CS/ECE 4501 / CS 6501 / ECE 6782 **Due by Sep 22nd** $11:59:59$**PM**

Problem Set I

1. (20%) Prove the properties of convolution. For all continuous function f, g, and h, the following axioms hold. **Please make sure to lay out all key steps of your proof for full credits.**

   - Associativity: $(f * g) * h = f * (g * h)$
   - Distributivity: $f * (g + h) = f * g + f * h$
   - Differentiation rule: $(f * g)' = f' * g = f * g'$
   - Convolution theorem: $\mathcal{F}(g * h) = \mathcal{F}(g)\mathcal{F}(h)$, where $\mathcal{F}$ denotes Fourier transform

2. (25%) Frequency smoothing:

   (a) Compute Fourier transform of the given image `lenaNoise.PNG` by using `numpy.fft.fft2` in Python, and then center the low frequencies (e.g., by using `fftshift`).

   (b) Keep different number of low frequencies (e.g., $7^2, 15^2, 31^2$ and the full dimension), but set all other high frequencies to 0.

   (c) Reconstruct the original image (`ifft2`) by using the new generated frequencies in step (b).

   Submit the code and include the restored images with different number of low frequencies in your report.

3. (55%) Implement gradient descent algorithm for image denoising with total variation model explained in class. **All codes and a two-page report including problem description, a concrete optimization objective function, and experimental results (a denoised image and a convergence graph that generated by your best-tuned parameters) with discussions should be submitted.**

   - With the given image `Einstein.jpeg`, generate different noisy images with additive Gaussian noises at different variance ($\sigma = 0.001, 0.01, 0.1$).
   - Test your denoising program on the generated noisy images.
   - The forward / backward difference for computing image gradient is given in `Dx / Dxt`. Feel free to use it, or use the python function provided by `scipy.ndimage`.
   - The convergence graph is a plot of your objective function $E(u^k)$ along with all iterations.
   - A detailed class note of deriving total variation, computing gradient term, and gradient descent algorithm can be downloaded from Canvas.