units throughout; 2) Four points are necessary to specify a bar: the origin and the bar's corner points along the three bar axes. These are all referred to the global system; 3) The bar's total current and direction are also specified; and 4) the specific bars over which a total force, due to all other bars, is desired and the collections of bars between which mutual or self-inductances are desired can be specified.

The program was checked against standard results as for instance may be found in Grover [3]. In all cases agreement to 3 or more decimal places was obtained. The user must specify the number of grid points desired in the 3 spatial directions for a given bar over which an integral is to be performed. These must be chosen to produce the desired accuracy.

Figure 3 shows an example of a simple augmented rail gun configuration with turnarounds modeled on BUS3D. From output obtained using BUS3D, design considerations such as the maximum flux density within the barrel bore were found. Also, forces on the individual bars were found as shown in Table 1. The rail burst force (bar #1 and 3's R_x component) shown in Table 1 is comparable in magnitude to what is obtained using a 2^D electromagnetic finite element code which assumes indefinitely long rails. Structural bracing for the barrel can be designed and positioned knowing these forces. Calculated forces have also been used to design breech and muzzle mechanical connections. The self-inductance of the turnarounds and interconnections to the breech were calculated in order to incorporate their effects in an overall system study. The self-inductance of the rails and the mutual inductance between the rails and augments were found for the armature (bar #9) in the position shown at the end of the rails. These quantities vary with the position of the armature and the standard assumption is to assume a linear variation in order to perform force calculations, however, BUS3D can more accurately determine the forces on a projectile by directly calculating the force as a function of the projectile's position along the rail. Because skin effects are not calculated by BUS3D, these effects on such quantities such as inductances and forces, which are typically on the order of about 10%, are not accounted for by BUS3D.

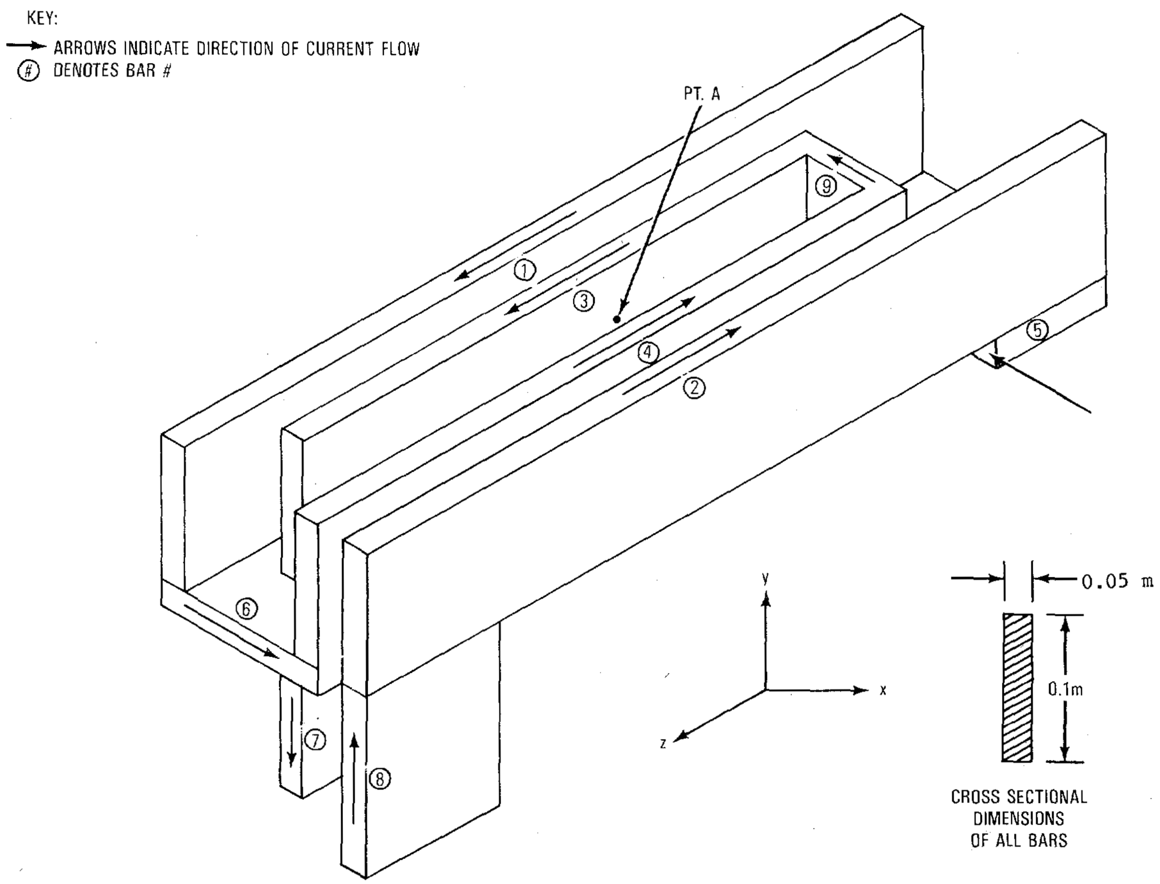

Fig. 3. BUS3D Simple rail gun model configuration

Table 1: BUS3D Rail Gun Model Results

B field @ pt. A bore center (Tesla): BX = 0.13E-02 BY = 11.13 BZ = 0.13E-03

Bar #	Current MA	bar length in Direction of Current Flow (M)	Force on Bar (N) Vector Components X	Y	Z
1	1.0	10.0	1.73E+06	-3.09E+05	0.00E+00
2	1.0	10.0	-3.59E+06	2.19E+04	0.00E+00
3	1.0	8.6	-5.23E+07	-1.35E+05	0.00E+00
4	1.0	9.3	5.51E+07	-1.82E+05	0.00E+00
5	1.0	0.25	0.00E+00	1.81E+02	-3.15E+05
6	1.0	0.19	0.00E+00	-1.96E+05	3.97E+05
7	1.0	1.0	-1.73E+04	0.00E+00	-3.34E+04
8	1.0	1.0	-1.01E+05	0.00E+00	6.59E+05
9	1.0	0.03	0.00E+00	-1.76E+02	-2.54E+05

Self inductance of turnarounds & breech interconnections (bars 5-8)	L = 0.110E-05 H
Self inductance of rails (bars 3 & 4)	L = 0.333E-05 H
Self inductance of augments (bars 1 & 2)	L = 0.692E-05 H
Mutual inductance of rails with augments (bars 3 & 4 with 1 & 2)	M = 0.264E-05 H

Conclusions

The code BUS3D is written to handle a variety of options; e.g., field and force calculations only, self- and mutual induction calculations only, etc. This makes it very flexible. The code handles up to 100 bars in each of up to 5 groups between which mutual inductances may be found.

As demonstrated in the rail gun example, BUS3D is very useful in designing busbar configurations for EML systems. Although a DC code, BUS3D is also useful in analyzing transient skin effects for rectangular bars by approximating the areas through which most of the current is expected to flow as thin rectangular slabs. BUS3D proves to be versatile and able to handle very complex arrangements of current-carrying bars in three dimensions.

References

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