5 Model

An instance consists of an underlying graph G, where each node represents a location. On each location there are two natural numbers n_r denoting how many black customers there are, n_g denoting how many white customers there are on this location. A customer can be served by any facility, that is placed on either the same location as the customer, or a location to which an edge exists from the source location. The set of all black customers is B, the set of all white customers is W. In the first phase, facilities can choose any of the location vertices on the underlying graph and place themselves on it. They are then potentially able to serve all customers placed on locations in their neighborhood. One customer is served by at most one facility. For the second phase of the game, the customers choose which facilities they enter. This induces a directed bipartite graph H of customers $C = B \cup W$ and facilities F with $H = (C \cup F, E)$ is induced. An edge $e_{ij} = (c_i, f_j)$ is an element of E if the facility f_j is in reach for the customer c_i. A customer c_i has utility

$$u_i = \left\{ \begin{array}{ll} -1, & \text{customer is in no facility} \\ ratio(c_i) \cdot \frac{1}{t}, & \text{for } 0 \leq ratio(c_i) \leq t \\ 1, & \text{for } ratio(c_i) > t \end{array} \right\}$$

, where $ratio(c_i)$ is the ratio of customers of his color in the facility in which he is. The utility of a facility is the number of customers it serves. At any step we require our customers to be impact blind, in the sense that while a customer is aware of having an impact on what the target facility ratio will change to, he can only see the abridged fraction and thus his impact could be any $\epsilon > 0$. E.g. a customer c_i would not move from a facility with ratio $\frac{3}{11} = 0.2727$ to a facility with ratio $\frac{1}{4} = 0.25$, although after him moving the ratio would turn to $\frac{2}{5} = 0.4$. But because the unreduced fraction behind $\frac{1}{4}$ could be $\frac{10}{40}$ the target ratio could become $\frac{11}{41} = 0.268 < \frac{3}{11}$. The only exception to this are empty facilities. Since both color ratios are 0 then, any customer can deduce that the target utility will be 1.
The social welfare function is then defined as the sum over the utility for each customer.

$$\text{social welfare} = \sum_{i=1}^{n} u_i, n = |C|$$

A strategy profile is a tuple (π_F, s_C), where $\pi_F : F \to 2^C$ is a mapping of facilities to locations (which are being characterized by the subset of customers in the range) and $s_C : C \to F$ a mapping of customers to the facility which serves the respective customer