Influence of Current Waveform on Rail-gun Launching Efficiency

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***Abstract* – In many factors affecting rail-gun's launching efficiency, magnetic energy stored in rail inductance and joule heat consumed in rail resistance are the key. To reduce magnetic energy loss, the mathematical expressions for rail-gun such as electromagnetic force, launching efficiency and breech voltage are given, and the relation between current waveform and launching efficiency is found. Theoretical analysis shows that the launching efficiency with constant current can not be greater than 50% due to magnetic energy remained in the rail; while pulse current is ideal current supply because the magnetic energy losses can be reduced remarkably. To reduce the rail resistance loss, the relation between the launching efficiency and current mode is analyzed through an example. Two distributed energy store (DES) modes, a single pulse current mode and a constant current mode are used in the example, and launching efficiencies with different current modes are compared by simulation. Results show that rail-gun launching efficiency with single pulse current is the highest since the rail resistance loss is the lowest among these current modes, and the efficiency with DES mode can not be improved effectively.**

I. INTRODUCTION

At present, methods for investigating rail-gun include experiment, theory analysis and numerical simulation. In experiment domain, the US is occupying the leading status, followed by Britain, France and Germany. They have made remarkable progress.

Experiment with selecting appropriate parameters is an important way to test system's performance and to check up rail-gun's parameters. Because the rail-gun's parameters are so many, it is impossible and unnecessary to do every experiment with every different parameter, and this will spend the huge financial resource and the physical resource. Therefore, in the system research process, theory analysis and numerical simulation are used widely. To research projects for rail-gun, one of the emphases is how to enhance rail-gun's launching efficiency. For this, many effective and concrete methods are used. One method is to change rail structure, whose aim is to enhance inductance gradient and to increase the electromagnetic force, such as enlargement mode rail-gun and coil artillery and so on. Another method is to change current waveform in order to improve rail-gun's efficiency, like constant current and pulse current etc. The third method is to change current introducing into rail-gun's position to reduce energy loss, such as distributed energy store method (DES) and multi-section rail-gun etc. The DES was put forward by Marshall R A in 1979, and has being researched by many organizations. Reference [6] and [7] show that the DES can enhance the rail-gun's system efficiency and it is proved by experiment. But in literature [8], through theory and simulation, authors believe that the DES method can not improve effectively.

Regarding the two contradictory views, this paper introduces the concept of launching efficiency firstly, then carries on theoretical analysis and model simulation through an actual rail-gun example with given parameters, and discusses the influence of the current waveform and the DES method on the launching efficiency.

II. RAIL-GUN OPERATING CURRENT MODE

Usually, two current modes are used to drive the rail-gun system. They are pulse current mode and constant current mode respectively.

(1) Pulse current mode. Current increases rapidly from zero to maximum value, and then reduces to zero. In this case, no energy is stored in rail-gun inductance, so the system efficiency is high. The current curve is shown in Fig.1, and this mode is usually applied in DES system and multi-section rail-gun.

(2) Constant current mode. Current is approximate constant, which curve is shown in Figure 2. This current supply mode is applied in the coil rail-gun, inductance stored energy mode rail-gun widely.

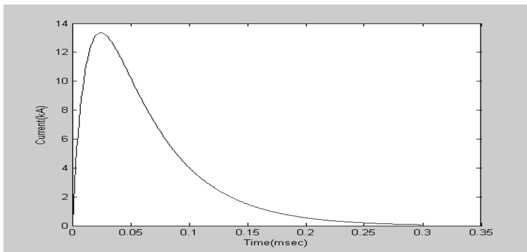

Fig.1. the current curve for pulse

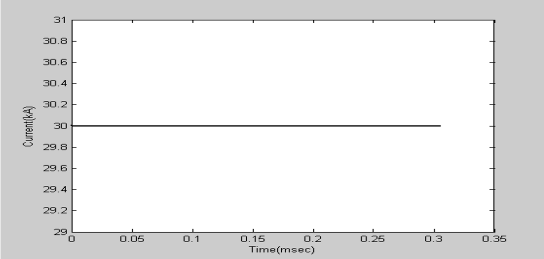

Fig.2. the current curve for constant

III. RAIL-GUN EFFICIENCY ANALYZING

According to the electromagnetism theory, when rail-gun's current passes through armature, the armature is forced to move along the orbital direction by the electromagnetic force. At the same time, an induced voltage is produced at armature's terminals, which is expressed as

$$\varepsilon = \frac{d\psi}{dt} = \frac{d(LI)}{dt} = L\frac{dI}{dt} + I\frac{dL}{dt} = V_e + V_b \tag{1}$$

Where V_e is the usual inductor voltage produced when charging the inductor to a given energy state. V_b is the so-called back voltage produced when performing mechanical work on the inductor, e.g., changing its shape or location in space.

Define system's electric power P_E as the product of the rail-gun's current and the induced voltage, which is:

$$P_E = \varepsilon I = V_e I + V_b I \tag{2}$$

The electromagnetic force may be expressed as

$$F_L = \frac{dE}{dx} = \frac{d\left(\frac{1}{2}LI^2\right)}{dx} = \frac{1}{2}\frac{dL}{dx}I^2 = \frac{1}{2}L^{'}I^2 \tag{3}$$

Where F_L is an electromagnetic force; E is system storage energy. $L^{'}$ is the rail inductance gradient. x is the armature movement displacement.

Rail-gun's efficiency includes system efficiency and launching efficiency. This paper takes the capacitive rail-gun as an example, defining them respectively as

$$\eta_s = \frac{E_P}{E_C} \times 100\% \tag{4}$$

$$\eta_L = \frac{E_P}{E_i} \times 100\% \tag{5}$$

Where E_p is projectile's kinetic energy; E_c is the total energy stored in capacity; E_i is the actual energy imported to rail-gun body, which may be expressed as

$$E_i = \int_0^\tau IU_i dt \tag{6}$$

Where τ is the time of the projectile moving in barrel; U_i is the breech voltage, which may be obtained by measuring muzzle voltage, or calculated by following mathematical expression

$$U_i = \frac{d}{dt}[LI] + RI + V_{arc} = L^{'}vI(t) + L\frac{dI(t)}{dt} + RI + V_{arc} \tag{7}$$

Where V_{arc} is called as the arc voltage, which is the contact voltage between the armature and rail. Replace (6) with (7) result, the E_i can be rewritten as

$$E_i = \int_0^\tau IU_i dt = \int_0^\tau L^{'}vI^2 dt + \int_0^\tau LIdI + \int_0^\tau RI^2 dt + \int_0^\tau V_{arc} Idt \tag{8}$$

The launching efficiency is the ratio of the projectile kinetic energy and actual energy imported to rail-gun body, which reflects the system's actual use factor. E_i includes the projectile kinetic energy, the mechanical energy loss, the magnetic energy loss and joule heat loss.

The mechanical energy loss is produced by the projectile movement, which is mainly the friction loss and the aero dynamical resistance loss. Under gram level projectile condition, with small caliber and normal atmospheric pressure, the rail-gun's mechanical energy loss is 2-3 times of projectile kinetic energy. Taken some methods, such as pulling out the vacuum the barrel and reducing the friction coefficient between projectile and rail as far as possible, the mechanical energy loss can drop to 3%～5％ of the projectile kinetic energy.

The magnetic energy loss is mainly the energy stored in the rail distributed inductance, which is determined by the rail current. For example, using constant current to drive rail-gun can reduce the rail length, but the magnetic energy is equal to projectile's kinetic energy, therefore the efficiency is lower. While providing rail-gun with pulse current, the partial magnetic energy may be taken back. Especially, if the electric current terminal drops to zero, the magnetic energy loss may be reduced to zero completely.

The joule heat loss includes two parts: armature resistance loss and rail resistance loss. To armature, different type's armature has different joule heat loss. For the plasma armature and solid one, as the former contact voltage is much larger than the later one, the former joule heat losses are much larger, which are approximately 25-50% of the sum of the projectile kinetic energy and the mechanical energy loss. To rail, the rail resistance heat loss is inevitable, which is determined by the length and material etc. of the rail.

The system efficiency reflects the rail-gun system's total efficiency. Besides above energy losses, E_c also includes the circuit's energy loss, like switch energy loss, transmission line energy loss, the energy stored in the circuit distributed inductance, capacitor excess energy and so on. Obviously, the system efficiency is much lower than the launching efficiency.

This paper mainly analyzes rail-gun's launching efficiency based on the following supposition:

1) Neglecting the friction.
2) Neglecting the air drag.
3) Neglecting the skin effect of the electric current.

Then the rail-gun's launching efficiency is:

$$\eta_L = \frac{E_P}{E_i} = \frac{E_P}{E_P + E_r + E_L} = \frac{1}{1 + \frac{E_r}{E_P} + \frac{E_L}{E_P}} \tag{9}$$

Where η_L is launching efficiency; E_r is the rail and armature's distributed resistance energy loss, which is expressed as

$$E_r = \int_0^\tau (R_r + R_a) I^2 dt \tag{10}$$

E_L is the energy stored in the rail distributed inductance, which is expressed as

$$E_L = \frac{1}{2} L I^2 \tag{11}$$

E_P is the armature (including projectile) kinetic energy, Which is expressed as

$$E_P = \frac{1}{2}(m_a + m_p)(v^2 - v_0^2) \tag{12}$$

Now, we analyze the launching efficiency according to two kinds of current wave-forms.

(1) Constant current mode

In this mode, the current is constant, so

$$E_P = \int F_L dx = \int \frac{1}{2}\frac{dL}{dx} I^2 dx = \frac{1}{2} L I^2 = E_L \tag{13}$$

Namely, the armature's kinetic energy is equal to the rail inductance stored energy. The launching efficiency η_L is

$$\eta_L = \frac{E_P}{E_i} = \frac{E_P}{E_P + E_r + E_L} = \frac{1}{1 + \frac{E_r}{E_P} + \frac{E_L}{E_P}} = \frac{1}{2 + \frac{E_r}{E_P}} \tag{14}$$

The energy loss of the rail and armature resistance expressed as

$$E_r = \int_0^\tau (R_r + R_a) I^2 dt = I^2 \int_0^\tau (R_r + R_a) dt$$

Where, R_r is the rail resistance and R_a is the armature resistance. Different type's armature, the resistance is different. For plasma armature, $R_a >> R_r$ so R_r may be neglected; while for solid one, $R_a << R_r$ we can ignore R_a.

Because we only discuss the solid armature in the paper, above equation may be rewritten as:

$$E_r = \int_0^\tau (R_r + R_a) I^2 dt \approx I^2 \int_0^\tau R_r dt = I^2 \int_0^\tau (2xR') dt$$

$$E_{r\max} = 2xR' I^2 \tau \tag{15}$$

Where x is the armature moving displacement; R_r is the rail resistance, and $R_r = 2xR'$, R' is the rail resistance increment. Rewrite (1) as

$$P_E = I\varepsilon = I(V_b + V_e) = I\left(L\frac{dI}{dt} + I\frac{dL}{dt}\right) \tag{16}$$

For constant current, $dI/dt = 0$, therefore the above equation may be simplified as

$$P_E = I\varepsilon = I(V_b + V_e) = I^2\frac{dL}{dt} = I^2\frac{dL}{dx}\frac{dx}{dt} = I^2 L' v \tag{17}$$

The expression of the armature kinetic energy may be rewritten as

$$E_P = \int_0^\tau (\alpha P_E) dt = \alpha \int_0^\tau (I^2 L' v) dt = \alpha \frac{1}{2} L' I^2 v_{\max} \tau \tag{18}$$

Where P_E is system's electric power; α is the coefficient of electric power. One part of P_E is converted as motion, and another part is transformed as magnetic energy. If the current keeps constant, according to the (13), then $E_P = E_L$, and $\alpha = 0.5$. Therefore, the launching efficiency's expression may be more written as

$$\eta_L = \frac{1}{2 + \frac{E_r}{E_P}} = \frac{1}{2 + \frac{2xR' I^2 \tau}{\alpha \frac{1}{2} L' I^2 v_{\max} \tau}} = \frac{1}{2 + \frac{8R' x}{L' v_{\max}}} = \frac{1}{2 + \left(\frac{8R'}{L'}\right)\frac{x}{v_{\max}}} \tag{19}$$

(2) Pulse current mode

When the rail-gun system uses the pulse current, system's magnetic energy is reduced gradually, which part is transformed into armature's kinetic energy. If the pulse current is reduced to zero, then no any magnetic energy is stored. That is $E_L = 0$. The launching efficiency expression may be given as

$$\eta_L = \frac{E_P}{E_i} = \frac{E_P}{E_P + E_r + E_L} = \frac{1}{1 + \frac{E_r}{E_P}} \tag{20}$$

As the pulse current is not constant, the kinetic energy expression can be written as

$$E_P = \frac{1}{2}mv^2 = \frac{1}{2}(mv)v = \frac{1}{2}v\int_0^{\tau} F_L dt = \frac{1}{2}vL'\int_0^{\tau} i^2 dt \quad (21)$$

The expression of the rail and armature's distribution resistance energy losses are given as

$$E_r = \int_0^{\tau} (R_r + R_a) i^2 dt \approx \int_0^{\tau} i^2 (2xR') dt$$

When the rail-gun uses the pulse current, the electromagnetic force is not constant, but the projectile velocity and the displacement are increase gradually. If rail length is reasonable, the velocity achieves the maximum with displacement simultaneously. Therefore, the expression of the rail resistance energy losses can be given as

$$E_{r\max} = \int_0^{\tau} i^2 (2xR') dt = 2xR' \int_0^{\tau} i^2 dt \quad (22)$$

The system launching efficiency expressed as

$$\eta_L = \frac{E_P}{E_i} = \frac{E_P}{E_P + E_r + E_L} = \frac{1}{1 + \dfrac{E_r}{E_P}} = \frac{1}{1 + \dfrac{8R'x}{L'v_{\max}}} = \frac{1}{1 + \left(\dfrac{8R'}{L'}\right)\dfrac{x}{v_{\max}}} \quad (23)$$

Where, x is maximum displacement, and $v_{\max}$ is maximum velocity.

A conclusion can be obtained from (19) and (23), when some parameters of the rail-gun are given, such as rail material, barrel size, armature type etc.; the launching efficiency is affected by current waveform. For constant current, projectile's acceleration keeps constant in the whole operation process, the velocity increases quickly with shorter rail length; the shortcomings are that the energy stored in rail inductance is too much to the launching efficiency is low. Therefore, in the actual experiment, pulse current is widely used to replace the constant one. According to different position that pulse current fed into rail-gun, the pulse current's application mode includes the single pulse current mode and distributed energy store mode (DES).

DES is the method which current introduces into at points along the rail, which contains two modes: zero acceleration mode and current threshold mode. The former operation process is: when the first level current drops to zero, the second level current starts to provide the rail-gun, such-and-such; The principle of the latter is: when the first level current reduces to a definite value, the second level current starts to energize the system. DES provides rail-gun current in several positions along barrel, whose goal is to reduce the rail's resistance heat loss and enhance the launching efficiency. Whether this mode can enhance the efficiency, reference [6], [7] and [8] hold inconsistent viewpoint. The following example compares the launching efficiency in different current supply modes through simulation and calculation.

IV. EXAMPLE SIMULATIONS AND ANALYSIS

According to (3), the velocity and displacement of armature (including projectile) may be written as

$$\frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{1}{2(m_p + m_a)} L'i^2 \quad (24)$$

$$E_{PM} = \frac{1}{2}mv_{\max}^{\ 2} \quad (25)$$

Where, E_{PM} is the maximum of projectile kinetic energy; $v_{\max}$ is the projectile's maximal velocity.

$$E_S = \frac{1}{2}CU_0^2 \quad (26)$$

Define the system efficiency as

$$\eta_S = \frac{E_p}{E_s}$$

Firstly, according to (24), we calculate the armature's velocity and displacement, and then the projectile's launching efficiency in different current waveform may be computed by (19) and (23).

In all steps, the armature's velocity and displacement is two key values. Because (24) is a second-order differential equation, and the mathematical expression of current is also complicated, it is very difficult and complex to use the conventional analysis method to calculate the velocity and displacement. We use Simulink software to simulate the process. The simulating flow chart is showed in fig.3, and simulation parameters are shown in Table I.

Four current supply modes are used in the simulating process. They are zero acceleration mode, current threshold mode, single pulse current mode and constant current mode respectively. And their initial energy is equal, only different in voltage. 12kV voltage value is used in the zero acceleration and the electric current threshold mode, and the voltage in single pulse current mode is 18kV. In constant current mode, voltage value takes the average value of the single pulse current mode. Simulation results are shown in table II.

TABLE I
SIMULATION PARAMETERS

Resistance gradient $R'(\Omega/m)$	2×10^{-4}
Inductance gradient $L'(\mathrm{H}/m)$	5×10^{-7}
Arc Voltage $V_{arc}(\mathrm{V})$	50
Projectile Mass $m_p(\mathrm{g})$	30
Armature Mass $m_a(\mathrm{g})$	1
Capacitance $C(\mathrm{F})$	0.06
Circuit inductance $L_0(\mathrm{H})$	5×10^{-7}
Circuit resistance $R_0(\Omega)$	10^{-4}
Charging Voltage $U_0(kV)$	12

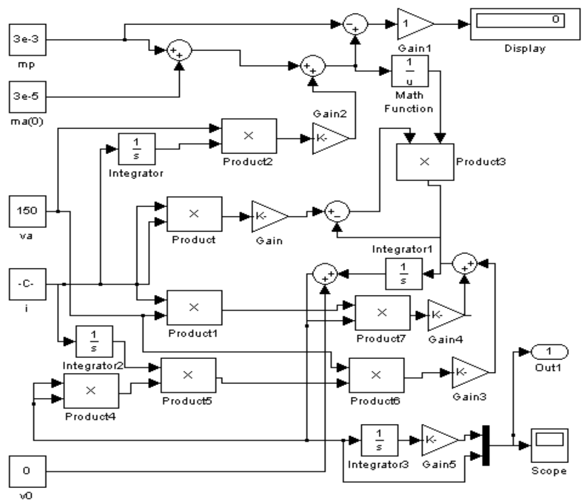

Fig.3. simulating flow chart using simulink

TABLE II
SIMULATION DATA

Current Mode	Current ($10^5 A$)	Velocity (m/s)	Time (msec)	Launching Efficiency (%)
single pulse	7.07	850.8	0.8	38.5
zero acceleration	5	852	2.0	30.3
current threshold	5	874.7	2.0	31.3
constant current	2.5	726.2	1.4	24.2

V. CONCLUSION

The simulated data in table 2 show: among these current modes, the launching efficiency with constant current mode is lowest, and single pulse current mode is the highest. The launching efficiencies with two DES current modes are approximately equal, so they can not improve the launching efficiency effectively. We think that the DES reduces magnetic energy losses, but current operation time is also lengthened, and the rail length also increases correspondingly. This leads to the rail resistance heat losses increasing massively.

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